HETEROGENEITY IN INFLATION PERSISTENCE AND MONETARY POLICY IN A MONETARY UNION

SÉVERINE MENGUY¹

ABSTRACT:
In the context of a new Keynesian macroeconomic model, this paper studies the monetary policy that should be conducted by the common central bank of a monetary union. In the event of inflationary supply shocks, the optimal monetary policy should be all the more contractionary as the inflation inertia increases, in order to obtain the same average price level. Moreover, the increase in interest rates should be all the more accentuated as the shock affects a country which has a higher inflation inertia, and as the heterogeneity in inflation persistence between the countries of the monetary union increases.

KEYWORDS: New-Keynesian model, monetary union, monetary policy, inflation persistence, structural heterogeneity.

JEL CLASSIFICATION: E52, E58, F41.

¹ EconomiX, Université de Paris X-Nanterre, bâtiment K, 200 Avenue de la République, 92001 Nanterre Cedex, France. tel: (00 33) 1 40 97 59 14. E-mail: severine.menguy@orange.fr.
I would like to thank the participants to the GDRE in Orléans in June 2009, as well as the anonymous referee of the Brussels Economic Review, for their helpful comments. Naturally, I am alone responsible for the errors and omissions that could remain in this paper.
HETEROGENEITY IN INFLATION PERSISTENCE AND MONETARY POLICY IN A
MONETARY UNION

INTRODUCTION

In the most recent economic literature, the New-Keynesian models are the favorite framework to study optimal monetary policy. These models are all characterized by nominal price rigidities and rational expectations. Thus, they have the advantage of combining optimal behavior with rigidities avoiding the Lucas critique. Nevertheless, most of these models only concern monetary policy, ignoring the presence of fiscal policies, and they often relate to a closed economy setting. In contrast, the current paper aims at explaining the determinants of monetary policy for the stabilization of various shocks, but in the context of a monetary union. Within the framework of these New-Keynesian models, one important subject of interest is then the influence of inflation inertia on optimal monetary policy. For example, Söderström (2002) and Moessner (2005) show that the central bank should respond more aggressively to cost-push or real interest rate shocks when the degree of inflation inertia is uncertain. Indeed, when the dynamics of the variables are uncertain, when inflation and output are further away from target, the uncertainty about their future development is greater. So, the central bank should be more aggressive to push inflation closer to target, in order to reduce uncertainty about the future development of inflation. Furthermore, Benigno and Lopez-Salido (2002) analyze various inflation targeting policies in presence of heterogeneity in inflation persistence among various regions in the Euro area. So, they find that targeting Euro-wide inflation is not optimal: the best option is to assign a greater weight to inflation volatility in the countries with a greater degree of inflation inertia. Therefore, the central bank should target a price index that assigns a relatively larger weight to the countries where price developments are stickier. Indeed, for a similar monetary policy, the rigid country bears a higher cost in its adjustment to a macroeconomic shock. In these conditions, by overweighting this more rigid country, the monetary policy ensures that the flexible country responds to a higher degree to the shock, thus making a stronger contribution to the overall adjustment needed in the economy.

In a close center of interest, the aim of this paper is to underline the importance of inflation inertia in the definition of optimal monetary policy. We show that inflation risks are higher, and therefore that monetary policy must be all the more contractionary, as inflation persistence increases in the case of inflationary supply shocks. Furthermore, our paper sheds light on the consequences of the heterogeneous degrees of inflation persistence between the countries of a monetary union. We show that in order to obtain the same inflation level, the common interest rate must be higher and the common monetary policy more contractionary if an inflationary supply shock affects a country with a higher degree of inflation inertia, and all the more as the heterogeneity increases between the member countries.

The rest of the paper is organized as follows. The first section describes our model. The second section underlines the importance of the parameter representing inflation persistence, whereas the third section defines the optimal inflation rate. The fourth section determines the levels of inflation and output, and the fifth section defines the optimal interest rate, in particular in case of an inflationary supply shock. Finally, the conclusion.
1. Setup

We use a New Keynesian Dynamic Model in open economy. However, we do not develop the underlying micro-economic structure of the model, and we use a set of plausible values for the parameters. The variables (except the interest rate) are expressed in logarithms and refer to deviations from the steady state values. Our modeling is very standard, and quite similar to Garretsen et al. (2007) or Fornero et al. (2007), for example.

Let us assume a closed monetary union consisting of two countries: (i) and (j). The structural parameters of these countries are fully identical, except for the inflation persistence which can diverge between the two member countries. Indeed, our aim was to study the consequences of heterogeneity in inflation persistence all other things being equal. In particular, we suppose that the two countries have exactly the same size. Naturally, if one member country of a monetary union is very small in comparison with its partners, the central bank should probably not overweight this country, even if the inflation persistence is much higher in this country. But this question would be beyond the scope of the current paper.

Furthermore, each variable or parameter (x) has an average or symmetrical component: \( x = \frac{1}{2}(x_i + x_j) \), and a differential or asymmetrical component: \( \bar{x} = \frac{1}{2}(x_i - x_j) \). We also assume that these components are independently distributed and that the covariance between them is nil.

The individual and global demands - (IS) curves - are as follows:

\[
\begin{align*}
y_{i,t} &= a_1y_{i,t-1} + (1 - a_1)E_t(y_{i,t+1}) - a_2[i_t - E_t(\pi_{i,t+1} - \pi^*)] \\
&\quad + a_3g_{i,t} + a_4y_{j,t} + a_5(p_{i,t} - p_{i,t}) + d_{i,t} \quad (1a)
\end{align*}
\]

\[
(1 - a_4)y_{j,t} = a_1y_{j,t-1} + (1 - a_1)E_t(y_{j,t+1}) - a_2[i_t - E_t(\pi_{j,t+1} - \pi^*)] + a_3g_{j,t} + d_{j,t} \quad (1b)
\]

\[
(1 + a_4)\bar{y}_t = a_1\bar{y}_{t-1} + (1 - a_1)E_t(\bar{y}_{t+1}) + a_2E_t(\bar{\pi}_{t+1}) + a_3\bar{g}_t - 2a_5\bar{p}_t + \bar{d}_t \quad (1c)
\]

With, in period (t) and for the country (i): \( y_{i,t} \): output; \( p_{i,t} \): price level; \( \pi_{i,t} = p_{i,t}p_{t-1} \): inflation rate; \( g_{i,t} \): fiscal balance (there is a deficit if \( g_{i,t} > 0 \)); \( d_{i,t} \): positive demand shock (white noise); \( \pi^* \): equilibrium real interest rate; \( i_t \): short term nominal interest rate. \( E(.) \) denotes the expectations operator.

Traditionally, in New-Keynesian models, aggregate demand is driven by the optimizing behavior of households, which maximize an intertemporal utility function. Moreover, output depends on past output, because of ‘habit formation’ in consumption decisions. But it also depends on expected future output, because rational agents can maximize their decisions intertemporally and smooth their consumption. Besides, in empirical estimations, authors generally find that the values of the forward and backward parameters approximately sum to one. Furthermore, aggregate demand also depends on the real interest rate, because of the intertemporal substitution of consumption. Nevertheless, in the economic
literature, this interest rate channel seems controversial (a2 is low), perhaps because monetary policy operates through other channels (asset prices, exchange rate, credit, wealth effect). Finally, aggregate demand also depends on net government spending, on a demand shock, and on the net exports, the latter being an increasing function of the foreign output and price competitiveness of the country.

The supply function takes the form of a ‘new Keynesian Phillips curve’:

\[
\pi_{t,1} = b_{1,1}\pi_{t-1,1} + (1 - b_{1,3})E_t(\pi_{t+1,1}) + b_{2,1}y_{t,1} + b_{3,1}\pi_{t,1} + s_{t,1}
\]

(2a)

\[
(1 - b_{3})\pi_t = b_{1}\pi_{t-1} + \tilde{b}_{1}\pi_{t-1} + (1 - b_{1})E_t(\pi_{t+1}) - \tilde{b}_{1}E_t(\pi_{t+1}) + b_{2}y_{t} + s_{t}
\]

(2b)

\[
(1 + b_{3})\pi_t = \tilde{b}_{1}\pi_{t-1} + b_{1}\pi_{t-1} - \tilde{b}_{1}E_t(\pi_{t+1}) + (1 - b_{1})E_t(\pi_{t+1}) + b_{2}\tilde{y}_{t} + s_{t}
\]

(2c)

With \((s_{i,t})\): inflationary supply shock (white noise).

Indeed, in the new Keynesian models, aggregate supply results from the behavior of firms that set prices for their products so as to maximize profits in a monopolistic competition setting. Lagged inflation has been motivated in the literature by the presence of partial price indexation or of rule-of-thumb price-setters. So, inflation depends on past inflation, expressing the inertia in price adjustment or adaptive expectations, but it also depends on expectations about future prices, because of learning effects. Besides, the output gap expresses the demand-pull factor and the tensions on the utilization of the productive capacities, whereas imported prices (the so called ‘pass through’) express the cost-push factor affecting inflation. Finally, \((s_{i,t})\) captures an inflationary shock unrelated to excess demand or to cost-push factors (mark-up, etc.). As formerly mentioned, in order to focus on the consequences of the heterogeneity between the inflation persistence parameters of the member countries of a monetary union \((b_{i,1})\), we assume here that all other structural parameters are fully identical between the two countries.

Let's also underline that \((b_{2})\) is related to the degree of price stickiness in the economy: more stickiness or price rigidity in the labor market implies a lower value of \((b_{2})\). However, the traditional New Keynesian Phillips Curves introduce generally real marginal costs as demand-pull indicator. Theoretically, this variable can be replaced by measures of cyclical pressures (output gaps) only if the labor markets are frictionless, if there is no capital stock, or if consumption and worked hours are proportional. More concretely, Domenech et al. (2001) find that a measure of the output gap performs better in econometrical estimations than real marginal costs. Nevertheless, in most studies, the empirical estimations of the coefficient \((b_{2})\) are often insignificant, or even negative... In fact, Jondeau and Le Bihan (2005) find that the real Unit Labor Cost specification with a single lag and lead combined with a large forward looking component is relevant in the US and the UK. Conversely, the output gap specification with three lags and leads and a low degree of forward lookingness would provide a better fit for continental
Europe. In any case, the choice of the forcing variable does not seem to affect the degree of inflation persistence ($b_{i,1}$).

Finally, below is, according to some econometrical studies, the calibration that we can retain for the parameters of our model (we have retained average values in comparison with those obtained by former studies).

**Table 1. Calibration of our parameters**

<table>
<thead>
<tr>
<th>Study</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coenen – Wieland (2002)</td>
<td>0.08</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Djoudad – Gauthier (2003)</td>
<td>0.52</td>
<td>0.03</td>
<td>0.62</td>
<td>0.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dornhein et al. (2001)</td>
<td>0.57</td>
<td>0.10</td>
<td>0.46</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Faruqee (2006)</td>
<td>0.61</td>
<td>0.23</td>
<td>0.6</td>
<td>0.25</td>
<td>0.20</td>
<td>0.46</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>Fornero et al. (2007)</td>
<td>0.61</td>
<td>0.23</td>
<td>0.6</td>
<td>0.25</td>
<td>0.20</td>
<td>0.46</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>Gagnon – Ihrig (2001)</td>
<td>0.61</td>
<td>0.23</td>
<td>0.6</td>
<td>0.25</td>
<td>0.20</td>
<td>0.46</td>
<td>0.08</td>
<td>0.05</td>
</tr>
<tr>
<td>Leith – Malley (2002)</td>
<td>0.22</td>
<td>0.06</td>
<td>0.3</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sahuc (2002)</td>
<td>0.44</td>
<td>0.06</td>
<td>0.54</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smets (2003)</td>
<td>0.57</td>
<td>0.3</td>
<td>0.48</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smets – Wouters (2002)</td>
<td>0.57</td>
<td>0.3</td>
<td>0.48</td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Our calibration</strong></td>
<td><strong>0.6</strong></td>
<td><strong>0.1</strong></td>
<td><strong>0.6</strong></td>
<td><strong>0.25</strong></td>
<td><strong>0.1</strong></td>
<td><strong>0.4</strong></td>
<td><strong>0.1</strong></td>
<td><strong>0.1</strong></td>
</tr>
</tbody>
</table>

2. **Inflation Persistence**

Inflation persistence is defined by the Inflation Persistence Network (IPN) as the delay by which inflation converges towards its long run value following a shock which has led inflation away from it. More precisely, the IPN defines three kinds of persistence. First, ‘intrinsic persistence’ corresponds to our parameter ($b_{i,1}$) as it relates to nominal rigidities and to the way wages and prices are set. Secondly, ‘expectations-based persistence’ ($1-b_{i,1}$) is related to the perception and the credibility for the public of the inflation target of the monetary authority. Finally, ‘extrinsic persistence’ ($b_2$) is inherited from persistent fluctuations in the economic environment and in the inflation-driving real variables (see Altissimo et al. (2006)).

In general, ‘hybrid’ monetary policy models, taking into account both backward and forward inflation determinants, are more consistent with empirical data as well as with the concrete gradual response of inflation to shocks than the traditional forward looking models. In fact, Smets and Wouters (2002) or Leith and Malley (2002) show that there is more inertia in the price-setting behavior in Europe than in the US. Nevertheless, even in Europe, a forward looking behavior seems dominant: the coefficient on expected future inflation substantially exceeds the coefficient on lagged inflation ($b_{i,1} < 0.5$). Besides, other econometrical studies show that additional inflation lags would be insignificant.

In various countries, the labor markets (real wage rigidities) have an important role in generating various levels of inflation persistence. Nevertheless, studies disagree on the levels of inflation persistence and even on the ranking of the countries! For example, in the current monetary policy regime, Altissimo et al. (2006) find that inflation persistence is only moderate in the Euro Area, and may even have fallen over the last decade. Indeed, today, inflation expectations are largely anchored by the price objective of the European Central Bank (ECB). Therefore, the importance of past inflation is less essential to form these expectations, and actual inflation...
developments are less persistent. However, Cecchetti and Debelle (2005) show that the change in the inflation process at least since the 1990s is essentially due to a decrease in its mean, the decline in the persistence of the process being of lesser importance. Indeed, in Europe, monetary policy has been gradually more focused on achieving low inflation, which has increased the credibility of the central bank, and which in turn has anchored inflation expectations. Nevertheless, inflation persistence seems to remain quite high in certain categories of goods: food, housing and transportation, whereas it appears to be low for alcohol, tobacco, furniture and health, and even negative for communication, recreation, clothing, restaurants and education (Altissimo et al., 2006).

In this context, the estimations of the degrees of inflation persistence \( (b_{i1}) \) are very heterogeneous, in the economic literature. Generally, Rumler (2005) finds that price rigidity and persistence are lower in open economies, indicating that when firms face more variable input costs, they tend to adjust their prices more frequently. But the estimations and even the classification of the countries according to this parameter differ widely. Table 2, below, gives the various estimated values for the heterogeneity degree:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.38</td>
<td>-0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>0.21</td>
<td>-0.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>0.42</td>
<td>0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.45</td>
<td>0.09</td>
<td>0.30</td>
<td>0.45</td>
</tr>
<tr>
<td>Germany</td>
<td>0.46</td>
<td>-0.20</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>Greece</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.41</td>
<td>0.44</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>-0.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.33</td>
<td>0.06</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td></td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.20</td>
<td>0.21</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U. Kingdom</td>
<td></td>
<td>0.59</td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>EU</td>
<td>0.40</td>
<td>0.30</td>
<td></td>
<td>0.37</td>
</tr>
</tbody>
</table>

Regarding the particular situation of the New Member States (NMS) in Europe, Franta et al. (2007) show that statistical measures assuming a constant mean deliver higher persistence estimates for these countries. On the contrary, employing a time-varying mean leads to similar or lower inflation persistence for the NMS compared to the Euro Area countries. These results signify that expectations and the implicit inflation target of the central bank have been the main cause of inflation persistence in the NMS since 1993. Nevertheless, backward-looking behavior may still be a more important component accounting for inflation dynamics in Hungary and in the Czech Republic than in the current Euro Area countries.
3. Determination of the Optimal Inflation Rate

To solve our model, we must now detail the preferences of the economic authorities. Regarding these preferences, Muscatelli et al. (2003) directly estimate monetary and fiscal policy rules, without deriving them from loss functions. Domenech et al. (2001) or Djoudad and Gauthier (2003) also estimate Taylor type interest rate rules, whereas Garretsen et al. (2007) or Fornero et al. (2007) estimate fiscal rules. Nevertheless, in the current paper, we want to derive the optimal economic policies from the loss functions of economic authorities. Therefore, we consider that these loss functions take the following form, respectively for the central bank (L_M) and the government (L_i) of the country (i):

\[ L_M = \frac{1}{2} [c^M y_t^2 + \pi_t^2] \quad (3) \]

\[ L_i = \frac{1}{2} [c^i y_t^2 + \pi_t^2] \quad (4) \]

In these loss functions, the output target is given by the potential output level; so, the authorities aim at a zero output gap (excluding the possibility of a systematic inflation bias). The target for the inflation rate is also zero. To simplify the modeling, we also assume that the instruments of economic policy, the interest rate (it) for the central bank and the public expenditures (g_i, t) for the government (i), are without cost for the economic authorities. In particular, we assume that the public deficits and surpluses are equilibrated on average, without considering the important question of the financing of the deficits and of the increase in the level of indebtedness. Thus, the budget constraint, as for example the one introduced in Europe by the Stability and Growth Pact, is supposed to be non-binding in the monetary union.

Now, let us assume that the inflation rate can be expressed as follows, where \( c_{i,1} \) measures the speed of convergence to the inflation target under the optimal policy:

\[ \pi_{t,i} = c_{i,1} \pi_{t-1,i} + c_{i,2} \pi_{t-1,i} + c_{i,3} s_{t,i} + c_{i,4} s_{j,t} \quad with \ 0 < c_{i,1} < 1 \quad (5a) \]

\[ \pi_t = (c_1 + c_2) \pi_{t-1} + (\bar{c}_1 - c_2) \bar{\pi}_{t-1} + (c_3 + c_4) s_t + (c_3 - c_4) \bar{s}_t \quad (5b) \]

\[ \bar{\pi}_t = (\bar{c}_1 + c_2) \pi_{t-1} + (c_1 - c_2) \bar{\pi}_{t-1} + (\bar{c}_3 + c_4) s_t + (c_3 - c_4) \bar{s}_t \quad (5c) \]

Thus:

\[ E_t(\pi_{t+1}) = c_{i,1} \pi_{t,i} + c_{i,2} \pi_{j,t} \quad (6a) \]

\[ E_t(\pi_{t+1}) = (c_1 + c_2) \pi_t + (\bar{c}_1 - c_2) \bar{\pi}_t \quad (6b) \]

\[ E_t(\bar{\pi}_{t+1}) = (\bar{c}_1 + c_2) \pi_t + (c_1 - c_2) \bar{\pi}_t \quad (6c) \]

We consider the equilibrium in the case of ‘time consistent’ discretion: the economic policies are the one that the authorities have an interest in conducting, if the agents’ expectations are rational (taken as given) and if they can’t surprise these agents in an unexpected way. Furthermore, we assume that the economic authorities play simultaneously, and interact in a kind of Nash equilibrium. Nevertheless,
within the framework of our equations, the preferences of the economic authorities (\(c^M_i\), \(c^{Gi}\) and \(c^{Gj}\)) are not independent. These preferences are related to the parameters of our model and interdependent (there is no conflict), if they want to be mutually consistent (see Appendix B)…

Each country (i) independently pursues an optimal fiscal policy. It minimizes its loss function (4) with respect to the fiscal instrument (\(g_i\)); its optimal fiscal rule is thus (see Appendix A):

\[
c^{Gi}_i m_{yi/gi} y_{i,t} + b_2 m_{pi/gi} \pi_{i,t} = 0
\]  
(7)

with:

\[
m_{yi/gi} = [(1 - c_{i,1} + b_{i,1} c_{i,1})(1 - c_{i,1} + b_{i,1} c_{i,1} + b_2 a_5 - b_2 a_2 c_{i,1}) \\
- (b_3 + c_{i,2} - c_{i,2} b_{i,1})(b_3 + b_2 a_5 + c_{i,2} - b_{i,1} c_{i,2})]
\]

\[
m_{pi/gi} = (1 - c_{i,1} + b_{i,1} c_{i,1} + b_3 a_4 + b_2 a_5 - b_2 a_2 c_{i,1} + a_4 c_{i,2} - a_4 c_{i,2} b_{i,1})
\]

Besides, the central bank minimizes its loss function (3) with its instrument, the interest rate (i); the optimal monetary policy rule is thus (see Appendix A):

\[
c^M m_{y/r} y_t + b_2 m_{p/r} \pi_t = 0
\]  
(8)

with:

\[
m_{y/r} = b_2[(2 a_5 - a_2 c_{i,1})(1 - c_{i,1} - c_{i,2} + c_{i,1} b_{i,1} + c_{i,2} b_{i,1} - b_3) \\
+ (2 a_5 - a_2 c_{i,1})(1 - c_{i,1} - c_{i,2} + c_{i,1} b_{i,1} + c_{i,2} b_{i,1} - b_3)] \\
+ 2(1 + a_4)[(1 - c_{i,1} + b_{i,1} c_{i,1})(1 - c_{i,1} + b_{i,1} c_{i,1}) - (b_3 + c_{i,2} - b_{i,1} c_{i,2})](b_3 + c_{i,2} - b_{i,1} c_{i,2})] > 0
\]

\[
m_{p/r} = (1 + a_4)[(2 - 2 c_t + 2 c_2 + b_{i,1} c_{i,1} + b_{i,1} c_{i,1} - b_{i,1} c_{i,2} - b_{i,1} c_{i,2} + 2 b_3) + 2 b_2(2 a_5 - a_2 c_{i,1})] > 0
\]

In this framework, as mentioned by Clarida et al. (1999), the values of the parameters in equations (5) depend on the basic parameters of our model. However, the unique stable solutions for (\(c_{i,1}\)) comprised between 0 and 1 are then the following (see Appendix B):

- If \(b_{i,1} = b_{i,1} \rightarrow 0\): \(c_{i,1} = c_{j,1} = \frac{c^M (1-b_3)}{2[b_2 + c^M (1-b_3)]} (\forall c^{Gi}) \) and \((\forall c^{Gj})\)
- If \(b_{i,1} = c_{i,1} \rightarrow 0\): \(c_{i,1} = c_{j,1} = \frac{c^M (1-b_3)}{[b_2 + c^M (1-b_3)]} (\forall b_{i,1})\)
- If \(b_{i,1} = c_{i,1} \rightarrow 0\): \(c_{i,1} = c_{j,1} = \frac{c^M (1-b_3)}{[b_2 + c^M (1-b_3)]} (\forall b_{j,1})\)
- If \(c^M = 0\): \(c_{i,1} = c_{j,1} = 0\)
If $b_{i,1} = b_{j,1} \neq 0$:

$$c_{t,1} = c_{f,t,1} = \frac{-b_{i,1} \sqrt{b_{i,1}^2 + (2b_{i,1} - 1)^2}}{2(1 - b_{i,1})(2b_{i,1} - 1)}$$

If $b_{i,1} = (1-k)b_{j,1}$; $b_{j,1} = (1+k)b_{1}$ and $k \neq 0$: we can find $c_{j,t} = f(c_{i,t})$ and then $c_{i,t}$

In this last expression, we assume that $(k)$ represents the heterogeneity between the member countries of the monetary union. Indeed, when $k=0$, these countries are fully identical. On the contrary, if they are structurally heterogeneous, the inflation inertia is $[2k/(1-k)]\%$ higher in country $(j)$ than in country $(i)$.

4. **OPTIMAL LEVELS OF INFLATION AND OUTPUT**

Temporally, demand or supply shocks occur first in the economy. Then, the monetary and budgetary authorities react to these shocks, and finally, we obtain the optimal levels of inflation and output after stabilization. Nevertheless, as (5a) to (5c) define the expressions of the optimal inflation levels, our model must be solved backwards. That is to say, we will define the optimal economic variables in this section, before the determination of the optimal economic policies which induce these results in the next section.

Thus, (5a) and the resolution of our model [see (m), (n) and (p) in Appendix B] imply the following inflation rates:

$$\pi_{t,t} = c_{t,1}\pi_{t,t-1} + (2b_{j,1} - c_{j,t})\pi_{j,t,t-1} + \frac{c_{i,t}}{b_{j,1}}s_{j,t} + (2c_{1} - \frac{c_{j,t}}{b_{j,1}})s_{j,t}$$  \hspace{1cm} (9a)

$$\pi_{t} = c_{1}(b_{i,1}\pi_{t,t-1} + b_{j,1}\pi_{f,t,t-1} + s_{j,t} + s_{j,t})$$  \hspace{1cm} (9b)

Therefore, after the conduct of stabilization policies by the economic authorities, (9a) shows that the optimal national inflation rate is an increasing function of the lagged national inflation and of the exogenous national inflationary supply shocks ($s_{j,t}$). It is a more ambiguous function of the lagged imported inflation (pass through) and of the foreign inflationary supply shocks ($s_{j,t}$). Furthermore, according to (9b), if monetary policy is optimal, the average inflation rate only depends on the average inflation persistence in the monetary union (on $c_{1}$), regardless of its level in the particular country affected by an inflationary supply shock. Indeed, a strict inflation targeting policy from the ECB ($c^{M}=0$ which implies $c_{i}=0$) allows for the perfect stabilization of the average inflation rate ($\pi_{t}=0$). On the contrary, the more the central bank aims at stabilizing output ($c^{M}>0$), the less contractionary the monetary policy is and the higher the inflation rates. In this last case, Fig.1 represents the average inflation rate according to the degree of inflation persistence, after an inflationary supply shock (the graphs are all made assuming the numerical calibration of the parameters in Table 1). Thus, it appears that the average inflation rate is always an increasing function of the average inflation persistence in the countries of the monetary union ($b_{i}$).
Furthermore, the heterogeneity between the degrees of inflation inertia in the member countries (a higher $k$) mainly appears to increase slightly the average inflation rate in the monetary union for small values of inflation persistence ($b_1<0.5$), which is empirically the most likely situation (see section 3). On the contrary, this heterogeneity would reduce slightly the average inflation rate for high values of inflation persistence ($b_1>0.5$).

**FIG.1 SUPPLY SHOCKS AND AVERAGE INFLATION RATE**

Besides, putting equation (9b) into (8), the optimal average output is as follows:

$$y_t = - \frac{b_2 m_{p/r}}{e^{M_{v/r}}} (b_{i,1} \pi_{t-1} + b_{j,1} \pi_{t-1} + s_{i,t} + s_{j,t})$$

(10)

And thus:

$$E_t(y_{t+1}) = - \frac{b_2 m_{p/r}}{e^{M_{v/r}}} E_t(\pi_{t+1}) = - \frac{b_2 c s_{p/r}}{e^{M_{v/r}}} (b_{i,1} \pi_{t-1} + b_{j,1} \pi_{t-1})$$

(11)

So, equation (10) shows that the consequences on the level of output and on inflation of demand shocks ($d_t$) and of shocks on the real interest rate ($r^*$) can be fully compensated by the monetary and budgetary authorities if they are not constrained in the use of their instruments. Indeed, these variables ($d_t$) and ($r^*$) don’t intervene at all in the optimal expressions for inflation (9) and for output (10). In contrast, the inflationary and recessive consequences of the supply shocks can only be avoided if $c_1=0$, that is to say if the inflation targeting policy of the
monetary authority ($c^M=0$) is perfectly credible. Otherwise, the heterogeneity between the degrees of inflation inertia in the member countries (a higher $k$) mainly appears to increase slightly the recessive consequences of these inflationary supply shocks.

**FIG.2. SUPPLY SHOCKS AND AVERAGE OUTPUT**

5. **OPTIMAL INTEREST RATE**

Now, we can find the optimal interest rate by combining equation (1b) with (9a), (9b), (10) and (11). Indeed, we have:

\[
\dot{i}_t =
\frac{a_2}{a_2} y_{t-1} + \left\{ 2(b_{1j} c_{1j} + b_{2j} c_{2j}) \left[ 1 - \frac{b_2(1-a_3) m_{y/r}}{a_2 c^M m_{y/r}} \right] + \frac{b_2(1-a_4) m_{p/r}}{a_2 c^M m_{y/r}} b_{1j} \right\} c_{1j} \pi_{i,t-1} \\
+ \left\{ 2(b_{1j} b_{1j} c_{1j} + b_{2j} c_{2j}) \left[ 1 - \frac{b_2(1-a_3) m_{y/r}}{a_2 c^M m_{y/r}} \right] + \frac{b_2(1-a_4) m_{p/r}}{a_2 c^M m_{y/r}} b_{1j} \right\} c_{1j} \pi_{i,t-1} \\
+ r^* + \frac{1}{a_2} d_t + \left\{ 2(b_{1j} c_{1j} - b_{1j} c_{1j} + c_{1j}) \left[ 1 - \frac{b_2(1-a_3) m_{y/r}}{a_2 c^M m_{y/r}} \right] + \frac{b_2(1-a_4) m_{p/r}}{a_2 c^M m_{y/r}} b_{1j} \right\} c_{1j} \pi_{i,t} \\
+ \frac{a_3}{a_2} g_t + \left\{ 2(b_{1j} c_{1j} - b_{1j} c_{1j} + c_{1j}) \left[ 1 - \frac{b_2(1-a_3) m_{y/r}}{a_2 c^M m_{y/r}} \right] + \frac{b_2(1-a_4) m_{p/r}}{a_2 c^M m_{y/r}} b_{1j} \right\} c_{1j} \pi_{i,t} 
\]

(12)
Therefore, positive demand or real interest rates shocks imply an initial increase in output and inflation in the monetary union (see Appendix A). More contractionary monetary (a higher interest rate \( i_t \)) and fiscal (less public spending: \( g_{it} \)) policies, according to equation (12), can then fully stabilize these shocks, as their impact is null on the final optimal levels of inflation and output in (9) and (10). Indeed, if the variation in the policy instruments is without cost for the economic authorities, (9a) and (10) show that these shocks can perfectly be compensated by counter-cyclical economic policies. However, if asymmetrical demand shocks can only be stabilized by the decentralized budgetary authorities, the stabilization of symmetrical demand shocks remains ambiguous in the framework of our model. Whether they are stabilized by the monetary or budgetary authorities depends on their respective negotiation powers, on the possibility for one authority to be in a situation of Stackelberg leader, and eventually also on the relative costs related to the utilization of their instruments. In particular, constraints on the use of the budgetary instrument for the governments can be related to the risk of increasing budgetary deficits and public debt. For example, the Stability and Growth Pact, in Europe, limits the budgetary deficits to 3% of GDP, and asks for a budgetary position of “close to balance or in surplus” in the medium term. Regarding the monetary authority, the constraints on the use of the monetary instrument are related to the dangers for the stability of the financial system and for the incentive to private savings if interest rates were highly volatile.

Inflationary supply shocks increase prices, but they have a more ambiguous effect on output in the monetary union (see Appendix A). More contractionary monetary (a higher interest rate \( i_t \)) and fiscal policies, according to equation (12), can then only stabilize incompletely these shocks. Indeed, equation (9) and (9a) show that inflationary tensions remain in the monetary union, whereas the recession can’t be avoided in equation (10).

More precisely, if there is no inflation inertia \((c_1 \to 0)\), that is to say if the monetary policy of inflation targeting \((c^M \to 0)\) is perfectly credible, inflation and output are perfectly stabilized and no variation in interest rates is thus necessary. Indeed, the agents believe in the monetary policy of strict inflation targeting, and therefore the anticipated future inflation is null. Otherwise, inflation and recession can’t be avoided in the case of inflationary supply shocks. For example, if the central bank also aims at stabilizing output \((c^S \to 0)\), then the monetary policy is generally all the more contractionary as the inflation persistence increases. Indeed, the inflationary risks are accentuated if the endogenous inflation persistence \((b_i)\) is high, as inflation returns then more slowly to equilibrium. So, as mentioned by Clarida et al. (1999), if endogenous inflation persistence is higher, the monetary policy must react more strongly to supply shocks, since any disturbance not eliminated today will continue and will imply more output contractions in the future. The monetary policy is thus more contractionary, since a potentially higher inflation inertia would imply that the effect of cost-push shocks persists for longer, possibly requiring greater output contractions in the future. In the same way, Altissimo et al. (2006) show that in the case of a cost-push shock, a smaller degree of inflation persistence reduces the contractionary policy necessary to respond to this shock, as the agents reduce their expectations of future inflation.
Fig. 3. Supply shocks and optimal interest rate

\[ b_{i,t} = (1-k)b_{i} \quad ; \quad b_{j,t} = (1+k)b_{j} \]

\[ \frac{\partial \pi_t}{\partial s_{it}} \]

\[ \frac{\partial \pi_t}{\partial s_{jt}} \]
However, the advantage of our model is also to show the consequences of the heterogeneity between the degrees of inflation persistence in the countries of a monetary union on the optimal monetary policy. As mentioned by Bofinger et al. (2005), monetary policy can be quite destabilizing in the case of an inflationary supply shock. Indeed, such a shock initially has inflationary consequences which lower the real interest rate and then automatically reduce the recessive consequences of the shock. In this framework, a contractionary monetary policy and an increase in interest rates may amplify the recessive consequences of the shock. So, according to our numerical calibration, the increase in interest rates is very limited if the shock affects mainly a country (i) which has the lower inflation inertia; for example, with our numerical calibration, there would be no variation in interest rates if $b_{i1}=0.21$ and $b_{j1}=0.35$. Monetary policy can then even be expansionary for small values of $b_1<0.5$, and all the more as the heterogeneity in inflation persistence increases between the countries of the monetary union.

On the contrary, monetary policy is much more contractionary if the shock affects a country (j), which has the higher inflation persistence; indeed, the central bank has interest in overweighting this country where prices are stickier. Moreover, monetary policy is then all the more contractionary as the heterogeneity between the two countries (k) increases. For example, with our numerical calibration, if $b_{i1}=b_{j1}=0.35$, then $i_1=2.31(s_{i1}+s_{j1})$, but if $b_{i1}=0.18$ and $b_{j1}=0.52$, then $i_1=8.14s_{j1}$, despite the same average inflation inertia ($b_1=0.35$) in both cases. Intuitively, the common central bank should thus give a higher weight to the countries of the monetary union where the inflation inertia is the highest, as inflationary risks and the danger of their long term persistence is the highest in these countries.

**CONCLUSION**

Our New Keynesian model allows underlining some important results, regarding optimal monetary policy in a monetary union. The monetary and budgetary authorities could stabilize demand shocks or shocks on the real interest rate if they were not constrained in the use of their policy instruments. On the contrary, inflationary supply shocks have inflationary and recessive consequences which depend on the inflation persistence in the countries of the monetary union. This degree of inflation inertia is a parameter which is quite difficult to estimate in econometrical studies, but which seems to diverge between countries, even between the members of the Economic and Monetary Union (EMU). However, evaluating this inflation persistence would be useful in the framework of our model, as it largely contributes defining the optimal monetary policy. In particular, the question of inflation persistence differentials in the Euro area is very important today. Indeed, if there are huge differences, the countries which have a higher inflation inertia would have to struggle harder to reduce inflation to its long run value. Therefore, in the context of an inflationary shock on oil prices for example, the consequences of a common monetary policy on the economic situation of the various countries would be very heterogeneous.

In our model, it appears that the inflation risks are higher and that the monetary policy must be all the more contractionary as the inflation persistence increases in the case of inflationary supply shocks. However, the contribution of our paper is mainly to study the implication, for the optimal common monetary policy, of the
heterogeneity between the inflation inertia in the various member countries of a monetary union. In particular, this would be very useful to shed light on the policy of the ECB in the framework of the EMU. Therefore, we show that the common interest rate needs to be higher and the common monetary policy more contractionary if an inflationary supply shock mainly affects a country which has a higher degree of inflation inertia. For example, the ECB should accentuate its increase in interest rates if an inflationary supply shock were to affect the less open countries of the monetary union, like Italy or France, which have usually higher levels of inflation persistence. Furthermore, the monetary policy should then be all the more contractionary as the heterogeneity in the inflation persistence parameters between the countries of the monetary union increases, as this heterogeneity seems to accentuate the inflationary and recessive consequences of the shock. Nevertheless, we have to recognize that this optimal monetary policy may create the wrong incentive for some countries to carry out the structural changes that could reduce their goods and labor market rigidities...

REFERENCES


HETEROGENEITY IN INFLATION PERSISTENCE AND MONETARY POLICY IN A MONETARY UNION


APPENDIX A. INFLATION AND OUTPUT

By combining (1a) and (2a), and with (6a), we obtain:

\[ D\pi_t = b_2(1 - c_{i1} + b_{i1}c_{i1} + b_2a_4 + b_2a_5 - b_2a_2c_{i1} + a_4c_{i2} - a_4c_{i2}b_{i1}) \]

\[ + [a_1y_{i1t-1} + a_2g_{i1t} + d_{i1} + (1 - a_1)E_t(y_{i1t+1})] \]

\[ + b_2(a_4 - a_4c_{i1} + b_{i1c_{i1}a_4} + b_3 + b_2a_5 + c_{i2} - c_{i2}b_{i1}) \]

\[ [a_1y_{i1t-1} + a_2g_{i1t} + d_{i1} + (1 - a_1)E_t(y_{i1t+1})] \]

\[ + (1 - a_3^2)(1 - c_{i1} + b_{i1}c_{i1}) + b_2a_5(1 - a_4) - b_2a_2(c_{i1} + a_4c_{i2})(b_{i11}\pi_{i1t-1} + s_{i1}) \]

\[ + (1 - a_3^2)(b_3 + c_{i2} - b_{i1}c_{i2}) + b_2a_5(1 - a_4) + b_2a_2(c_{i1} + a_4c_{i2})(b_{i11}\pi_{i1t-1} + s_{i1}) \]

\[ - b_2a_5(1 - a_4)(1 - c_{i1} + b_{i1}c_{i1}) + b_2a_5(1 - a_4) - b_2a_2(c_{i1} + c_{i2})(b_{i11}\pi_{i1t-1} + s_{i1}) \]

\[ - b_{2a_5}(1 + a_4)(1 - c_{i1} + b_{i1}c_{i1} + c_{i2} - b_{i1}c_{i2} + b_3) + 2b_2a_5 - b_{2a_2}c_{i1}(i - r^*) \]

\[ D = (1 - a_3^2)(1 - c_{i1} + b_{i1}c_{i1})(1 - c_{i1} + b_{i1}c_{i1}) - (b_3 + c_{i2} - b_{i1}c_{i2})(b_{i1} + c_{i2} - b_{i1}c_{i2}) \]

\[ - b_2a_5[2c_i(1 + b_2a_4) + 2c_i(a_4 + b_3) + 2b_2a_5(c_i + c_2) - (c_{i1}c_{i1} - c_{i2}c_{i2})(2 - 2b_4 + a_2b_2)] \]

\[ + b_2a_5(1 - a_4)[2(1 - b_3 - c_1 - c_2) + b_{i1}(c_1 + c_2) + b_{i1}(c_1 + c_2)] \]

\[ 2D\pi_t = f(a, b, c)[a_1y_{i1t-1} + a_2g_{i1t} + d_{i1} + (1 - a_1)E_t(y_{i1t+1})] \]

\[ + f(a, b, c)[a_1y_{i1t-1} + a_2g_{i1t} + d_{i1} + (1 - a_1)E_t(y_{i1t+1})] \]

\[ + f(a, b, c)(b_{i11}\pi_{i1t-1} + s_{i1}) + f(a, b, c)(b_{i11}\pi_{i1t-1} + s_{i1}) - f(a, b, c)(p_{i1t-1} - p_{i1t-1}) \]

\[ - b_{2a_5}(1 + a_4)(2 - 2c_1 + 2c_2 + b_{i1}c_{i1} + b_{i1}c_{i1} - b_{i1}c_{i2} - b_{i1}c_{i2} + 2b_3) \]

\[ + 2b_2(2a_5 - a_2c_1))(i_t - r^*) \]
Where \( f(a,b,c) \) are functions of the parameters of the model available upon request from the author.

Then, with (2a), we have:

\[
\begin{align*}
Dy_{t,t} &= \left(1 - c_{i,1} + b_{i,1}c_{i,1}\right)\left(1 - c_{i,1} + b_{i,1}c_{i,1} + b_{2}a_{5} - b_{2}a_{2}c_{i,1}\right) - (b_{3} + c_{i,2} + b_{2}a_{5} + c_{i,2} - b_{i,1}c_{i,2})\left[a_{1}y_{t,t-1} + a_{3}g_{t,t} + d_{t,t} + (1 - a_{1})E_{t}(y_{t,t+1})\right] + f(a, b, c) [a_{1}y_{t,t-1} + a_{3}g_{t,t} + d_{t,t} + (1 - a_{1})E_{t}(y_{t,t+1})] + f(a, b, c) (b_{i,1}c_{i,1} + s_{i,1}) + f(a, b, c) (b_{i,1}c_{i,1} + s_{i,1}) - f(a, b, c) (p_{i,t-1} - p_{i,t-1}) - a_{2}(1 + a_{4})\left(1 - c_{i,1} + b_{i,1}c_{i,1}\right)\left(1 - c_{i,1} + b_{i,1}c_{i,1}\right) - (b_{3} + c_{i,2} - b_{i,1}c_{i,2}))(b_{3} + c_{i,2} - b_{i,1}c_{i,2}))\left(i_{t} - r^*\right)

- b_{2}a_{2}(2a_{5} - a_{2}c_{i,1})(1 - c_{i,1} + c_{i,1}b_{i,1}) - (2a_{5} - a_{2}c_{i,1})(c_{i,2} - b_{i,1}c_{i,2} + b_{3})(i_{t} - r^*)

2Dy_{t} = f(a, b, c) [a_{1}y_{t,t-1} + a_{3}g_{t,t} + d_{t,t} + (1 - a_{1})E_{t}(y_{t,t+1})] + f(a, b, c) [a_{1}y_{t,t-1} + a_{3}g_{t,t} + d_{t,t} + (1 - a_{1})E_{t}(y_{t,t+1})] + f(a, b, c) (b_{i,1}c_{i,1} + s_{i,1}) + f(a, b, c) (b_{i,1}c_{i,1} + s_{i,1}) - f(a, b, c) (p_{i,t-1} - p_{i,t-1}) - a_{2}(1 + a_{4})\left(1 - c_{i,1} + b_{i,1}c_{i,1}\right)\left(1 - c_{i,1} + b_{i,1}c_{i,1}\right) - (b_{3} + c_{i,2} - b_{i,1}c_{i,2}))(b_{3} + c_{i,2} - b_{i,1}c_{i,2}))\left(i_{t} - r^*\right)

- b_{2}a_{2}(2a_{5} - a_{2}c_{i,1})(1 - c_{i,1} - c_{i,2} + c_{i,1}b_{i,1} + c_{i,2}b_{i,1} - b_{3})

+(2a_{5} - a_{2}c_{i,1})(1 - c_{i,1} - c_{i,2} + c_{i,1}b_{i,1} + c_{i,2}b_{i,1} - b_{3}))(i_{t} - r^*)

APPENDIX B. DETERMINATION AND STUDY OF THE PARAMETERS (C)

If we put (8) in (2b), if we use (6b) and (6c), and then, if we replace \( (i) \) by its value in (5b), we have:

\[
\begin{align*}
[c^{M}m_{y/r}[1 - b_{3} - (1 - b_{3})(c_{1} + c_{2}) + \overline{b_{1}}(c_{1} + c_{2})] + b_{3}^{2}m_{p/r}] \\
[(c_{1} + c_{2})\pi_{t-1} + (c_{1} - c_{2})\overline{\pi_{t-1}} + (c_{3} + c_{4})s_{t} + (c_{5} - c_{4})\overline{s_{t}}] \\
c^{M}m_{y/r}[b_{1}\pi_{t-1} + \overline{b_{1}}(\pi_{t-1}) + s_{t}] + c^{M}m_{y/r}(1 - b_{3})(c_{1} - c_{2}) - \overline{b_{1}}(c_{1} - c_{2})\overline{\pi_{t}}
\end{align*}
\]
Then, by identification, we have:

\[
\begin{align*}
\{ c^M m_{y/r} [1 - b_1 - (1 - b_1)(c_1 + c_2) + \bar{b}_1 (\bar{c}_1 + \bar{c}_2)] + b_2^Z m_{p/r} \}(c_1 + c_2) &= c^M m_{y/r}, b_1 \quad (a) \\
\bar{b}_1 (\bar{c}_1 - \bar{c}_2) &= \bar{b}_1 (c_1 + c_2) \quad (1 - b_1) (\bar{c}_1 - \bar{c}_2) = \bar{b}_1 (c_1 - c_2)
\end{align*}
\]

Which imply:

\[
\begin{align*}
c_2 &= (2b_1 - 1)c_1 \quad (b) \quad \bar{c}_2 = \bar{c}_1 - 2c_1 \bar{b}_1 \quad (c)
\end{align*}
\]

\[
(\bar{c}_3 + \bar{c}_4) = \frac{(c_1 + c_2)}{b_1} = 2c_1 \quad \bar{c}_3 = \bar{c}_4
\]

Which imply:

\[
(\bar{c}_{i,3} + \bar{c}_{j,4}) = (c_{j,3} + c_{i,4}) = 2c_1 \quad (d).
\]

(a), (b) and (c) imply:

\[
c^M m_{y/r} \{2c_1[1 - b_3 - c_1(2b_1 - 2b_{11}b_{1,1}) + 2\bar{b}_1 \bar{c}_1] - 1\} + 2b_2^Z m_{p/r} c_1 = 0 \quad (e)
\]

with: \( m_{p/r} = 2(1 + a_4)[1 - 2c_1(1 - b_1 + b_{11}b_{1,1}) + b_3] + 2b_2(2a_5 - a_2c_1) \)

\[
m_{y/r} = (1 - b_3)m_{p/r} + 2[(1 + a_4)(1 - 2c_1 + 4b_1 c_4 - 2c_1 b_{11}b_{1,1} + b_3) + b_2(2a_5 - a_2c_1)] \left(2b_{11}b_{1,1}c_1 - \hat{c}_{i,1} \right)
\]

If we put (7) in (2a), if we use (6a), and then, if we replace \((n_{i,j})\) by its value in (5a), we have:

\[
\begin{align*}
&\{[c^G m_{y/g}]_i \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} \{c^G m_{y/g} \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} - c^G c^G m_{y/g} m_{y/g} \left(b_3 + c_{i,2} - b_{i,1} c_{i,2}\right) \left(b_3 + c_{j,2} - b_{j,1} c_{j,2}\right) [c^G m_{y/g}]_i + b_2^Z m_{p/g}]_i = 0 \quad (i)
\end{align*}
\]

Then, by identification, we have:

\[
\begin{align*}
&\{[c^G m_{y/g}]_i \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} \{c^G m_{y/g} \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} - c^G c^G m_{y/g} m_{y/g} \left(b_3 + c_{i,2} - b_{i,1} c_{i,2}\right) \left(b_3 + c_{j,2} - b_{j,1} c_{j,2}\right) [c^G m_{y/g}]_i + b_2^Z m_{p/g}]_i = 0 \quad (f)
\end{align*}
\]

\[
\begin{align*}
&\{[c^G m_{y/g}]_i \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} \{c^G m_{y/g} \left(1 - c_{i,1} + b_{j,1} c_{j,1}\right) + b_2^Z m_{p/g}]_i \} - c^G c^G m_{y/g} m_{y/g} \left(b_3 + c_{i,2} - b_{i,1} c_{i,2}\right) \left(b_3 + c_{j,2} - b_{j,1} c_{j,2}\right) [c^G m_{y/g}]_i + b_2^Z m_{p/g}]_i = 0 \quad (g)
\end{align*}
\]

139
Which imply:
\[
c^{i,j}_1 b_{1,j} m_{1,j}/g_j \left(b_3 + c_{i,2} - b_{i,2} c_{i,2}\right) = b_{i,1} c_{i,2} \left[c^{o,j}_1 m_{y,j}/g_j \left(1 - c_{j,1} + b_{j,1} c_{j,1}\right) + b^2_{j} m_{p,j}/g_j\right] \quad (h)
\]
and:
\[
(c_{i,1} c_{i,2} - c_{i,2} c_{j,2}) \left(b_3 + c_{j,2} - b_{j,2} c_{j,2}\right) = b_{j,1} c_{j,2} \quad (i)
\]
\[
c_{i,3} = \frac{c_{i,1}}{b_{i,1}} \quad (j) \quad c_{i,4} = \frac{c_{i,2}}{b_{i,1}} \quad (k)
\]
Equation (h) defines \((c^{o,j}_i)\) and \((c^{o,j}_j)\) according to the parameters of our model. Equation (e) defines \((c^M)\) according to the parameters of our model. Equation (i) defines the values for \((c_{i,1})\) and \((c_{j,1})\).

(d), (j) and (k) imply:
\[
c_{i,2} = b_{j,1} c_{i,1} - (1 - b_{j,1}) c_{j,1} \quad \text{and} \quad c_{j,2} = b_{i,1} c_{j,1} - (1 - b_{i,1}) c_{i,1} \quad (m)
\]
\[
c_{i,3} = \frac{c_{i,1}}{b_{i,1}} \quad c_{j,3} = \frac{c_{j,1}}{b_{j,1}} \quad (n)
\]
\[
c_{i,4} = c_{i,1} - (1 - b_{j,1}) \frac{c_{j,1}}{b_{j,1}} \quad \text{and} \quad c_{j,4} = c_{j,1} - (1 - b_{i,1}) \frac{c_{i,1}}{b_{i,1}} \quad (p)
\]

- If \(c^M=0\): (e) implies: \(m_{y,i}/c_1 = 0\)

\(c_1=0\), as \(m_{y,i}=0\) implies: \(c_1 = \frac{[2b_2 a_3 + (1+a_3)(1+b_3)]}{b_2 a_2 + 2(1+a_4)(1-2b_1 + b_{1,1})}\) which is higher than 1 with plausible values of the parameters.

- If \(b_{i,1}=b_{j,1}=0\): we have: \(c_{i,1} = c_{j,1}\) and with (i): \(c_1 = c_{i,1} = c_{j,1}\) \((\forall c^{Gi})\) and (e) implies: \(m_{y,i}/m_{y,i}(1-b_3)\) and \(c^M(1 - b_3)\left(2 c_1 (1 - b_3) - 1\right) + 2 b^2_{j} c_1 = 0\)

- If \(b_{i,1}=0\) and \(c_{i,1}=0\): we have: \(c_{i,2} = 0\) \((\forall b_{j,1})\), \((\forall c^{Gi})\) and (e) implies: \(m_{y,i}/m_{y,i}(1-b_3)\) and \(c^M(1 - b_3)\left(c_{i,1} (1 - b_3) - 1\right) + b^2_{j} c_{i,1} = 0\)

- If \(b_{i,1}=0\) and \(c_{i,1}=0\): we have: \(c_{j,2} = 0\) \((\forall b_{j,1})\), \((\forall c^{Gi})\) and (e) implies: \(m_{y,i}/m_{y,i}(1-b_3)\) and \(c^M(1 - b_3)\left(c_{j,1} (1 - b_3) - 1\right) + b^2_{j} c_{j,1} = 0\)

- If \(b_{i,1}=-b_{j,1}\): we have: \(c_1 = c_{j,1}\)

(i) implies: \(c_{i,1} = c_{j,1} = \frac{-b_{3} b_{3} + 2 b_{3} (2 b_{1,1} - 1)}{2(1-b_{1,1})(2 b_{1,1} - 1)}\)
• If \( b_{i,j} = (1-k)b_1 \) and \( k \neq 0 \): we have: \( b_{i,j} = (1+k)b_1 \) and (i) implies:

\[
c_{j,1} = \frac{c_{i,1}(1-b_{j,1}b_{j,2}+2b_{j,1}b_{j,3})(b_{j,1}-b_{i,1})+b_{i,1}b_3 \pm \sqrt{\Delta}}{2b_{i,1}(1-b_{j,1})(b_{j,1}-b_{i,1})}
\]

\[
\Delta = [c_{i,1}(1 - b_{j,1} - b_{j,1})(b_{j,1} - b_{i,1}) + b_{i,1}b_3]^2 - 4c_{i,1}b_{i,1}b_3^2(1 - b_{i,1} - b_{j,1})(b_{j,1} - b_{i,1}).
\]

Then, by replacing \( (c_{j,1}) \) in (i), we can obtain \( (c_{i,1}) \).