Nonlinear Mean Reversion in EMS Exchange Rates

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Abstract:
Time series evidence on exchange rates has been unable to reject the random walk hypothesis. A simple structural model that accounts for target zone nonlinearities provides conclusive evidence of mean reversion in EMS exchange rates.

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Keywords: Mean reversion, target zone, random walk.

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NONLINEAR MEAN REVERSION IN EMS EXCHANGE RATES

INTRODUCTION
Exchange rate modeling has been a theoretical and empirical conundrum. Structural and time series approaches have proven unreliable in tracking the data out-of-sample. The random walk is a beguiling empirical approximation for the exchange rate at short horizons.\(^2\)

Unpredictability in asset price movements is often regarded as the hallmark of an efficient market. It is true that currencies trade in a very thick market. Daily volume in the global foreign exchange markets, according to the most recent survey by the Bank of International Settlements,\(^3\) routinely exceeds 3 trillion U.S. dollars. Speculators arbitrage away small differentials in the blink of an eye. Is the random walk the logical outcome of market efficiency or have empirical studies failed to detect some hidden structure? In specific, might nonlinearity be the key? The answer, at least for key European currencies prior to the Euro, appears to be yes.

We study a time period from 1987 to 1992 where European Money System (EMS) currencies fluctuated in narrow exchange rate bands known as target zones. Despite the confinement of the exchange rate, Anthony and MacDonald (1998) and Pattachis (2002) find that traditional unit root tests fail to detect mean reverting behavior in the data.

One perspective on this failure is econometric: unit root tests have low power against this alternative. This explanation is considered by Kanas (1998) and Psaradakis (2001). Both find size distortions in unit root tests in the presence of Markov switches.

My approach is more structural. I model the nonlinearity in the exchange rate theoretically. I also take into account the known dates of the Markov switches in the system. Using theory and events in tandem provides a straightforward rejection of a random walk for the exchange rate.

A theoretical literature, beginning with Krugman's (1991) seminal paper, has been developed to describe target zone exchange rate behavior. In Krugman's model, the bands induce a nonlinear relationship between the exchange rate and fundamentals. A second type of nonlinearity also arises in models like Bertola and Svensson's (1993) that allow for devaluation risk. Beetsma (1995) and Rangvid and Sorensen (2001) show that, even in the new generation of target zone models, the evidence against the unit root is very weak though.

My paper utilizes a model of Markov switches originally developed in Mizrach (1995). I specify a parsimonious nonlinear model where the risk varies with the exchange rates' divergence from parity. The model weights the data according to the probability of realignment, highlighting states where the reversion to parity is likely. This structural approach decisively rejects the unit root in exchange rates in favor of mean reversion within the band.

\(^2\) For a recent survey of exchange rate prediction failures see Kilian and Taylor (2003).
\(^3\) See the final results of the 2007 survey at http://www.bis.org/publ/rpfxf07t.htm
The paper is organized as follows: Section 1 begins with a preliminary examination of the unit root question. Section 2 looks at the second generation target zone models that have been devised. Section 3 develops the Markov model for realignments, and this structure is tested in Section 4. A summary and conclusions are in Section 5.

1. A PRELIMINARY EXAMINATION OF THE UNIT ROOT QUESTION

This section begins with an exploratory empirical analysis. In part 1.1, I briefly describe the data and institutional setting. In part 1.2, I perform some conventional unit root diagnostics, and in 1.3, I repeat these tests, incorporating regime changes. In 1.4, I survey several nonlinear and nonparametric investigations into the unit root debate.

1.1. INSTITUTIONAL SETTING AND DATA DESCRIPTION

In March of 1979, seven European countries, Belgium, Denmark, France, Germany, Ireland, Italy and the Netherlands, agreed to join a system of managed exchange rates known as the Exchange Rate Mechanism (ERM). Rates floated freely within target zone bands around an European Currency Unit (ECU) central parity. When threatened, central banks were supposed to intervene to defend the weak currency. In practice, devaluations were frequent. From 1979 to 1987, there were 12 episodes. In the period 1987-1992, following the Basel-Nyborg agreement, the target zones firmed. There was only a single devaluation during this time period. By mid-1992, with the signing of the Maastricht treaty, many were confidently predicting a smooth path to a single currency. These hopes were crushed only a few months later in the crisis of September 1992. By mid-1993, the bands were widened to ±15%, and the Lira suspended from the mechanism until 1996.

I confine the work herein to the French Franc (FF), Italian Lira (IL), and German Deutschemark (DM) for conciseness. I convert daily nominal ECU spot rates, using the DM as a base, into Franc/Mark (FF/DM) and Lira/Mark (IL/DM) exchange rates, since these were the rates implicitly targeted by the ERM.

My sample spans September 14, 1987 to September 11, 1992, providing nearly 1,300 daily observations. During this time, there were no devaluations of the Franc and only one, January 8, 1990, for the Lira. Given the confinement of the exchange rate to narrow trading ranges, one might expect there to be strong evidence against the unit root in this sample.

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4 The bands were ± 2.25% for all the countries in the sample except for the Italian Lira, which until January 7, 1990, had a ± 6.0 band.
1.2. UNIT ROOT ANALYSIS OF THE SPOT EXCHANGE RATE

In this section, I treat the data quite naively and confront them as a time series analyst might. Ignoring the many changes in regime and other potential nonlinearities, I do straightforward univariate tests of the unit root hypothesis. I take log 2nd differences, one month in daily data, of the spot rate, denoted $\Delta s_{t+22}$ in Table 1, and regress on the log of the current levels, $s_t$. I correct for the overlapping observations using Newey-West estimates of the standard errors. A coefficient significantly less than zero would reject the null hypothesis of a unit root.

<table>
<thead>
<tr>
<th>French Franc</th>
<th>Constant</th>
<th>$s_t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_{t+22}$</td>
<td>0.378</td>
<td>-0.310</td>
<td>0.170</td>
</tr>
<tr>
<td>(2.73)</td>
<td></td>
<td>(-2.73)</td>
<td></td>
</tr>
<tr>
<td>$\Delta x_{t+22}$</td>
<td>0.003</td>
<td>-0.304</td>
<td>0.170</td>
</tr>
<tr>
<td>(2.68)</td>
<td></td>
<td>(-3.16)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Italian Lira</th>
<th>Constant</th>
<th>$s_t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_{t+22}$</td>
<td>0.860</td>
<td>-0.130</td>
<td>0.044</td>
</tr>
<tr>
<td>(2.73)</td>
<td></td>
<td>(-1.19)</td>
<td></td>
</tr>
<tr>
<td>$\Delta x_{t+22}$</td>
<td>0.004</td>
<td>-0.304</td>
<td>0.072</td>
</tr>
<tr>
<td>(1.51)</td>
<td></td>
<td>(-1.95)</td>
<td></td>
</tr>
</tbody>
</table>

The dependent variable in the first line is the 2nd difference of the log of the spot rate with the DM. The second line uses the deviation from central parity. Standard errors are calculated using the Newey-West correction with 22 lags. 22 observations before and after each realignment are excluded when a regime change takes place. $t$-statistics are in parentheses. Each equation is estimated over the period 14-Sep-1987 to 11-Sep-1992.

As evidenced by the table, even conventional $t$-statistics (which are biased towards rejecting the unit root) often cannot refute the null hypothesis of a random walk for the Franc and the Lira. The Franc has a $t$-statistic of -2.73. The Lira’s is even smaller, -1.19. Both are below the Dickey-Fuller critical values ($|t|>2.86$) in a one-sided 5% test, $>3.43$ at 1%). This table confirms the univariate and multivariate evidence in Anthony and MacDonald (1998) and Ma and Kanas (2000). Ignoring structural properties of the data, the spot exchange rate appears well characterized empirically by a random walk.
1.3. INCORPORATING REGIME CHANGES

Rose and Svensson (1995) argue persuasively that the simple unit root tests are misspecified. They look at the deviation from parity

\[ x_t = s_t - c_t \]  

where \( c_t \) is the log of the central parity in Deutschemark (DM). I follow their lead and incorporate regime change dummy variables at each of the devaluations. For my sample, this requires two dummies for the Lira, but only a single constant term for the Franc.

The second lines of Table 1 repeat the empirical exercise of part 2.2 with the dummies. The dependent variable is the log 22nd difference of the deviation from central parity, denoted \( \Delta x_{t+22} \). Estimation is done using OLS with Newey-West standard errors, with the Bartlett window again set at 22 lags. For the Franc, I can apparently reject the unit root using the Dickey-Fuller critical values at the 5% level.

Because of the regime changes, non-normality and serial correlation in the data, I was concerned that the standard Dickey-Fuller critical values might not be appropriate. To address this issue, I did a bootstrapping exercise. I regenerated 500 samples of \( x_t \) using the coefficients in Table 1, substituting a unit root (a zero coefficient) for the lagged dependent variable. For the error terms, I sampled with replacement from the original residuals. In all cases, I again used the Newey-West correction.

The resulting critical values were somewhat larger than usual. A 5% one-sided test requires a \( t \)-statistic of -3.84 for the Franc, and -5.06 for the Lira. A 1% test requires -4.59 and -4.71, respectively.

With the bootstrap critical values, the evidence against the unit root appears more fragile.\(^5\) For both currencies, one cannot reject at typical confidence levels. This confirms the findings in Beetsma (1995) and Rangvid and Sorensen (2001) who look at perfectly and imperfectly credible target zone models.

1.4. NONLINEAR AND NONPARAMETRIC APPROACHES

Several papers have investigated the possibility that nonlinearity might be the real cause of our inability to predict the exchange rate. Diebold and Nason (1990), Meese and Rose (1990, 1991) and Mizrach (1992), however, are generally unable to improve upon the random walk in forecasting the exchange rate using

\(^5\) It might just be that the unit root tests have low power. With a fully credible \( \pm 2.25\% \) target zone, Froot and Obstfeld (1991) can only reject the unit root in 25 years of data 12.4% of the time. Devising a more powerful test, robust to nonlinearity, is the objective of Section 4.
nonparametric techniques. Engel and Hamilton (1990) have found long swings in the U.S. dollar and forecast using a switching model. Their nonlinear time series model, however, only beats the random walk with drift. Without the drift term, the random walk does better out-of-sample.

Kilian and Taylor (2001), Taylor, Peel and Sarno (2001) and Sarno (2004) find much stronger evidence of predictability in a nonlinear vector error correction framework. These papers, however, find no evidence of short horizon predictability and do not consider the structural implications of a target zone.

Time series analysis, whether linear or nonlinear, seems unable to make a decisive rejection of the unit root at a daily frequency. I investigate whether a structural model will do better below.

2. TARGET ZONE MODELS WITH STOCHASTIC DEVALUATION RISK

The seminal paper in the theoretical target zone literature is due to Krugman (1991). Krugman's model has been extended in numerous papers, including contributions by Bertola and Caballero (1992), Bertola and Svensson (1993), Froot and Obstfeld (1991), Rangvid and Sorensen (2001), and Svensson (1991). This second generation of target zone models has relaxed Krugman's assumption that the target zones are perfectly credible. Part 2.1 looks at implications for the exchange rate process, and part 2.2 examines interest differentials.

2.1. THE EXCHANGE RATE PROCESS

In Krugman's (1991) model, the (log of) the spot exchange rate, \( s(t) \), is assumed to evolve (in continuous time) with a fundamental, \( f(t) \), and a term proportional to the expected depreciation rate,

\[
s(t) = f(t) + \alpha E_s[ds(t)]/dt. \tag{2}
\]

In the standard monetary interpretation of the model, the fundamental is assumed to have two components,

\[
f(t) = m(t) + v(t), \tag{3}
\]

where \( m(t) \) is the (log of) the domestic money supply, and \( v(t) \) are velocity shocks. For tractability, the shocks are assumed to be an exogenous Brownian process with drift \( \mu_v \) and standard deviation \( \sigma_v \),

\[
dv(t) = \mu_v dt + \sigma_v d\omega_v(t), \tag{4}
\]

and \( \omega_v(t) \) is a standard Wiener process.
To model realignment of the central parity, Bertola and Svensson (BS, 1993) treat the change in central parity as a jump process. The exchange rate is assumed to be regulated on the symmetric interval $[s, s]$.

Using the identity (1), re-write (2) as

$$x(t) + c(t) = f(t) + \alpha E_x[dx(t) + dc(t)]/dt.$$  \hfill (5)

The expected change in the central parity has two components, the probability of realignment and the change in the parity when a realignment occurs. Assume that in the finite interval $dt$, the probability of realignment is $p(t)$. Let the exchange rate jump be a random variable, $q(t)$. BS incorporate the two variables, defining a stochastic devaluation process, $\{g(t)\}$.

$$E_x[dc(t)]/dt = (p(t)E_x[q(t)]dt)/dt = g(t).$$  \hfill (6)

Substituting into (5),

$$x(t) = f(t) - c(t) + \alpha g(t) + \alpha E_x[dx(t)]/dt,$$

$$= h(t) + \alpha E_x[dx(t)]/dt.$$  \hfill (7)

BS then show that if $g(t)$ is a Brownian process $(\mu_g, \sigma_g)$, the composite term $h(t)$ is also a Brownian process,

$$dh(t) = \mu_h + \sigma_h d\omega(t),$$  \hfill (8)

where $\mu_h = \mu + \alpha \mu_g$ and $\sigma_h = \sqrt{\sigma_g^2 + \alpha \sigma_g^2}$. BS then obtain a solution analogous to Krugman's in the case of a perfectly credible target zone,

$$x(h(t)) = h(t) + \alpha \mu_g + \lambda_1 \exp[\lambda_1(h(t))] + \lambda_2 \exp[\lambda_2(h(t))]$$  \hfill (9)

$\lambda_1$ and $\lambda_2$ satisfy the characteristic equation, and the “smooth pasting conditions” determine the constants of integration.

I now turn to the implications of this model for interest differentials.
2.2. INTEREST DIFFERENTIALS

Consider nominal pure discount bonds maturing at date \( t + \tau \). Let \( i_t^r \) denote the home currency interest rate and let \( i_t^{r*} \) denote the foreign rate. Define the \( \tau \)-period interest differential,

\[
\delta_t^\tau = i_t^r - i_t^{r*}. \tag{10}
\]

I will assume\(^6\) that the uncovered interest parity condition holds,

\[
\delta_t^\tau = E_t[\Delta s_t] / \tau. \tag{11}
\]

Using the identity in (5), (11) can be written as

\[
\delta_t^\tau = E_t[\Delta x_t + \Delta c_t] / \tau. \tag{12}
\]

Assume that there are two states of the world, \( j = 0, 1 \), with 1 indicating a devaluation of the central parity. A realignment occurs during the interval \( t + \tau \) with probability \( p_t^r \).

Re-write 12 as

\[
\delta_t^\tau = \{1 - p_t^r\}(E_t[\Delta x_t | j = 0] + E_t[\Delta c_t | j = 0]) + p_t^r(E_t[\Delta x_t | j = 1] + E_t[\Delta c_t | j = 1]) / \tau. \tag{13}
\]

The second conditional expectation will drop out of (13) since, with no devaluation, the change in the central parity is zero. For state \( j=1 \), I reverse definition (1) to include the spot rate

\[
\delta_t^\tau = \{(1 - p_t^r)E_t[\Delta x_t | j = 0] + p_t^r E_t[s_t | j = 1]\}. \tag{14}
\]

I specify this model in the next section. Econometric issues are left to Section 4.

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\(^6\) Svensson (1992) works out a representative agent asset pricing model where the spot exchange rate takes Poisson jumps with constant intensity. He concludes that “disregard[ing] the risk premium seems warranted, at least for narrow target zones.”
3. A MARKOV MODEL WITH ENDOGENOUS PROBABILITY OF DEVALUATIONS

To implement 14 econometrically, I make several identifying assumptions. I begin with the expected deviation from parity given no realignment, \( E[x_{t,r} \mid j = 0] \). While this expectation is the solution to a nonlinear partial differential equation, Svensson (1991) shows that it is well approximated by a linear autoregression for many reasonable parameter values. I then assume

\[
E[x_{t,r} \mid j = 0] = \beta_1 x_r.
\]  

(15) is critical for the unit root question. If the exchange rate within the band is a mean reverting process, one should expect that \( \beta_1 < 0 \), indicating that a positive (negative) deviation from central parity will lead to a smaller (in absolute value) positive (negative) deviation in the future. On the other hand, if \( x_r \) is well approximated by a random walk, \( \beta_1 \) should be close to zero.

For the expectation of the spot rate conditional on realignment, I assume that agents anticipate the authorities will allow for some real depreciation

\[
E_r[x_{t,r} \mid j = 1] = \beta_2 + \beta_3 r_t,
\]  

(16) where \( r_t \) is the log of the real exchange rate. If the spot rate tends to be unaffected by realignments, both parameters should be zero. \( \beta_2 \) will measure the anticipated accommodation of real appreciation by the central bank.

My last identifying assumption concerns the devaluation risk. I let the probability of realignment be a function of the distance from the lower boundary of the target zone,

\[
\gamma_1 + \gamma_2 (s_t - \underline{s})/\overline{s} = \gamma z_t,
\]  

(17) where \([\underline{s}, \overline{s}]\) are bounds of the band. If I make (the rather plausible) assumption that devaluations are likely to occur closer to the top (the weak) edge of the band, then \( \gamma_2 > 0 \). To keep the probability on the \([0,1]\) interval, I make a probit transformation,

\[
p_r^t = \int_{-\infty}^{\gamma z_t} (\sqrt{2\pi})^{-1} \exp (-t^2/2) \, dt = \Phi (\gamma z_t).
\]  

(18) Making these substitutions leads to the econometric specification,

\[\text{footnote}{Rose and Svensson (1995) obtain this estimate using the univariate autoregression in Table 1. By including regime specific dummy variables though, they supply the representative agent with an ex ante knowledge of the new stochastic process following a regime shift. I thank Lars Svensson for this point.} \]
\[ \delta_t' = \left[ \beta_1 (1 - \Phi (\gamma z_t)) x_t + \beta_2 \Phi (\gamma z_t) + \beta_3 \Phi (\gamma z_t) r_t \right] / \tau \]  

(19)

which I estimate in Section 4.

4. ESTIMATION AND HYPOTHESIS TESTING

In part 4.1, I estimate the nonlinear model (19). In part 4.2, I develop Wald tests to answer the unit root question.

4.1. NONLINEAR LEAST SQUARES ESTIMATION

The interest rates are annualized 1-month, Euromarket rates, \( i_{t}^{1/12} \), in decimal form, from France, Italy and Germany. I treat Germany as the foreign country and denote its' rate with an asterisk. For the dependent variable in (19), I set

\[ \log (1 + i_{t}^{1/12}) - \log (1 + i_{t}^{1/12}*) \]  

(20)

To construct the real exchange rate variables, I made a daily price level series by interpolating from monthly figures on consumer prices. I transform the spot rate into a real rate by dividing by the relative price levels. I then normalize the series with the rate from March 13, 1979 as a base. To coincide with the interest rate data, the probit specifies the probability of a realignment in the next 22-days (one month in daily data),

I minimized the sum of squared residuals,

\[ \sum_{t=1}^{T} \left[ \delta_t^{1/12} - f(\beta, \gamma, r_t, z_t, x_t) \right]^2 \]  

(21)

using an iterative procedure, the Davidon-Fletcher-Powell algorithm. I corrected the standard errors using the Newey-West estimator. I again used 22 lags to account for the overlap in the data. Results are in Table 2.

<table>
<thead>
<tr>
<th>( \delta_t^{1/12} )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>SSR</th>
<th>Wald</th>
</tr>
</thead>
<tbody>
<tr>
<td>Franc</td>
<td>-0.314</td>
<td>0.036</td>
<td>0.289</td>
<td>-1.910</td>
<td>1.447</td>
<td>0.202</td>
<td>40.34</td>
</tr>
<tr>
<td>Lira</td>
<td>-0.079</td>
<td>0.064</td>
<td>0.140</td>
<td>-1.500</td>
<td>1.241</td>
<td>0.161</td>
<td>60.07</td>
</tr>
</tbody>
</table>

The dependent variable is the 1-month interest differential as defined in (20). The structural parameters are given in (15) to (19). The Wald statistic (23) is distributed \( \chi^2(1) \).

\( \beta_1 \) is significantly different from zero in both regressions, at -0.314 for the Franc and -0.079 for the Lira. Even with the bootstrapped critical values, a standard
Dickey-Fuller test would reject the unit root as both t-statistics are above six. An asymptotically efficient test, robust to target zone nonlinearities, is still required to answer the question definitively though.

4.2. WALD TESTS OF THE UNIT ROOT

Allowing for both heteroscedasticity and autocorrelation in the errors, denote the asymptotic distribution of the parameters as

\[ \sqrt{T} \left( \hat{\beta} - \beta \right) \sim N(0, \sigma^2 \Omega^{-1}), \]

where \( \Omega \) is a symmetric, positive-definite matrix. Consider a \( q \)-vector of restrictions on the parameters, \( H(\beta) \). If we have estimated the covariance matrix of the \( \beta \)'s in a consistent manner, it follows that

\[ W = H(\hat{\beta}) \left[ \sigma^2 \left( \frac{\partial H(\hat{\beta})}{\partial \beta} \right) \Omega^{-1} \left( \frac{\partial H(\hat{\beta})}{\partial \beta} \right)^T \right]^{-1} H(\hat{\beta}) \sim \chi^2(q). \]

In this case, I test \( H_0 : \beta_1 = 0.0 \), so \( q = 1 \). The Wald statistics in Table 2 are quite large, 40.34 for the Franc and 60.07 for the Lira. These safely reject the unit root at better than the 99.9% level.

In contrast to the time series test, the structural model produces powerful evidence in favor of mean reversion within the band. The rejections are even stronger for the Lira than the Franc. Apparently market participants know when the currency is likely to be successfully returned to the center of the band, and when a realignment is likely to come.

The probit Markov model endogenously weights the data by the credibility agents assign to the central parity. Where agents expect the parity to be defended, the model reality identifies mean reversion in EMS exchange rates.

5. CONCLUSION

A simple nonlinear model with endogenous devaluation risk leads me to decisively reject the unit root for EMS exchange rates. Previous studies, using linear and nonlinear time series estimators, have had difficulty identifying the mean reverting behavior. My results indicate that a small amount of structural information can be very useful in explaining exchange rate movements. I speculate that other areas of the applied unit root literature might also benefit from a more structural approach.
REFERENCES


