FINANCIAL INSTABILITY AND ECONOMIC CYCLES:
A MODEL OF BANKING CRISIS

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ABSTRACT:
After the recent cross-border financial crisis, this paper aims to develop a new framework in order to portray the dynamics of current banking systems. In a dynamic model, international banks adopt different strategies of risk according to the economic cycle phases. It describes a mechanism by which even cautious entities are urged on adopting risky behaviors to remain competitive and attract capital. Such a new framework based on an uncommon (positive) approach is completed by simulations demonstrating that this process inexorably leads to a banking liquidity crisis, hence the importance of banking regulations for financial stability.

JEL CLASSIFICATION: E44; F34; F47; G21.

KEYWORDS: Banking, Liquidity, Panic, Interbank Market.

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INTRODUCTION

The question that is broached in this paper refers to the complex interaction of risks that points out the need for a macroeconomic overview. Actually, even though “The macromonetary policy side of central banking activities has made enormous progress, in operational success, in practical procedures and in theoretical understanding”, Goodhart (2004) does not really see “an equivalent success on the financial stability side [...] Those working on this side need to take some new directions. First, we need a better, systemic model of financial fragility, notably a model which incorporates default as a central, essential element” (p. 13). A few years later, after a systemic crisis especially for advanced and emerging countries, one clearly goes on underlining the significance of far better collaborative instruments for the stabilization of financial markets.

Do the whole malfunctioning systems and markets present the same relevance to implement stabilizing reforms? We refer to Greenspan (2008), who explains that “Forecasters’ concerns should be not whether human response is rational or irrational, only that it is observable and systematic”. Actually, the multiplicity of aspects of each crisis forces to intervene upstream, where identified mechanisms are still straightforward and systematic. According to Demirgüç-Kunt and Detragiache (1998), a banking crisis occurs if the banking system holds more than ten percent of non-performing assets, and/or in case of big scale nationalizations, so that the cost of bailouts would reach more than two percent of the GDP. Well, in the past, banking crises often preceded currency crises; recently, the banking crisis has preceded the speculative funds’ problems as well as a decline in stock markets. It legitimates the need for providing a new theoretical foundation likely to describe the specificity of banking activity.

So, we build a model of banking crisis based on a positive approach, both isolating us from the literature and providing an interesting way of portraying what is observable and systematic in order to find solutions for financial stability: investors are unable to distinguish the risk level of distant entities, but the renewal of loans depends on the – well-known – fraction of risky entities in the host economy. Moreover, we define new (hyperbolic) attraction functions calibrating the strategy shifts in the model and driving its dynamics. Intuitively, each type of bank tends to adopt a riskier (more cautious) strategy in period of non-crisis (of crisis) and if the differential profit is high (low), hence a new degree of risk of the whole banking system. Simultaneously, such profits define a new ability, for each bank, to attract more or less international capital, ultimately supporting the financial instability hypothesis (Minsky, 1977).

After reviewing the literature, we study the internal dynamics of the banking sector, taking into account the banking strategies to maximize profit, both in terms of international indebtedness and domestic portfolio choice. In our framework, two different behaviors (cautious vs risky) are affected by a widespread euphoria, so that we observe the reinforcement of the internal instability of (integrated) markets. This trend toward a risky equilibrium stresses the need for a better preclusion of risks, which is summarized in the conclusion.
1. RELATED LITERATURE

The banking activity is subject to liquidity risks. A lack of confidence, which elicits massive withdrawals (i.e. self-fulfilling forecasts), can be modelled as the consequence of a “sunspot” influencing beliefs even though it contains no information upon the bank or its domestic economy (Diamond and Dybvig, 1983; Schotter and Yorulmazer, 2009). At the international level, the liquidity risk takes new forms (Geithner, 2007; Lopez, 2008) and the non-renewal of short term loans by lenders replaces domestic depositors’ withdrawals. The interbank market improves the information spread and the risks’ diversification, while favouring the access to liquidity, so that it should represent a better preclusion from risks (Cocco et al., 2009; Dinger and von Hagen, 2009); but it can suddenly stop: in this case a crisis is more likely to be contagious, even systemic (Iori et al., 2006, Nier et al., 2008).

Actually, assets and liabilities are interdependent. Purchases and sales of financial securities by banks provide the capital markets with liquidity since traditional bank intermediation is coupled with a market intermediation, with a feedback: banks’ liquidity depends, in turn, on the securities market (Franck and Krausz, 2007; Johnson, 2008). Sales of assets by distressed institutions depress their market price and reduce the available liquidity for sound banks (Acharya and Yorulmazer, 2007). As a result, banking crises do not automatically arise from a run or a panic, and liquidity can be globally abundant but at the same time vanishing locally3 (Devriese and Mitchell, 2006).

Obstfeld et al. (2008) deal with financial stability through global imbalances. Conversely, in our model we wish to underline the incidence of borrowers’ behavior (the only limit for liabilities is linked to earnings). The behavior of private competitors can reinforce banking procyclicality, e.g. an active management of balance-sheets reinforces the intrinsic procyclicality of the leverage (Adrian and Shin, 2008)4. So our dynamic study stresses the importance of cycles on liquidity growth: as in the model of Goodhart and Huang (1999), the value of banks is defined by their debts. However, we do not only introduce the relative weight of each type of bank, but we also compute the changes in the borrowed amounts (the domestic financial attractiveness and then the potential financing of growth).

Models of bank run emphasize the effects of a negative information spread on assets as (random) future returns (Allen and Gale, 2000). A lot of models (e.g. Ennis and Keister, 2006; Goldstein and Pauzner, 2005) aim to determine the equilibrium that occurs. There is determinism in the triggering and progress of crises but it is often

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3 Tirole (2008) proposes a useful distinction between micro and macroeconomic liquidity: the first one characterizes assets with low transaction costs or small amounts of informational asymmetries on resilient and deep markets; whereas the macroeconomic liquidity also implies assets to be tradable as a reserve during emergency cases, by providing cash.

4 The procyclicality of leverage is simply linked to a wealth effect. But if banks actively manage their balance-sheet, they increase the leverage in the sense of prices, so that securities purchases increase again. Thanks to this financial accelerating, Adrian and Shin (2008) demonstrate that the aggregate liquidity corresponds to the rate of growth of financial sector’s balance sheets. More generally, the Basel Agreements and the “fair value” rule of accountability equally limit the ability of banks to smooth the needs of the private sector, which is an essential function of banking intermediation (the only one in the model of Diamond and Dybvig, 1983).
identified ex post, while crises are relatively unpredictable: in our model, capital withdrawals are partially exogenous (conversely to Corsetti et al., 2006), since we should try not only to foresee but also anticipate the different possibilities of crises, in case of – sooner or later – triggering. Bastidon et al. (2008), Corsetti & al. (2006), Goodhart and Huang (1999), Ratnovski (2009) and Rochet and Vives (2004) provide models of financial stability (including borrowers) which are likely to define optimal bailouts. Each time, banks can make suboptimal liquidity choices and the (asymmetric) information is pivotal (e.g. for the uniqueness of equilibrium). The disappearance of liquidity depends upon the intrinsic uncertainty (assets’ quality), and upon the strategic uncertainty (forecast behavior of other depositors). We propose a new way of portraying the observable reality. At the beginning, the framework looks like the one of Diamond and Dybvig (1983), widely re-used in the (above) literature, with simplified banks’ balance sheets and a progress within three periods. Then, a dynamics is driven through singular behavioral and informational hypotheses, following a positive approach likely to highlight the current instability.

2. THE MODEL

2.1. BANKING INTERMEDIATION AND LIQUIDITY RISK: FRAMEWORK

Let a small open economy be constituted by a set of banks \( i = \{1, \ldots, j\} \) whose behavior is observed during a period \( T \) subdivided in three subperiods \( (t_0), (t_1) \) and \( (t_2) \).

First \( (t_0) \), each bank’s liabilities are formed by initial allocation \( E_i \) (equity) and international short term debt \( D_i \). At the same time, each one chooses the composition of its assets: a fraction \( \alpha_i \) is invested in a domestic risky technology \( (E_i + D_i) \) with \( 0 \leq \alpha_i \leq 1 \), the other fraction \( (1 - \alpha_i) \) is invested in a liquid asset \( M_i \). This portfolio choice indicates the risk aversion of each bank \( i \):

- Investment \( I_i \) is illiquid; it yields a return \( R \) at \( (t_2) \), but a possible advance disruption at \( (t_1) \) incurs higher liquidation costs \( \kappa \) (per unit of investment), so that:

\[
RI_i / (1 + \kappa) < RI_i
\]

- \( M_i \) is free from yield, but available at \( (t_1) \) without any cost.

At the interim step \( (t_1) \), the current liability \( D_i \) of each bank implies the payment of an interest rate \( ^6 r_i \). In case of distress, a fraction \( x_i \) \( (0 \leq x_i \leq 1) \) of international investors does not renew its lending; so two cases arise:

\[ M_i \geq r_i D_i \] in order to preclude a systematic illiquidity at \( (t_1) \). Moreover, \( R \) exceeds the liquidity cost (debtor interest rate) expected for \( (t_1) \) and \( (t_2) \):

\[ R > 2 r_i D_i. \]

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5 “Over the past half-century, the American economy was in contraction only one-seventh of the time. But it is the onset of that one-seventh for which risk management must be most prepared” (Greenspan, 2008).

6 \( M_i \geq r_i D_i \) in order to preclude a systematic illiquidity at \( (t_1) \). Moreover, \( R \) exceeds the liquidity cost (debtor interest rate) expected for \( (t_1) \) and \( (t_2) \): \( R > 2 r_i D_i \).
Reimbursements correspond to a fraction $\beta_i$ of available liquidity: $\beta_i(M_i - r_iD_i) = x_iD_i$ with $0 \leq \beta_i \leq 1$.

Each bank $i$ reimburses investors by bearing distress costs, because it resorts to the disruption of a fraction $z_i$ of the illiquid investment:

$$M_i - r_iD_i + \frac{z_iRI_i}{1 + \kappa} \geq x_iD_i$$

Bank $i$ faces the shock ($\beta_i = 1; 0 < z_i \leq 1$);

$$M_i - r_iD_i + \frac{RI_i}{1 + \kappa} < x_iD_i$$

It is unable to face the shock and defaults ($z_i = 1$).

Then, at $(t_2)$ the renewed part of debt $D_i$ implies again the payment of the interest rate $r_i$. This third subperiod is characterized by the recovery of $RI_i$ for the fraction $(1 - z_i)$ of illiquid investment, and banks clear debts and interests. The firm is solvent as soon as earnings are positive; it is profitable if earnings exceed the amount $E_i$ given at the onset. Here, we assess bank resilience by evacuating $E_i$; liabilities are wholly due at $(t_1)$; consequently a profitable firm just presents a positive profitability (proof 1).

### 2.2. Resilience to Liquidity Shocks for Two Types of Banks

The earnings each bank aims to achieve are equal to the amount of interest $2r_iD_i$ offered to investors for the whole period $T$. Such a purpose implies to get a return $4r_iD_i$ (which will be equally allocated to both the bank and its creditors) from the initial debt $D_i$. We derive a relationship linking $r_i$ with the fraction $\alpha_i$ invested in illiquid investment (proof 2):

$$\alpha_i = 4r_i/(R - 1) \quad (1)$$

There is no run ($x = 0$) for threshold $A_i$. Let $\eta_i$ denote profitability, the amount of profit $\Pi_i$ is:

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7 As in the model of Rochet and Vives (2004), it corresponds to an illiquidity situation (hence a foreclosure, not automatically a bankruptcy). We note: $R/(1 + \kappa) < 1$.

8 This assumption aims to define the changes in strategies. Although another amount does not substantially change the dynamic of the model, this one is representative and permits to simplify it.

9 As numerous banks are concerned and non renewals are related, it is possible to employ the term "panic".
Next, thresholds $A_i$, $B_i$, $C_i$, $D_i$ form the segments $[A_iB_i]$, $[B_iC_i]$ and $[C_iD_i]$ (proof 3) and give the computations of respective profitability curves $\eta_{i1}$, $\eta_{i2}$ and $\eta_{i3}$ (proof 4). When the run expands, the bank reimburses by making use of its liquid asset $M_i$, not $I_i$:

$$\eta_{i1} = r_ix + 2r_i$$

(3)

From peak $B_i$, banks run out of liquidity, hence a partial advance liquidation of investment $I_i$:

$$\eta_{i2} = (r_i - \kappa)x + \kappa + \left(\frac{2R - (R - 1)\kappa - 6}{R - 1}\right)r_i$$

(4)

Finally, from threshold $C_i$, banks are much less profitable than they hoped:

$$\eta_{i3} = r_ix + r_i\left(2 - \frac{4\kappa Rr_i}{(R - 1)(1 + \kappa)}\right)$$

(5)

The run is total for point $D_i$.

A risky portfolio implies a compensation, as a risk premium (conversely to Allen and Gale, 2000). Therefore, the amount offered to investors directly depends on the bank’s exposure. We define $i \in \{1, 2\}$. If $\alpha_2 = 0.92$ (type 2 bank’s liquidity ratio reaches 8%) and if $\alpha_1 = 0.7$ (type 1 banks are more cautious)\(^\text{10}\), we obtain $r_1$ and $r_2$:

<table>
<thead>
<tr>
<th>Type of bank</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>0.7</td>
<td>0.92</td>
</tr>
<tr>
<td>$r_i = \frac{\alpha_i}{4}(R - 1)$</td>
<td>3.5%</td>
<td>4.6%</td>
</tr>
</tbody>
</table>

As the liquidity risk of type 2 banks is higher ($\alpha_2 \geq \alpha_1$), they offer a higher return to their investors ($r_2 \geq r_1$). We suppose the yield of illiquid investment is 20% ($R = \ldots$

\(^{10}\) Before Mc Donough ratio, the Cooke ratio fixes at 8% the required reserve ratio. During the 1950s, liquid assets commonly equalled 30% of total asset of British commercial banks (bonds and current sovereign debt, Tim Congdon, Financial Times, September 2007, cited by Goodhart, 2008). So, we point out the fact that crisis risk is the consequence of structural mutations of banking activity too.
1.2), and its liquidation value in $(t_1)$ equals to 70% of this amount ($\kappa = 0.3)^{11}$. The graph 1 presents the banks’ ability to face a liquidity shock.

**FIGURE 1. THE IMPACT OF RUNS ON BANKING PROFITABILITY**

\[ \text{Profitability} \]

\[ \text{Run} \]

$A_i$, $B_i$, $C_i$ and $D_i$ represent different turning-points of profitability. Actually, as withdrawals do not affect the illiquid investment ($x \leq x_i^B$), they raise profitability: they alleviate interests offered for costly (free for yield) liquid reserves, so banks face the shock easily. Beyond $x_i^B < x < x_i^C$, the tilt indicates a sharp downturn in profitability induced by the liquidation of a growing part of the illiquid investment. Even though 8% of the debt is invested in liquid asset, type 2 banks incur advance liquidation costs as soon as 3.4% of debt is not rolling over (26.5% for type 1 banks). The solvency of type 1 banks is effective as less than 56% are rolling over ($\eta_{12}(0.56) = 0$), 40% for type 2 banks. Beyond a high level of run ($x_1^C = 91\%$ and $x_2^C = 88\%$), the bank would prefer to cancel its debt completely ($x = 1$) at $(t_1)$, to minimize losses by avoiding the payment of related interest rates (dotted area). Here, the inability to reimburse creditors at $(t_1)$ leads to compulsory closure, hence this situation cannot occur at $(t_2)$ and even less after period $T$.

11 These values correspond to simulations proposed (and justified) by Corsetti et al. (2006). Even though yields rarely reach 20%, the liquidation cost often exceeds 30%. These amounts are less important for our findings than the fact that liquidation costs exceed yields.

12 Here the graph also represents bank profit because the debt $D_i$ is normalized to 1 for each (type of) bank.
2.3. Banking Strategies and International Attractiveness

After a period \( T \) including any value of run \( x \), type 1 and 2 banks reveal a different profitability, in accordance with their prior portfolio choice. This impacts the amount \( D_1 + D_2 \): banks attract more or less new capital flows \( \Delta D_{i,T} \). We note \( \rho \) the fraction of profitability each bank keeps, and \( \mu \) the coefficient of capital attraction related to this kept profitability\(^\text{13} \). Variation of attracted capital at the end of a period \( T \):

\[
\Delta D_{i,T} = \rho \mu \eta_{i,T} D_{i,T}
\]  

(6)

At the same time, this impacts the ratio \( D_2 / D_1 \): a fraction of each type of banks switches its strategy/type, keeping its new borrowing ability \( (D_{i,T} + \Delta D_{i,T}) \). This shift depends on the ability to yield (or not) the purpose \( 2r_iD_i \), and on the effective difference of profitability between the two categories: when there is no run, banks are likely to become more risky to raise earnings, especially if competition is strong. Conversely, capital withdrawals constitute an incentive to cautious behaviors. We note \( \tau_i \) the fraction of banks \( j \) (measured by their borrowing capacity \( [1 + \rho \mu \eta_{i,T}]D_{i,T} \) ) effectively shifting towards type \( i \) banks:

\[
D_{i,T+1} = (1 + \rho \mu \eta_{i,T})D_{i,T} + (1 + \rho \mu \eta_{j,T})D_{j,T} \tau_i - (1 + \rho \mu \eta_{i,T})D_{i,T} \tau_j
\]  

(7)

The purpose of banks is to maintain their profit level and their market share (measured by the attracted fraction of total international lending), hence the ratio \( \left( H^N_{i,T} \right) \) of expected profitabilities for the \( N_i \) periods \( T \) that constitute their respective time horizon:

\[
H^N_{i,T} = \left( \frac{1 + \rho \mu \eta_{i,T}(0)}{1 + \rho \mu \eta_{j,T}(0)} \right)^{N_i}
\]  

(8)

The type shifts depend on the difference between this target ratio (for \( x=0 \)) and the ratio of effective profitabilities for the \( N_i \) periods. Assuming that \( \bar{N}_i = p_i + q_i \), this ratio of effective probabilities is defined:

- by observed profitabilities for the \( p_i \) past periods:
  \[
  \prod_{T=0}^{p_i} \left( \frac{1 + \rho \mu \eta_i}{1 + \rho \mu \eta_j} \right)
  \]
- and by expected profitabilities for the next \( q_i \) periods.

We note \( H^9_{i,T} \) this expected profitability, corresponding to the one obtained by maintaining the same risk strategy if the run stops, or, equiprobably, if it is the same as the one we observe in period \( T \) (unpredictability):

\text{13} In our modelling, benefits correspond to financial profitability, i.e. the profit per unity of equity \((\text{return on equity})\). Hence banking strategies consisting in raising the leverage.
\[ H_i^{\sigma} = \frac{1}{2} \left( 1 + \rho \mu_i \eta_1(0) \right)^{\gamma_i} + \frac{1}{2} \left( 1 + \rho \mu_i \eta_2(0) \right)^{\gamma_i} \]  

The (normalized to 1) ratio of effective profitabilities \(^{14}\) (proof 5) \( H_i^{\sigma} \) derives from our specification:

\[ H_i^{\sigma} = \frac{p}{\prod_{\tau=0}^{\rho} \left( \frac{1 + \rho \mu_i}{1 + \rho \mu_2} \right)} \left[ 1 + \frac{1 + \rho \mu_i}{1 + \rho \mu_2} \right]^{\sigma_i} + \frac{1}{2} \left( 1 + \rho \mu_i \eta_2(0) \right)^{\gamma_i} \]  

We note \( \tau_{i\phi} \) the percentage of banks going towards type \( i \) at the break even \( (H_i^{N_j} = H_i^{N_j} \text{ as } x=0) \), and \( \tau_i \) the attraction coefficient of banks \( j \) towards type \( i \). It describes the form of the attraction functions:

\[ \tau_1 = \begin{cases} -\frac{\tau_{1\phi}}{\left( H_i^{N_j} - H_e^{N_i} \right)^2} & \text{if } H_i^{N_j} < H_e^{N_i} \\ \frac{1}{\left( H_i^{N_j} - H_e^{N_i} \right)^3} & \text{if } H_i^{N_j} \geq H_e^{N_i} \end{cases} \]  

\[ \tau_2 = \begin{cases} -\frac{\tau_{2\phi}}{\left( H_i^{N_j} - H_e^{N_i} \right)^2} & \text{if } H_i^{N_j} < H_e^{N_i} \\ \frac{1}{\left( H_i^{N_j} - H_e^{N_i} \right)^3} & \text{if } H_i^{N_j} \geq H_e^{N_i} \end{cases} \]  

After periods without bank run, it becomes timely to adopt a riskier strategy: risk aversion decreases, so the attraction towards type 2 somewhat grows linearly for each period \( T+1 \) if \( x_T = 0 \). The variable \( \text{trend} \) describes this supplement of bank 1 joining type 2; we then mitigate this effect: \( \tau_{2\max} \) is the maximum of banks 1 joining type 2. In period \( T \), if \( x > 0 \):

\[ \tau_{2\phi_T} = \tau_{2\phi_{T-1}} (1 - x) \]

Two cases arise for \( x = 0 \):

\[ \tau_{2\phi} = \begin{cases} \tau_{2\phi} + \text{trend} & \text{if } \tau_{2\phi} < \tau_{2\phi_{\max}} \\ \tau_{2\phi_{\max}} & \text{if } \tau_{2\phi} = \tau_{2\phi_{\max}} \end{cases} \]

\(^{14}\) For example, it is possible to favor the anticipation by substituting (via \( \varphi \)) periods \( T+2 \), \( T+3 \) to \( T-(p-1) \), \( T-(p-2) \).
2.4. INTERBANK LINKAGES AND ISSUES FOR A WHOLE PERIOD "T"

The inclusion of insolvency entails three cases:

- \( \eta_2 > 0 \): the process continues for the whole banking system.
- \( \eta_2 < 0 \) and \( \eta_1 < 0 \): the process stops because of a "financial collapse".
- \( \eta_2 < 0 \) and \( \eta_1 > 0 \): there are another two hypotheses.

Actually, if \( \eta_1 \geq -\eta_2 \), banks 1 are responsible for the default of banks 2; they grant an interbank loan to them. The effectiveness of this mechanism is complete as well as banks 1 earnings compensate banks 2 losses. This debt, held by banks 2, is infra-periodic, insofar it does not affect the attraction coefficients. But if \( \eta_1 < -\eta_2 \), the process stops for a variable part of banks 2 (ruling out), as an idiosyncratic shock.

The ratio \( D_1 / D_2 \) rises, because \( D_1 \) does not change and \( D_2 \) decreases: in other words, the banking system is sounder but the productive economy is affected by both the downturn in financing and advance liquidation of productive investments.

For this, we define \( s = (\eta_1 D_1) / (\eta_2 D_2) \), and simplify it:

\[
\begin{align*}
    s > 1 & \quad \text{then} \quad s = 1 \\
    0 < s < 1 & \quad \text{then} \quad s = s \\
    s < 0 & \quad \text{then} \quad s = 0 \\
\end{align*}
\]

\[
s = \min \left[ \max \left( \frac{\eta_1 D_1}{-\eta_2 D_2}, 0, 1 \right) \right]
\]

(13)

Subsequently:

\[
D_2 = \begin{cases} 
    D_2 (1 + \rho \mu \eta_2) & \text{if } \eta_2 \geq 0 \\
    D_2 s & \text{if } \eta_2 < 0 \\
    D_2 & \text{if } s = 1 \\
    0 & \text{if } s = 0 
\end{cases}
\]

\[
D_2 s = \begin{cases} 
    D_2 s & \text{if } s = 1 \\
    \eta_1 D_1 & \text{if } s = 0 
\end{cases}
\]

If \( s < 0 \), any interbank loan exists: no bank or both categories are faced with insolvency risks.

From (8) and (9), there is still:

\[
D_1 = D_1 (1 + \rho \mu \eta_1)
\]

\[
D_{1T+1} = D_1 T \left(1 - \tau_2\right) + D_{2T} \tau_1
\]

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But new values of $D_2$ result in:

$$D_{2, T+1} = D_{2, T} \left(1 - \tau_i\right) + D_{1, T} \tau_2$$

In a dynamic framework, the changing structure of the banking system impacts in turn the next runs, even though remaining equally distributed towards groups of banks. These runs are becoming stronger if banking system is essentially risky. Conversely, a lot of cautious entities reduce their magnitude. Investors are unable to gather specific information on private banks, but they know the global degree of risks of the host country, therefore they are prone to stop lending if banks 2 are relatively more numerous:

$$x = x \ln \left(1 + \frac{D_2}{D_1}\right)$$  \hspace{1cm} (14)

### 2.5. Dynamic Set Up

The intra-sectoral banking evolution is measured so as to take into account the new weight of the banking system, thanks to the total debt after $n$ periods, and the new relative weight of each group of banks (Figure 2).

**Figure 2. Dynamics of the Banking System**

We assume $\rho = 0.1$ and $\mu = 5$. Then $\tau_{i\phi} = 0.1$, which means 10% of each group of banks ($\tau_{1\phi} = \tau_{2\phi}$) switches its strategy in $T+1$ if the ratio of expected profitabilities equals the one of effective profitabilities in $T$. As every bank $i$ aims to achieve the same yield $2r_i$ offered to creditors, the new borrowing capacity of each one is $(1 + \rho \mu 2r_i)D_{i, T}$ (Appendix 6). Any limitation is imposed to the sum of loans $D$: we emphasize the behavior of borrowers in the context of a liquidity crisis associated with a globally liquid situation. Whatever the situation of a bank in $T+1$
(which depends notably on a forecast resting upon only one period: \( p = 1 \)), it will be able to borrow\(^{15}\), from (6):

\[
D_{i,t+1} = (1 + r_i)D_{i,t}
\]

We reformulate the (normalized to 1) expected profitability for \( N_i \) periods (for \( x=0 \)):

\[
H_{i,N}^{N_i} = \left( \frac{1 + \rho \mu \eta_1(0)}{1 + \rho \mu \eta_2(0)} \right)^N_i = \left( \frac{1 + r_i}{1 + r_2} \right)^N_i \quad (8')
\]

We continue with the former values given for \( \alpha_i \), \( r_i \), \( R \) and \( \kappa \). The time horizons \( N_i \) modify the respective attraction coefficients. More precisely, this horizon is all the shorter, as a bank chooses a risky strategy in order to maximize its profit (table 2).

<table>
<thead>
<tr>
<th>( N_i )</th>
<th>Attraction to banks 2 (( i = 1 ))</th>
<th>Attraction to banks 1 (( i = 2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( H_{i,10} = \left( \frac{1 + r_1}{1 + r_2} \right)^{10} = 0.725 )</td>
<td>( H_{i,5} = \left( \frac{1 + r_1}{1 + r_2} \right)^{5} = 0.852 )</td>
</tr>
</tbody>
</table>

The Figure (3) highlights the form of the attraction functions\(^{16}\) \( \tau_1 \) and \( \tau_2 \).

\(^{15}\) \( D_{i,t+1} = (1 + 0.1 \times 5 \times 2r_i)D_{i,t} \).

\(^{16}\) Graphes (1) and (3) only represent situations where \( D_i = D_2 \). Moreover, after many calm periods, the variable trend can intensify the attraction function towards type 2 banks (Appendix 7).
2.6. Simulation and Interpretation

After a crisis occurrence, the market seems to regulate itself, especially with free changes of categories. But how does the banking market evolve without run, as if debt were denominated in the long term? By transposition of market “myopia” about the future, its short memory fast leads to a very risky equilibrium. Even though individual risks are compensated by expected returns, they cause adverse effects, namely both an increase in the likelihood of a crisis and a greater fragility. Over many years, the graph (4) shows the evolution of a market whose feature is, at the onset of the process, an identical size of the groups of banks.

Figure 4. Non Crisis: Performance but Not Resilience
The debt of type 2 banks increases dramatically for 36 periods, from 50 to 65% of the sector (for 1 cautious entity, we find 1.86 risky entities). The trend extends, when simulating a second process beginning with this ratio: the fraction moves from 65 to 71.8% and the debt reaches 12.855 (i.e millions of euros). We can extrapolate an excessive weakening. Even if the domestic aim is to attract more capital flows $D$, it is also to obtain their renewal.

Because there is no run, we can observe graphically the euphoric upsurge (attraction to type 2) that reinforces the cost of liquidity detention (an opportunity cost) comparatively to profitable investments: in a competitive world, such an evolution stresses the need for adopting, in turn, risky behaviors (Minsky, 1977). Domestic profits increase faster and faster as the leverage is becoming largely preferred to liquidity buffer. So investors are more numerous, but their expectations include the growing default risk ($\Delta D_2/D_2$). This spiral jeopardizes the market and inexorably leads to a crisis, all the more as it occurs late (not automatically self-fulfilling prophecies, but mainly a fundamental-based crisis). The framework is likely to capture an effective dynamics on banking markets, in which disaster myopia (Guttentag and Herring, 1986) first depends on free (unbiased) competition (not on the “originate and distribute” model of lending).

As a finding, the will of pegging banking systems implies to compensate the growth of the opportunity cost, which is higher when euphoria settles. First, reducing the frequency of the cycle would be useful: a countercyclical (floor) liquidity ratio would slow down the upsurge. Actually, if the difference $(\alpha_2 - \alpha_1)$ lowered, the differential of profit earned by type 2 banks would decrease too, so that their relative growth would not be as explosive. Second, we also need to reduce the magnitude of the cycle (the accelerating attraction towards risky strategies), as could do a progressive tax rate: the incentive for cautious entities would be measured in terms of additional market shares ($\Delta D_1/D_1$), and it could permit to better “share out the burden of rescues” (Goodhart, 2004, p.8) in case of crisis.

**CONCLUSIONS**

By means of singular behavioral assumptions and thanks to a new framework based on Goodhart’s observations, we run simulations which do not appear in the model of Bastidon et al. (2008). And conversely to Corsetti et al. (2006) who highlight catalytic finance through endogenous non-renewal of loans, we try to demonstrate that the private strategies elicit both a fragilisation of banking systems and a higher likelihood of panic, by deteriorating macroeconomic fundamentals altogether. So we assert that the internal functioning of banking markets is unstable, and emphasize

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17 “Banks’ holdings of liquid assets not only protects other commercial banks, it also protects the monetary authorities, and helps them to maintain systemic stability. The more liquid assets a bank has, the longer it can sustain adverse clearings [...] It is liquid assets, not capital, that provides time in crises” (Goodhart, 2004, p. 10). Note that the model could be extended, i.e by differentiating market and credit risks, by linking the debt value to possible exchange rate fluctuations, or by better specifying the correlations of banks balance-sheets.
the fact that banking fragility concerns developed (OECD) countries as well as emerging countries. This is all the more true as the integrated monetary markets are relevant to highlight usually domestic mechanisms. This outcome remains true with a market close to pure and perfect competition (except information asymmetry): atomistic entities without “too-big-to-fail” firms and free shifts of groups for banks. It concerns either investment or commercial banks, whose business is the management of assets and liabilities and maturity imbalances.

In our framework, the recurrence of crises and the succession of booms and busts can be explained by a deficit of regulation. Promoting an efficient stabilization of finance is possible, but a simple liquidity ratio would not be sufficient. Such a standard should at least be countercyclical, so as to slow down the expansion phase of the cycle. Furthermore, we need to amend the game in which cautious behaviors mean suboptimal choices, by reducing the additional profit raised by entities weakening the domestic economy: "One difficult problem is that much of the dubious financial-market behavior that chronically emerges during the expansion phase is the result not of ignorance of badly underpriced risk, but of the concern that unless firms participate in a current euphoria, they will irretrievably lose market share" (Greenspan, 2008). So the wide attractiveness for profit opportunities hiding risk matters is not a problem of "greedy" behaviors but the effect of a missing regulation. A progressive tax could improve the situation by reducing the attraction towards risky strategies, if implemented before the euphoria and included in expectations. Beyond the numerous factors highlighted by the crisis, there is no dead end but rather happy prospects for financial stability, provided historical recurrences are taken into account (Aglietta, 2008), i.e. the absence of long run overview for private markets and operators. Free-market supporters will underscore the need for international competitiveness.
REFERENCES


Greenspan, A., 2008. “We will never have a perfect model of risk”, The financial times, March 16th.


Appendixes

Proof 1. Conditions for Saving $E_i$ for Later

We note $\Lambda_i$ the liquidity remaining available in $(t_1)$:

$$\Lambda_i = (1 - \alpha_i)(E_i + D_i) - r_i D_i.$$ 

The run that exactly exhausts liquid reserves for a bank $i$ reaches $xD_i = \Lambda_i$. Therefore, the threshold value from which any additional $x$ elicits a positive liquidation cost is

$$x = \left[(1 - \alpha_i)(E_i + D_i) - r_i D_i\right]/D_i = \lambda_i.$$ 

We obtain the threshold $\alpha_i$ by expressing $E_i$ as a proportion $\gamma$ of $D_i$ (i.e. $E_i = \gamma D_i$). As $\alpha_i < 1$, $(r_i + \lambda_i)/(\gamma + 1) < 1$; and as $r_i > 1$, $\gamma$ must ascertain the following condition: $\gamma < \lambda_i$, either: $E_i/D_i < \Lambda_i/D_i$. But $x = \Lambda_i/D_i < 1$: we can make abstraction of $E_i$ provided that it is significantly inferior to $D_i$; in our model: $E_i << D_i$. By evacuating $E_i$ we choose a flow study instead of a stock study (patrimonial, as balance sheet) to better highlight the liquidity risk.

Proof 2. The Link Yield/Risk

Each bank $i$ anticipates, from debt $D_i$ and without run, to earn in $(t_2)$ the amount $R\alpha_i D_i + (1 - \alpha_i)D_i$ to reimburse $D + 2r_i D_i$ and get itself a net profit $2r_i D_i$. Hence the Eq. (1): $\alpha_i = 4r_i/(R - 1)$.

Proof 3. Profitability Thresholds According to Runs and Portfolio Choices

By recognition, each point $A_i, B_i, C_i, D_i$ corresponds to a level of profitability. The profitability for a period $T$ also depends upon stages $(t_1)$ and $(t_2)$:

$$\Pi_i = \eta_{it} D_i = \left(\eta_{it_1} + \eta_{it_2}\right)D_i.$$ 

When the balance sheet is balanced at $(t_1)$, we have $\eta_{it_1} = 0$; provided that $\eta_{it} = \eta_{it_2}$.

$A_i$ corresponds to the absence of run: $x_i^{A} = 0$ and $z_i = 0$; hence

$$\Pi_{it_1}(x_i^{A}) = \beta_i \left(1 - \alpha_i\right)D_i - r_i D_i = 0$$ 

and $\beta_i = r_i/(1 - \alpha_i)$. With both

$$\Pi_{it_2}(x_i^{A}) = R\alpha_i D_i + (1 - \alpha_i)(1 - \alpha_i)D_i - (1 + r_i)D_i$$ 

and (1), we obtain:

$$\Pi_{it_2}(x_i^{A}) = 2r_i D_i = \Pi_{it}(x_i^{A}).$$

$B_i$ corresponds to the run that equals liquid reserves:

$$x_i^{B} = M_i - r_i D_i = \Lambda_i/D_i > 0$$ 

and $z_i = 0$, hence: $\beta_i = 1$. As

$$\Pi_{it_1}(x_i^{B}) = x_i^{B} D_i - (1 - \alpha_i)D_i + r_i D_i = 0$$ 

, so $x_i^{B} = 1 - \alpha_i - r_i$ (3a). With
\[ \Pi_{ii} (x_i^B) = R\alpha_i D_i - (1 - x_i^B)(1 + r_i)D_i \] and (1) and (3a), we have:

\[ \Pi_{ii} (x_i^B) = \left( -\frac{R + 3}{R - 1} r_i^2 + 3r_i \right)D_i = \Pi_{ii} (x_i^B). \]

\[ C_i \] corresponds to the disruption of the project: \( z_i = 1 \).
As \( \Pi_{ii} (x_i^C) = x_i^C D_i - (1 - \alpha_i)D_i + r_i D_i - RI/(1 + \kappa) = 0 \), so

\[ x_i^C = \left( R/(1 + \kappa) - 1 \right)\alpha_i + 1 - r_i \quad (4a). \]

With \( \Pi_{ii} (x_i^C) = - (1 - x_i^C)(1 + r_i)D_i \) and (1) and (4a), we have:

\[ \Pi_{ii} (x_i^C) = \left( \frac{3[R - (1 + \kappa)] - \kappa R}{(1 + \kappa)(R - 1)} \right)D_i = \Pi_{ii} (x_i^C). \]

\( D_i \) corresponds to the non renewal of all lendings: \( x_i^D = 1 \); therefore:

\[ \Pi_{ii} (x_i^D) = \left( \frac{R\alpha_i}{1 + \kappa} - \alpha_i - r_i \right)D_i \]. So, from (1):

\[ \Pi_{ii} (x_i^D) = \left( \frac{3[R - (1 + \kappa)] - \kappa R}{(1 + \kappa)(R - 1)} \right)D_i = \Pi_{ii} (x_i^D). \]

From (2), the profit permits to compute the profitability of banks for each threshold:

\[ \eta_i (x_i^A) = 2r_i \]
\[ \eta_i (x_i^B) = -\frac{R + 3}{R - 1} r_i^2 + 3r_i \]
\[ \eta_i (x_i^C) = \frac{3[R - (1 + \kappa)] - \kappa R}{(1 + \kappa)(R - 1)} r_i \]
\[ \eta_i (x_i^D) = \frac{3[R - (1 + \kappa)] - \kappa R}{(1 + \kappa)(R - 1)} r_i \]

**PROOF 4. THREE CURVES OF PROFITABILITY**

We write again \( x_i^B \) and \( x_i^C \) by substituting \( \alpha_i \) with its value given by (1). As \( \{1;2;3\} \) indicate the respective curves of profitability formed by segments \([A_iB_i], [B_iC_i], [C_iD_i]\), we are able to compute tilts \( a_{ij} \) and y-intercepts \( b_{ij} \) of curves \( \eta_i \), whose form is \( \eta_i = ax_i + b_i \):
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$\alpha_1 = \frac{\eta_i(x_i^a) - \eta_i(x_i^b)}{x_i^a - x_i^b} = r_i$  

$b_{11} = \eta_i(x_i^a) - a_{11} x_i^a = 2 r_i$

$a_2 = \frac{\eta_i(x_i^c) - \eta_i(x_i^b)}{x_i^c - x_i^b} = r_i - \kappa$  

$b_{12} = \eta_i(x_i^c) - a_{12} x_i^c = \kappa + \left( \frac{2 R - (R - 1)\kappa - 6}{R - 1} \right) r_i$

$a_3 = \frac{\eta_i(x_i^o) - \eta_i(x_i^e)}{x_i^o - x_i^e} = r_i$  

$b_{13} = \eta_i(x_i^o) - a_{13} x_i^o = r_i \left( \frac{4 \kappa R r_i}{(R - 1)(1 + \kappa)} \right)$

**Proof 5. Profitability Ratio and Normalization To 1**

Gains or losses actually made for each period $T$ constitute the terms of a geometric sequence whose form is $U_N = U_M (1 + q)^{N-M}$. Hence the expected relative profitability for $N_i$ periods, and for two (types of) banks $i = \{1, 2\} : 
\left[ \left( 1 + \rho \mu \eta_i(0) \right) / \left( 1 + \rho \mu \eta_2(0) \right) \right]^{N_i}$

We normalize to 1 the differences of profitability by dividing the ratio by the maximal spread of $\eta_1$ and $\eta_2$:

$H_c^{N_i} = \left( \left( 1 + \rho \mu \eta_i(0) \right) / \left( 1 + \rho \mu \eta_2(0) \right) \right)^{N_i} (10)$

**Proof 6. Numerical Example**

Each bank keeps 10% of its profit $\eta D_i$ (from Eq. (2): 10% $2r_D = 0.2 r_D$; e.g. 90% of the added value – banking net product – is charged in autofinancing, taxes and dividends). These residual 10% give the bank’s ability, in the next period, to borrow a new additional amount: the quintuple (without run: $r D_i$). Let the six months interest rate be five percent. A bank borrows €1 million on January the 1st. On June the 30th, the loan is renewed and on December the 31st, the bank has to pay €100,000 to investors for the whole year. Our hypothesis means the bank also wishes to earn €100,000, hence a sum of €200,000 expected for this year. Two cases:

- The target is reached. Shareholders and lenders absorb €200,000. The next year, the bank is more attractive and its total debt becomes €1,100,000 (additional capital is equal to: $\rho \mu 200,000 = 0.1 \times 5 \times 200,000 = 100,000$). A type switch is unlikely.
- The target is not reached, because of non renewals on June 30th. Again, either the amount earned is at least €100,000 (additional capital attracted next year diminishes and a type switch is probable), or it is inferior (the bank defaults).
PROOF 7. SHORT MEMORY OF PRIVATE ENTITIES AND GROWING INTEREST FOR RISKY STRATEGIES

After several periods without crisis, the maximum attraction rate towards type 2 banks ($\tau_{2\phi_{\max}}$) is fixed at 25% (dotted curve, in case of adequacy between expected and observed profits). If a crisis occurs, we foresee a rapid droop of the euphoria: $$\tau_{2\phi} = \tau_{2\phi^{-1}}(1 - x).$$