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## **Controlling firms through the Majority Voting Rule**

**Ariane Chapelle and Ariane Szafarz**

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**ULB**

# Controlling firms through the Majority Voting Rule

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## **Abstract**

Pyramids, cross-ownership, rings and other complex features inducing control tunnelling are frequent in the European and Asian industrial world. Based on the matrix methodology, this paper offers a model for measuring integrated ownership and threshold-based control, applicable to any group of interrelated firms. In line with the theory on pyramidal control, the model avoids the double counting problem and sets the full-control threshold at the conservative - but incontestable - majority level of 50% of the voting shares. Any lower threshold leads to potential inconsistencies and leaves unexplained the observed high level of ownership of many dominant shareholders. Furthermore, the models lead to ultimate shareholders' control ratios consistent with the majority voting rule. Finally, it is applied to the Frère Group, a large European pyramidal holding company known for mastering control leverages.

## 1. Introduction

While common in many countries, pyramids, cross-ownership, and dual shares systems make the controlling shareholders hard to identify at first sight. Precise measure instruments are thus required but, surprisingly, the literature still offers no consensus on this topic. Therefore, findings obtained by different research teams are not always comparable. Methodological discrepancies mostly concern the threshold for full control over a firm, the extent of corporate pyramids, the taking into account of cross-holdings and reciprocal shares, etc.

Starting from the classical input-output matrix approach for indirect ownership computation, this paper builds a model leading to control (voting rights) over ownership (cash-flow rights) ratios. Its originality is twofold. First, the threshold for full control is introduced at the very beginning of the specification (we produce a counter-example showing that an *ex post* threshold may induce misleading conclusions). Second, the control threshold is fixed at 50%, i.e., at the incontestable majority level. The presentation of the model is accompanied all the way by a simple example and further illustrated by a real-life application.

The majority voting threshold (50% of the voting rights) for full corporate control is in line with the theoretical literature. Indeed, Bennedsen and Wolfenzon (2000) make it explicit that a winning control coalition requires at least half of the total voting power. Also, the papers on partial benefits of control and coalitions of voters, often derived from Zwiebel's (1995) important contribution, stress that 50% is the only acceptable threshold for control (Earle et al., 2004), even if it is shared among several owners acting jointly. In the case of joint ventures, Hauswald and Hege (2003) show that the optimal ownership clustering for rent extraction by the majority owner lays around 50% of the equity stakes. Furthermore, the 50%

threshold mechanically avoids counterintuitive results, like the existence of two controllers for the same company, as obtained by some authors.

The empirical literature contrasts with the theoretical models by frequently taking control thresholds lying below 50% of the voting rights. The 20% level, probably inspired by the influential paper by La Porta et al. (1999) is chosen, for instance, by Claessens et al. (2000). This rule of thumb is mainly justified by the ownership dispersion observed in some countries. In practice, such thresholds are used for explaining the presence of private benefits of control. Actually, this is not in contradiction with our view since in our model, the 50% threshold is only required for full control, i.e. for fully expropriating all other shareholders of the company. However, partial control benefits may appear at smaller voting right levels. Following our specification, the control ratio of a shareholder who controls more than she owns is larger than one, meaning that she is in a position to reach potential benefits. However, with less than 50% of the voting rights this position remains contestable, for instance by a hostile takeover.

As a matter of fact, holders of large blocks of shares benefit from control premia. Barclay et al. (2001) show that stock blocks trading prices include a premium reflecting anticipated private benefits. On the Milan stock exchange, Zingales (1994) estimates that the control of a corporation is priced more than 60% of the equity value, due to the possible exploitation of a dominant position and to the dilution of minority property rights. On the Polish stock market, Trojanowski (2003) finds that block premia increase with the dispersion of the voting rights. Dyck and Zingales (2004) observe that the value of control, relative to the share price, varies largely across countries, with a striking negative minimum of  $-4\%$  in Japan and a maximum of  $65\%$  in Brazil. The puzzling result concerning Japan, a country with very complex

ownership patterns, stresses the need for a rigorous model. In particular, the 50% control threshold would make this result impossible.

Since the paper by Berle and Means (1932), the literature on ownership and control has much evolved. It is now recognized that the voting right structure of a company has a strong influence on its corporate governance in several respects. The expropriation of minority shareholders and the tunneling of profits outside the group have been analyzed by, e.g., La Porta et al. (1999), Bebchuk (1999), Bertrand et al. (2002), Bertrand and Mullainathan (2003), Claessens et al. (2002). The race for corporate control is particularly developed in countries or companies facilitating the access to private benefits (Bebchuk, 1999). Franks and Mayer (1996) identify four types of corporate control: large shareholders, complex ownership patterns, bank control in widely held companies, and blocks of shares. Moreover, Roehner (2005) shows that cross-ownership relationships influence the stock price dynamics, especially during crises.

For Italy, Barca (1996) exhibits pyramids and shareholders' coalitions while Bianchi and Casavola (1996) focus on their impact on growth. Bianco et al. (1996) link separation between ownership and control to the firm efficiency. Thesmar (2001) provides empirical evidence on French firms. Wolfenzon (1999) shows that pyramids tend to develop in countries with low protection of minority shareholders. For Belgium, Chapelle (2001) and Renneboog (2000) provide examples of pyramidal structures and estimate the level of separation between ownership and control in the nineties. ECGN (2001) displays the ownership characteristics of continental Europe ownership, exploring the means of enhancing control in European Corporations.

Cross-ownership is also efficient to increase the opacity of a control pattern and to interlock control among a limited group of companies. Cross-ownership structures play, for instance, a prominent role in Scandinavian countries (Bohren and Norli, 1997, Eckbo, 1997). These features are typical of Japanese corporations, grouped for decades in keiretsus, and characterized by a high voting rights concentration (Hopt et al., 1998). This might explain why the models designed for taking into account cross-shareholdings and indirect ownership were first applied to Japanese firms (Flath, 1992; Hoshi and Ito, 1991). Attig and Gadhoom (2003) find that, in Canada, the separation between ownership and control is even more pronounced than in East Asia and Western Europe. Control issues also arise in emerging economies, especially in South-East Asia (Claessens et al., 2000; Faccio et al., 2001).

Our model is mainly designed for interlocked complex features and pyramidal empires. It is based on the matrix methodology proposed by Flath (1992), Hoshi and Ito (1991), Baldone et al. (1998) and Brioschi et al. (1997) to compute shareholders' integrated ownership. This approach was also followed by Huber and Ryll (1989), and Ellerman (1991) starting from an accounting view of the firm. However, the inclusion of the majority threshold drastically modifies the results and allows for properly defining control ratios. Indeed, previous work was mainly devoted to indirect ownership, not to control (but the terminology was sometimes misleading).

The control ratio of an ultimate owner's in a firm is obtained by dividing his integrated voting power (including the effects of the majority threshold) by his integrated ownership share. It expresses the leverage of control exerted by this shareholder. Control ratios are relevant only for the ultimate shareholders, i.e., those with no controlling shareholder. However, in practice the identification of the ultimate shareholders depends on the corporate coverage of the

model. Therefore, we suggest an easily workable definition based on the use of a parametric threshold that can be sized according to the empirical needs.

The paper is organised as follows. Section 2 summarises the integrated ownership matrix approach and introduces the no-double counting principle. Section 3 presents the model for the integrated control under the majority rule. Control ratios for ultimate shareholders are presented in Section 4. In Section 5, the model is applied to the Frère Group, a large European pyramidal holding company known for mastering control leverages. Section 7 concludes.

## 2. Direct and integrated ownership

Consider a set of  $n$  firms. Let  $a_{ij}$  be the share of cash-flow rights that firm  $i$  holds in firm  $j$ . The  $n$ -square matrix  $A = (a_{ij})$  thus represents the *direct cross-ownership* in the set. Allowing that the ownership of the firms can be partly unidentified, we have:

$$\sum_{j=1}^n a_{ij} \leq 1, \text{ for } i = 1, \dots, n.$$

*Integrated ownership* is the sum of all direct and indirect participations. Its determination is useful for consolidation (accounting perspective) as well as for corporate governance concerns. It typically appears through a recursive scheme: firm  $i$  has cash-flow rights in firm  $j$  and firm  $j$  has cash-flow rights in firm  $k$ . In this simple case of a two-step chain, the indirect ownership share of firm  $i$  in firm  $k$  is obtained by multiplying  $a_{ij}$  by  $a_{jk}$ . More generally, the matrix approach to integrated ownership (Flath, 1992) involves the matrix, denoted  $Y = (y_{ij})$ , obtained by multiplying all transitive direct ownership shares:

$$Y = \sum_{\alpha=1}^{\infty} A^{\alpha} = (I - A)^{-1} A, \tag{1}$$



where the  $\alpha^{\text{th}}$  term of the series,  $A^\alpha$ , stands for all possible chains of length  $\alpha$ .

Indirect ownership does not give rise to additional cash-flow rights. For this reason, it is sometimes called control. However, in our view, control considerations go far beyond integrated ownership due to the impact of majority shareholders (see next section).

The computation of integrated ownership is not obvious because the simple sum of direct and indirect participations suffers from a double counting problem. Indeed, when adding up all levels of ownership, the same shares are considered at each ownership level. Therefore, the sum  $y_{ij}$  of all shares held by direct and indirect shareholders of a company often exceeds 100%.

Starting from an accounting perspective, Huber and Ryll (1989) derive an integrated ownership<sup>1</sup> matrix  $V = (v_{ij})$  cleaned from double counting.

*Definition:*

The  $n$ -square matrix  $V = (v_{ij})$  of *integrated ownership* is given by:

$$V = [Diag(I - \bar{A})] Y = [Diag(I - \bar{A})] (I - A)^{-1} A, \quad (2)$$

where:  $\bar{A} = (a_j)$  and  $a_j = \sum_{i=1}^n a_{ij}$ .

The column sums in  $V$  never exceed 100%. Indeed,  $a_j$  is the sum of all shares held in firm  $j$  by the firms belonging to the set and the diagonal matrix  $Diag(I - \bar{A})$  gives the direct shares held outside this set. Pre-multiplying matrix  $Y$  by this diagonal matrix is equivalent to

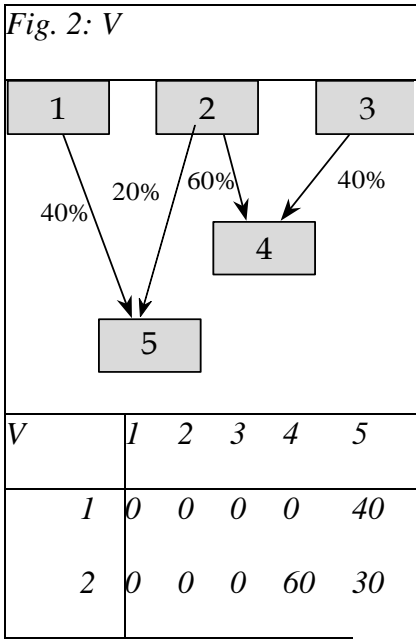
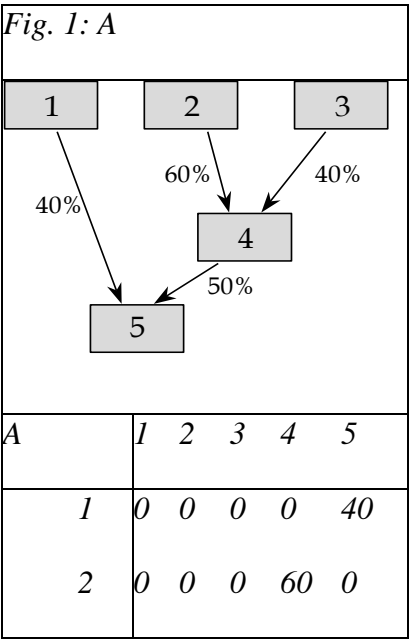
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<sup>1</sup> Actually, these authors consider indirect ownership as control.

selecting the shares held at successive levels by the external shareholders. Therefore, the shareholdings coming from intermediary levels of ownership are cancelled, i.e., multiplied by zero.

This approach eliminates the double counting. Indeed, the total holdings are the same for direct (matrix *A*) and integrated (matrix *V*) ownerships. The difference lies in the allocation of the ownership pieces to the company's shareholders. Integration takes away the cash-flow rights from the intermediate firms and attributes them to the corresponding ultimate shareholders in due proportions.

Fig. 1 and 2 illustrate the matrix algebra on a set of 5 firms counting 3 ultimate shareholders. Fig. 1 gives the original situation corresponding to matrix *A*. The direct cash-flow rights of firm 4 in firm 5 (50% in matrix *A*) lead to indirect ownership of firms 2 and 3 in firm 5 (respectively, 30% and 20%, in matrix *V*), as shown in Fig. 2. Thus, the shares of the intermediate corporation, firm 4, are reallocated to its ultimate shareholders.



3	0	0	0	40	0
4	0	0	0	0	50
5	0	0	0	0	0
<i>Total</i>	0	0	0	100	90

3	0	0	0	40	20
4	0	0	0	0	0
5	0	0	0	0	0
<i>Total</i>	0	0	0	100	90

### 3. The majority control model

Control is a matter of voting power. Therefore, any issue related to controlling shareholders must start from voting rights. Frequently, all company's stocks give rise to identical voting power (sometimes for legal reasons), making direct ownership coincide with direct voting rights. But, this may not be the case: Some shares may be associated to multiple votes or, on the opposite, to no vote at all. On this point, national habits differ largely. For instance, multiple voting rights are forbidden in Belgium, hardly used in France, Portugal and Spain, but widely spread in Germany (Biebuyck et al., 2005). Different categories of shares are then traded at different prices, like in South Africa (Barr et al., 1997), and the spread conveys information on the value of corporate votes (Zingales, 1995).

Let  $b_{ij}$  be the share of voting rights that firm  $i$  holds in firm  $j$ . The  $n$ -square matrix  $B = (b_{ij})$  thus represents the *direct control shares* in the set of  $n$  firms. If cash-flow rights and voting rights are identical, then  $b_{ij}$  is equal to  $a_{ij}$  for all  $i$ 's and  $j$ 's.

*Effective control* is defined with reference to the majority threshold. A shareholder having more than 50% of the voting rights of a company is said to hold its *full control*, i.e., 100% of effective control. This rule is consistent with the Bebchuk et al. (2000) view that 50% of voting rights are necessary for formal control in pyramidal structures. However, the debate on

this issue is linked to the observed ownership concentration, which typically exhibits a large variability in time and space. But, even in an environment of dispersed ownership, partial control is always open to challenge through takeovers, for instance. Furthermore, even if the 50% threshold may look too conservative for several situations, it is the smallest threshold that cannot lead to theoretical inconsistencies such as a full control exerted by two different shareholders<sup>2</sup>.

Several papers consider that a shareholder controls a company with only 20% of the voting rights, a level suggested by La Porta et al. (1999). According to these authors, the threshold can even be lowered to 10% when the ownership is more dispersed, like in Anglo-Saxon countries. Analysing Asian countries, Claessens et al. (2000) follow the arguments by La Porta et al. (1999) to justify their choice of the 20% and 10% thresholds. They, nevertheless, agree that this limit may vary according to the country legislation.

Bennedsen et al. (2003) examine the distribution of ownership and the implied allocation of control in small closely held companies, where control is generally very concentrated. They state that, although an ownership stake of 25% might bring ultimate control when the rest of the ownership distribution is much dispersed, such a stake brings almost no control in closely held companies, where a shareholder can reach 55% of the votes. This argument is not only applicable to small firms. Indeed, many European listed companies display an ownership concentration similar to the one of closely held corporations (ECGN, 2001). Working on European companies, Renneboog (2000) and Köke (2000) share the belief that the voting power of 50% plus one vote is needed to achieve full control.

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<sup>2</sup> Some authors allow for control exerted by two shareholders. For instance, Faccio and Lang (2002) state that “If a firm has two owners with 12% control rights, then we say that the firm is half controlled by each owner at the 10% level threshold ...”. According to our definition, half control is impossible. Either a firm is fully controlled by a single owner, or there is no full control of this firm.

Actually, the threshold of 20%, for instance, follows from an empirical rule of thumb that might be relevant for specific countries and specific periods of time, but has no specific formal justification. In case of concentrated ownership, it cannot hold. On the opposite, the 50% voting power threshold benefits from the legal enforcement in corporate strategic. By nature, it allows *a priori* at most one controlling shareholder per company. Moreover, the majority rule rationalizes the actual existence of shares larger than 50%. Indeed, if full control were possible at a lower level, why would a rational shareholder hold 50% of the voting rights in a single company and, consequently, loose potential gains from diversification? Stylized facts on European data (ECGN, 2001) confirm that the median largest voting block in listed companies lies above 50% in Austria, Belgium, Germany and Italy (but under 10% in the US and the UK).

Effective control is obtained by applying the *majoritization rule* to the matrix of voting rights, namely, matrix  $B$ . A voting share  $b_{ij} > 0.5$  is turning into absolute control (100%), meaning that firm  $i$  holds full control over firm  $j$ . Moreover, since the other shareholders of firm  $j$  are expropriated from control, their effective control over firm  $j$  is set equal to zero. This definition of effective control formalizes the expropriation faced by the minority shareholders, largely documented in the literature (Shleifer and Vishny, 1995; Zingales, 1994,1995; La Porta et al., 1999).

*Definition:*

The  $n$ -square matrix  $C = (c_{ij})$  of *effective control* is given by:

$$c_{ij} = \begin{cases} 1 & \text{if } b_{ij} > 0.5 \\ 0 & \text{if } \exists k \neq i : b_{kj} > 0.5. \\ b_{ij} & \text{otherwise} \end{cases} \quad (3)$$

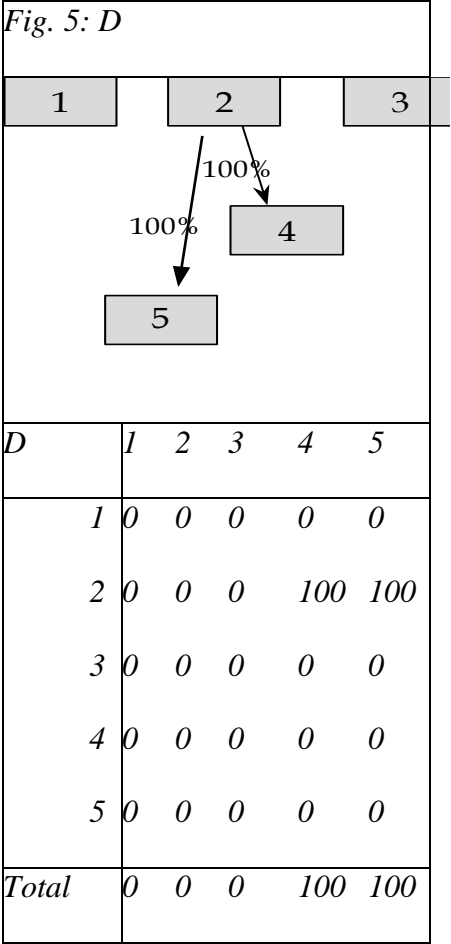
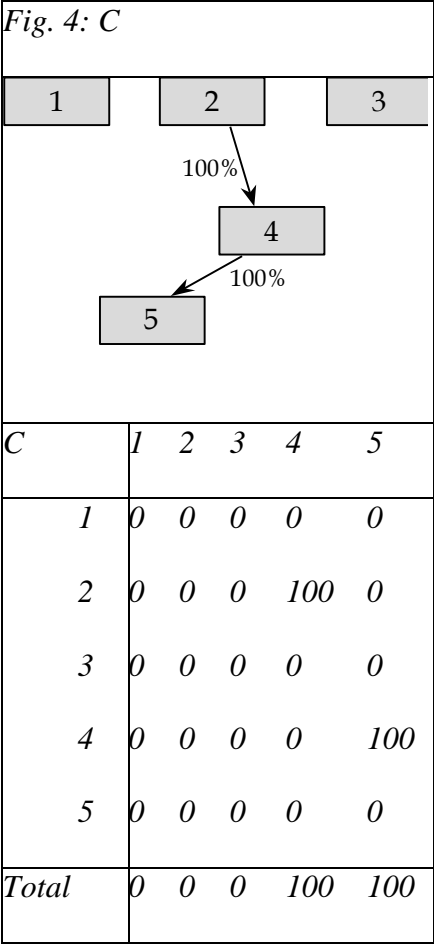
Next, the transition from effective control to *integrated control* is similar to the transition from direct ownership to integrated ownership.

*Definition:*

The  $n$ -square matrix  $D = (d_{ij})$  of *integrated control* is given by:

$$D = [Diag(I - \bar{C})] (I - C)^{-1} C \tag{4}$$

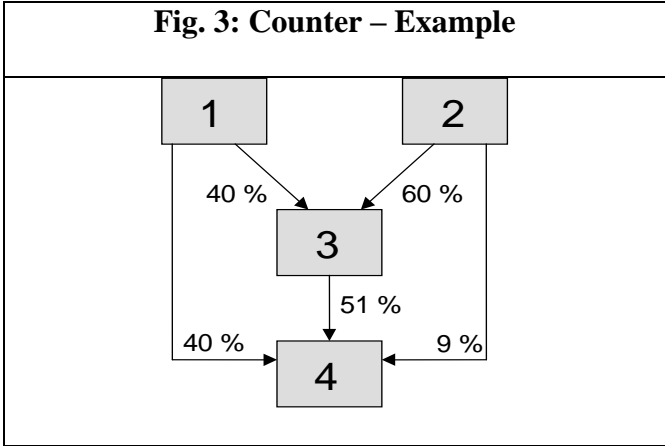
where:  $\bar{C} = (c_j)$  and  $c_j = \sum_{i=1}^n c_{ij}$ .



Following the example provided by Fig. 1 and assuming that  $A = B$  (all shares are identical), Fig. 4 exhibits the effects of applying the majoritization rule to matrix  $A$ , leading to matrix  $C$  of effective control. Firms 1 and 3 are expropriated from their shares of control, respectively,

over firm 5 and firm 4. Fig. 5 illustrates the computation of matrix *D* of indirect control. It appears that firm 2 controls firm 5 through firm 4. The elimination of the arrow linking firm 4 to firm 5 is a consequence of the no double counting principle, attributing to the ultimate shareholder, here firm 2, the whole control exerted by its branches.

Logically, the majoritization rule is applied to matrix *B*, thus before any matrix transformation. In this respect, the specification diverges from Ellerman's (1991) model which incorporates the majority rule at the integrated ownership level (matrix *V*). Fig. 3 provides a counter-example showing that such an *ex post* majoritization may induce misleading results. The illustration includes four firms among which two ultimate shareholders, firms 1 and 2. Applying the majority rule to the coefficient of integrated ownership would lead to the conclusion that firm 1 controls firm 4 since its indirect share of  $40\% \times 51\% = 20.4\%$  is added to its direct share of 40%, making its integrated ownership of 60.4% larger than the 50% threshold. However, firm 2 is the actual controlling shareholder of firm 4. Indeed, with more than 50% of the voting rights, firm 2 controls the intermediate firm 3, which in turn for the same reason controls firm 4. This inconsistency disappears if the majority rule is applied to the effective control matrix *C*.



#### 4. Ultimate shareholders' control ratios

Control ratios are defined with reference to matrix  $V$  of integrated ownership and matrix  $D$  of integrated control.

##### *Definition*

The *control ratio* of firm  $i$  in firm  $j$  is given by:

$$r_{ij} = \frac{d_{ij}}{v_{ij}}. \quad (5)$$

This definition is fully consistent with the previous setting. A control ratio is thus obtained by dividing integrated control by *integrated ownership* and not by *direct ownership* as frequently assumed in papers on ownership and control.

Theoretically, control ratios may be computed for any pair of firms belonging to the original set. However, they are meaningful only for the ultimate shareholders, i.e., the firms exhibiting no known voting shareholder in the system (Becht and Chapelle, 1997). An *ultimate firm* is thus characterised by an empty column in matrix  $B$ .

##### *Definition*

Firm  $j$  is *ultimate* in the system if:

$$\sum_{i=1}^n b_{ij} = 0. \quad (6)$$



There are typically two kinds of ultimate shareholders. The first kind includes the firms whose shareholders are too small to be identified. Such firms have a 100% float. Alternatively, ultimate shareholders can be families, individual investors, or even foundations. Pyramids led by individuals are common, e.g., in Italy (Bianchi and Casavola, 1996) and in Belgium (Becht et al., 2001). Foundations are frequent in the Netherlands (Kabir et al., 1997). In both cases, ultimate shareholders are the ones who really control the corporate decisions, whereas the firms sitting at the lower levels down the pyramids act as their agents.

The previous definition of ultimate shareholders might look radical because it excludes all identified owner, even those with a very weak voting right. It can therefore present some drawbacks when applied to real cases. For instance, a large corporation leading a pyramid could be excluded from the ultimate shareholders class because it depends on another firm through a tiny participation. Even more confusing is the case of a subsidiary a few levels down the pyramid which is holding a small voting share in its top parent company. Such cross-ownerships prevent the identification of the head of the pyramid as the ultimate shareholder. Therefore, we also propose a softened version relying on the introduction of a specific threshold.

### *Definition*

Firm  $j$  is  $T$ -ultimate if:

$$\sum_{i=1}^n b_{ij} \leq T, \quad \text{where } T \in [0, 50\%) \quad (7)$$

The  $T$ -threshold can range from 0 to 50% according to the wished strength of the condition<sup>3</sup>.

The  $T$ -ultimate shareholders may be not only individuals and firms with a 100% float, but also

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<sup>3</sup> This definition refers the ultimate firm's shareholders, while Faccio and Lang (2002) link the "ultimate" qualification for a firm to its own shareholdings.

“non-dominated” firms, that is, firms that are not fully controlled, at least in the set of firms under consideration.

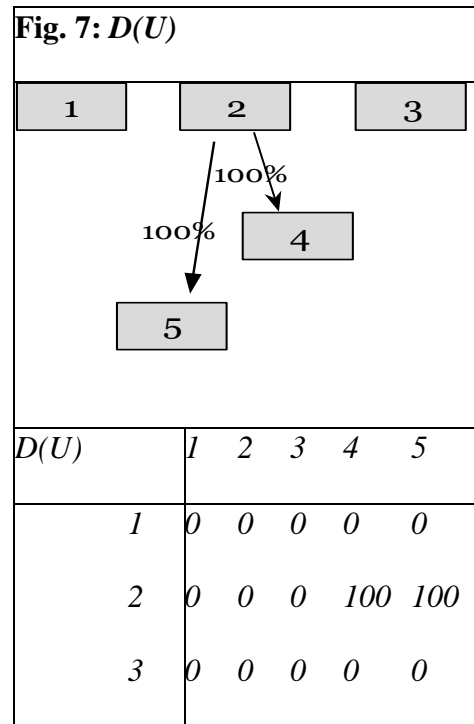
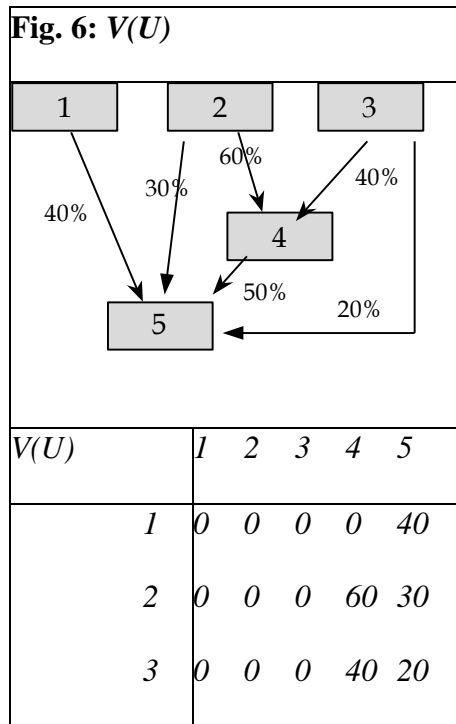
The  $T$ -threshold is very different from - but as important as - the 50% threshold used in the majoritization rule. It influences the main characteristics of the industrial groups, in particular their size and the number and identity of the ultimate shareholders. For example, a group appearing as a single pyramid for  $T = 0$  can be split into smaller groups for larger values of  $T$ . Unfortunately, researchers are often missing this point because their databases are subject to external restrictions, like geographical delimitations or incomplete ownership declarations. Therefore, the extent of the pyramids is generally not tested according any formal definition, but rather dictated by the available information (sometimes coming from the leading firm of the pyramid).

Control ratios quantify the voting leverage exercised by an ultimate shareholder on the various levels of the pyramid. All other things being equal, the longer the pyramid, the higher the control ratio and the potential expropriation of minority shareholders. This is consistent with Wolfenzon's (1999) model which predicts that expropriation is a prominent feature of pyramidal structures.

For any fixed value of the  $T$ -threshold, the selection of the  $T$ -ultimate shareholders (set  $U$ ) is performed on the matrix  $B$  of *direct control*. Then, the corresponding rows of  $V$  and  $D$ , forming, respectively, the matrices  $V(U)$  and  $D(U)$  display the integrated ownership and control of the ultimate shareholders. Note that  $V(U)$  and  $D(U)$  are no longer square matrices.

Following the example starting from Fig. 1, we observe that, whatever the  $T$ -threshold, there are three ultimate shareholders in the set:  $U = \{1, 2, 3\}$ . The three first rows in matrices  $V$  and

$D$  lead to two new  $3 \times 5$  matrices, namely  $V(U)$  for the ultimate shareholders' integrated ownership (Fig. 6) and  $D(U)$  for the ultimate shareholders' integrated control (Fig. 7).



In a general setting, assuming that the  $T$ -ultimate shareholders are the  $p$  first firms of the set, i.e.,  $U = \{1, \dots, p\}$ , then  $V(U)$  is a  $p \times n$  matrix with:

$$[V(U)]_{ij} = v_{ij}, \quad \text{for } i = 1, \dots, p; j = 1, \dots, n; \quad (8)$$

and  $D(U)$  is the  $p \times n$  submatrix of  $D$  given by:

$$[D(U)]_{ij} = d_{ij}, \quad \text{for } i = 1, \dots, p; j = 1, \dots, n. \quad (9)$$

The determination of control ratios (Eq. (5)) only applies to the  $(i, j)$ 's for which  $v_{ij} \neq 0, i \in U, j \notin U$ , i.e., when ultimate shareholder  $i$  holds cash-flow rights in the non-ultimate company  $j$ .

In our example, the relevant ultimate shareholders' control ratios are synthesized by the following matrix:

$$\begin{pmatrix} / & / & / & / & 0 \\ / & / & / & 1.67 & 3.33 \\ / & / & / & 0 & 0 \end{pmatrix}. \quad (10)$$

The existing entries of this matrix correspond to the non-zero entries of  $V(U)$ . The control ratio is equal to zero when the ultimate shareholder  $i$  has no control on the company  $j$  while having some ownership in it. The ratios of  $1.67=100/60$  and  $3.33=100/30$  represent the control leverages of firm 2 in, respectively, firm 4 and firm 5.

Finally, various synthetic control ratios obtained by averaging individual ratios may lead to specific economic interpretations:

- Control ratios *per firm* express the average separation between ownership and control exercised by all shareholders in a given company. A high ratio will typically arise for a firm sitting at the end of the ownership chain, or far from its ultimate controllers in the organisation chart;
- Control ratios *per shareholder* quantify the voting leverage of the ultimate shareholders in their whole portfolio. In this case, a *weighted* average based on the firms' market capitalisation might better reflect the shareholders' voting power relatively to their investment.

Tables 2, 3 and 4 display different synthetic ratios for the set of firms defined in Fig. 1.

<i>Table 2</i>	<i>Average Control Ratios per Firm</i>
Firm 4	0.83
Firm 5	1.11
Gross Average	0.98

<i>Table 3</i>	<i>Average Control Ratio per Owner</i>
Shareholder 1	0
Shareholder 2	2.5
Shareholder 3	0
Gross Average	0.83

<i>Table 4</i>	<i>Weighted Average Control Ratio per Owner</i> <i>(Hyp.: Market capitalisation of firm 4: 100,</i> <i>Market capitalisation of firm 5: 500)</i>
Shareholder 1	0
Shareholder 2	3.05
Shareholder 3	0
Gross Average	1.02

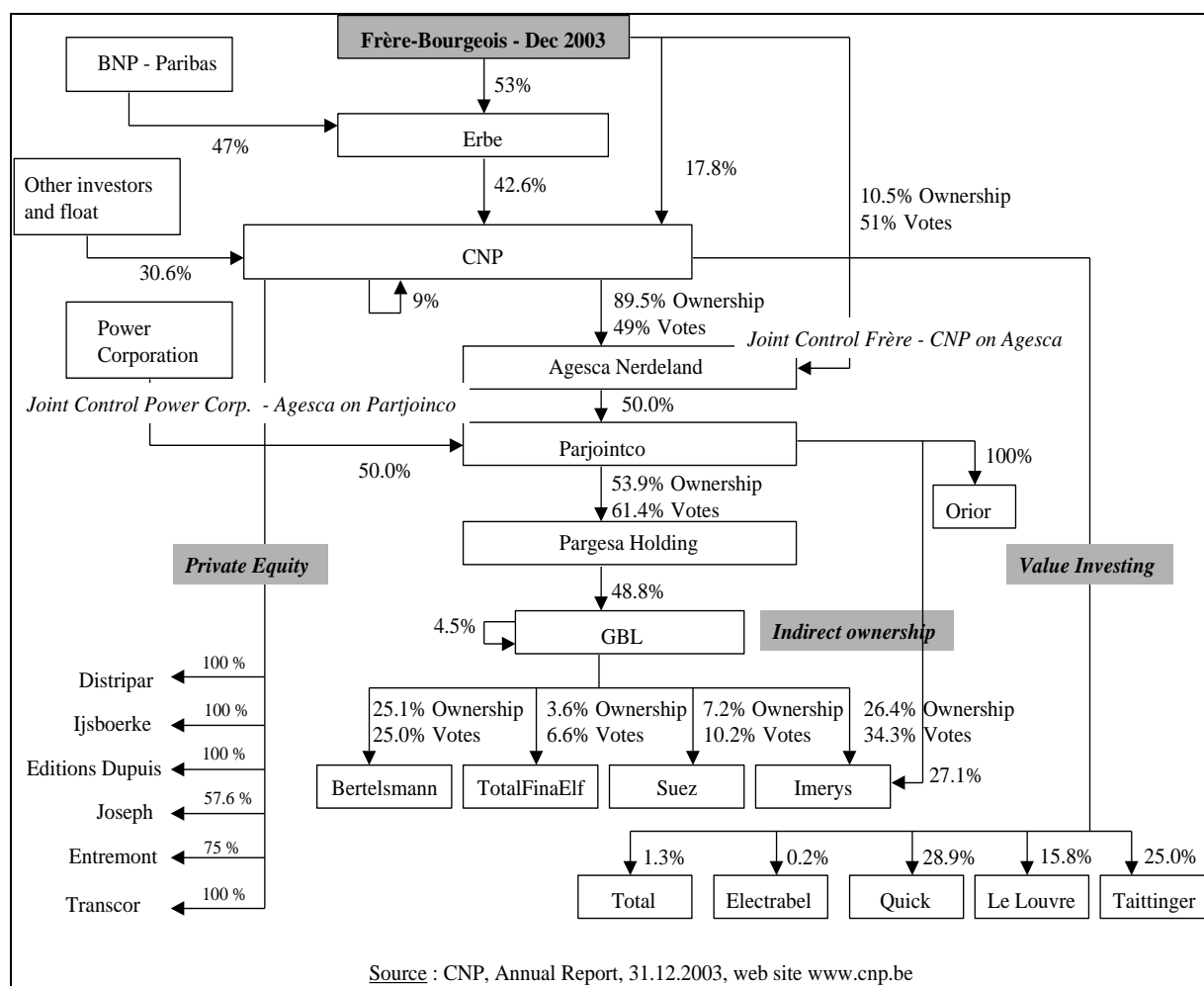
According to Table 2, the ultimate shareholders of firm 4 control on average 17% stakes less than they actually own, whereas those of firm 5 control on average 11% more than they own. From Table 3, it appears that shareholder 2 is the only one who controls the firms she owns with a leverage of 2.5. Table 4 gives the weighted control ratios per owner, assuming a market capitalisation of 100 for firm 4 and 500 for firm 5. The control ratio of owner 2 rises then from 2.5 to 3.05, since this investor has a higher control ratio in the largest firm. Finally, a global index averaging all control ratios in the system provides a synthetic view on the leverages at work in the economy.

## 5. Application to a prominent European pyramid: The Frère Group

The Frère Group (Fig. 8) is the largest Belgian corporate pyramid. Its founder Albert Frère, an important businessman in Europe, is known for his ability to master control leverages (Fralon, 1997, Delvaux and Michielsen, 1999). Therefore, this group offers an adequate ground for applying our model. The control ratios are provided by Table 5.

The group is dominated by the Frère-Bourgeois company which is privately held and controlled at 100% by Albert Frère and his family. It is composed of a cascade of 22 shareholdings counting up to 7 ownership levels. The head of the structure counts three levels. Besides the leading company, the family is controlling by the pyramid *via* the firm Erbe in partnership with the French bank BNP-Paribas. Then, the holding company CNP has institutional minority owners and dispersed shareholders. The Frère family holds the full majority control over these three levels, while benefiting from outside capital: 47% brought to Erbe by BNP-Paribas, and 30.6% brought to CNP by the market. Even at this early stage of the pyramid, the family group has built high control leverages. Indeed, its control ratio is 2.5 in CNP (see Table 5). In other words, Frère controls 2.5 times more than he owns – and paid for – in CNP (full control with 40% of integrated ownership).

Fig. 8 – The Frère Group in December 2003



Moving down the pyramid, the portfolio of CNP is divided into three categories of assets characterized, respectively, as value investing, private equity, and indirect shareholdings. The value investing component is made of minority holdings, mostly in luxury goods (Taittinger) and food retail (Quick). Private equity refers to local companies in UK, Belgium and France in various sectors (food, cosmetics, clothing, editing, and transportation). In these two groups, the control leverage is equal to the CNP ratio, since either the companies are fully owned (in private equity), or they represent minority shareholdings (in value investing). Joseph (UK clothing) and Entremont (French cheese) are two exceptions: these companies are controlled without full ownership, leading to higher control ratios, respectively, 4.3 and 3.3.

The third category includes the indirect investments passing through Pargesa and GBL. The situation there is more complex, and the control ratios are much higher. Indeed, Frère and CNP jointly control Agesca Nederland, a Dutch holding company, which in turn jointly controls Partjointco. Up to this point, according to the majority voting rule, the Frère Group fully controls all the firms. The control ratios are: 1.9 in Erbe, 2.5 in CNP, and 2.1 in Agesca (Table 5).

Table 5: Control Ratios of the Frère Group in December 2003

FRERE Portfolio 2003				
Firms	Integrated Ownership	Integrated Control	Control Ratio	
Erbe	53,0	100,0	1,9	
<b>CNP</b>	40,4	100,0	2,5	
Agesca	46,6	100,0	2,1	
Parjointco	23,3	50,0	2,1	
Pargesa	12,6	50,0	4,0	
Orior	12,6	50,0	4,0	
<b>GBL</b>	6,1	50,0	8,2	
Indirect Investing	Average		3,5	
Bertelsmann	1,5	12,5	8,1	
TotalFinaElf	0,2	3,3	14,9	
Suez	0,4	5,1	11,5	
Imerys	12,5	50,0	4,0	
Value Investing	Average		9,7	
Total	0,5	1,3	2,5	
Electrabel	0,1	0,2	2,5	
Quick	11,7	28,9	2,5	
Le Louvre	6,4	15,8	2,5	
Taittinger	10,1	25,0	2,5	
Private Equity	Average		2,5	
Distripar	40,4	100,0	2,5	
Ijsboreke	40,4	100,0	2,5	
Editions Dupuis	40,4	100,0	2,5	
Joseph	23,3	100,0	4,3	
Entremont	30,3	100,0	3,3	
Transcor	40,4	100,0	2,5	
Min			1,9	
Max			14,9	
Mean	Global Average		3,4	



Partjointco is a company set up to seal the agreement between the Frère family and Power Corporation, a large Canadian holding company controlled by Paul Desmarais Sr. and his family. Frère and Desmarais are partners since 1990. Their convention lasting until 2014 concerns Pargesa, its subsidiaries, and its strategic interests (CNP, 2003). Pargesa Holding, incorporated in Switzerland, is the holder of a portfolio of industrial participations in Europe, either directly or through GBL. In order to stick to the assumptions of the model, we do not incorporate the consequences of this agreement in the control ratios. In this way, we underestimate them.

The successive ownership layers lead to impressive control ratios in the indirect investing portfolio, close to 10 on average, and reaching a maximal value of 14.9 in TotalFinaElf, a large petroleum company. Note that due to the exact split of control shares between the two shareholders of Partjoinco, Frère's control level in these companies remains at 50% at this stage and down in the structure. With any lower control threshold (even 49.9%), the control ratio of the Frère group (but also of the Desmarais group) would be multiplied by at least 2, transforming this strict 50% shareholding into a full control.

Frère's minimal control ratio lies above one, indicating that his integrated control always dominates his integrated ownership. The structure combines several means of separation between ownership and control: pyramiding, majority rules, shares without voting right, and own shares increasing the control of outside shareholdings, like in CNP and GBL, where the 48.8% ownership share is associated to a majority voting right of 51%, implied by the 4.5% of own shares. On average, the control ratio of Frère-Bourgeois in 2003 equals 3.4.

## 6. Concluding remarks

Based on the majority voting rule, this paper provides a model designed for the valuation of the control ratios (integrated control over integrated ownership) exerted by the ultimate shareholders of a set of companies. These ratios are defined per stake, per firm and per shareholder.

What is the adequate threshold for full control over a firm? The debate on this matter opposes the conservative view that 50% of the votes are required to the assessment that lower shares, like 20%, are sufficient to drive the firm policy. While this paper argues in favor of and uses the 50% threshold, the theoretical model remains applicable to any other control threshold.

Nevertheless, if full control is assumed to be reachable below 50%, then the occurrence of multiple non-colluding controlling shareholders cannot be avoided. Besides, Continental Europe and Asia offer plenty of examples of highly concentrated ownership structures, showing that the case for the majority threshold is not only theoretical but does also correspond to the stylized facts in several regions of the world.

By combining the majority-based control and the no-double counting rule, our model solves the underestimation of control typically observed in earlier applications of the matrix methodology. In this sense, it offers an alternative to the combination of underestimated control coefficients and theoretically unjustifiable low thresholds.

A related issue concerns the definition of the ultimate shareholders (the " $T$ -threshold"). According to the strictest definition, an ultimate shareholder is a company or an individual with no identified owner in the system under consideration ( $T=0$ ). However, the

determination of the ultimate shareholders is less obvious to address in practice. Indeed, due to restrictions inherent to the data collection, it may be impossible to take into account all the links existing among the firms. Some ultimate shareholders involved in the group of interest may also be missing. Furthermore, choosing  $T = 0$  may unreasonably extend the perimeter of industrial groups because of very small participations. Therefore, small positive values for  $T$  may reveal more realistic in the empirics. Further research could investigate the robustness of the control ratios with respect to the variations of both the full control threshold and the ultimate shareholder  $T$ -threshold.

Besides the choice of thresholds, the literature is not unanimous on how control is to be measured. The matrix approach followed in this paper is challenged, for instance, by the so-called *weakest link* method (Claessens et al., 2000). However, the matrix design used, e.g., for consolidation remains in our view the most adequate for determining consistently integrated ownership and integrated control. Like in the classical input-output framework, it is the ideal tool for dealing with transitive relationships whatever their amplitude. Any other method would at some point disconnect control measurement from ownership measurement, making it hard to build coherent control ratios.

Whatever the methodology they use, all authors agree that a control threshold is needed. This paper has shown that this threshold must be introduced at the very beginning of the procedure. Otherwise, one can get inconsistent results attributing full control to a wrong shareholder. This finding encompasses our model. It could be seen as a basic requirement for any formal determination of control.

While the issue of controlling coalitions remains beyond the scope of this paper, it might be seen as a privileged domain for further applications. Section 5 has presented the situation of

the Frère Group which offers an insightful example of a fifty-fifty coalition. This case leads to the challenging question of how such well-defined, but still temporary, agreements should be considered as far as control ratios are concerned.

Finally, in line with Nicodano and Sembenelli (2000), we believe that the use of control ratios would carry the drawback of neglecting ownership structure characteristics in the analysis of control rents. Moreover, precise information on control shares should help regulators and small shareholders' advocates arguing their case. As a matter of facts, the conservatism of the majority rule makes the results from our model incontestable regarding the minority shareholders' expropriation, if any.

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