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A Two-Period Model

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Does Ambiguity Aversion Raise the Optimal Level of Effort? A two-period model*

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Abstract

I consider two-period self-insurance and self-protection models in the presence of ambiguity that either affects the loss or the probabilities, and analyze the effect of ambiguity aversion. I show that in most common situations, ambiguity prudence is a sufficient condition to observe an increase in the level of effort. I proposes an interpretation of the model in the context of climate change, such that self-insurance and self-protection are respectively seen as adaptation and mitigation efforts a policy-maker should provide to deal with an uncertain catastrophic event, and interpret the results obtained as an expression of the Precautionary Principle.

Keywords: Non-expected utility, Self-protection, Self-insurance, Ambiguity prudence, Precautionary Principle.

JEL Classification: D61, D81, D91, Q58.

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1 Introduction

Self-insurance and self-protection are two risk management tools used to deal with the risk of facing a monetary loss when market insurance is not available. In both situations, a decision maker (DM) has the opportunity to furnish an "effort" to modify the distribution of a given risk. In the terminology of Ehrlich and Becker (1972), the effort in a self-insurance model corresponds to the amount of money invested to reduce the size of the loss occuring in the bad state of the world, while in a self-protection model (also called prevention model), the effort is the amount invested to reduce the probability of being in the bad state. Computing the optimal level of effort is relatively straightforward in standard self-insurance and self-protection models, it is simply given by equalizing the marginal benefit and the marginal cost of the effort furnished.

Those two models have received an important attention in the literature these last few years, but this literature has, until now, generally only focused on simple one-period, two-state models that remain in the expected utility framework. In particular in those setups, the effect of risk aversion has been shown to have different impacts depending on the model considered: it always increases the optimal demand for self-insurance, but has an ambiguous effect on the demand for self-protection[1] (Dionne and Eeckhoudt (1985); Briys and Schlesinger (1990)). It is therefore necessary to have access to wider information like the level of the initial optimal probability of loss (Jullien, Salanié, and Salanié (1999)), or to make further assumptions on the utility function (Dachraoui et al. (2004); Eeckhoudt and Gollier (2005)) to obtain a general result from the analysis of risk aversion.

If those single period, risky models are well adpated to describe a certain number of situations as such as the road problem[2], they seem to be too restrictive in at least two dimensions to describe a great number of other issues efficiently. First, there exist many situations requiring self-insurance or self-protection in real life in which the decision to

[1]Since an increase in effort in this model does not correspond to a decrease in risk in the sense of Rothschild and Stiglitz (1970).

[2]A driver must go through a road with little visibility and has the choice between two options: either intall an airbag system or drive very fast. The airbag is costly but reduces the loss in case of crash, while going faster decreases the probability of accident by reducing the time of exposure, but increases the oil consumption.

invest and the realization of uncertainty do not take place at the same point in time. Sometimes a long period of time may elapse between these two events, giving rise to the necessity of taking into account intertemporal considerations. To illustrate this assertion, think for example of an individual living for two periods. This individual faces a risk of heart attack when he becomes old and has the choice, when he is young, to practise sport or not. Doing sport is costly but has the advantage of either reducing the probability of a heart attack with which a potentially important fixed loss is associated, or of reducing the severity of an attack that happens with a fixed probability (think for example of lowering recovery costs due to better physical condition). In this example, it is clear that many years may separate the moment the effort decision is taken and the moment the uncertainty realizes. It therefore seems unappropriate to use a one-period setting to model such a problem. In a recent paper (Berger (2010)), I extended the analysis of self-protection and self-insurance to a two-period environment. I found that the single-period results could be easily extended in the self-insurance case, while in the most usual situations concerning self-protection (i.e. events characterized by low probability of accident occurence), the notion of prudence plays a central role[3].

The second limitation of the models described above is that they remain in the expected utility framework, and are therefore unable to deal with other kinds of uncertainty besides *risk*. Indeed, in almost all the self-insurance and self-protection models studied in the literature, only risky situations are analyzed, meaning that the probabilities associated with the different outcomes are all known with certainty. In particular, all those models implicitly assume the absence of any kind of ambiguity or equivalently assume that agents are ambiguity-neutral (and therefore behave as subjective expected utility maximizers in the sense of Savage (1954)). In many real problems however, the nature of the uncertainty considered cannot be limited to risk. Think for example of environmental economics problems, more specifically to climate change policy issues. Those problems are characterized by a high level of uncertainty that is especially due to our imperfect scientific knowledge: we do not know exactly for example what the benefits associated with a reduction in en-

[3] In particular, I show that prudent and risk-averse expected utility maximizers exert more effort than the risk-neutral agent.

vironmental damage are, or we do not know precisely what the costs for human beings of an increase in average temperatures are. This thus means that, when considering the right thing to do to preserve our environment, the decision making process has to carried out without knowing the probabilities associated with the random events perfectly. Instead, what the decision maker usually has at his disposal is a panel of predictions coming from different scientific models or confidence intervals. As noted by Lange and Treich (2008), the role of the decision maker is therefore to aggregate the findings coming from competing models, and this is generally not done by taking the average value to end up with a single probability distribution, as it is the case in the (subjective) expected utility framework. Indeed, as first shown by Ellsberg (1961), and later confirmed by a number of experimental studies (see Einhorn and Hogarth (1986), Viscusi and Chesson (1999), and Ho et al. (2002) among others), the uncertainty about the probabilities of a random event (called ambiguity) often leads the decision maker to violate the axiom of reduction of compound lotteries in the sense that it makes him over-evaluate less desirable outcomes (this corresponds to ambiguity aversion). It is therefore important to take this individual behavior into account when considering problems in the presence of ambiguity.

In this paper, I present two models of self-insurance and self-protection that do not suffer from these two limitations. For things to remain tractable, each model takes the form of a simple two-period model that incorporates the theory Klibanoff, Marinacci, and Mukerji (2005, 2009) developed to deal with ambiguity. The timing of the decision process is very simple: in first period, a DM chooses the level of effort he wants to exert to either reduce the probability of being in the bad state in second period (self-protection), or to reduce the loss associated with this bad state (self-insurance). Moreover, I introduce in the model the possibility for the state of the world or the loss to be ambiguous.

What I have in mind when elaborating such a model is the climate change issue and the instruments we have at our disposal to fight the global warming phenomenon. In particular, self-protection and self-insurance may be respectively seen as mitigation and adaptation efforts[4] in the sense that the former refers to action taken to permanently eliminate or re-

[4]The definitions given in the IPCC Third Assesment Report are the following. Mitigation: An an-

duce the risk of climate change to human life, while the latter refers to the ability to adapt to climate change to moderate potential damage or to cope with the consequences. If for simplicity, I consider there are only two states of the world, one "good" state in which the temperatures do not increase too much and in which the economic activities are not really altered, and one catastrophic "bad" state which corresponds to a very unfavorable situation in which the economic environment is deeply damaged[5], the situation corresponds exactly to the self-protection or self-insurance model. In that view, it becomes clear that the two limitations I discussed above that are imposed by the one-period, expected utility model need to be overcome. The various IPCC reports strenghten this position by noting that "climate change decision-making is not a once-and-for-all event", "rather it is a process that will take place over decades" (Halsnæs et al. (2007)), and by making a clear distinction between risk and uncertainty. For example they mention that "in most instances, objective probabilities are difficult to estimate", and "where we cannot measure risks and consequences precisely, we cannot simply maximize net benefits mechanically". This thus means that in such a context the traditional decision making process consisting in "choosing the policy set that maximizes the expected (monetary) value of the outcomes might not be appropriate" and that "non-conventional criteria might be required to make robust decisions" (Halsnæs et al. (2007)).

The objective of this paper is to analyze the effect the uncertainty about the probabilities has in the decision making process. In particular, I give an answer to the following question: Does the ambiguous nature of an outcome lead an ambiguity averse decision maker to exert more effort than another who does not take this ambiguity into account? In the context of climate change, this question could be assimilated to the one referring to the Precautionary Principle: Should the lack or the incompleteness of information lead

thropogenic intervention to reduce the sources or enhance the sinks of greenhouse gases. Adaptation: Adjustment in natural or human systems in response to actual or expected climatic stimuli or their effects, which moderates harm or exploits beneficial opportunities. (IPCC (2001))

[5]Think for example of situations resulting from an abrupt climate change such as the disintegration of the West Antartic Ice Sheet that could raise the sea-level by 4-6 meters, the increase in the frequency of droughts or of important tropical cyclones that could have extreme ecological and agricultural consequences (see Meehl et al. (2007)). Note that those situations are generally characterized by a very low probability of occurence.

environmental policy to be precautionary in the sense of favoring more intense intervention? In other words, I investigate whether economic theory is consistent with the claim defined in the 1998 Wingspread Statement on the Precautionary Principle that stipulates

> "when an activity raises threats of harm to human health or the environment, precautionary measures should be taken even if some cause-and-effect relationships are not fully established scientifically".

This paper therefore proposes an economic interpretation of the Precautionary Principle that is different from the one proposed by Gollier, Jullien, and Treich (2000). In what follows, I show that this principle should be respected in the most usual situations, and that additional immediate prevention or adaptation measures should be made if scientific knowledge is imperfect.

This paper is organized as follows. Before going into the details of the model, I describe in Section 2 the notions of ambiguity and ambiguity aversion. In Section 3, I extend the standard one-period models of self-insurance and self-protection to a two-period framework in which either the state of the world, or the loss may be ambiguous. I then provide a comparative statics analysis to see the role played by ambiguity aversion. Finally I conclude in Section 4.

2 Ambiguity and attitude towards ambiguity

The concept of ambiguity is now widely accepted to refer to situations in which the probabilities of a random event are not known with certainty. To be more precise, it refers to situations in which the probability of a given outcome is not objectively known. Instead, what the decision maker (DM) has at his disposal is a subjective prior distribution of what this probability might be. In what follows, I express this idea by considering that the probability an outcome realizes is not unique but consists in a set of probabilities depending on an external parameter for which the DM has prior beliefs. In that sense, the uncertainty about the relevant priors makes the probabilities of the outcomes uncertain and in turn creates ambiguity. Ambiguity therefore modifies the final probability distributions and thus has an impact on the ex ante evaluations of the possible alternatives. Imagine for example a (unambiguous) lottery with n possible outcomes. For all $i = 1, ..., n$, the outcome z_i associated with probability q_i is known with certainty by any DM, making the situation risky. In that situation it is possible to use the economic theory of decision making under risk to evaluate the possible outcomes. If instead, the probability associated with the outcome z_i is not known with certainty but is given by $q_i(\tilde{\theta})$ where $\tilde{\theta}$ is a random variable whose distribution depends on the DM's particular beliefs, the outcome z_i is ambiguous and the probability that it realizes is no longer unique. Instead, it consists in a set of probabilities, each one being associated with a given probability reflecting the DM's beliefs. Since by definition the probabilities of the different ouctomes must sum up to one given any value of the parameter $\tilde{\theta}$, at least two outcomes must be ambiguous if θ is stochastic, making the situation ambiguous.

In what follows, I consider that ambiguity is introduced in such a way that a DM manifesting ambiguity neutrality behaves exactly as a savagian expected utility maximizer. However, he is *ambiguity averse* if he dislikes any mean-preserving spread[6] affecting the probabilities of a given lottery.

To study the effect of ambiguity in the context of self-protection and self-insurance analytically, I use the theory developed by Klibanoff, Marinacci, and Mukerji (2005, 2009) (KMM). Even if their model does not remain in the expect utility framework, it has the advantage of being fairly tractable and enables to consider the effects of ambiguity[7], ambiguity attitude and risk attitude separately. In the general framework KMM developed, the decision maker has a prior distribution (called a second order belief), over the set of possible distributions of the random variable (first order beliefs). Preferences are represented by the following expression taking the form of an "expected utility of an expected utility"

$$W = \phi^{-1} \left\{ E_\mu \phi \left\{ E_{\pi(\tilde{\theta})} u(\tilde{z}) \right\} \right\}$$

[6]Remark that the spread is taken around the "mean" corresponding to the savagian subjective beliefs.

[7]Even if this notion embodies in fact both objective ambiguity and ambiguity perception, an individual characteristic.

where μ is is the DM's subjective prior distribution over the set Θ of possible parameters θ, and hence over the set of possible probabilities π of the random outcome $\tilde{z}$. As noted by Klibanoff, Marinacci, and Mukerji (2005), μ therefore reflects the *"subjective relevance of a particular π as the "right" probability"*. In this representation, u is a classical vNM utility function representing risk preferences over possible payoffs and from now on, I will denote by $\mathcal{U}$ the set of expected utilities obtained with different values of θ: $E_{\pi(\theta)}u(\tilde{z})$. What is new in the form W above is the presence of the increasing function ϕ reflecting the DM's attitude towards ambiguity[8]. To be more precise, if ϕ is concave, we say that the individual is ambiguity averse in the sense that he dislikes any mean-preserving spread in the space of $\mathcal{U}$, while when ϕ is linear, the individual is said to be ambiguity neutral[9]. To see this, note that under concavity of ϕ, the *certainty expected utility equivalent W* is unambiguously lower than the expected utility $E_\mu E_{\pi(\tilde{\theta})}u(\tilde{z}) \equiv E_\pi u(\tilde{z})$, which does no longer depend on θ, since the linearity of ϕ imposes the reduction between μ and π in the support of μ, and therefore makes the preferences equivalent to the ones of a DM in the subjective expected utility framework of Savage (1954).

In a way very similar to Pratt (1964)'s well-known result comparing different degrees of risk aversion, Klibanoff, Marinacci, and Mukerji (2005) also show that it is possible to order individuals according to their degree of ambiguity aversion.

Theorem 1 (Klibanoff, Marinacci, and Mukerji (2005)) *Let A and B be two DMs whose families of preferences share the same probability distribution μ and the same vNM utility function u. If ϕ_A and ϕ_B are twice continuously differentiable, then A is more ambiguity averse than B if and only if for every $x \in \mathcal{U}$*

$$-\frac{\phi_A''(x)}{\phi_A'(x)} \geq -\frac{\phi_B''(x)}{\phi_B'(x)}.$$

Analogously to risk theroy, the ratio $\alpha(x) = -\frac{\phi''(x)}{\phi'(x)}$ is called the *coefficient of absolute*

[8] Note that for simplicity, I assume that ϕ is only defined for non-negative values. Any element of $\mathcal{U}$ must therefore be non-negative, which should not be a problem since any positive affine transformation of u represents the same preferences over risky situations. KMM consider for example the unique continuous, striclty increasing function u with $u(0) = 0$ and $u(1) = 1$ that represents any given preferences.

[9] See Proposition 1 in Klibanoff, Marinacci, and Mukerji (2005).

ambiguity aversion at x. As mentioned by KMM in their second theorem, the function ϕ_A can therefore be obtained by taking an increasing concave transformation h of function ϕ_B: $\phi_A(.) = h(\phi_B(.))$. Intuitively, this means that if the DMs A and B have the same function u, and the same ambiguity perception, agent A will dislike all ambiguous situations that B dislikes, independently of the function u.
A particular case often studied in this literature is the case of constant absolute ambiguity aversion (CAAA) characterized by the functional form $\phi(x) = (1 - \exp(-\alpha x))/\alpha$. Besides being easily tractable[10], this specification has the advantage of encompassing both the two extreme cases of ambiguity neutrality (if $\alpha \to 0$) and therefore the standard expected utility model, and the maxmin utility model of Gilboa and Schmeidler (1989) which may be seen as a limiting case of CAAA assuming infinite ambiguity aversion ($\alpha \to \infty$).

In this paper, I consider separately ambiguity on different elements of the self-protection and self-insurance models. In their simplest versions, those models consider only two states of the world: a loss and a no-loss state, each one being associated with a given probability. In what follows, I analyze the case in which the loss probability is not perfectly known but is ambiguous, before considering the case of ambiguity about the possible loss.

3 Ambiguous self-protection and self-insurance models

According to the terminology first adopted by Ehrlich and Becker (1972), a self-protection model is a model in which the DM has the opportunity to invest a part of his wealth to reduce the probability of facing the bad state of the world. In that sense, it can be distinguished from the self-insurance model in which the effort furnished by the DM redistributes income across states by reducing the size of the loss associated with the bad state. The properties of these two models have been widely studied in the literature, especially in unambiguous, one-period environments (Dionne and Eeckhoudt (1985); Briys and Schlesinger (1990); McGuire, Pratt, and Zeckhauser (1991); Sweeney and Beard (1992); Jullien, Salanié, and Salanié (1999); Chiu (2000); Dachraoui, Dionne, Eeckhoudt,

[10]Note that α is the index of concavity of the function, or similarly the *coefficient of absolute ambiguity aversion*.

and Godfroid (2004); Eeckhoudt and Gollier (2005)).

In what follows, I extend the standard models in two directions. First, I introduce ambiguity on several elements of the models and analyze the effect on the optimal level of effort. Second, I extend the existing one-period models generally used in this literature to a two-period environment in which the decision to invest is taken in the first period, while uncertainty materializes in the second period.

As mentioned in the introduction, such specifications enable to interpret these models as climate change models, in which self-protection corresponds to mitigation effort, and self-insurance to adaptation effort. In particular, imagine that the decision maker is a government that has to choose the percentage of GDP to invest today to fight, or deal with global warming. In the case of self-protection, this government represents a big country (i.e. a country responsible for a large part of global greenhouse gas emissions) such that the action it undertakes now has an impact on the average global temperatures in the future[11]. Think for example to the United States or China. On the contrary in the self-insurance model, the government is the one of a small country, whose preventive action alone cannot affect global temperatures. For example take the Netherlands. If there are no possibilites of coordination with other countries, the only action this government can take is to invest today in technologies that enable its population to survive and go on its economic activity even if a catastrophic event happens. A good example of possible action is the construction of dikes to protect against coastal surges resulting from a potential sea-level raise.

The general two-period expected utility model can be written as follows: a decision-maker chooses the level of self-protection x and of self-insurance y that maximizes his intertemporal expected utility (IEU)

$$IEU = u(w_1 - x - y) + p(x)U(w_2 - L(y)) + (1 - p(x))U(w_2).$$

[11] As noted by Schneider et al. (2007), "recent research indicates that human influence has already increased the risk of certain extreme events such as heatwaves and intense tropical cyclones. There is high confidence that a warming of up to 2°C above 1990-2000 levels would increase the risk of many extreme events, including floods, droughts, heatwaves and fires, with increasing levels of adverse impacts and confidence in this conclusion at higher levels of temperature increase".

In this formulation, w_i is the wealth in the begining of period $i = 1, 2$, u and U represent respectively the first and second period utility functions, $p(x)$ is a decreasing function representing the probability of bad state, and $L(y)$ is a decreasing loss function. The problem is assumed to be concave. Remark that the utility function U incorporates both attitude towards risk and towards intertemporal substitution, but that for IEU to remain in the expected utility framework, we need the coefficient of relative risk aversion to be the reciprocal of the elasticity of intertemporal substitution.

The objective in this paper is to determine whether the optimal levels of effort x^* and y^* are altered by the introduction of ambiguity in the model. However, since insurance and protection may be substitutes, it is impossible to consider them simultaneously while making a comparative statics analysis. I therefore consider separately the models of self-insurance and self-protection, and rewrite the problem as follows

$$\max_e u(w_1 - e) + p(e)U(w_2 - L(e)) + (1 - p(e))U(w_2). \tag{1}$$

In this formulation, e denotes the effort furnished by the DM, and it is easy to recover the standard two-period models of self-insurance (when $p(e) = p$ for all e) and of self-protection (when $L(e) = L$ for all e). Note also that the problem is convex if we have $p''(e) \geq 0$ and $L''(e) \geq 0$, meaning that the marginal benefits of self-protection and of self-insurance are decreasing in the level of effort. Finally remark that since the investment decision takes place in first period and the potential loss in second period, the marginal cost of effort will never be affected by the introduction of ambiguity on either the loss or on the probabilities. Therefore, to see whether the introduction of ambiguity aversion does raise the optimal level of effort, I only have to check whether it increases the marginal benefit.

3.1 Ambiguous states or probabilities

Imagine now that the probability of being in the bad state is not anymore known with certainty to be $p(e)$ but is instead ambiguous. For example, think that the climate scientists of a country are able to compute the law linking the effort to the probability of catastrophe occurence, but that there exists an external parameter that the government cannot control for (it can be the actions taken by governments of other countries, or the natural evolution of the environmental system). In an unambiguous world, the value of this parameter would be know objectively. However, if the world is ambiguous, what the government has at his disposal is only a (subjective) probability distribution over the possible values of this parameter.

The way we transpose this assumption in the economic model is the following. The probability of loss depends on the value of a parameter θ and is given by $p(e, \theta)$. Ambiguity may therefore be interpreted as a multi-stage lottery. A first lottery determines the value of the parameter θ, and a second one determines whether a loss occurs or not. Figure 1 illustrates this situation in the special case of two possible parameters θ_1 and θ_2 associated with probabilities μ and $1 - \mu$.

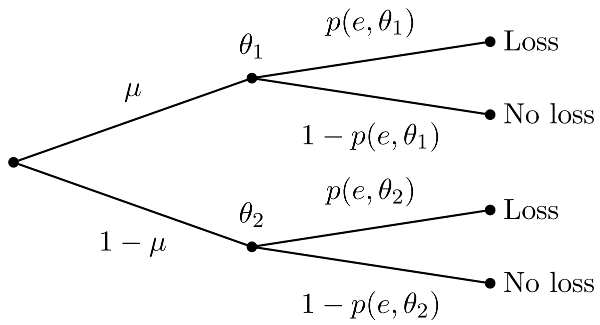

Figure 1: Ambiguous states

Similarly, imagine that the DM, instead of thinking that loss will occur with known probability $p(e)$, believes that different laws $p_i(e)$ exist, but cannot distinguish with certainty the probabilities q_i attached to each of them. In a unambiguous world, q_i would be known and it would be easy for an agent to compute the probability of loss occuring by averaging the different laws. The presence of ambiguity however makes it impossible for an ambiguity averse individual to behave this way. Instead, the probability of facing the law $p_i(e)$ is given by $q_i(\tilde{\theta})$ and each individual has a prior distribution over the possible θ's. To give the intuition, imagine the following situation. Our government receives two different studies coming from two climate scientists, a famous professor and a young post-doc researcher. Each of them has computed his own law $p(e)$ and those two are not exactly similar. The government has now to make a decision about e on the basis of those studies. The problem is that it does not know the intrinsical value of each of them, and can therefore not weight correctly each probability law. If the government were sure that the famous professor had less chance to be mistaken, it could for example give a weight 3/4 to his study and compute an "average" law useful to make the decision. However, this is not the case because some government advisors follows this view, while others think that the famous professor is becoming old and outmoded, and that more importance should be given to the promising young resarcher. How can the government deal with this ambiguous situation?

I assume that all the technical conditions are respected and that I can rank the n possible laws from the most to the less favorable, implying that $p_i(e) \leq p_{i+1}(e)$ for all e and for all $i = 1, ..., n$. The probability of suffering from a loss therefore directly depends on the ambiguity parameter θ and is given by $\text{Prob}[\text{Loss}|\tilde{\theta} = \theta] = \sum_{i=1}^{n} q_i(\theta)p_i(e)$, while the expected probability

$$E\left[\text{Prob}[\text{Loss}]\right] = E\left[\sum_{i=1}^{n} q_i(\tilde{\theta})p_i(e)\right] = \sum_{i=1}^{n} q_i p_i(e) = p(e)$$

does not. The multi-stage lottery illustrating the way ambiguity enters the model is then the following: a first lottery determines the value of the prior, a second one determines the law linking the effort to the probability of loss, and a third one determines whether the loss occurs or not. Figure 2 illustrates this situation when there are only two laws and two priors.

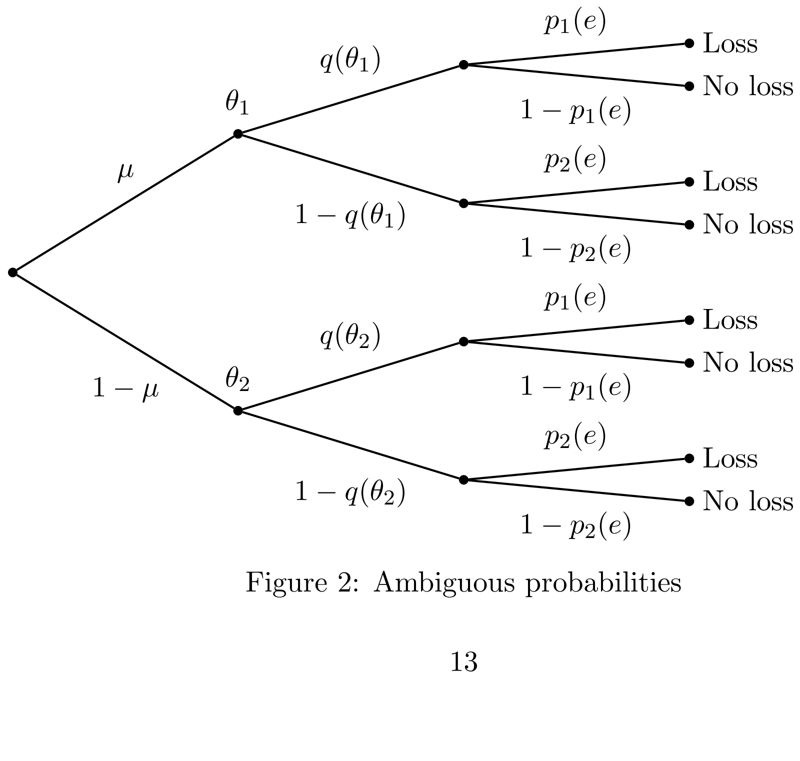

Figure 2: Ambiguous probabilities

It is easy to see that the two situations described above are identical. Since the parameter θ representing ambiguity is fully determined after the first stage of the lottery, the decision maker will be able to use the axiom of reduction of compound lotteries to merge the second and third stages. We then recover the lottery we had in the case of ambiguous states. In the illustrations presented in Figures 1 and 2, this means for example that the probability $p(e, \theta_1)$ in the first case will correspond to $q(\theta_1)[p_1(e) - p_2(e)] + p_2(e)$ in the second.

Under ambiguity aversion, the problem faced by the decision maker is the following

$$\max_e u(w_1 - e) + \phi^{-1} \left\{ E\phi \left\{ p(e, \tilde{\theta})U(w_2 - L(e)) + (1 - p(e, \tilde{\theta}))U(w_2) \right\} \right\}. \tag{2}$$

For simplicity, I assume that the problem is convex such that an interior solution always exists. The only novelty introduced in this formulation is the presence of the increasing concave function ϕ representing ambiguity aversion, and of E which is the expectation operator taken with respect to the distribution of θ, conditional on all information available during the first period. If the individual is ambiguity neutral (ϕ is linear), it is easy to see that we are back to a problem like the original problem (1)[12]. In that case, the optimal level of effort e^* is the solution of the first-order condition (FOC)

$$-u'(w_1 - e^*) - p'(e^*)[U(w_2) - U(w_2 - L(e^*))] - L'(e^*)p(e^*)U'(w_2 - L(e^*)) = 0.$$

Remark that the first term of this expression is negative and represents the marginal cost of effort, while the second and the third terms are both positive and represent the marginal benefits of self-protection and of self-insurance respectively.

To see whether ambiguity aversion raises the optimal level of effort, I simply compute the FOC under ambiguity aversion and evaluate it at e^*. A positive result implies an increase in the level of effort due to aversion towards ambiguity.

By letting $V(e, \theta) \equiv p(e, \theta)U(w_2 - L(e)) + (1 - p(e, \theta))U(w_2)$ be the second period expected

[12] For simplicity, assume that beliefs are objectively unbiased in the sense that it is possible to know the true law $p(e)$, and that this law corresponds exactly to the one considered by any savagian expected utility maximizer.

utility for a parameter θ and effort e, and by denoting by V_e the first derivative of this function with respect to variable e, it is easy to see that it is the case if and only if

$$\frac{E\left[\phi'\{V(e,\tilde{\theta})\}V_e(e,\tilde{\theta})\right]}{\phi'\left\{\phi^{-1}\left\{E\phi\{V(e,\tilde{\theta})\}\right\}\right\}} \geq EV_e(e,\tilde{\theta}). \tag{3}$$

The interpretation of this condition is simple. Since ambiguity only affects variables in second period, the marginal cost of effort, which takes place in first period, is unaffected and the condition indicates that the marginal benefit of protection or insurance must be higher under ambiguity aversion. The following lemma is useful to pursue.

Lemma 1 *Let ϕ be a three times differentiable function reflecting ambiguity aversion. If ϕ is DAAA (Deacreasing Absolute Ambiguity Aversion) then $E\phi'\{\tilde{x}\} \geq \phi'\left\{\phi^{-1}\left\{E\phi\{\tilde{x}\}\right\}\right\}$.*

Proof Saying that ϕ is DAAA is equivalent to say that $-\phi'$ is more concave than ϕ or equivalently that $-\phi'''/\phi'' \geq -\phi''/\phi'$. Since $(\phi')^{-1}$ is a decreasing function, the proof follows from the fact that the certainty equivalent of function ϕ is bigger than that of function $-\phi'$. ∎

Corollary 1 *If ϕ exhibits CAAA (constant absolute ambiguity aversion), then $E\phi'\{\tilde{x}\} = \phi'\left\{\phi^{-1}\left\{E\phi\{\tilde{x}\}\right\}\right\}$.*

Using this lemma, it is easy to see that the condition (3) is equivalent to the condition

$$\operatorname{cov}\left(\phi'\{V(e,\tilde{\theta})\}, V_e(e,\tilde{\theta})\right) \geq 0 \tag{4}$$

in the case of constant absolute ambiguity aversion, and that it is always respected in the case of decreasing absolute ambiguity aversion[13] if conditon (4) holds true. The expressions $V(e,\tilde{\theta})$ and $V_e(e,\tilde{\theta})$ must therefore covary negatively in θ since ϕ' is a decreasing function under ambiguity aversion. From now on, I also assume that $p_\theta(e,\theta) > 0$ for all levels of effort, meaning that the probability of being in the bad state is higher for higher values

[13] Note that this assumption does not seem too restrictive when modeling individuals preferences. Gierlinger and Gollier (2010) for example mentioned that "DAAA is a reasonable property of uncertainty preferences".

of parameter θ[14]. As it will become clear, this assumption ensures that the different curves $p(e, \theta)$ for different values of θ do not cross, which would indicate that the level of ambiguity could be decreasing in e up to a given value of effort and then increasing. This is equivalent to say that the order among the different θ's must be the same when e evolves.

This being said, remark that the second period expected utility $V(e, \theta)$ is decreasing in θ. A sufficient condition to observe a higher level of effort under ambiguity aversion than under ambiguity neutrality therefore simply becomes that $V_e(e, \theta)$ is increasing in θ. The analytical form of this expression is the following

$$V_e(e, \theta) = -p_e(e, \theta)[U(w_2) - U(w_2 - L(e))] - L'(e)p(e, \theta)U'(w_2 - L(e)). \qquad (5)$$

Remember that it represents the marginal benefit of effort, conditional on the value of the parameter θ. Since the probability of accident does not depend on the effort furnished in the self-insurance model, it is easy to see that $V_{e\theta}(e, \theta) \geq 0$, and therefore condition (4) is respected, in the case of self-insurance.

To analyze the self-protection model, the crucial element is the sign of $-p_{e\theta}(e, \theta)$, such that a positive sign implies a higher level of prevention due to ambiguity aversion. Graphically, since by assumption we also know that the probability of catastrophe is a decreasing function of effort ($p_e(e, \theta) < 0$ for all θ), we just need to know whether the different curves representing the probability of bad state as functions of effort are getting closer when the level of protection increases.

Examples To illustrate what preceeds, consider two particular forms of loss probability function. To keep things simple, both are linear in the ambiguity parameter θ, the first is additive and takes the form $p(e, \theta) = p(e) + \theta$, while the second is multiplicative and is written $p(e, \theta) = \theta p(e)$.

In the additive case, we simply have $V_e(e, \theta) = -p'(e)[U(w_2) - U(w_2 - L(e))]$ and it is easy to see that an increase in θ has no effect on $V_e(e, \theta)$. This thus means that the level of self-protection is exactly the same for any individual manifesting constant absolute ambiguity

[14]Alternatively, the same result could be obtained by assuming $p_\theta(e, \theta) < 0$. In that case, the inequalities following would just be reversed.

aversion. In particular an ambiguity neutral individual ($\alpha = 0$) and a maxmin expected utility maximizer à la Gilboa and Schmeidler (1989) ($\alpha = \infty$) both choose to self-protect precisely the same way. On the contrary, if the individual manifests decreasing absolute ambiguity aversion, he will still always choose a higher level of protection.

On the contrary, if for a reason or another, ambiguity is made smaller when the effort is higher[15] the slope $-p_e(e, \theta)$ will be higher for higher value of θ and hence condition (4) will hold true. This is the case with the multiplicative form described above. In this case we have $V_e(e, \theta) = -\theta p'(e)[U(w_2) - U(w_2 - L(e))]$, and it is clear that an increase in θ will have a positive impact on V_e, therefore that any individual manifesting non-increasing absolute ambiguity aversion chooses a higher level of protection.

Figure 3 illustrates the situation when there are two possibles θ: θ_1 and θ_2, and when the ambiguous loss probability is linear in θ.

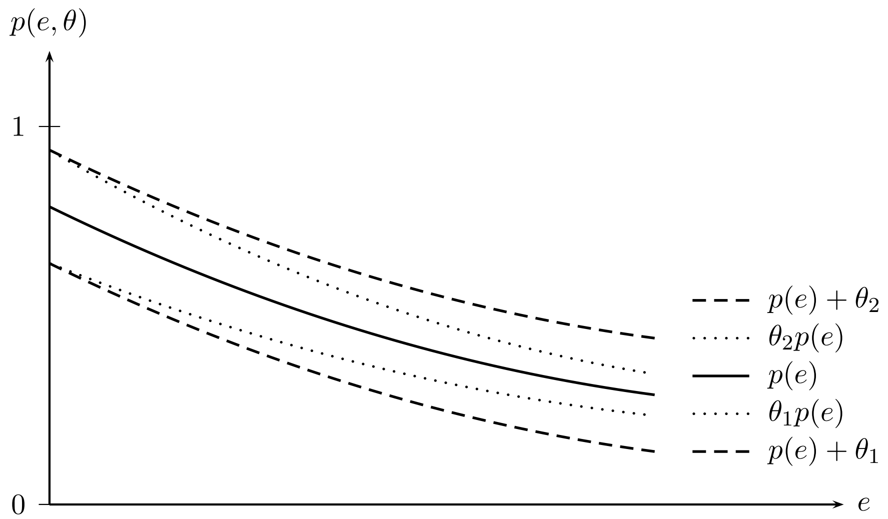

Figure 3: Linear ambiguous loss probability

As we can see when θ increases, going from θ_1 to θ_2[16], different scenarios are possible. In the additive case, the slopes of the two dashed lines are exactly the same for any given level of effort. Ambiguity in this case is therefore constant for any level of effort. On the contrary, with the multiplicative form it is easy to see that the slope of the dotted curve

[15]Imagine for example that some uncertainties decrease with the effectiveness of mitigation.

[16]Remark that in this example, the DM associates the same prior belief to each value of θ, such that $\theta_2 = -\theta_1$ in the additive case, and $\theta_2 = 2 - \theta_1$ in the multiplicative case.

for any given level of effort is higher with θ_2 than with θ_1. Intuitively, this corresponds to a situation in which ambiguity decreases with the effort furnished and the condition (4) is therefore respected.

The intuition behind those two examples is simple. In the absence of ambiguity, we know that a key determinant of the optimal level of self-protection is the slope of $p(e)$. Now when ambiguity is introduced, the DM does not know exactly in which situation he is: if $\mu = 1/2$, he has one chance over two to be confronted to probability $p(e, \theta_1)$, and one chance over two to have $p(e, \theta_2)$. If the individual is ambiguity neutral, this situation does not affect him and the decision is taken by considering the expected law $p(e)$. However, if the agent is ambiguity averse, he will overevaluate the less desirable outcome (i.e. the law $p(e, \theta_2)$) and hence take a decision by considering a law *somewhere above* the line $p(e)$. In the special case of infinite ambiguity aversion, corresponding to the maxmin model of Gilboa and Schmeidler (1989), the DM takes his decision by considering the worst sceniario $p(e, \theta_2)$.

The study of these two particular cases in which the probabiliy is linear in the parameter θ emphasizes the differences there are between the one and the two-period models. In the single period model, when both the marginal cost and the marginal benefit of self-protection are affected by the introduction of ambiguity, it is indeed impossible to sign the effect ambiguity aversion has on the optimal prevention, even when the probabilties are linear in the ambiguity parameter. In particular in that situation, the DM will always choose to reduce his demand of self-protection if the probability law is additive (see the Appendix A) while he will choose a higher level of protection if the probability law is multiplicative (Snow (2011)). As explained by Alary et al. (2010), this unability to obtain a general result is due to the fact that both the marginal cost and the marginal benefit of self-protection increase under ambiguity aversion. The net effect hence depends on which one is more affected. In the two-period model analyzed in this paper however, ambiguity aversion only affects the marginal benefit, making possible to draw general conclusions.

In reality however, it is clear that there is no reason for $p(e, \theta)$ to be linear in θ. We a priori have no information on the way the ambiguity parameter enters the loss probability.

The only information the DM has at his disposal is the way his beliefs evolve when the level of effort is altered. If for example a DM finds a situation less ambiguous when the level of effort is high than when it is low, we have more chance to be in a situation like the multiplicative one than like the additive one.
If I do not consider the most implausible case in which ambiguity increases with the level of effort, the results of this section can be summarized by the following proposition.

Proposition 1 *In the two-period model of self-protection described above, an individual manifesting decreasing absolute ambiguity aversion (DAAA) will always choose a higher level of effort than an expected utility maximizer if the state of the world is ambiguous. Moreover, in the two-period model of self-insurance, this individual will always choose a strictly higher level of effort than an expected utility maximizer if the state of the world is ambiguous.*

In the context of climate change, this means that taking into account the presence of ambiguity and the attitude agents generally manifest towards it, leads to an increase in the mitigation and adaptation efforts if, as proposed by Halsnæs et al. (2007), the desirability of preventive efforts is measured not only by the reduction in the expected (average) damages, but also by the value of the reduced uncertainties that such efforts yield. This result is in line with the IPCC report which claims that

> Some uncertainties will decrease with time – for example in relation to the effectiveness of mitigation actions and the availability of low-emission technologies, as well as with respect to the science itself. The likelihood that better information might improve the quality of decisions (the value of information) can support increased investment in knowledge accumulation and its application, as well as a more refined ordering of decisions through time. (Halsnæs et al. (2007))

As mentioned earlier, the idea of ambiguous state is equivalent to the idea of having different probabilities which are themselves ambiguous. In the case of self-protection, the Proposition 1 can therefore be restated as follows for ambiguous probabilities.

Corollary 2 *If the probabilities of being in each state of the world are ambiguous and the effect ambiguity is not made higher when effort increases, an individual manifesting DAAA in a two-period situation like the one described above will always choose a level of self-protection higher than the one chosen by a (savagian) expected utility maximizer.*

It is easy to prove this assertion using the covariance rule. For that, remember that I ranked the n possible laws from the most to the less favorable one, implying that $p_i(e) \leq p_{i+1}(e)$ for all e and for all $i = 1, ..., n$. A sufficient condition to observe a higher level of self-protection, is then that $-p'_{i+1}(e) \geq -p'_i(e)$ for all $i = 1, ..., n$ and for all levels of effort e. Graphically, this means that the curves representing the probability laws must come closer as the level of effort increases. The intuition behind this is simple, even if it is not the level of ambiguity in stricto sensu (represented by the parameter θ) that diminishes with effort, this is the difference between the possible ambiguous outcomes that becomes smaller when e increases. Ambiguity has therefore a less important impact on final wealth, which is valuable for any ambiguity averter.

In the next section, I turn to the analysis of models in which the possible loss is ambiguous.

3.2 Ambiguous loss

Imagine now another ambiguous situation in which the probabilities laws are perfectly known by the DM, but in which there exists n different loss situations $L_i(e)$ for $i = 1, ..., n$, each of them being associated with a probability $q_i(\tilde{\theta})$. This ambiguity may again result from the inability to obtain one single estimate for the economic impact of a catastrophic climate change, due for example to insufficient scientific knowledge. The DM therefore has at his disposal a range of values that he associates to some probabilities[17].

Again, I assume that the agent has a prior distribution over θ, and that I can rank the n possible laws, say from the less to the most favorable, implying that $L_i(e) \geq L_{i+1}(e)$ for all e and for all $i = 1, ..., n$. As before, the way ambiguity is introduced can be naturally interpreted as a multi-stage lottery. The schedule is the following: a first stage determines whether the DM suffers from a loss or not, a second determines the value of the prior

[17]For example, Stern (2007) estimates the total loss for a high-climate scenario with non-markets impacts and risk of catastrophe to be between 2.9% and 35.2% in GDP per capita in 2200 (see Figure 6.5 in Stern (2007) for more details).

and a third one determines the size of the loss. Note that we could alternatively have a lottery in which the value of the prior is determined in the first stage, the size of the loss in the second, and the possibility of suffering from the loss or not in the third one. If it is clear that those two situations have different interpretations, interestingly, as I show in the Appendix C, they both lead to the same comparative statics results when investigating the effect of ambiguity aversion. I will therefore concentrate on the first of the two lotteries described above, which seems relatively better adapted to describe situations in real life. To illustrate the situation, imagine a self-protection model in which there are two possible losses L_1 and L_2. The DM is therefore confronted to the lottery illustrated in Figure 4.

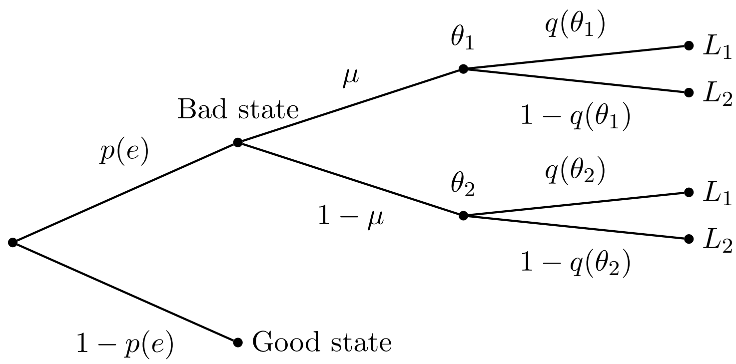

Figure 4: Ambiguous loss, case 1

We can for example think to the situation of a big country which knows that, by reducing its greenhouse gas emissions by investing an amount e of its GDP in mitigation, is able to reduce the probability of an extreme increase in average temperatures to $p(e)$. However, if temperatures still drastically increase, leading to the partial or complete disintegration of the West Antartic Ice Sheet, it is not clear whether the sea-level increases by 3 or by 6 meters, making the destructive power of the catastrophic event ambiguous.

In a situation with ambiguous loss like the one described above, the problem faced by an ambiguity averse DM is the following

$$\max_e u(w_1 - e) + (1 - p(e))U(w_2) + p(e)\phi^{-1} \left\{ E\phi \left\{ \sum_{i=1}^{n} q_i(\tilde{\theta})U(w_2 - L_i(e)) \right\} \right\}. \qquad (6)$$

I again assume that this maximization problem is convex. In this formulation, the nota-

tions are the same as before, and I denote by $W(e, \theta) \equiv \sum_{i=1}^{n} q_i(\tilde{\theta})U(w_2 - L_i(e)), theexpectedutilityobtainedif$
If the individual is ambiguity neutral, he will choose a level of effort e^* that satisfies the condition

$$-u'(w_1 - e^*) - p'(e^*)\left[U(w_2) - \sum_i q_i U(w_2 - L_i(e^*))\right] + p(e^*)\sum_i q_i U'(w_2 - L_i(e^*))(-L_i'(e^*)) = 0.$$

As it is now usual, the first term is the marginal cost of effort while the second and third terms represent the marginal benefits of self-protection and self-insurance respectively. As mentioned earlier, only the marginal benefit is affected by the introduction of the function ϕ characterizing the attitude towards ambiguity. It is therefore easy to see that ambiguity aversion raises the optimal level of effort compared to ambiguity neutrality if

$$\begin{aligned} & p'(e)\phi^{-1}\left\{E\phi\left\{W(e,\tilde{\theta})\right\}\right\} \\ +p(e)(\phi^{-1})'\left\{E\phi\left\{W(e,\tilde{\theta})\right\}\right\} E\phi'\left\{W(e,\tilde{\theta})\right\} & \sum_i q_i(\tilde{\theta})U'(w_2 - L_i(e))(-L_i'(e)) \\ \geq p'(e)\sum_i q_i U(w_2 - L_i(e)) + p(e) & \sum_i q_i U'(w_2 - L_i(e))(-L_i'(e)). \end{aligned} \tag{7}$$

This will always be the case in a self-protection model. To see this, remember that $L_i'(e) = 0$ for all i and all e when self-protection is considered, and use Jensen's inequality to show that ambiguity aversion increases the optimal level of self-protection when the loss is ambiguous.
In the self-insurance model, $p'(e) = 0$ and we can use the lemma under DAAA such that we only have to check whether the following inequality is respected

$$\frac{E\phi'\{W(e,\tilde{\theta})\}\sum_i q_i(\tilde{\theta})U'(w_2 - L_i(e))(-L_i'(e)))}{E\phi'\{W(e,\tilde{\theta})\}} \geq E\sum_i q_i(\tilde{\theta})U'(w_2 - L_i(e))(-L_i'(e)). \tag{8}$$

If this is the case, or equivalently if cov $\left(\phi'\{W(e,\tilde{\theta})\}, \sum_i q_i(\tilde{\theta})U'(w_2 - L_i(e))(-L_i'(e))\right)$ is positive, ambiguity aversion also increases the optimal level of self-insurance for an individual manifesting non-increasing absolute ambiguity aversion. This condition is not easy to handle with, however it is very close to the one needed to observe ambiguity prudence (see the Appendix B), and it is therefore still possible to draw some general conclusions.

Remark for example the role played by the slope $-L_i'(e)$ in the expression above. As it was the case when being confronted to ambiguous probabilities, the way the curves $L_i(e)$ are relative to each others has an importance. If they come closer when the level of effort increases (i.e. if $-L_{i+1}'(e) < -L_i'(e)$ for all e), the discrepancy between the possible losses becomes smaller when the individual exerts more effort, so that the importance ambiguity has on the final wealth distribution is relatively lower. This situation will give an ambiguity averter extra incentive to increase his level of effort. On the contrary, if the difference between the possible loss is independent of e (i.e. if the curves are parallel), this extra benefit of increasing effort is null and we do not have to take into account the terms $-L_i'(e)$ in the covariance expression above.

To illustrate what preceeds, imagine there are only two ambiguous outcomes, say $L_1(e)$ and $L_2(e)$ associated respectively to the big and the small catastrophe. We need to check that $U(w_2 - L_2(e)) - U(w_2 - L_1(e))$ and $U'(w_2 - L_2(e))(-L_2'(e)) - U'(w_2 - L_1(e))(-L_1'(e))$ have opposite signs for inequality (8) to be respected under DAAA. This will be the case if $-L_1'(e) \geq -L_2'(e)$. If the loss functions are parallel ($-L_1'(e) = -L_2'(e)$), this result is due to the fact that that the marginal utility is increasing with the size of the loss. To make things concrete, imagine that if the West Antartic Ice Sheet begins to melt, we can either face an increase of the sea-level of 3 or of 6 meters. If a small coastal country does not make any investment to construct dikes, it is confronted to the following situation: under the first scenario, half area would be under water, while if the second scenario materializes, all the country would be flooded. On the contrary, if all the coast of this country is equipped with dikes, the first scenario will have no impact and the second will only put half the country under water.

On the other hand, if effort reduces the difference between the two losses ($-L_1'(e) > -L_2'(e)$), an additional effect appears which makes the marginal benefit under L_1 higher than the one under L_2. To think to such a situation imagine that the houses of a city would be completely destroyed by a big storm if the city is not protected, while only the roofs would be broken if the city undergo a small storm without protections. Now if the houses are fully barricaded, only the roofs may be potentially destroyed by a big storm while the houses will not be affected at all by a small storm.

Remark that in a more general context, when there are more than two ambiguous outcomes, it is not necessarily obvious that ambiguity aversion raises the optimal level of self-insurance. It is easy to see that if the loss functions are parallel, condition 8 is respected if the individual is ambiguity prudent. This will also be the case if the loss functions are coming closer, since a reinforcing effect enters the formula.

The following proposition summarizes the results obtained in this section.

Proposition 2 *In the two-period model of self-protection, an individual manifesting non-increasing absolute ambiguity aversion*[18] *always chooses a higher level of effort than a (subjective) expected utility maximizer if the loss is ambiguous.*

Moreover, in the two-period model of self-insurance, an individual manifesting ambiguity prudence always chooses a higher level of effort than a (subjective) expected utility maximizer if the different losses functions are such that $-L_i'(e) \geq -L_{i+1}'(e)$.

4 Conclusion

This paper has proposed an economic interpretation of the Precautionary Principle based on the concept of ambiguity aversion. I showed that the scientific uncertainty, characterized by ambiguous probabilities of occurence of a catastrophic event or by ambiguous loss associated with this event, should induce a public decision maker to take stronger measures today to adapt to or to prevent the formation of such an event in the future. This conclusion is similar to the one obtained by Gollier, Jullien, and Treich (2000) but results from a different modelization in which ambiguity and attitudes agents manifest towards it are specifically taken into account. Through a simple economic model, I gave a justification to the idea that the decision-making tools of risk assessment in the expected utility framework are not appropriate in the presence of scientific uncertainties. In consequence, the statement stipulating that "when an activity raises threats of harm to human health or the environment, precautionary measures should be taken even if some cause-and-effect relationships are not fully established scientifically" (January 1998 Wingspread

[18]Remark that for this version of ambiguous loss, ambiguity aversion is a sufficient condition, while for the version presented in Appendix C, non-increasing absolute ambiguity aversion is needed.

Statement on the Precautionary Principle) should be respected.

To arrive to that conclusion, I used a two-period model of self-protection and self-insurance under KMM preferences to show that in most usual situations, the level of effort chosen by an agent manifesting decreasing ambiguity aversion is always higher than the one chosen by an ambiguity neutral individual when the state of the world or the loss is ambiguous. This conclusion may equivalently be interpreted as the impact on the optimal level of effort resulting from the existence of scientific uncertainty. To make things concrete, I assimilated this model to the climate change policy problem consisting in finding the optimal level of adaptation (actions helping human and natural systems to adjust to climate change) or mitigation (actions reducing net carbon emissions and limiting long-term climate change) efforts a government should provide. Indeed those measures, that can be seen respectively as self-insurance and self-protection, concern both long time horizon and deeply uncertain situations, and can therefore not be analyzed using traditional models in the expected utility framework. I chose to move away from the single period model traditionally used in the self-protection and self-insurance literature to make a clear distinction between the cost of effort (which is decided today and is therefore not ambiguous) and the benefit this effort yields in the future (which is ambiguous because of the presence of different scientific uncertainties). In this view, my model distinguishes from the recent, papers of Snow (2011) and Alary, Gollier, and Treich (2010) who consider ambiguity also on the cost of effort, since they consider single period models. The drawback of this approach is that those models are therefore not able to draw general conclusions because of the conflicting effect ambiguity aversion has on marginal benefit and marginal cost. Alternatively, remark that one could obtain the same results I present in this paper by considering a single-period model in which the cost of effort would be non-monetary and not affected by ambiguity.

In the portfolio of measures existing to respond to climate change, there is also a third instrument that the IPCC describes as "the research on new technologies, on institutional designs and on climate and impacts science, which should reduce uncertainties and facilitate future decisions" (Fisher et al. (2007)). This instrument on its own is not studied explicitely in this paper, but it should be clear that an effort whose only effect is to modify the ambiguity level would always be desirable for an ambiguity averter if, as assumed throughout this paper, the decision problem is convex. However, when the desirability of the preventive effort measured by the reduction in the loss probability is also measured by the value of the reduced uncertainties that this effort yields, I showed that ambiguity aversion always increases the effort desired.

Overall this paper therefore contributes to a better incorporation of uncertainty in policy design, which constitutes undoubtedly one of the more important areas in economics in general and in environmental economics especially. In future research, I plan to introduce into the picture the possibility of learning to see how the evolution of ambiguity over time affects the decision-making process.

Appendix

Appendix A. One-period model with ambiguous states

In the one-period model with ambiguous states, ambiguity aversion always leads the DM to decrease his investment in self-protection if the ambiguous probability law is additive in θ. To see this consider the following general ambiguous problem:

$$\max_e \phi^{-1}\left\{E\phi\left\{(p(e)+\tilde{\theta})u(w-e-L)+(1-p(e)-\tilde{\theta})u(w-e)\right\}\right\}.$$

The associated FOC can be written as:

$$\frac{E\phi'\left\{g(e,\tilde{\theta})\right\}(-(p'(e))[u(w-e)-u(w-e-L)]}{E\phi'\left\{g(e,\tilde{\theta})\right\}((p(e)+\tilde{\theta})u'(w-e-L)+(1-p(e)-\tilde{\theta})u'(w-e))}=1$$

where $g(e,\theta)\equiv(p(e)+\theta)u(w-e-L)+(1-p(e)-\theta)u(w-e)$. The numerator is the marginal benefit of self-protection and the denominator represents the marginal cost. To see if ambiguity aversion raises the optimal level of effort, we need to check whether the following inequality is verified

$$\frac{E\phi'\left\{g(e,\tilde{\theta})\right\}}{E\phi'\left\{g(e,\tilde{\theta})\right\}((p(e)+\tilde{\theta})u'(w-e-L)+(1-p(e)-\tilde{\theta})u'(w-e))}$$
$$\geq\frac{1}{E((p(e)+\tilde{\theta})u'(w-e-L)+(1-p(e)-\tilde{\theta})u'(w-e))},$$

meaning that the ratio marginal benefit/marginal cost of protection is higher under ambiguity aversion. This is the case if

$$\mathrm{cov}_{\tilde{\theta}}\left(\phi'\{g(e,\tilde{\theta})\},(p(e)+\tilde{\theta})u'(w-e-L)+(1-p(e)-\tilde{\theta})u'(w-e)\right)\leq 0,$$

but since both elements are increasing in θ, by the covariance rule we can conclude that ambiguity aversion decreases the optimal level of effort.

Appendix B. The meaning of ambiguity prudence

As noted by Gollier (2001), it is widely believed that the agents are prudent in the sense that the *uncertainty* affecting future incomes raises current savings. From Leland (1968) and Kimball (1990) among others, it is well known that a DM is "risk prudent" if the addition of an uninsurable pure risk to his future wealth raises his optimal level of saving. This is the case if the marginal utility of future consumption is convex.
Now if the the future wealth (or labor income) is ambiguous, under which conditions does ambiguity aversion increase the optimal level of saving? In other words, are individuals "ambiguity prudent" in the sense defined below.

Definition An agent is *ambiguity prudent* if the presence of ambiguity on his future wealth raises his optimal saving.

To answer those questions, consider a simple two-period consumption-saving problem under ambiguity, in which the DM with a time additive utility function, faces ambiguous labor income in its second period of life. The risk-free gross rate of return R and the labor incomes w_1 and w_2 are exogeneously given[19]. In first period, the DM receives labor income w_1 and decides his levels of consumption c_1 and saving s. In second period, the agent disposes of his capital income Rs and of an ambiguous labor income $w_2 + \tilde{z}$ (similarly as before, w_2 is the mean of the second period labor income if the situation is not ambiguous, z_i is associated with probability $p_i(\tilde{\theta})$ for all $i = 1, ..., n$[20], and all the previous properties hold). The agent's decision problem is

$$\max_s u(w_1 - s) + \phi^{-1}\left\{E\phi\left\{\sum_i p_i(\tilde{\theta})U(w_2 + z_i + Rs)\right\}\right\}$$

where u and U are the first and second period vNM utility functions, and ϕ represents attitude towards ambiguity. Under ambiguity neutrality, the optimal level of saving s^* is implicitly given by

$$u'(w_1 - s^*) = \sum_i p_i U'(w_2 + z_i + Rs^*)R \tag{9}$$

[19]Labor supply is assumed to be inelastic such that labor income corresponds to pure wealth coming from the sky.

[20]Without loss of generality, assume that the uncertain outcomes are such that $z_1 < z_2 < ... < z_n$.

which is assumed to hold at some interior value s^{*}[21]. This implies that an ambiguity averse DM will always choose a higher level of saving than the ambiguity neutral one if

$$\frac{E\phi'\left\{\sum_i p_i(\tilde{\theta})U(w_2+Rs^*+z_i)\right\}\sum_i p_i(\tilde{\theta})U'(w_2+Rs^*+z_i)}{\phi'\left\{\phi^{-1}\left\{E\phi\left\{\sum_i p_i(\tilde{\theta})U(w_2+Rs^*+z_i)\right\}\right\}\right\}} \geq \sum_i p_i U'(w_2+Rs^*+z_i). \tag{10}$$

Proposition 3 *An agent is ambiguity prudent if inequatlity 10 is respected.*

The uncertainty affecting future income may therefore have different impacts on current saving. It increases the willingness to save in the standard expected utility framework if the future income is risky and the agent is risk prudent, and it also raises further current saving if the future outcome is ambiguous and the agent is ambiguity prudent.
Using the lemma described in the main body of the paper, we know that if ϕ is CAAA, condition 10 is equivalent to

$$\operatorname{cov}_{\tilde{\theta}}\left(\phi'\left\{\sum_i p_i(\tilde{\theta})U(w_2+Rs^*+z_i)\right\}, \sum_i p_i(\tilde{\theta})U'(w_2+Rs^*+z_i)\right) \geq 0, \tag{11}$$

and that condition 10 is implied by condition 11 in the case of DAAA. Hence, using the covariance rule, this condition will be verified if the two terms entering the formula are both either increasing or decreasing in θ. In particular, it will be the case if the conditional second period expected utility and marginal utility have opposite signs.

Appendix C. Equivalence between different kinds of ambiguous losses

As explained in the main body of the paper, when the loss is ambiguous several situations with different timings may be possible. Considering the multi-stage lottery describing ambiguity, we could for example also have a situation in which the value of the prior is determined in the first stage, the size of the loss in the second, and the possibility of suffering from the loss or not in the third one. In analogy to the example presented in Figure 4, this situation would be described by the lottery presented in Figure 5.

[21] See Gierlinger and Gollier (2010) for the specific convexity conditions of the problem.

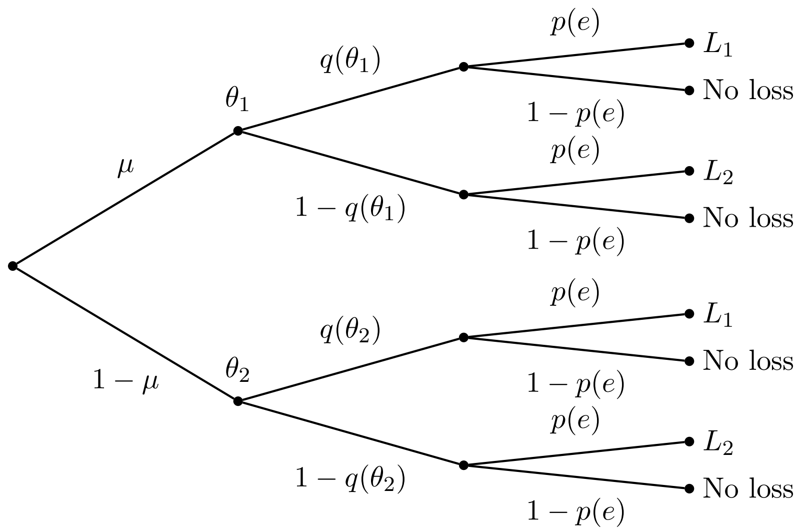

Figure 5: Ambiguous loss, case 2

Remark that stages 2 and 3 may be merged to one unique stage with three possible outcomes L_1, L_2 and 'No Loss' since ambiguity does not affect this part of the lottery. Equivalently, we could have a situation in which the first stage determines the value of the prior, the second stage determines whether a loss occurs or not, and the third one determines the size of the loss. Those equivalent situations may be interesting on their own if we are able to find a reality to which they correspond. A disposition like the one presented in Firgure 5 corresponds for example to a situation in which a government is confronted to two possible ecological disasters: a important flood and a violent storm. The probability of being confronted to each of them is ambiguous, and in each situation, the disaster may be avoided with the same probability $1-p(e)$.

The general decision problem that kind of situation refers to can be written as follows

$$\max_e u(w_1 - e) + \phi^{-1}\left\{E\phi\left\{\sum_{i=1}^{n} q_i(\tilde{\theta})p(e)U(w_2 - L_i(e)) + (1-p(e))U(w_2)\right\}\right\}.$$

The associated FOC is therefore

$$-u'(w_1 - e) + (\phi^{-1})'\left\{E\phi\left\{h(e,\tilde{\theta})\right\}\right\} E\phi'\left\{h(e,\tilde{\theta})\right\} h_e(e,\tilde{\theta}) = 0,$$

where $h(e,\theta) \equiv p(e)\sum_{i=1}^{n} q_i(\theta)U(w_2 - L_i(e)) + (1-p(e))U(w_2)$ and hence $h_e(e,\theta) = -p'(e)\left[U(w_2) - \sum_i q_i(\theta)U(w_2 - L_i(e))\right] + p(e)\sum_i q_i(\theta)U'(w_2 - L_i(e))(-L_i'(e))$. Evaluating as usual this condition at the optimal level of effort chosen by an ambiguity neutral DM and using the lemma leads to the conclusion that non-increasing ambiguity aversion raises the optimal level of effort if the condition

$$\mathrm{cov}_{\tilde{\theta}}\left(\phi'\{h(e,\tilde{\theta})\}, h_e(e,\tilde{\theta})\right) \geq 0 \tag{12}$$

is satisfied. In the self-protection case, it is easy to see that this will always be the case while in the self-insurance model, the same conditions as the ones mentioned in the body of the paper must be respected.

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