Control consolidation with a threshold: An algorithm

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JEL Classifications: G32, C67, L22
Keywords: Ownership, Corporate control, ultimate shareholder.

CEB Working Paper N° 06/007
May 2006
Control consolidation with a threshold:

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May 2006

Abstract

Control tunnelling over firms can be reached through pyramids, cross-ownership, and other complex features. This phenomenon is frequent in Europe and in Asia. However, the theoretical literature has not yet converged toward a well-defined and robust measurement of integrated control that takes into account the threshold for control as applied in practice. Based on graph theory, this paper aims at filling this gap and proposes a new algorithm for evaluating the control tunnelling exerted by the firms' ultimate shareholders. Then, the paper discusses the various forms of control existing next to voting shares, like multiple voting rights, board representation and active monitoring, before suggesting ways to include them into the modelling of control.

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¹ We are very grateful to Yves Crama from the Université de Liège, and Bertrand Mareschal and Marc Levy from the Université Libre de Bruxelles for the suggestions they made on earlier versions of this paper.
1. Introduction

According to the popular wisdom, either you control something or you do not. Control is not meant to be a partial right. It is merely an exclusive privilege that allows for taking all important decisions related to the controlled entity.

In the corporate world, as prescribed by law, the shareholders' control is exerted through their voting rights in the General Assembly. Although the literature offers no consensus$^2$ on the level of voting rights necessary to capture these benefits, the existence of such a control threshold is largely recognized.

Voting practices vary across the world, in politics as well as in corporations (Holly, 2001). International corporate governance principles state that “All shareholders of the same class should be treated equally” (OECD, 2004). But the Principles do not take a position on the concept of “one share/one vote” (OECD Principle IIA.1). While deviations from the one-share-one-vote principle seem to affect negatively the value of the firm and its efficiency (Jensen and Warner, 2000), they are unlikely to harm minority holders, as the lack of control rights is compensated by the lower share price at the flotation (Hart, 1988).

Non-voting shares, dual classes of shares, multiple voting rights, golden shares, etc. are ownership modalities that contradict the one-share-one-vote rule and allow for concentrating

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$^2$ Most theoretical contributions set the control threshold at 50% of the voting rights (Hauswald and Hege, 2003; Earle et al., 2004; Chapelle and Szafarz, 2005), in particular in the context of controlling coalitions (Bennedsen and Wolfenzon, 2000). Contrastingly, the empirical literature often considers lower control thresholds - typically 20% of voting rights, even 10% in case of dispersed ownership - (La Porta et al., 1999; Claessens et al., 2000; Faccio and Lang, 2002).
control, leading to more potential private benefits (Zingales 1994, La Porta et al., 1999; Bebchuk, 1999; Bertrand et al., 2002; Barclay et al., 2001; Dyck and Zingales, 2004).

As a matter of facts, multiple voting shares used to be allowed in countries like Germany, France, UK, US and particularly common in Scandinavia. However, the EU Takeover Directive (Directive 2004/25/EC, planned to be implemented in May 2006) by allowing hostile takeover bids while discouraging certain incumbents defenses mechanisms (Nilsen, 2004) has led to a decline in the occurrence of multiple voting rights (Goergen et al., 2005). Issue of shares with multiple voting rights is outlawed since 1998 in Italy, Spain, the UK and Germany³ and it has always been forbidden in Belgium (Faccio and Lang, 2002). Among countries respecting the one share/ one vote principle the most, we find: Belgium (100%), Germany (97%) and UK (88%), (Deminor, 2005).

Pyramidal structures inducing indirect ownership constitute another way to depart from the one-share-one-vote principle. They separate ownership from control by using chains of successive participations starting at the top by an ultimate shareholder (Renneboog, 2000; ECGN, 2001, Köke 2004). Because of the embedded structure of pyramids, the direct voting rights which ignore leverages underestimate the control power of some dominant shareholders. Therefore, the correct measurement of actual control shares is based on some consolidation techniques adding up direct and indirect voting rights. Along this line, "full control" over a firm is associated to holding more consolidated voting shares than a given threshold.

³ Prior to this, German firms could be authorized to issue shares with multiple voting rights, like Rwe AG did (x20 voting rights at the end of 1996) or Siemens AG (x6 voting rights) (Faccio and Lang, 2002).
The literature offers various methods for consolidating control matrices (Flath, 1992; Ellerman, 1991; Huber and Ryll, 1989). They are mainly based on input-output matrix algebra (Brioschi et al., 1991; Baldone et al., 1997) applied to ownership data. After the consolidation, the controlling shareholders are identified by comparing the resulting entries of the control matrix to a given threshold. Any value larger than the threshold is then interpreted as the sign that the corresponding firm is dominated by the corresponding shareholder.

In this paper, we show that consolidating before applying the control threshold may lead to inconsistencies. Therefore, any control threshold needs to be introduced in the algorithm before consolidating. Consistently, a recursive approach based on recursive control (control chains) on the one hand, and additive control (summing up of voting shares) on the other hand is proposed. The algorithm converges in a finite number of steps. At the end, exclusive ultimate controllers are identified.

The paper is organised as follows: Section 2 presents the formal definitions of control relationships. Section 3 details the algorithm. Section 4 concludes.
2. Definitions and assumptions

Let \( S = \{ F_1, \ldots, F_n \} \) be the finite set of firms under consideration and \( a_{ij}, i = 1, \ldots, n; j = 1, \ldots, n \), the share of voting stakes of firm \( i \) in firm \( j \). The \( n \)-square matrix \( A = (a_{ij}) \) thus represents the direct voting powers in \( S \). By definition, we have:

\[
\sum_{i=1}^{n} a_{ij} \leq 100\%, \quad j = 1, \ldots, n.
\]  

(2.1)

This inequality is strict when some shareholders of firm \( i \) are unidentified or lie outside of \( S \). Therefore, the system of firms may be linked to outside firms, which is the most realistic situation.

Control may be direct or indirect. Direct control is immediately readable from matrix \( A \). Indirect control will result from transitive mechanisms which in turn require some matrix algebra. We first define direct control.

The control relationship is assumed to be exclusive. Indeed, the very meaning of controlling a firm refers to the possibility to enforce unilaterally its main management policy decisions. Except in some rare cases of a joint contractual control, this corresponds to the actual stylized facts in the corporate world. Control is therefore defined with respect to a fixed rule, valid for all firms in the set of interest.
A firm is said to control another one if it reaches the control threshold denoted by $T_c$. In practice, the most obvious threshold corresponds to the majority voting rule $T_c = 50\%$. However, as a matter of fact, lower thresholds - like 20\% or 30\% - are often used in the empirical literature, due to high dispersion of voting rights among shareholders. Therefore, we opt for a general definition where this threshold remains parametric, but still constrained in such a way (Hypothesis 1) that the definition of control (Definition 1) is consistent.

**Hypothesis 1:** $0.5 \geq T_c \geq T_{c_{min}} = \max_{j=1,\ldots,n} \{a_{(2)j}\}$, where $a_{(k)j}$ denotes the firm $F_j$’s $k^{th}$ largest shareholder.

**Definition 1:** Firm $F_i$ directly controls firm $F_j$ ($F_i \succ F_j$) if $a_y > T_c$.

The consistency of Definition 1 follows from the assumption (Hypothesis 1) that $T_c$ is such that there is at most one controlling shareholder for any firm in $S$. Moreover, a threshold $T_c$ does always exist since, for all $S$, $T_{c_{min}} \leq 50\%$. Thus the maximal threshold, i.e., 50\% is admissible in any situation. In the case where a firm in $S$ has two owners sharing equally the total control this maximal value is the only possibility for $T_c$. Otherwise, the interval $[T_{c_{min}}, 0.5]$ includes more than one element and values of $T_c$ lower than 50\% are also admissible. Note that $T_{c_{min}}$ is endogenously determined since it depends on the control power of all the second largest shareholders in the set.
Definition 1 may look quite restrictive because it uses the same control threshold for all firms, i.e., the size of the largest second shareholder of the set, plus one vote. An extension to firm-specific thresholds is theoretically conceivable but would make the algorithm harder to handle. The ultimate shareholders are the firms in $S$ that cannot be controlled in $S$.

**Definition 2:** $F_i \in S$ is an **ultimate shareholder (or an ultimate firm)** in $S$ if

$$\sum_{j=1}^{n} a_{ij} < T_c .$$

The set of ultimate shareholders in $S$ is denoted $US$ (with $US \subset S$).

Only ultimate firms are candidate for achieving full control over the other firms in $S$. However, some non-ultimate firms may act as control vehicles for ultimate ones.

### 3. The algorithm

#### 3.1. First step: $A \rightarrow B$

Put into an operational fashion, the exclusiveness of control states appears in the first step of the procedure: Any entry of $A$ corresponding to a direct control is replaced by one. Consequently, the other voting shares in the controlled firm offers no more control power, and is thus set equal to zero. Namely, matrix $B$ of **direct control** in $S$ is built in the following way: If $F_i \succ F_j$ ($F_i$ directly controls $F_j$), then $b_{ij} = 1$, and all other control shares in firm $F_j$ vanish:
\[ B = \begin{pmatrix} b_{ij} \end{pmatrix} \text{ is the } n\text{-square matrix defined by: } b_{ij} = \begin{cases} 1 & \text{if } F_i \succ F_j \\ 0 & \text{if } \exists k \neq i : F_k \succ F_j \\ a_{ij} & \text{otherwise} \end{cases} \]

3.2. Second step: \( B \rightarrow C^{(1)} \)

Control over firms is not restricted to direct control. Two non-mutually-exclusive mechanisms leading to indirect control are possible: Transitivity and summing up of voting shares. These two possibilities will be included successively.

A new matrix, \( C^{(1)} \), is built from \( B \) in order to acknowledge for transitive - or chain - control exerted by ultimate shareholders. Only ultimate shareholders are admissible for chain control. Moreover, the intermediate firms, i.e., those acting as control leverages for their ultimate parent, are deterred from their control shares. Note however, that non-ultimate non-controlled firms keep unchanged their control shares. \( C^{(1)} = \begin{pmatrix} c^{(1)}_{ij} \end{pmatrix} \) is the \( n\)-square matrix defined by:

\[ c^{(1)}_{ij} = \begin{cases} 1 & \text{if } F_i \in US \text{ and } \exists F_{k_1}, F_{k_2}, \ldots, F_{k_p} \in S : F_i \succ F_{k_1} \succ F_{k_2} \succ \ldots F_{k_p} \succ F_j \\ 0 & \text{if } \exists F_k \in US, k \neq i, \exists F_{k_2}, \ldots, F_{k_p} \in S : F_k \succ F_{k_2} \succ \ldots F_{k_p} \succ F_j \\ b_{ij} & \text{otherwise} \end{cases} \]
For all \( F_i \in S \), the set \( \Lambda_{i}^{(1)} \) includes the firms that are already identified as exclusively controlled by \( F_i \):

\[
\Lambda_{i}^{(1)} = \{ F_j \in S : c_{ij}^{(1)} = 1 \}.
\]

3.3. Third step: \( C^{(1)} \rightarrow D^{(1)} \rightarrow C^{(2)} \)

Cross-participations are another way to get indirect control. However, the summing up of control powers coming out distinct sources, whether direct or indirect, is less obvious to formalize. Indeed, all minority participations are not to be taken into account.

For instance, suppose that \( T_c = 50\% \) and that a shareholder, say \( F_1 \), has direct control shares of 45\% in \( F_2 \) and of 40\% in \( F_3 \). Further assume that \( F_3 \) has a 55\% control in \( F_2 \) and is directly controlled by \( F_4 \) (see Figure 1). The arithmetic share of \( F_1 \) in \( F_2 \) is equal to \( 45\% + (40\% \times 55\%) = 57\% \) and lies above \( T_c = 50\% \). However, \( F_4 \succ F_2 \) and \( F_3 \succ F_2 \) imply that \( c_{42} = 100\% \). Therefore, by transitivity, the actual control on \( F_2 \) belongs to \( F_4 \), and not to \( F_1 \).

![Figure 1](image-url)
Let us now consider a different scheme, displayed in Figure 2. Let $F_1$ have a direct voting power of 60% in $F_3$, implying that $F_1 \succ F_3$, and also 40% in $F_2$. Assume that $F_3$ holds a 20% control share in $F_2$. Now, there is no loss of control since $F_1 \succ F_3$, and $F_1$ operates with $40\% + 20\% = 60\% > T_c$ in $F_2$, and benefits from full control.

It follows from these examples that the summing up of control shares is legitimate only for direct shares and shares held by controlled firms. Matrix $D^{(1)} = (d^{(1)}_{ij})$ formalizes this principle:

$$d^{(1)}_{ij} = \begin{cases} 
1 & \text{if } c^{(1)}_{ij} + \sum_{k \in F_j^{(i)}} c^{(1)}_{ik} \geq T_c \\
0 & \text{if } \exists k \neq j : c^{(1)}_{ik} + \sum_{l \in F_j^{(i)}} c^{(1)}_{il} \geq T_c \\
c^{(1)}_{ij} & \text{otherwise}
\end{cases}$$

At this stage, additive control is taken into account. However, this process may introduce new entries equal to unity. Therefore, a new detection of chains is required, leading to matrix $C^{(2)} = (c^{(2)}_{ij})$:

\footnote{Note that in this example: $T^\text{min}_c = 0.45$.}
\[
c^{(2)}_y = \begin{cases} 
1 & \text{if } F_i \in US \text{ and } \exists F_{k_1}, F_{k_2}, \ldots, F_{k_p} \in S : d^{(1)}_{k_1} = d^{(1)}_{k_2} = \cdots = d^{(1)}_{k_p} = 1 \\
0 & \text{if } \exists k \in US, k \neq i, \exists F_{k_2}, \ldots, F_{k_p} \in S : d^{(1)}_{k_2} = \cdots = d^{(1)}_{k_p} = 1 \\
d^{(1)}_y & \text{otherwise}
\end{cases}
\]

Again, for all \(F_i \in US\), the set \(\Lambda^{(2)}_i\) includes the firms now identified as exclusively controlled by \(F_i\):

\[
\Lambda^{(2)}_i = \left\{ F_j \in S : c^{(2)}_{ij} = 1 \right\}.
\]

If \(\exists F_i \in US : \Lambda^{(1)}_i \neq \Lambda^{(2)}_i\), then the next step is required. Otherwise, the algorithm is over.

### 3.4. The recursive procedure:

A recursive procedure is to be implemented in order to fully capture all controls at work in \(S\). It is summarized in figure 3.

\[
A \to B \to C^{(1)} \to \begin{bmatrix} \Lambda^{(1)}_1 \\ \vdots \\ \Lambda^{(1)}_{n^*} \end{bmatrix} \to D^{(1)}, C^{(2)} \to \begin{bmatrix} \Lambda^{(2)}_1 \\ \vdots \\ \Lambda^{(2)}_{n^*} \end{bmatrix} \to \text{test} : \begin{bmatrix} \Lambda^{(2)}_1 \\ \vdots \\ \Lambda^{(2)}_{n^*} \end{bmatrix} = \begin{bmatrix} \Lambda^{(1)}_1 \\ \vdots \\ \Lambda^{(1)}_{n^*} \end{bmatrix} \text{ etc.}
\]

where \(n^* = \#US\)

The transition from \(C^{(h)}\) to \(D^{(h)}\) and from \(D^{(h)}\) to \(C^{(h+1)}\) are given by:

\[
d^{(h)}_y = \begin{cases} 
1 & \text{if } c^{(h)}_y + \sum_{k \neq y} c^{(h)}_{yk} \geq T_c \\
0 & \text{if } \exists k \neq j : c^{(h)}_{jk} + \sum_{k \neq j} c^{(h)}_{yk} \geq T_c \\
c^{(h)}_y & \text{otherwise}
\end{cases}
\]

\[
c^{(h+1)}_y = \begin{cases} 
1 & \text{if } F_i \in US \text{ and } \exists F_{k_1}, F_{k_2}, \ldots, F_{k_p} \in S : d^{(h)}_{k_1} = d^{(h)}_{k_2} = \cdots = d^{(h)}_{k_p} = 1 \\
0 & \text{if } \exists F_k \in US, k \neq i, \exists F_{k_2}, \ldots, F_{k_p} \in S : d^{(h)}_{k_2} = \cdots = d^{(h)}_{k_p} = 1 \\
d^{(h)}_y & \text{otherwise}
\end{cases}
\]
The sets $\Lambda_i^{(h+1)}$, $F_i \in US$, are given by:

$$\Lambda_i^{(h+1)} = \{F_j \in S : c_i^{(h+1)} = 1\} \supseteq \Lambda_i^{(h)}.$$ 

The procedure stops at step $H$ if: $\forall F_j \in US : \Lambda_i^{(H)} = \Lambda_i^{(H-1)}$ [$= \Lambda_j$]. Due to the finite number of firms in the set, the algorithm is convergent. Moreover, since ultimate firms are by definition uncontrolled ones, the number of step is $\leq n - n^*$. 

The final sets $\Lambda_i$, $i \in US$, give the firms exclusively controlled by each ultimate shareholder. These sets may reveal the existence of very indirect control which were not visible at all on the original matrix $A$. Besides, the final $C^{(H)}$ provides the full picture about the control powers in the whole set of firms.

Note, however, that some chains involving non-ultimate firms might remain missing from this picture because the stopping rule is built only on ultimate control. In order to go further in this direction, one would need to impose at least the additional assumption that the voting-share graph (corresponding to Matrix $A$) is a tree (i.e., there exists no ring). Indeed, control in rings is not defined properly (any firm controls the others and vice-versa). Legal statements often prohibit this form of indirect self-control.
4. Conclusion

This paper proposes a new algorithm for evaluating the control tunneling exerted by the firms' ultimate shareholders. It is based on a proper measurement of integrated control that allows for any consistent threshold level. In this way, it generalizes previous theoretical and empirical work restricted to specific thresholds like 10%, 20% and 50% of the shares. Moreover, the minority shareholders' expropriation is fully accounted for.

The algorithm starts from the voting rights repartition among the shareholders and not from the number of held shares. Therefore, the model can accommodate the presence of special features, like for instance golden shares, provided that their presence is correctly acknowledged in the data set. Indeed, such peculiarities do only change the granularity of the allocation of the percentages of votes.

This paper focuses on voting rights at the General Assembly as a way to exercise control over a company. However, other control channels are also at work in the corporate world: external influence on board members, board member representation, etc. As a matter of facts, through official representation at the board level, minority shareholders may benefit from a much larger say in the process than their voting share arithmetics would account for. For instance, the Belgian businessman Albert Frère is the largest shareholder of the international French-based utility company Suez but only controls 12,3% of the voting rights (with 7,1% of the capital) of Suez via the holding GBL. Even being a minority shareholder, Frère holds two seats out of eight on

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5 Suez Annual Report 2004
6 that he controls jointly with Mr Desmarais
the Suez Board to represent his interests; one occupied by himself as vice-president, the other by his partner Paul Desmarais Jr (Biebuyck et al., 2005).

To avoid the predominance of blockholders in the board, corporate governance recommendations insist on the role of independent board members, that is, non-executive directors without any links either with the management or with the block shareholders. Nevertheless, in most systems, shareholders appoint and remove the board, and the board needs shareholders approval for any major strategic decision. Thus, the votes in the Generally Assembly remain the surest source of power in corporations and the algorithm presented in this paper may be interpreted as reflecting both the voting power of the shareholders and their controlling power through acquainted board members.

Monitoring the company management, even without formal representation at the board level, is third way of exercising control. Pension funds and large institutional shareholders typically fit into this category, by keeping a close look on the value of their share investment and on the management of the firm. Financial officers and executive board members dedicate an increasing amount of time briefing institutional investors on the financial statements and strategic prospects of their company. While Wahal (1996) finds no clear impact of pension funds on firm performance, Del Guercio and Hawkins (1999), with more recent data, show the heterogeneity of

7 Suez Annual Report 2004
8 See the European Corporate Governance Institute (www.ecgi.org) for full texts of the corporate governance recomendations.
9 Investor sight benchmark, 2005 on financial communications of 162companies in 10 countries: CEO’s spend twice as much time communicating on the results than in 2004. Available on www.investorsight.com
pension funds activism and their impact on the firm. Moreover, according to Smith (1996), CalPERS, one of the most active pension funds, could be at the origin of the Corporate Governance awareness\textsuperscript{10} in the US.

Possible extensions of our algorithm may thus concern the inclusion of phenomena like the presence on the board and shareholders' activism. The challenge in this direction lies obviously in creating the adequate quantification of these alternative channels of control.

Another potential extension could deal with shareholders' coalitions. Indeed, major influence of minority shareholders often requires the coalition of several parties toward a single decision. Winning coalitions for control have been studied in the literature (Zwiebel, 1995; Gomes and Servaes, 2000; Bennedsen and Wolfenzon, 2000). However, a group can be controlling on one decision type, but not on all. Therefore, the collusion of several parties in a specific context or on a specific point is not equivalent to having several controllers in a firm. Taking account of coalitions may thus be seen as a context-specific issue which departs from the understanding of control as an exclusive prerogative.

Technically, the model concerns the matrix of direct voting powers in a given set of firms. The way this set is composed is not considered here. However, the selection of firms in the set follows from an implicit choice and the definition of ultimate shareholders depends upon this

\textsuperscript{10} In some cases, even without board representation, the pressure of the pension funds can be so hard that it pushes management out of office. This happened recently, when the CEO of the Deutsche Börse has been forced to resign, in May 2005, under the pressure of the pension funds shareholders that strongly disapproved the takeover attempt of the Deutsche Börse over the London Stock Exchange (Financial Times, May, 09, 2005). This decision may be seen as the output of a coalition of several pension funds.
selection. Therefore, the selection process of firms could be endogenised starting from a much larger set.

Finally, our algorithm may be viewed as a building block of a larger model linking corporate governance features to other firms' characteristics such as performance, private benefits of control, or managers' behavior. In this perspective, our results offer a measure of integrated control power that does not suffer from ad hoc - and often inconsistent - assumptions deriving control from ownership data.
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