Isotope shift in the electron affinity of beryllium

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Abstract. The study of the isotope shift in the electron affinity is interesting for probing correlation effects. Experiments that allow this property to be measured are rare, being difficult to realize, while accurate calculations remain a challenge for atomic theory. The present work focuses on the theoretical estimation of the isotope shift in the electron affinity of Be (2s2p $^3P^o$), using correlated electronic wave functions obtained from multiconfiguration Hartree-Fock (MCHF) and interaction configuration (CI) variational calculations. The reliability of the correlation models is assessed from a comparison between the observed and theoretical electron affinities, and between theoretical isotope shift values for the 2s2p $^3P^o - 2s^2 1S$ transition of neutral beryllium. The sign and the magnitude of the difference between the mass polarization term expectation values obtained for the neutral atom and the negative ion are such that the resulting isotope shift in the electron affinity is “anomalous”, corresponding to a smaller electron affinity for the heavier isotope.
1. Introduction

Negative ions are atomic systems of growing interest owing to the continuous development of experimental techniques. The review paper by Anderson et al [1] reveals the enormous progress realized in this field since the previous survey articles [2, 3]. These spectacular experimental developments are still on their way, leading to the possibility of measuring accurately electron affinities [4], sometimes for different isotopes [5], or cross sections for the electron impact detachment [6]. It is well known that electron correlation plays a crucial role in the calculation of properties of negative ions [7]. A review of many-body effects in negative ion photodetachment can be found in [8]. The comparison of theory and experiment through the isotope shift in the electron affinity is of particular interest for probing correlation effects that affect both properties. Experiments that allow this effect to be measured are difficult to realize while accurate calculations remain a challenge for atomic theory.

The electron affinities of atomic hydrogen and deuterium have been determined by Lykke et al [9] using tunable-laser threshold-photodetachment spectroscopy. The isotope shift in the electron affinity has been found to be $3.2 \pm 0.7 \text{ cm}^{-1}$, confirming the predicted theoretical shift of $3.6 \text{ cm}^{-1}$ calculated from the expectation values of Pekeris [10]. Drake [11] estimated this shift with a spectacular accuracy, $3.613037 \text{ cm}^{-1}$ correct to the figures quoted, including the finite mass and recoil corrections to the relativistic and QED terms up to order $\alpha^3 \text{Ry}$, as well as the nuclear volume effect. The positive isotope shift corresponds to a larger electron affinity for deuterium than for hydrogen, the normal mass shift effect being reinforced by the specific mass shift correction.

The isotope shift in the electron affinity between $^{37}\text{Cl}$ and $^{35}\text{Cl}$ has been determined from tunable-laser photodetachment spectroscopy measurements by Berzinsh et al [12]. Observation also reveals a “normal” isotope shift (i.e. an electron affinity larger for the heavier isotope) of $0.22(14) \text{ GHz}$, of which $-0.51(14) \text{ GHz}$ is due to the specific mass shift. As shown by the pioneer many-body calculations presented in the same work [12], a theoretical estimation of the isotope specific mass shift contribution in the electron affinity of such a large system, was considered beyond the (by then) possible state-of-the-art computational techniques. A conclusion made by these authors was that theory, predicting a specific mass shift of $+0.50 \text{ GHz}$, having the correct order of magnitude but the wrong sign, could not lead to a quantitative description due to the omission of higher-order correlation effects. It was then suggested to attempt more precise experiments on other negative ions, with $S^-$ as a possible candidate, or to investigate “few-electron” systems, like Li$^-$, for which more accurate calculations could be performed. In this line, a new determination of the Li electron affinity has been reported by Haeffler et al [13] using a state selective photodetachment spectroscopic method but the measurements were unfortunately limited to the $^7\text{Li}$ isotope, shedding no light on the isotope shift in the electron affinity.

A satisfactory agreement between theory and observation was found for the isotope shift in the oxygen electron affinity. Valli et al [5] measured the electron affinities of $^{18}\text{O}$ and $^{16}\text{O}$ isotopes by using the photodetachment microscopy technique from which a negative isotope shift in the electron affinity of oxygen was found. This “anomalous” character of the isotope shift, corresponding to a smaller electron affinity for the heavier isotope, was explained from the variational ab initio calculations by Godefroid and Froese Fischer [14] of the expectation value of the mass polarization term for both the neutral atom and the negative ion. The theoretical ab initio specific
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mass shift contribution to the electron affinity, was indeed found to be negative, and sensitively larger in absolute value than the (positive) normal mass shift contribution. This fact was confirmed through the measurements of the $^{17}$O electron affinity by Blondel et al [15] owing to the high sensitivity of the photodetachment microscopy experiments that revealed electron images even for the rarest isotope $^{17}$O$^-$ ion.

Another interesting testcase for a comparison of theory with observation could be the isotope shift in the electron affinity of beryllium. Atomic spectroscopy experiments on unstable beryllium isotopes become indeed possible owing to the ISOLDE facility at CERN [16]. Within this context, the feasibility of some experiments allowing the determination of the nuclear quadrupole moment of $^7$Be from the observed hyperfine structure of the negative ion $^7$Be$^-$ ($2s2p^2\,^2P_J$) has been investigated recently [17]. In the present work we report the first ab initio calculations of the isotope shift in the electron affinity of Be ($2s2p\,^3P_o$) for various isotopic pairs involving the $^{11,10,9,8,7}$Be isotopes.

The necessary theoretical background for understanding the relevant parameters monitoring the isotope shift in the electron affinity is presented in section 2. The variational MCHF method and the CI correlation models, based on the concept of the orbital and configuration active spaces, are briefly described in section 3. The results are reported in section 4. In section 4.1, the theoretical evaluation of the electron affinity, is studied, as a quality test of the wave functions describing both the neutral atom and the negative ion. In section 4.2, the reliability of the correlation models is further assessed, for the neutral atom, by a comparison of the theoretical isotope shift values for the $2s2p\,^3P_o - 2s^2\,^1S$ transition, evaluated with the present MCHF/CI wave functions, with the results of Chung and Zhu [18] using the Full-Core Plus Correlation (FCPC) method. In section 4.3, we study the corresponding convergence of the specific mass shift parameters allowing the estimation of the isotope shift in the electron affinity.

2. Theory

The isotope shift of an energy level arises from the addition of two effects, the mass shift and the field shift. The former accounts for the nuclear motion while the latter is due to changes in the nuclear charge distributions. The extended charge correction is known to be weak for light atoms [19] and is not considered in the present work.

The mass shift for the energy level of an $N$-electron atom with a finite nuclear mass, can be derived by treating the mass polarization term

$$\frac{1}{M} \sum_{i<j}^N p_i \cdot p_j$$

as a small perturbation [20], where $p_i$ is the momentum of the $i$th electron. Keeping only the first-order specific mass shift (SMS) correction, the mass shift has the form [21, 22, 23]

$$E_M - E_\infty = -\frac{\mu}{M} E_\infty + \frac{\mu}{m_e M + m_e} \frac{1}{\langle N \sum_{i<j}^N \Psi_\infty \Psi_\infty \rangle}$$

(1)

where $\mu = m_e M/(m_e + M)$ is the reduced mass. $E_\infty$ and $\Psi_\infty$ are respectively the (negative) eigenvalue and eigenfunction of the infinite nuclear mass problem. The two
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terms of the right-hand-side of this equation are known, respectively, as the normal mass shift (NMS) and the specific mass shift (SMS). Expressing energy, masses and linear momentum in atomic units, equation (1) can be rewritten as

\[ E_M = \frac{M}{1+M} E_\infty + \frac{M}{(1+M)^2} S_{\text{SMS}}, \]

where

\[ S_{\text{SMS}} = -\left\langle \mathbf{\Psi}_\infty \left| \sum_{i<j} \nabla_i \cdot \nabla_j \right| \mathbf{\Psi}_\infty \right\rangle, \]

is often referred to as the specific mass shift parameter.

According to equation (2), the electron affinity of a given beryllium isotope, \( ^M\text{Be} \), is given by

\[ E_a^M = E_a(\text{Be}) - E_a(\text{Be}^-) = \frac{M}{1+M} E_\infty^a + \frac{M}{(1+M)^2} \Delta S_{\text{SMS}}, \]

where \( E_\infty^a \) is the beryllium electron affinity calculated with an infinite nuclear mass, and where

\[ \Delta S_{\text{SMS}} = S_{\text{SMS}}(\text{Be}) - S_{\text{SMS}}(\text{Be}^-). \]

The isotope shift in the electron affinity is the difference of the electron affinities between two different isotopes of masses \( M \) and \( M' \),

\[ \Delta E_a^{MM'} = E_a(\text{Be}) - E_a(\text{Be}^-) = \Delta E_a^{\text{NMS}}(M-M'\text{Be}) + \Delta E_a^{\text{SMS}}(M-M'\text{Be}) \]

with, according to (4),

\[ \Delta E_a^{\text{NMS}}(M-M'\text{Be}) = \left[ \frac{M}{1+M} - \frac{M'}{1+M'} \right] E_\infty^a \]

and

\[ \Delta E_a^{\text{SMS}}(M-M'\text{Be}) = \left[ \frac{M}{(1+M)^2} - \frac{M'}{(1+M')^2} \right] \Delta S_{\text{SMS}} \]

Adopting the convention that the isotope \( M \) is heavier than the isotope \( M' \), \( (M > M') \), the mass factor of (6) is always positive. Since \( E_\infty^a \) of Be \((2s2p^3P^o)\) is definitely positive, corresponding to a bound negative ion, the normal mass shift contribution \( \Delta E_a^{\text{NMS}} \) to the isotope shift in the electron affinity is also positive. On the contrary, the mass factor in equation (7) is always negative \( \dagger \). If \( \Delta S_{\text{SMS}} < 0 \), the SMS and the NMS contributions will add up, giving rise to a normal isotope shift. On the contrary, if this quantity is positive, the specific mass shift contribution to the isotope shift in the electron affinity counteracts the normal mass shift. In such a negative interference case, the specific mass shift contribution to the isotope shift in the electron affinity will depend on the balance between the NMS and SMS contributions. In some situations, the magnitude of the SMS effect can be large enough to produce an “anomalous” isotope shift, i.e. a smaller electron affinity for the heavier isotope.

\( \dagger \) the mass factor of eq. (7) can be rewritten as \((M-M')(1-MM')/[(1+M)(1+M')]^2\) and is negative for any physical situation \((MM' > 1)\), with the convention \( M > M' \) and remembering that the nuclear masses are expressed in atomic units.
3. Computational strategy

The infinite nuclear mass eigenfunctions $\Psi_\infty$ are calculated using the MCHF and CI methods. An MCHF wave function $\Psi_\infty$ is expanded in terms of configuration state functions (CSF) $\{\Phi_i\}$ having the same $LSM_LM_S\pi$ symmetry but arising from different electronic configurations ($\gamma_i$).

$$\Psi(\gamma LSM_LM_S\pi) = \sum_{i=1}^N c_i \Phi(\gamma_i LSM_LM_S\pi).$$

The CSF’s are built on a basis of one-electron spin-orbital functions

$$\phi_{nlm_lm_s} = \frac{1}{r} P_{nl}(r) Y_{lm_l}(\theta, \phi) \chi_{m_s}.$$ 

In the MCHF procedure both the sets of radial functions $\{P_{nl_i}(r)\}$ and mixing coefficients $\{c_i\}$, are optimized to self-consistency by solving numerically and iteratively the multiconfiguration Hartree-Fock differential equations for the former and the configuration interaction (CI) problem for the latter [24].

The active space (AS) method [25, 26] is used for building the CSFs expansion (9). The core-core correlation effect being very important on the specific mass parameters [27] of Be states, we have used a full correlation model for generating the CSF. We considered for the first step, i.e. the MCHF optimization of the one-electron orbitals, all the simple (S) and double (D) excitations from the Hartree-Fock reference configuration to an increasing active set of orbitals from $n = 2$ up to $n = 10$, with the angular momentum limitation $l_{max} = 4$ corresponding to $g$-orbitals for $n \geq 6$. The obtained active spaces are noted “$n$” for $n < 6$ and “$ng$” for $n \geq 6$.

Once the radial functions have been determined, a configuration interaction calculation is performed over a set of configuration states. In the present work, the configuration lists used in the CI calculations have been produced by merging the configuration subspace created from single and double excitations to the “10g” active set (SD[10g]) with another subspace generated by allowing further triple and quadruple excitations (TQ[x]) to smaller orbital active sets (x). This merging of CSF lists is denoted hereafter by the union “∪” symbol. The limited population constraint [28, 29] “at least three electrons with $n \leq 4$” was adopted in this last step in order to keep the size of the multiconfiguration expansions manageable. The number of configuration state functions used in the MCHF/CI expansions (8) is denoted NCSF.

4. Results

4.1. The Be electron affinity

In a recent paper [17], the relevant parameters for the calculation of the Be $(2s2p^3P^o)$ electron affinity and the Be$^-$ $(2s2p^2 4P_{1/2,3/2,5/2})$ hyperfine structure have been discussed. In the present approach we used the same largest set of one-electron orbitals optimized from the 10g-SD-MCHF expansions but extended the CI configuration space considered by Nemouchi et al [17] by including the triple and quadruple excitations up to the extra layer “8g”.

Table 1 shows the total energies of Be $(2s2p^3P^o)$ and Be$^-$ $(2s2p^2 4P)$, together with the electron affinity of Be $(2s2p^3P^o)$ as a function of the SD-MCHF and SDTQ-CI expansions. As it can be seen from monitoring the electron affinity value with the CI configuration spaces, the convergence has not been achieved, even with the
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largest CI calculations. However, it has been somewhat improved relatively to [17] and it was worthwhile to add the extra layer in the CI space. The corresponding value of 0.2876 eV (with 263 450 CSF for the negative ion) is larger than the theoretical estimation of Olsen et al [30] but the convergence trend is in line with the larger theoretical value obtained by Hsu and Chung [31] and with the most recent experimental value [32] taking into account their quoted uncertainty. Table 1 undoubtedly reflects the high reliability of the used correlation models and the good quality of the wave functions that are used for estimating the isotope shift parameters discussed in section 4.3.

4.2. The isotope shift of the $2s2p^3P^o - 2s^2 1S$ transition

The lack of experimental data and other theoretical results on the isotope shift in the electron affinity of beryllium does not allow us to make any comparison. However, in addition to the analysis of the electron affinity itself as a good accuracy indicator (see section 4.1), we are using the $^{10-9}$Be and $^{9-8}$Be isotope shifts calculations as another test of reliability for the neutral system. Isotope shifts of the $1s^22s2n\ell - 1s^22s^2$ transitions have been evaluated by Chung and Zhu [18] using the Full-Core Plus Correlation (FCPC) method. From the good agreement between this theory and experimental data [33] obtained for the $2s3d 1D - 2s^2 1S$ transition found in this work, one can infer the reliability of the FCPC results.

By using the MCHF and CI methods we have studied the convergence of the specific mass parameters for the states Be ($2s^2 1S$) and Be ($2s2p^3P^o$) from which we deduced the $^{10-9}$Be and $^{9-8}$Be isotope shifts for the $2s2p^3P^o - 2s^2 1S$ transition, using the following formula:

$$M - M' \Delta\nu = \left\{ \frac{M}{1 + M} - \frac{M'}{1 + M'} \right\} [E(2s2p^3P^o) - E(2s^2 1S)]$$

$$+ \left\{ \frac{M}{(1 + M)^2} - \frac{M'}{(1 + M')^2} \right\} [S_{\text{SMS}}(2s2p^3P^o) - S_{\text{SMS}}(2s^2 1S)] \times k$$

In this expression atomic masses, energies and specific mass parameters are in atomic units and $k = 6.579\times 10^6$ is the conversion factor calculated from the recommended values of the fundamental constants found on the NIST website [34] to get the frequency shift in GHz. The isotopes masses have been taken from the compilation of Audi and Wapstra [35].

The results are given in Table 2. Although the 10g-SD-MCHF results for the IS almost reproduce the one-configuration Hartree-Fock values, it should be realized from the comparison of the individual state contributions that this agreement is fortuitous, the electron correlation effects for both states being crucial. The triple and quadruple excitations increase the transition isotope shift by 11%, arising from a larger effect on the ground state than on the excited state level isotope shift. The results obtained with the largest CI configuration space are in good agreement with the FCPC values calculated by Chung and Zhu [18]. This observation alone gives us confidence in our correlation models used for the four-electron system.

4.3. The isotope shift on the electron affinity

We give in Table 3 the values of the specific mass parameters for Be$^-$ ($2s2p^4P$) and Be ($2s2p^3P^o$) as well as the normal mass shift ($\Delta E_N^{\text{NMS}}$), the specific mass
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shift ($\Delta E_{\text{SMS}}^a$) and the isotope mass shift in the electron affinity ($\Delta E_a$) for $^{11}\text{Be}$ and $^9\text{Be}$. The isotopes masses have been taken from the compilation of Audi and Wapstra [35].

As we can see through Table 3, the negative normal mass shifts found in the Hartree-Fock approximation and in the SD-MCHF correlation model up to the $n = 3$ active set, are coherent with the corresponding negative electron affinities given in Table 1. The convergence of the normal mass shift indeed strictly follows, as it should (see equation 6), the convergence trend of the electron affinity. Electron correlation has to be included beyond this active set for getting a correctly bound negative ion as observed, i.e. a positive electron affinity from which a positive normal mass shift is deduced.

The three HF rows of Table 3 correspond, respectively, to the use of the Hartree-Fock method on the neutral atom (HF(Be)), on the negative ion (HF(Be$^-$)), and on the separately optimized states (HF) for the neutral atom and the negative ion. The sign of $\Delta S_{\text{SMS}}^a$ is positive in all the calculations that we have performed. It can be understood in the single-configuration Hartree-Fock approximation, adopting the same orbital basis set for describing both the negative ion and the neutral atom. We indeed know that the matrix elements of the mass polarization term (3) have the same angular part than the $k = 1$ exchange contributions to the electrostatic Coulomb interaction between the electrons [24]. The weight angular coefficients for the exchange integrals $G^1(nl, n'l')$ and the corresponding products of Vinti integrals $J(nl, n'l')J(n'l', nl)$ are then the same. Using this mapping, the specific mass shift parameters are given by

$$S_{\text{SMS}}(\text{Be}) = -\frac{1}{3}[J^2(1s2p) + J^2(2s2p)]$$

and

$$S_{\text{SMS}}(\text{Be}^-) = -\frac{2}{3}[J^2(1s2p) + J^2(2s2p)]$$

for, respectively, Be ($1s^22s2p\ 3P^o$) and Be$^-$ ($1s^22s2p^2\ 4P$). The resulting difference, only valid in a frozen orbitals model,

$$\Delta S_{\text{SMS}} = \frac{1}{3}[J^2(1s2p) + J^2(2s2p)],$$

has to be positive, producing a negative SMS isotope shift on the electron affinity, as discussed in section 2.

The third row of Table 3 illustrates the importance of the relaxation effects that have been captured by a separate optimization of the orbitals on the negative ion and the neutral atom. The SMS in the electron affinity is largely reduced by the relaxation and by the one-electron excitations implicitly contained in the Hartree-Fock approximation owing to Brillouin’s theorem [36].

The largest “10g” orbital spaces obtained from the SD-MCHF correlation model look complete from the convergence achieved in the SMS parameters. The triple and quadruple excitations included in the CI calculations affect considerably the specific mass shift operator mean value, more for the negative ion ($\simeq 10\%$) than for the neutral atom ($\simeq 2\%$). A satisfactory convergence, $10^{-3}$ and $10^{-4}$ respectively, is achieved for the specific mass parameters of Be$^-$ ($2s2p\ 4P$) and Be ($2s2p\ 3P^o$).

Columns 6 and 9 ($\Delta E_a$ for the two selected isotopic pairs) of table 3 reflect the importance of correlation effects on the isotope shift in the electron affinity. The SMS contribution to it ($\Delta E_a^{\text{SMS}}$) largely dominates the normal mass shift contribution
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$(\Delta E^\text{NMS}_a)$. The analysis of the SMS parameters themselves, for either the negative ion or the neutral atom, illustrates the crucial role of single and double excitations considered in the MCHF expansions. As observed above, the role of triple and quadruple excitations is not negligible, reaching 10% for the negative ion. While the large effects of single and double excitations on the SMS parameters are largely smoothed out when making their difference between the neutral atom and the negative ion, the effect of triple and quadruple excitations is reinforced, reaching up to 30% on the $\Delta E_a$ value.

The specific mass isotope shift in the electron affinity $(\Delta E^\text{aSMS}_a)$ is found to be about four times larger, in absolute value, than the normal mass shift isotope $(\Delta E^\text{aNMS}_a)$. Counteracting the normal mass shift contribution, the isotope shift found in the electron affinity is definitely negative, corresponding to an “anomalous” isotope shift.

5. Conclusion

We report large-scale MCHF and CI calculations of the isotope shift in the electron affinity of beryllium. The configuration expansions were generated with the systematic active space method by using a full correlation model. The calculated electron affinity is in good agreement with the recent experimental value. The “anomalous” character of the isotope shift found in the present study, corresponding to a smaller electron affinity for the heavier isotope, is not specific to beryllium. It has been already observed in the case of the electron affinity of oxygen [14, 15], on the contrary of the chlorine case [12]. Before revisiting theoretically the difficult case of chlorine for which the many-body calculations failed to reproduce observation, it would be interesting to investigate, both theoretically and experimentally, the isotope shift on the electron affinity of lighter systems such as lithium. We also hope that the present work, together with the theoretical work on hyperfine structures [17], will stimulate some new experiments on the negative beryllium ion for its various isotopes.

Acknowledgments

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15. King W H isotope shifts in atomic spectra 1994
Table 1. Total energies of Be (2s2p \(^3P\)) and Be\(^-\) (2s2p \(^4P\)) together with the Be (2s2p \(^3P\)) electron affinity (E\(_a\)) for different SD-MCHF (1st series) and SDTQ-CI (2nd series) configuration expansions. NCSF is the number of configuration state functions.

<table>
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<th>Active set</th>
<th>(E_{tot}) (a.u.)</th>
<th>NCSF</th>
<th>(E_{tot}) (a.u.)</th>
<th>NCSF</th>
<th>(E_a) (eV)</th>
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<td>-14.509 0277</td>
<td>1</td>
<td>-0.0673</td>
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<tr>
<td>5</td>
<td>-14.563 6590</td>
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<tr>
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</table>

SD[10g] \(\cup\) TQ[3] | -14.566 5738 | 4230 | -14.575 3383 | 6919 | 0.2385 |
SD[10g] \(\cup\) TQ[4] | -14.566 6494 | 5238 | -14.576 4941 | 11062 | 0.2679 |
SD[10g] \(\cup\) TQ[5] | -14.566 7212 | 7918 | -14.576 8124 | 35999 | 0.2746 |
SD[10g] \(\cup\) TQ[6g] | -14.566 7304 | 10598 | -14.577 0444 | 85976 | 0.2807 |
SD[10g] \(\cup\) TQ[7g] | -14.566 7336 | 13278 | -14.577 2259 | 161893 | 0.2855 |
SD[10g] \(\cup\) TQ[8g] | -14.566 7347 | 15958 | -14.577 3036 | 263450 | 0.2876 |

other theory [30] | 0.285(5) |
other theory [31] | 0.2891(10) |
ob. [32] | 0.29099(10) |
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Table 2. Specific mass shift parameters (in $a_0^{-2}$) for Be ($2s^2 \, 1S$), Be ($2s2p \, 3P^o$) and 10−9Be and 9−8Be isotope shifts (in GHz) for the $2s2p \, 3P^o - 2s^2 \, 1S$ transition.

<table>
<thead>
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<th>$S_{SMS}$ (Be ($2s^2 , 1S$))</th>
<th>$S_{SMS}$ (Be ($2s2p , 3P^o$))</th>
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<th>9−8Be</th>
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<td>0.429 415 56 0.255 429 27</td>
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<td>13.6297</td>
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<tr>
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<td>SD[10g]∪TQ[3]</td>
<td>0.460 699 35 0.260 441 31</td>
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<td>15.0869</td>
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<td>SD[10g]∪TQ[4]</td>
<td>0.461 811 78 0.261 024 27</td>
<td>12.0364</td>
<td>15.1394</td>
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<td>SD[10g]∪TQ[5]</td>
<td>0.462 014 14 0.261 297 61</td>
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<td>SD[10g]∪TQ[7g]</td>
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<td>Other theory[14]</td>
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<td>12.003</td>
<td>15.097</td>
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Table 3. Specific mass shift parameters (in \(a_0^{-2}\)) for the negative ion and the neutral atom, normal mass shift (NMS), specific mass shifts (SMS) and isotope shifts (all in cm\(^{-1}\)) in the electron affinity \(E_a\).

<table>
<thead>
<tr>
<th>AS</th>
<th>(S_{\text{SMS}})</th>
<th>(S_{\text{SMS}})</th>
<th>(\Delta E_a^{\text{NMS}})</th>
<th>(\Delta E_a^{\text{SMS}})</th>
<th>(\Delta E_a)</th>
<th>(\Delta E_a^{\text{NMS}})</th>
<th>(\Delta E_a^{\text{SMS}})</th>
<th>(\Delta E_a)</th>
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<td>HF(Be)</td>
<td>-0.42025584</td>
<td>-0.21012792</td>
<td>-0.006028</td>
<td>-0.511929</td>
<td>-0.517957</td>
<td>-0.006028</td>
<td>-0.798454</td>
<td>-0.807856</td>
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<tr>
<td>HF(Be-)</td>
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<td>-0.13400418</td>
<td>-0.006028</td>
<td>-0.326471</td>
<td>-0.332499</td>
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<td>-0.21012792</td>
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<td>-0.147041</td>
<td>-0.009403</td>
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<td>-0.229339</td>
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<td>-0.20894838</td>
<td>-0.006028</td>
<td>-0.143361</td>
<td>-0.149543</td>
<td>-0.009642</td>
<td>-0.223599</td>
<td>-0.233242</td>
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<td>0.27557716</td>
<td>0.001477</td>
<td>-0.121303</td>
<td>-0.122781</td>
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<td>0.27507521</td>
<td>0.015639</td>
<td>-0.198819</td>
<td>-0.183180</td>
<td>0.024393</td>
<td>-0.310097</td>
<td>-0.285794</td>
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<td>0.25941306</td>
<td>0.018687</td>
<td>-0.122398</td>
<td>-0.103711</td>
<td>0.029147</td>
<td>-0.190903</td>
<td>-0.161757</td>
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<tr>
<td>6g</td>
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<td>0.25724218</td>
<td>0.019577</td>
<td>-0.133484</td>
<td>-0.113907</td>
<td>0.030536</td>
<td>-0.208194</td>
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<tr>
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<td>0.25600781</td>
<td>0.020342</td>
<td>-0.132334</td>
<td>-0.111991</td>
<td>0.031729</td>
<td>-0.206400</td>
<td>-0.174671</td>
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<td>0.25562343</td>
<td>0.020694</td>
<td>-0.132862</td>
<td>-0.112168</td>
<td>0.032278</td>
<td>-0.207224</td>
<td>-0.174947</td>
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<tr>
<td>SD[10g]∪TQ[3]</td>
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<td>SD[10g]∪TQ[5]</td>
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<td>0.26129961</td>
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<td>-0.099031</td>
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<td>0.038350</td>
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<tr>
<td>SD[10g]∪TQ[6g]</td>
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<td>0.26141050</td>
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<td>-0.099399</td>
<td>-0.074269</td>
<td>0.039197</td>
<td>-0.155632</td>
<td>-0.118535</td>
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<tr>
<td>SD[10g]∪TQ[7g]</td>
<td>0.22104187</td>
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<td>-0.079322</td>
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