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THE WELFARE OF POORER OLDER PEOPLE IN BELGIUM AND THE NETHERLANDS
AN APPLICATION OF QUANTILE REGRESSION

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ABSTRACT :

On the basis of the SHARE data, we estimate age-conditional quantiles at the 5th, 10th, 15th, 20th, 25th and 50th percentile levels for equivalised gross income, net financial assets and consumption levels, as well as for a (remaining) lifetime utility indicator of people over the age of 50 in Belgium and the Netherlands. We conclude that the poor performance of Belgium in terms of older people's income, compared with the Netherlands, is mitigated when the consumption figures are considered. However, the picture presented by the wealth figures is more diverse: the median net financial assets position for the 55-70 age group in Belgium is higher than that in the Netherlands, but their relative ranking on the basis of the first quartile value is reversed for several age groups. In terms of lifetime utility, the two countries do equally well. Income and asset poverty do not seem closely related to consumption poverty. Remarkably, the income distribution of the poorer half of older people in Belgium turns out to be skewed to the left, but income inequality is lower in Belgium than in the Netherlands. The reverse holds true for consumption inequality. Wealth inequality is highly skewed to the right in both countries, but no clear conclusions can be drawn about their relative ranking in terms of inequality.

JEL CLASSIFICATION : I300, I310, C31, C13.

KEYWORDS : Welfare Measurement, Life Cycle Hypothesis, Lifetime Utility, Nonparametric Quantile Regression.

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INTRODUCTION

In this paper we want to make a comparison between the socio-economic welfare of older people (aged 50 years and older) in Belgium and the Netherlands. Our point of departure is that, even in the most simple approach, the welfare of a person at the present point in time cannot be judged independently from her situation in the past and from expectations about the future. A person's current economic welfare situation also depends partly on the amount of resources she has accumulated from the past, and partly upon future needs. Moreover, institutional arrangements and economic circumstances can have a considerable impact upon the life cycle pattern of some welfare indicators. Financing a public pension system requires a tax on earned incomes at some point(s) in time. It means that the net earned income of the generations that pay during the period when they are active will be lower than in a situation where no public pension system is available and all old age income is generated exclusively from private savings. Therefore, standard welfare economics argues for measures of welfare based on consumption rather than income, and usually measured by a money metric utility function (Diamond and McFadden, 1974). However, in an intertemporal context, welfare depends not only on current consumption, but also on future consumption possibilities. A measure of welfare should therefore take the latter into account (Ahlheim 1999). This means not only reintegrating current and past savings into the measures of welfare, but also somehow taking future incomes and transfers into account. When attention is focused on international comparisons, differences in future consumption possibilities depend upon differences in the evolution of consumption prices, interest rates, and differences in household needs, such as life expectancy and the size and composition of the households in which older people live.

This paper does not aim to find an all-encompassing measure of welfare. We will provide figures for commonly used measures of welfare such as (gross) income, wealth (a value of net financial assets) and expenditure on non-durable consumption goods. On the basis of these latter and expectations about future prices, interest rates, survival probabilities and household size and composition, we also calculate a lifetime utility indicator. Durable goods are excluded from the picture here, the main reasons being that it is by no means obvious how to value the possession of such goods and how to account for differences between owners and renters. As a result, we neglect welfare inequalities caused by differences in the use of such goods (housing being by far the most important example). On the other hand, the possession of such goods has repercussions on the remaining household budget available for spending on non-durable goods. For example, people who own the house in which they live and have no outstanding mortgage debts any more will have a larger budget available than people with the same income who have to rent their dwelling.

Our specific contribution lies in the way we present these figures. All measures of welfare are *age-specific*, because making direct welfare comparisons between people of different ages or, for the sake of the argument, between different moments in the life cycle of one particular person, may involve unrealistic assumptions. For example, suppose someone claims to be better off now than ten years ago. The reason for the statement may be that the person has a higher income now. If so, she is either failing to take into account the prospect she had ten years ago of a higher income, or she may not have had any expectation of this higher income ten years ago. If she has deliberately not taken this expectation into account, this may be because she wants to compare her current situation with the one ten years ago, irrespective of any future prospects at that time. She may further argue that these future prospects are completely irrelevant, since in the economy in which she lives there are no means to transfer income from one period to another. She may also contend that her current and past incomes are comparable since prices of consumption goods have not changed during the intervening period, and the set of consumer goods available now and then is identical too. Furthermore, nothing else in her world has changed. She still lives in exactly the same situation as ten years ago and her preferences have not changed over time, meaning that relative exchange values e.g. of health care for food and drink have remained the same. We can continue to add *ad hoc* assumptions in order to make the comparison sound. In the end, we believe, this would amount to the following question in order to determine wether that person is better off with the higher income now than her income of ten years ago: Would she have been better off ten years ago if she had earned then the same income that she earns now, *all else remaining equal*? Thus we arrive at a type of comparison which is, in the end, age-specific. When comparing the economic welfare of people of different ages in different economic environments (countries, say), we would like to maintain this logic. First, welfare comparisons should be limited to people of the same age. Then, the relative performance of economic environments or countries for broader age groups can be judged by somehow aggregating these age-specific comparisons. This suggestion stands in contrast to the usual practice of comparing countries using statistics that reflect the relative welfare (income, wealth, consumption) of old versus young. For example, this is the case with comparisons of poverty incidence ratios among older people using a country-specific poverty line (the percentage of older people falling below this poverty line), or when looking at replacement ratios for pension schemes (the ratio of pension income to earned income).

In this paper, we concentrate on the situation of poorer older people. To this end, we estimate age-conditional lower quantile values. In particular, median values, first quartile, quintile and decile levels, as well as 15th and 5th percentiles are calculated. Section 1 introduces the notion of quantile-based welfare comparisons. The appendix contains an explanation of the nonparametric quantile regression technique that is used in order to perform the requisite estimates of age-specific quantile values. In Section 2 we compare age-specific median values of income, wealth, consumption and lifetime utilities in Belgium and the Netherlands. Section 3 looks at the lower quantile levels and investigates whether people who belong to the $100p\%$ poorest in one dimension are likely to be among the poorer in other dimensions as well. In Section 4 some properties of the age-specific distributions of consumption, wealth and income are investigated. Section 5 summarises some of the results.

1. QUANTILE-BASED COMPARISONS

As argued in the previous section, comparing welfare levels of people of different ages is not straightforward. Hasty conclusions on the basis of a comparison of income, wealth, consumption or welfare levels of people at different phases of their life cycle may give a misleading picture. We argue in favour of developing methods that aggregate comparisons of *age-specific* statistics across different economic environments, rather than comparing different societies using statistics that measure their performance for old compared with young, such as replacement ratios for pensions, or older people's poverty incidence ratios. However, except where societies are so obviously different that we do not need any statistical computations at all to judge their relative performance, it will rarely be the case that *all* people of a specific age are better off in one society than in another. Therefore, a picture of the age-specific distributions of certain welfare indicators may be valuable. Measuring welfare, in any case, remains a highly normative issue. In order to limit divergence of opinion, we will use evaluation tools that presuppose only a low degree of measurability and few interpersonal comparability assumptions, though any choice in this respect remains questionable. More specifically, we will assume that only age-specific welfare *levels* can be measured, thus avoiding attributing any significance to the units in which these levels are expressed. It is also assumed that these welfare levels can be compared interpersonally. Quantile values satisfy these criteria. This can be shown as follows. Let F_Y be a given (absolutely continuous) distribution function of a variable Y, where $p = F_Y(y)$ means that $100p\%$ of the population obtains a value for Y lower than or equal to y. The $100p$-th quantile for that distribution is then the value q of Y below which lie $100p\%$ of the values obtained by the population:

$$q = Q\left(p, F_Y\right) \equiv F_Y^{-1}\left(p\right). \qquad (1)$$

It can then be seen that rankings obtained by comparing quantile values of one welfare distribution, say F_Y, to another one, say G_Y, are not affected by positive monotone transformations of the measure of welfare. Indeed, let $\phi(y)$ be such a positive monotone transformation of the measure of welfare Y. A positive monotone transformation is a change in the measurement units that preserves the ranking: for all possible values x and y of Y, it holds that $\phi(y) \geq \phi(x)$ if and only if $y \geq x$. For a given distribution F_Y of Y, the distribution of the positively monotonously transformed variable $\phi(Y)$, say $F_{\phi(Y)}$, is equal to F_Y: $F_{\phi(Y)}\left(\phi(y)\right) = F_Y\left(y\right)$ for all possible values y of Y. Consequently,

$Q\left(p, F_{Y}\right) \geq Q\left(p, G_{Y}\right)$ if and only if $Q\left(p, F_{\phi(Y)}\right) \geq Q\left(p, G_{\phi(Y)}\right)$.[1] More frequently used welfare statistics, such as (rank-ordered and weighted) means, inequality indices such as the Gini coefficient or poverty incidence ratios, do not satisfy this property. Hence, they require higher information prerequisites than the present approach: they at least need to be measurable up to a positive affine transformation.[2]

In the present contribution, we concentrate on the age-specific welfare levels of poorer people. More specifically, we compare the age-specific median welfare levels as well as the first quartile value, first quintile value, 15th percentile, first decile and 5th percentile in Belgium and the Netherlands for people between the ages of 50 and 90 years. We explore different dimensions of welfare: gross income, the stock of net financial assets, expenditure on non-durable consumption goods and a lifetime utility index. This latter can be derived from observations on current consumption expenditure, and can be interpreted as the present value of the expected amount of future consumption that would optimally fulfil intertemporal preferences, given the present value of the remaining lifetime income. The age-conditional quantile values are estimated by means of nonparametric quantile regressions, as proposed by Yu and Jones (1998). Further details on the estimation method can be found in the appendix.

All monetary values are expressed in adult equivalent units. The equivalence scale we use is adapted from the old League of Nations equivalence scales (see Dereymaeker, 1985). It is more sensitive to the age of young household members than the current OECD alternatives.[3] The use of these equivalence scales to transform household income, consumption expenditure or financial wealth into welfare of an individual living in the household, rests upon the assumption that less income per person is needed to generate the same individual welfare level when people are living together and when people living together in the same household are younger. Furthermore, the use of equivalence scales rests upon the assumption that welfare is equally distributed within the household. This last assumption (labelled 'the unitary household model') has been questioned in recent literature on collective household models (see e.g. Vermeulen, 2002). Thus our results with respect to the distribution of welfare among older people do not take intra-household inequalities into account.

From the SHARE database[4] we draw figures for gross annual income (2003), the

[1] A small *caveat* should be made here. We calculated *average* estimates of the quantiles for the five imputations available for the data that we use (*cf. infra*). This could (and on occasion did) cause rank reversals across countries, when changing units of measurement.

[2] A positive affine transformation ϕ of Y is of the form $\phi(y) = a + b \cdot y$, with $b > 0$.

[3] The first household member stands for one unit. Every additional household member of age 14 or older counts for 0.8 additional adult equivalent units, people aged 8-14 count for 0.7 units; 0.5, 0.4, 0.3 and 0.2 additional adult equivalents are counted for people aged 6-8, 4-6, 2-4 and younger than two respectively.

[4] The Survey of Health, Ageing and Retirement in Europe (SHARE) is a multidisciplinary and cross-national panel database of micro data on health, socio-economic status and social and family networks of more than 30,000 individuals aged 50 years or older, and contains some information on people living in the same household as those in the sample. It was first organised in 11 European countries in 2004. We

stock of net financial assets (end 2003) and annual expenditure on non-durable goods (2004 for the Netherlands, 2004/5 for Belgium). Gross incomes include personal income taxes and personal social security contributions. All household incomes are added together: earned incomes, pensions, other social security transfers and capital income. Single capital payments from occupational pension schemes are excluded. They show up in the net financial asset figures. The stock of net financial assets includes the value of all financial assets *minus* outstanding debts (exclusive of mortgage debt).[5] The data providers deliver a dataset with completed information on both, gross incomes and the stock of net financial assets, expressed in purchasing power parity (PPP) euros. When data were missing, they provide five imputations. The following analyses are based on averages from these five imputations, as advised by the data providing agency. Standard errors were not adapted for imputation error. We equivalised the amounts by means of the equivalence scale discussed in note 3. Before applying the equivalence scale, we worked out ages of the household members on 31 December 2003. The value equivalised in this way is then assigned to every household member. The quantiles are also made conditional on age at the end of 2003. We concentrate on the quantile values for ages between 50 and 90 years.

Expenditure on non-durable goods does not include housing payments such as rent or mortgage interest and repayments, but it does include expenditure on clothing and out-of-pocket medical expenditure. No imputed values are provided for this variable. In about 20% of the cases answers were missing. However, for the three subcategories of expenditure that are surveyed in more detail (food and non-alcoholic beverages at home, food and non-alcoholic beverages outside the home, telephone communication costs), imputed values are available. We used the sum of these subcategories of expenditure as an explanatory variable in a nonparametric regression of total non-durable expenditure, in order to impute values for total expenditure on non-durables when data were missing. Again, we equivalised the resulting amounts, this time using ages of household members at the moment when the survey took place, since we regard this as the moment to which the reported (monthly) expenditure figures apply. Consequently, the reported conditional quantile values for this variable are with respect to ages at the moment of the survey (2004 for the Netherlands, 2004/5 for Belgium).

Finally, on the basis of the current consumption expenditure, life expectancies and expected evolution of future prices and interest rates, a lifetime utility indicator was calculated. This is the discounted value of current and future consumption, when current consumption is interpreted as an element of an optimal path of consumption given expected future prices, interest rates and survival probabilities. The latter were calculated from life tables, while prices and interest rates were forecast on the basis of ten-yearly moving averages. A standard constant intertemporal elasticity of substitution utility function with uncertain lifetime was used (see Levhari and Mirman, 1977). We converted the values to the situation of a single.[6]

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used the data for Belgium and the Netherlands from release 2.0.1 of this first wave. See note * for acknowledgments with respect to the data collection sponsors.

[5] We refer the reader to Börsch-Supan and Jürges (2005) for further details on the survey.

[6] Further details on these calculations can be found in Capéau and Pacolet (2009).

2. INCOME, WEALTH, CONSUMPTION AND WELFARE

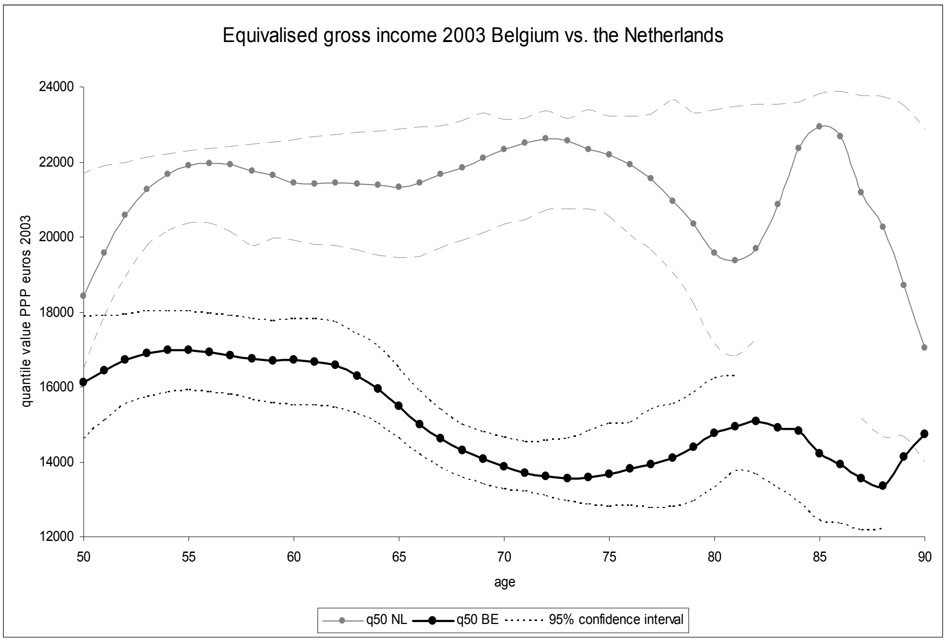
In Figure 1 we report the estimated age-conditional median values for equivalised gross income, net financial assets, non-durable consumption expenditure and lifetime utilities. The dashed lines represent upper and lower limits of the 95% confidence intervals. At several points, these estimates are missing, because the inverse rank test procedure suggested by Koenker (1994) did not give results. In some other cases, the limits come unreasonably close to the point estimates. This is because in these cases no confidence limits were available for all of the five imputations, so that the average confidence interval limit does not apply to all the values of the point estimates for which the average is taken. We will not calculate formal dominance statistics for the estimated curves, but limit ourselves to eyeball inference on the basis of upper versus lower confidence limits.

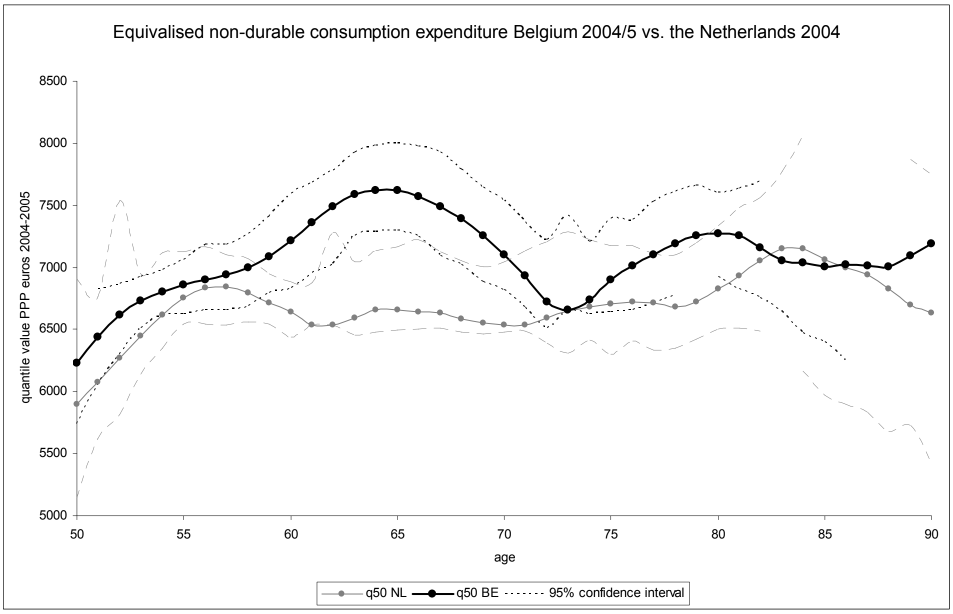
Usually, *net* incomes are used for the purpose of welfare evaluation. We did not follow this common practice mainly because no imputed data on aggregate net household incomes are available in SHARE release 2.0.1. On the other hand, *net* incomes neglect the welfare generated through the provision of public goods and services. Of course, using gross incomes as a proxy presupposes that the welfare generated by these public goods and services is in proportion with the personal income tax contributions. Moreover the inclusion of transfer payments such as social security contributions may further obviate such an interpretation.

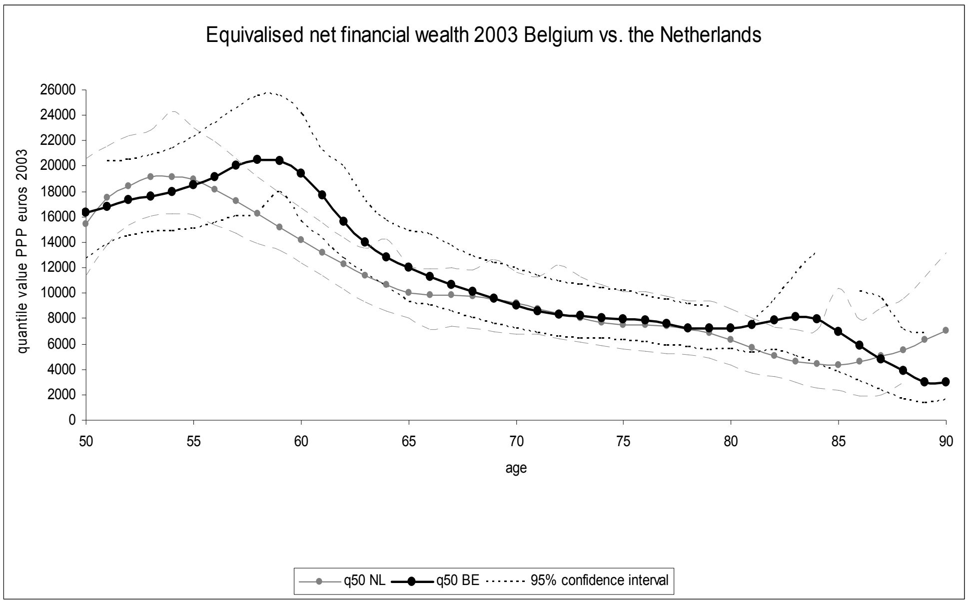
The median value of equivalised *gross* incomes is (significantly) higher in the Netherlands than in Belgium, for (almost) all ages. The maximum median income in the Netherlands is reached at age 85, equalling €24,950, while the difference from Belgian median income reaches its maximum at age 72, when it is almost €9,000. Belgian median income is highest at age 54, when it equals €16,975. Despite this large difference, this does not imply that mean incomes are lower too. An overall average of total equivalised household income yields €27,960 for Belgium and €29,400 for the Netherlands. On average, Belgian age-specific mean incomes are still substantially lower, but the difference is less striking than for the median values. This may indicate that in Belgium, age-specific income distributions are more unequal than in the Netherlands. We come back to this issue in Subsection 4.2.

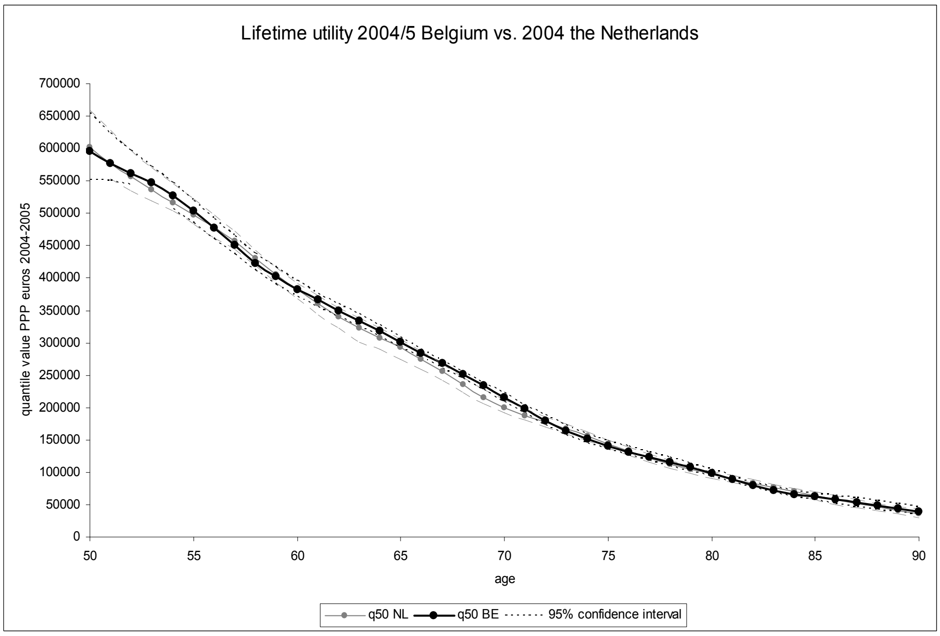
When we turn to consumption, the picture is almost reversed, although differences are less salient than in the case of income and not really significant. The median value of equivalised household expenditure on non-durable consumption is higher in Belgium than in the Netherlands for almost all ages. The only exception is for the ages 83-85. Belgian median value of consumption reaches its maximum of €7,620 at the age of 64-65 years, while in the Netherlands this maximum value is reached at ages 83-84, and equals €7,150. The minimum level in both countries is reached for the youngest people in the dataset, and equals €6,225 in Belgium and €5,895 in the Netherlands. In Belgium, however, the median consumption level starts to decrease again from 65 years onwards, reaching a value close to the minimum at 73 years and increasing again afterwards until the age of 80, after which it is almost constant. The median consumption level in the Netherlands seems to increase more or less continuously with age, with a little hump around the age of 57, and an almost flat section between ages of 65 and 80 years. This need not, however, to be attributed to a life cycle effect. We only have a cross-section of observations, so that people of different ages belong to different cohorts, and the observed pattern may thus be partly the consequence of cohort effects too.

FIGURE 1. AGE-SPECIFIC MEDIAN VALUE OF INCOME, WEALTH, CONSUMPTION AND LIFETIME UTILITY









What could explain the difference in rankings according to consumption and income? Possibly, in Belgium a larger part of older people's income is spent immediately, while in the Netherlands, older people are still saving more on average. At the level of the households sample, more than 17% of the Belgian households report a value of non-durable consumption expenditure that is higher than the gross income that is reported, while this proportion is less than 13% in the Netherlands. So these households are either depleting assets accumulated from the past or incurring debts in order to finance their current expenditure. More than 2% of the Belgian sample households have no or negative financial assets, while spending more than their gross income. In the Netherlands, this percentage is a little more than 1.7%. This might give rise to the hypothesis that older Belgian people, while *currently* earning less than their Dutch peers, have saved more *in an earlier phase of their life cycle*. Therefore, in terms of welfare, they are not as badly off as might be concluded from income figures. This hypothesis could be supported by looking at the wealth figures, since these figures reflect accumulated savings from the past. The age-specific median values of net financial assets held by an equivalent adult are higher in Belgium than in the Netherlands for people between 55 and 70 years old and roughly equal for people between 70 and 80 years old. But the differences are smaller than the income differences. The maximum difference is reached at the age of 59 years, and equals €5,235, which is only 60% of the maximum difference in age-specific median values of incomes. As with incomes, mean values of net financial assets are much higher than medians, coming close to, and, in Belgium, sometimes even exceeding the value of the third quartiles. The differences between Belgium and the Netherlands in terms of the mean are much greater too, and uniformly in favour of Belgium. In so far as outliers are trustworthy, this reveals a high level of inequality in wealth and might support the hypothesis that the high wealth figures for Belgian people aged 50 years or more, mainly relate to wealthier people, while the situation is less rosy for poorer older people.

A higher propensity to save in the past can be expected to be partly reflected in current incomes, since these include capital income. The extent to which this is the case depends on interest rates. The OECD statistics on annual interest rates on a ten-year bond for Belgium and the Netherlands were roughly equal for the period 1996-2007, but Eurostat statistics on a ten-year government bond's annual interest rates show substantial differences in the previous decade (1985-1995), with the interest rates up to 2.4 percentage points higher in Belgium than in the Netherlands. Older people's overall average household income from capital in Belgium (€3,275 per household) is more than double that in the Netherlands (€1,415 per household). But capital income only constitutes 6 to 7% of older people's overall household income in both Belgium and the Netherlands.

Differences in wealth and income figures can also be greatly affected by institutional factors. Public pensions in the Netherlands constitute only 17% of older people's household income, compared with 35% in Belgium. On the other hand, the Dutch occupational pension schemes contribute 10% of household income, while this figure is less than 1% in Belgium. However, in Belgium, occupational pensions are mostly paid out in the form of a one-off capital lump sum (see Berghman et al., 2007). They therefore show up in the wealth figures and capital income figures rather than in the occupational pension figures.[7]

[7] Some Belgian respondents may also erroneously regard their occupational pension income as part of the public pension.

For these reasons, the connection between consumption welfare and wealth figures is rather weak. A nonparametric regression[8] of equivalised consumption expenditure on equivalised income and wealth shows that consumption expenditure is almost constant in wealth for all income values in Belgium, while it is slightly U-shaped for lower incomes and increases mildly for higher incomes in the Netherlands. Though such a cross-section regression cannot serve as evidence against the life cycle hypothesis, it might indicate that current wealth figures are a poor indicator for consumption welfare, and even poorer than current income.

A substantial portion of household savings occurs through house ownership. Between 80 and 90% of Belgian people between 50 and 70 years old live in households in which one or more of the members own the house. For 50 to 100% of these people, the owner-occupied house no longer bears a loan charge. Similar figures for the Netherlands yield only 40 to 80% house ownership, and the mortgage has been paid off for less than half of the people living in owner-occupied houses.[9] House ownership may therefore be a more significant explanation than financial savings in an earlier phase of the life cycle for the difference in rankings according to income as compared to consumption expenditure. People without current housing charges may spend a larger part of their income on current consumption, and may therefore be better off in these terms, despite having lower incomes.

Current consumption figures neglect the influence of future prices, interest rates, life expectancy and household needs on welfare. We therefore also constructed a lifetime utility indicator. This reflects the monetary value (expressed in common euros) of the non-durable goods and services that a person is expected to consume during the rest of her life, on the assumption that she will spread out her budget optimally over expenditure during the rest of the life cycle, given the expected evolution of prices, interest rates, household composition and life expectancy. The age-specific median values of this lifetime utility indicator can be found in the bottom part of Figure 1. We converted these values into the consumption a single person would need in order to obtain the same welfare as a respondent, if she faced average survival probabilities, average inflation and interest rate evolution. Belgian and Dutch median welfare values turn out to be roughly equal. What might explain the fact that the median Dutch future consumption prospects are better than would be concluded from looking at the current expenditure figures only? The interest rates and life expectancies that we used for calculating these projections were roughly equal. Inflation figures in the Netherlands were lower in the period 2003-2007, and these figures gain more weight in future projections. Finally, the household size of Belgian older people is higher than in the Netherlands, and in particular, it decreases faster in the Netherlands than in Belgium.[10] As a consequence, the same future consumption level would generate less welfare for a person in the larger Belgian households. Or, similarly, the consumption a single adult would need to generate the

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[8] The results are available upon request from the authors.
[9] These figures might be biased to the disadvantage of ownership incidence in the Netherlands, because the target population of the Dutch sample includes people living in institutions for the elderly (old age homes), while they are not included in the Belgian target population. To confine the impact of this potential bias, we limited the comparison to people under the age of 70.
[10] This might also be a biased inference from the data, since the Dutch reference population includes people living in old age homes, whereas the Belgian one does not.

same lifetime utility as someone who will live in a larger household in the future, all else remaining equal, is lower than what she would need to generate the same welfare as someone in a household with the same lifetime budget but of smaller future size.

3. Aspects Of Poverty

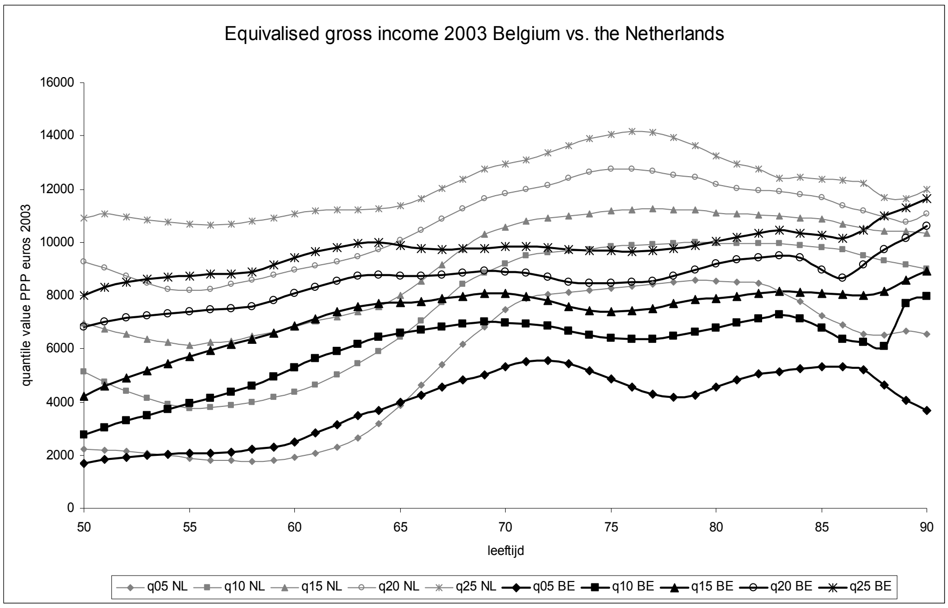
3.1. The Poorest Quarter

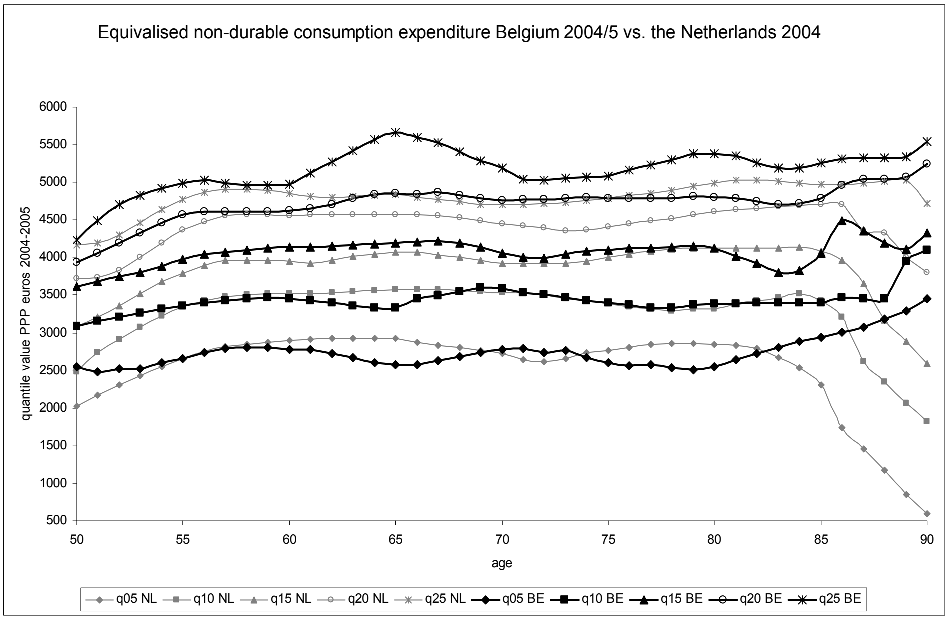
So far, we have focused attention exclusively on median values. In Figure 2, we investigate the level of some lower quantiles. The dominance of the Netherlands over Belgium in terms of median equivalised incomes is confirmed at the first quartile and quintile level. From the 15th percentile on, there is a tendency for the incomes of people aged 55-65 years in Belgium to dominate those of their Dutch peers. This may reflects the generosity of the Belgian early leavers' scheme ('*prépension*' in French, '*brugpensioen*' in Dutch). This is an increased unemployment benefit for older people who have been made redundant.

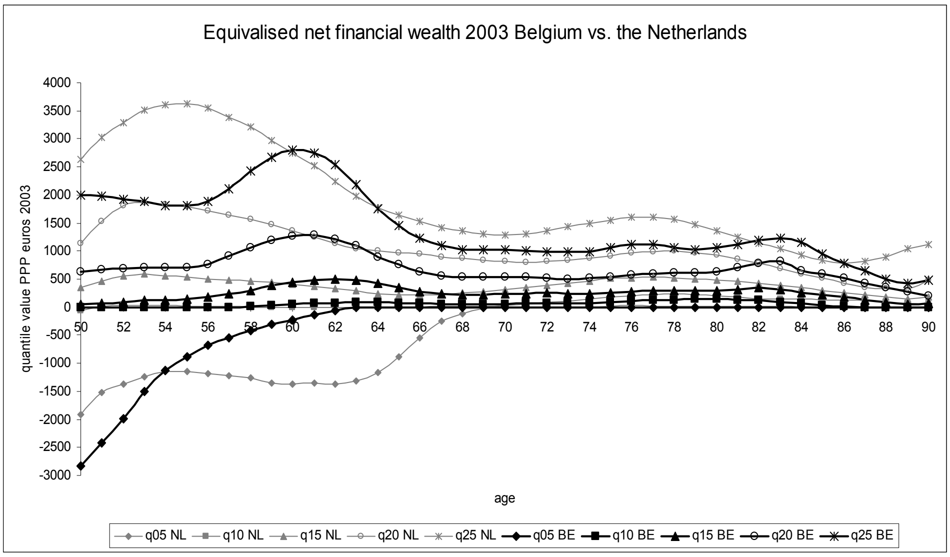
For people aged 70 to 80 years, the equivalised income level of the 5th quantile in the Netherlands exceeds that of the 15th percentile in Belgium, and comes even close to the first quintile level in Belgium. For people who have reached the legal retirement age (65 years), and who have low incomes, the Dutch income distribution seems to dominate the Belgian one.

Again, the picture is reversed when looking at the consumption figures. Belgian consumption levels dominate those of the Netherlands at the first quartile and quintile. There is a tendency for consumption levels to flatten out with ages when looking at lower percentile values, and the consumption levels of the poorest older people do not differ that much in the two countries. The lower values in the Netherlands at ages over 85 are statistically unreliable.

FIGURE 2. AGE-SPECIFIC VALUES OF THE 5TH, 10TH, 15TH, 20TH AND 25TH PERCENTILES






Equivalised net financial wealth 2003 Belgium vs. the Netherlands
quantile value PPP euros 2003
age
q05 NL
q10 NL
q15 NL
q20 NL
q25 NL
q05 BE
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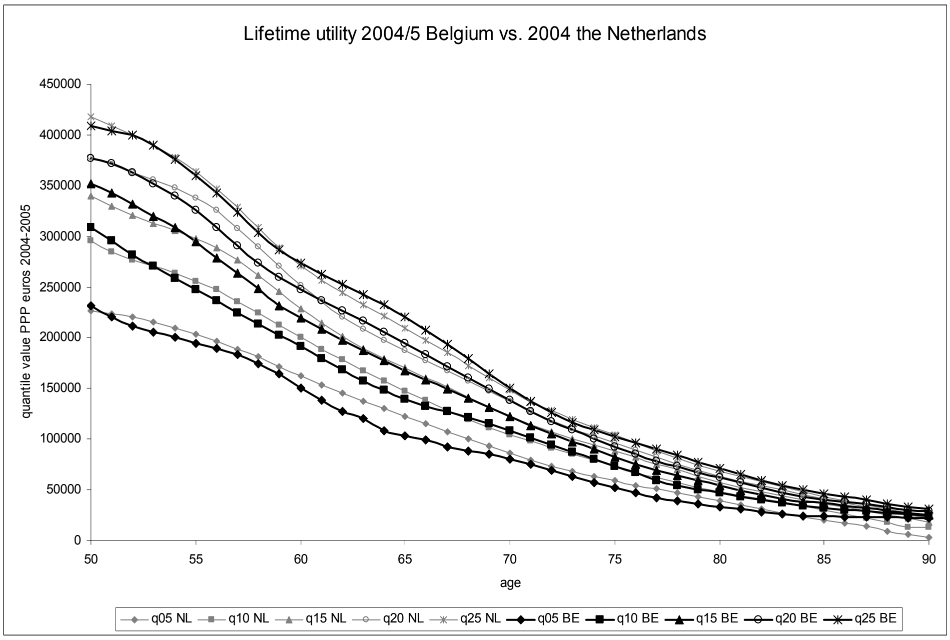
q25 BE


Lifetime utility 2004/5 Belgium vs. 2004 the Netherlands
quantile value PPP euros 2004-2005
age
q05 NL
q10 NL
q15 NL
q20 NL
q25 NL
q05 BE
q10 BE
q15 BE
q20 BE
q25 BE

The wealth picture of the first quartile and quintile values is different from that of the median level. There is in this case a tendency for the Dutch values to dominate the Belgian ones. The lowest decile in terms of wealth has almost no wealth, and the lowest 5% even has a negative amount of wealth at the younger end of the scale. Thus, for the poorer Belgian older people, the explanation of the difference between rankings on the basis of incomes and consumption levels in terms of higher accumulated savings from the past does not seem to hold – unless, of course, consumption poverty is negatively correlated with asset poverty. From the current analysis, for example, it cannot be inferred that people exhibiting low consumption are the same as those with low incomes and/or financial wealth. We come back to this issue in the next subsection.

Finally, the lower quantile values of lifetime utility remain, as was the case for the median, quite close to each other in Belgium and the Netherlands, with a tendency for the fifth percentile in the Netherlands to dominate the Belgian one. In fact, this mirrors what happens to the consumption figures. The median lifetime utilities in Belgium and the Netherlands are roughly equal, while the Belgian consumption figures dominate the Dutch ones. Since the latter also holds true for the first quartile and quintile, it stands to reason that one can expect their utility levels to draw closer together too. Since the consumption level of the first decile and fifth percentile in the Netherlands is already close to that in Belgium, and sometimes even higher, it can be expected that their utility levels will shift further to the advantage of the Netherlands. Actually, it can be shown that if an age-specific value of a consumption quantile in one country is higher than in another, and the corresponding quantile in terms of utility of the latter is greater than or equal to its value in the former country, it will hold for any other quantile of consumption for which the second country obtains a higher score, that it will also do better for this quantile in terms of the utility indicator. We will therefore not report any separate figures on inequality for the lifetime utility indicator in Section 4.

3.2. Dimensions Of Poverty

As was noted in the previous section, the analysis presented thus far does not allow us to detect whether *individuals* belonging to the lower part of the distribution in one dimension are also poor in another dimension. In Table 1 we report the frequency distribution of individuals across poverty in different dimensions. For example, a person with a lower stock of financial wealth than the first decile, but whose income, consumption and utility level exceeds their respective first decile values, will be classified as belonging to the group with the siglum PRRR, where the j -th entry P (for 'poor') stands for scoring a value lower than the decile (or another quantile) level in the j -th welfare dimension, and R (for 'rich') means having a value higher than or equal to the corresponding quantile. The sequence of the different dimensions is: wealth, income, consumption and utility. Afterwards we further aggregate the thus constructed subgroups into four main groups, depending on the *number* of dimensions in which people score a value lower than a certain quantile. For example, a person belonging to the second group has a value lower than the corresponding quantile value in at least two dimensions. The frequency distribution according to the latter specification is tested against the expected frequency distribution if the four dimensions of welfare were independently distributed. In that case, the frequency distribution across these groups would follow a binomial distribution $B(p,n,k)$, with parameters $p = F(Q(p,F))$, $n = 4$, and the probability of being poor in k dimensions, for $k = 0,1,\ldots,4$, is:

$$B(p,n,k) = \binom{n}{k} p^k (1-p)^{(n-k)}. \qquad (2)$$

We report the results for the first decile, the first quartile and the median in Table 1. Results for the other quantile values are available upon request. The analysis is at the individual sample level (without reweighting).

For all quantiles, poverty incidence in all four dimensions and being poor in none of the four dimensions are over-represented in both Belgium and the Netherlands, compared with what would be expected if the different dimensions in which we tried to quantify welfare were independent of one another. Conversely, being poor in only one dimension is under-represented in both countries. This might indicate that the different dimensions of poverty are not independent. However, part of this correlation is spurious, since the observed consumption values play a big role in the calculation of the utility indicator. This can also be seen from the different subcategories of those who are poor in two dimensions, the main subgroup being those who combine consumption poverty with a low level of utility, followed by those who combine income with asset poverty. The connection between consumption poverty and low income or wealth is much weaker than it is between utility and consumption poverty. The extent to which consumption poverty is more closely connected with asset poverty than with income poverty is inconclusive. For less severe poverty lines, consumption and income poverty seem more closely connected than consumption and asset poverty, while the reverse seems to be the case for lower poverty lines. These subsamples are small, however, and the differences so slight that firm conclusions can scarcely be drawn.

TABLE 1. INDIVIDUAL CORRELATION AMONG DIFFERENT MEASURES OF WELFARE

dimension	first decile			first quartile			median		
wealth inc. cons. welf.	freq.(%)		exp. freq. under hyp. of indep. dist.	freq.(%)		exp. freq. under hyp. of indep. dist.	freq.(%)		exp. freq. under hyp. of indep. dist.
country	BE	NL		BE	NL		BE	NL	
poor in 4 dimensions	0.27	0.10	0.01	2.96	2.24	0.39	12.59	14.27	6.25
P P P P	0.27	0.10		2.96	2.24		12.59	14.27	
poor in 3 dimensions	1.75	1.25	0.36	7.21	7.53	4.69	21.42	20.87	25.00
P P P R	0.05	0.03		0.43	0.72		1.72	1.91	
P P R P	0.03	0.00		0.32	0.33		2.13	1.68	
P R P P	1.10	0.62		3.07	2.50		8.13	7.13	
R P P P	0.57	0.59		3.39	3.98		9.45	10.16	
poor in 2 dimensions	6.97	7.13	4.86	14.99	17.69	21.09	24.46	25.38	37.50
P P R R	0.73	0.69		2.37	3.52		6.67	7.66	
P R P R	0.24	0.20		0.43	0.56		1.13	1.18	
P R R P	0.22	0.20		0.81	0.72		1.40	1.28	
R P P R	0.19	0.16		0.89	0.99		1.94	2.10	
R P R P	0.22	0.13		0.75	0.56		1.96	0.79	
R R P P	5.38	5.75		9.74	11.34		11.36	12.36	
poor in 1 dimension	13.59	15.52	29.16	24.41	23.90	42.19	20.80	19.33	25.00
P R R R	4.82	5.06		8.42	8.71		7.10	7.23	
R P R R	6.16	6.31		11.03	10.32		9.77	8.32	
R R P R	1.21	2.37		2.50	2.70		2.37	2.10	
R R R P	1.40	1.78		2.45	2.17		1.56	1.68	
not poor	77.42	76.00	65.61	50.43	48.65	31.64	20.72	20.15	6.25
R R R R	77.42	76.00		50.43	48.65		20.72	20.15	

Also, the differences between Belgium and the Netherlands in terms of the connection between the different dimensions of poverty are small. There is a tendency for the Belgian figures for those who are poor in none of the four dimensions to dominate those in the Netherlands, and poverty incidence in two dimensions seems to be slightly greater in the Netherlands.

These last figures do not allow us to draw any conclusions about the relative intensity of poverty in the two countries. For example, 2.2 to 3.0% of the older people seem to combine an income lower than the first quartile (whose value lies between €10,650 and €14,175 in the Netherlands, and between €8,010 and €11,655 in Belgium) with a low amount of financial wealth (lower than, depending on age, €780 to €3,630 in the Netherlands and €430 to €2,810 in Belgium), a low current consumption (lower than €5,000 to €5,600 or less than €16 of expenditure a day) and poor future consumption prospects. The cut-off levels are almost always lower for wealth, and always lower for income in Belgium than in the Netherlands, while they are higher for consumption. This group, though small in number compared to the total population, undoubtedly suffers from tough material life conditions. Representative samples from the population as a whole do not seem to be the most appropriate tool to use in order to gain more of an insight into the causes and consequences of the poor socio-economic welfare level of these people. They form outliers in the older population as a whole, and should perhaps form the subject of specially targeted studies.

4. Some Properties Of The Welfare Distribution Among The Poor

One of the advantages of quantile regression is that it produces a more detailed picture of the change in the shape of the distribution of the dependent variable at different values of the explanatory variable(s). In this section, some characteristics of the income, financial wealth and consumption distribution are presented. We concentrate on the welfare distribution *among the poor*, meaning that only the poorer half of the population is taken into account here. In Subsection 4.1 the results for quantile-based measures of skewness of the distribution are presented. Skewness measures the extent to which deviations from the central tendency of the distribution are bigger to the left or the right. The natural candidate for a central tendency measure of the poorer half of the distribution is the first quartile value. Therefore, an adapted version of the set of quantile-based measures of skewness, $QSK(p)$, from Hao and Naiman (2007, p.14), reads as:

$$QSK(p) = \frac{Q(.50-p,F)-Q(.25,F)}{Q(.25,F)-Q(p,F)} - 1 \quad \text{for } p \in (0,.25) \tag{3}$$

The measure is normalised such that it is positive if the deviation of the p-th percentile from the first quartile value (which represents a measure for deviations of the lower half of the distribution from the first quartile) is smaller than the deviation of the $(.50-p)$-th percentile (which is a measure for deviations of the upper half of the distribution from the first quartile) and vice versa. When the measure of skewness is positive (negative), the distribution is said to be skewed to the right (left), exhibiting a longer tail to the right (left) in the picture of the density function. Notice that not all quantile-based measures of skewness should agree in sign. A distribution can be skewed to the right on certain (symmetrical) parts of the support, and skewed to the left on other parts.

In Subsection 4.2 a set of quantile-based measures of inequality are presented. The inequality concept that has been used here is defined as follows:

Definition: *A distribution* F *is more unequal than a distribution* G *if there exists a value* $q \in (0,.5)$*, such that* $Q(p,F) < Q(p,G)$ *for all* $p \in (0,q)$ *and*

$Q(p,F) \geq Q(p,G)$ *for all* $p \in [q,.5]$.

Intuitively, one distribution is more equal than another, according to this concept, if there exists a point q in the distribution, such that all quantile values below that point are higher than in the other distribution, and the quantile values above or equal to that point are at least as high in the other distribution. The quantile values of the more equal distribution are thus higher than the corresponding ones for the more unequal distribution, for quantiles p lower than a value q, and do not exceed the corresponding quantiles of the more unequal distribution for p's greater than or equal to q. Therefore, the quantile function $Q(p,G)$ of the more equal distribution, G, intersects that of the more unequal distribution F, $Q(p,F)$, from above, and these curves intersect only once. The intersection point q or any weighted average of the values p lower than or equal to q could serve as a measure of this inequality concept, say $I(F,G)$:

$$I(F,G)=\int_0^q \Big(\chi\big(Q(p,F)-Q(p,G)<0\big)-\chi\big(Q(p,F)-Q(p,G)>0\big)\Big)\alpha(p)dp, \qquad (4)$$

where q is the intersection point and χ is the characteristic function: it assumes the value of 1 when the condition specified in its argument is satisfied, and is equal to 0 otherwise.

If the weighting function $\alpha(p)$ is constant, the associated inequality measure for the more unequal of two distributions is $\alpha \cdot q$, and $-\alpha \cdot q$ for the more equal one.

For a given set of $2T$ estimated quantile values, $\left(\hat{Q}(p_i,F),\hat{Q}(p_i,G);i=1,\ldots,T\right)$, the empirical equivalent to (4) equals:

$$\begin{aligned}\hat{I}(F,G) = & \sum_{i=1}^{k} \alpha(p_i)\,\mathrm{d}p_i \ \text{ if } \hat{Q}(p_i,F) < \hat{Q}(p_i,G) \quad i = 1,\ldots,k, \\ = & -\sum_{i=1}^{k} \alpha(p_i)\,\mathrm{d}p_i \ \text{ if } \hat{Q}(p_i,F) > \hat{Q}(p_i,G) \quad i = 1,\ldots,k,\end{aligned} \tag{5}$$

where the cutting point q lies in the interval $(p_k, p_{k+1}]$, $k < T$, $\mathrm{d}p_i \equiv p_i - p_{i-1}$ and $p_0 \equiv 0$.

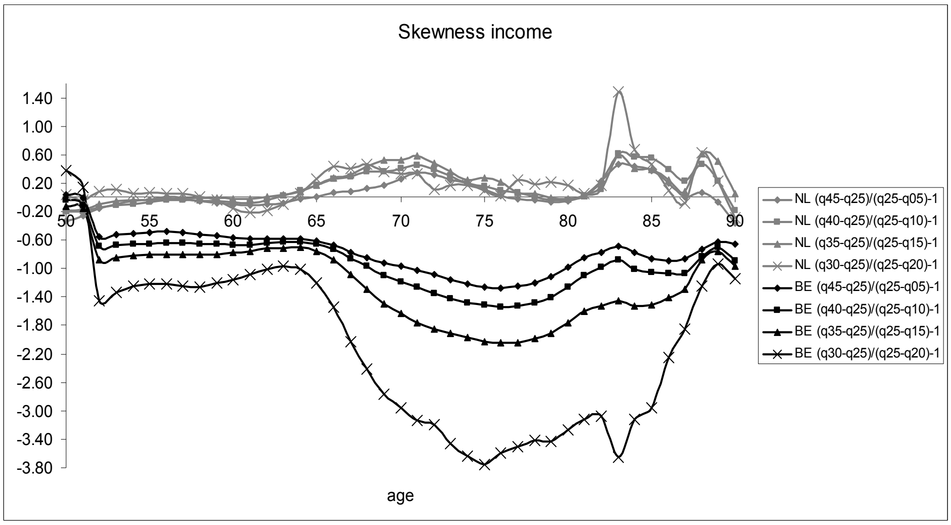
4.1. Quantile-Based Skewness

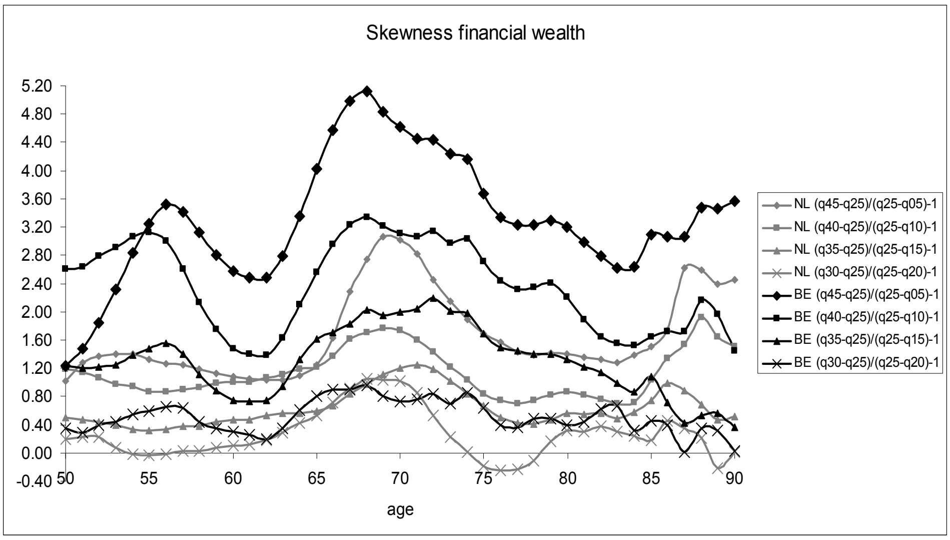
In Figure 3 we report the values of the age-specific measures of skewness as defined in equation (3) for income, financial wealth and consumption and values of p equal to $(0.05, 0.10, 0.15, 0.20)$.

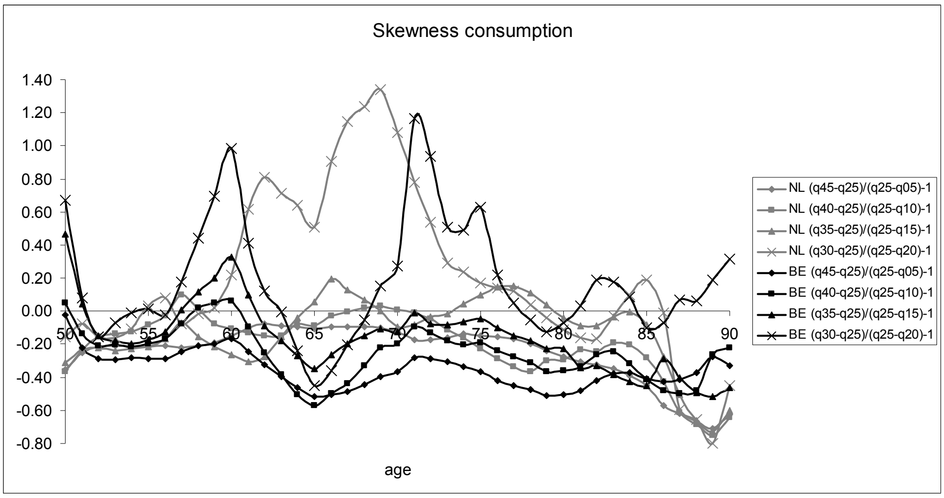
The income distribution among the poorer half of the older population is skewed to the *left* in Belgium, while it is skewed to the right in the Netherlands for people older than 65 years, and more or less symmetrical for those between 50 and 65 years old. This means that in Belgium, the distance between e.g. the first decile and the first quartile value of income is larger than the difference between the 40th percentile and the first quartile, while the reverse holds true for people older than 65 years in the Netherlands. The skewness of the income distribution in the Netherlands for people older than 65 years is greater than for those between 50 and 65 years old.

Even among the poorer half of the older population, the distribution of wealth is skewed to the right, and more so in Belgium than in the Netherlands. The picture is much more diverse for the consumption distribution among the poorer half. For the more extreme quantile values, there is a tendency for the distribution of consumption expenditure to be skewed to the left, irrespective of age. For values close to the first quartile and ages between 60 and 80 years old, the distribution is skewed to the right in the Netherlands, while a reverse tendency towards more skewness to the left seems to hold in Belgium for ages between 60 and 70 years old. In any case the shape of the consumption expenditure distribution seems much less stable across ages than those for incomes and wealth.

FIGURE 3. QUANTILE-BASED SKEWNESS MEASURES







4.2. Quantile-Based Inequality

Skewness and inequality are two different concepts. A perfectly symmetrical distribution may be more unequal than a skewed one, because the effect of a larger distance from the central tendency of the distribution may outweigh the effect of an unequally sized distance from the central tendency point for values below versus those above the central tendency point. Moreover, not all inequality concepts agree whether a poor-to-rich transfer below the central tendency point contributes more or less to inequality than a similar transfer above the central tendency point.

In Table 2, we report results from applying the inequality measure (5) for two sets of weights $\alpha\left(p_i\right)$. The first set (concept 1 in the table) is constant in the quantiles p_i: $\alpha\left(i\times.05\right)\mathrm{d}p_i=1$ for $i=1,2,\ldots,5$. The second set (concept 2 in the table) decreases in those values: $\alpha\left(.05\right)\mathrm{d}p_1=5$ and $\alpha\left(i\times.05\right)\mathrm{d}p_i=-1$ for $i=2,\ldots,5$. We calculated the inequality measures for each age, assigning a value of zero to the measure when the Belgian and Dutch distribution could not be compared according to the inequality concept that we use. Afterwards, the resulting values for Belgium were aggregated across age classes. These aggregated values are reported in the table. A positive value means that the age-specific distributions for a certain age class, in the aggregate, are more unequal in Belgium than in the Netherlands.

TABLE 2. QUANTILE-BASED INEQUALITY MEASURES

dimension	Income	wealth	consumption
age class	concept 1		
50-64	-26	-2	18
65-74	-2	5	10
75-84	0	41	8
50-90	-28	44	36
	concept 2		
50-64	-40	-10	36
65-74	-4	1	26
75-84	0	13	28
50-90	-44	4	90

Despite the age-specific income distributions being more symmetrical in the Netherlands than in Belgium, especially in the younger part of the population (50-64 and 65-74 years old), they are more unequal too, irrespective of the weights we used. This confirms results from Cantillon, Lefebure and Van den Bosch (2009) on the basis of SILC (the Survey on Income and Living Conditions), for a different income concept (net incomes), a completely different inequality concept (Atkinson inequality index) and a differently defined population (all older people instead of the poorer half). For the older part of the population (75-84 years old), the income distributions in Belgium and the Netherlands are not comparable in terms of inequality. Moreover, as far as only the lower part of the distribution is concerned, we could *not* confirm the conjecture made in Section 2 about the *overall* income distributions in Belgium being more unequal than in the Netherlands.

Even though the Belgian age-specific wealth distributions are almost unequivocally to be considered as more skewed to the right than their Dutch counterparts, the wealth distributions of the younger part of the population (50-64 years) in Belgium are less unequal than in the Netherlands.

Finally, despite the good performance of Belgium in comparison with the Netherlands as far as non-durable consumption expenditure of older people is concerned, inequality among poorer people in terms of consumption expenditure is higher in Belgium than in the Netherlands. Again, this may indicate that there is a small fraction of the population that suffers from severe deficiencies in terms of their ability to fulfil daily consumption needs, but that risks dropping out of the picture of large datasets, because their number is so small.

5. Conclusion

On the basis of the SHARE data, we calculated age-conditional quantiles at the 5th, 10th, 15th, 20th, 25th and 50th percentile levels for equivalised gross income, net financial assets and consumption levels, as well as for a (remaining) lifetime utility indicator of people over 50 years old in Belgium and the Netherlands. We conclude that the poor performance of Belgium in terms of older people's income, compared with the Netherlands, is mitigated when the consumption figures are considered, while the picture is more diverse in terms of wealth figures: the median net financial assets position is almost always higher in Belgium than that in the Netherlands, while their relative ranking on the basis of the first quartile value reverses for many ages. In terms of lifetime utility, the two countries do equally well. Generally, ranking reversals occur when one country is better for the younger part of the population than for the older and vice versa. In none of the dimensions do such ranking reversals seem to occur prominently in our dataset, except possibly for the poorest 5th percentile level.

How is it possible for people who are poorer in terms of income to do better in terms of consumption? One possible explanation stems from the life cycle hypothesis about saving behaviour: possibly older Belgian people have succeeded, for whatever reason, in saving more during an earlier phase of the life cycle, so that they can guarantee a higher consumption level than their current income would indicate. The high median value of financial wealth might confirm this. The wealth distribution is very unequal, so that the savings hypothesis might not hold for the poorer people. Over 2% of the households with older people in Belgium spend more on non-durable goods than they currently earn, while having no or a negative amount of financial assets. In the Netherlands, this figure is 1.7%. On the other hand, there is no clear evidence to say that consumption poverty is correlated with wealth, or with income poverty. Even so, there is a small fraction of older people who face deficiencies in consumption, have low incomes and have accumulated hardly any or even negative savings from the past. The extent to which this occurs more often in Belgium than in the Netherlands, depends on the poverty lines used in the different dimensions. There is also no clear evidence that people with low incomes in Belgium have accumulated a larger stock of financial wealth to finance their current expenditure than in the Netherlands.

A larger part of household savings in Belgium does however occur in the form of house ownership, and more older people live in owner-occupied houses without outstanding loan charges. This might help explain, more than financial wealth, why older Belgian people can spend a larger part of their income on current consumption goods than their peers in the Netherlands.

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APPENDIX

The age-conditional quantile values were estimated by means of nonparametric quantile regressions as proposed by Yu and Jones (1998). They start from the basic idea behind quantile regressions (Koenker and Bassett, 1978) that the p-th quantile of a(n absolutely continuous) distribution (function) F_Y, denoted by $Q\left(p, F_Y\right) \equiv F_Y^{-1}\left(p\right)$, can be expressed as the result of a linear programming problem:

$$Q\left(p,F_Y\right) = \underset{a\in\mathbf{R}}{\operatorname{argmin}} \; \mathrm{E}_{F_Y} \left\{(1-p)\left|Y-a\right|\chi\left(Y\leq a\right)+p\left|Y-a\right|\left(1-\chi\left(Y\leq a\right)\right)\right\}, \qquad \text{(A1)}$$

where $\chi\left(\cdot\right)$ is the characteristic function: it assumes the value of 1 when the condition specified in its argument is satisfied, and is equal to 0 otherwise.

Indeed, the right hand side of equation (A1) can be rewritten as:

$$p\int_a^{\infty}\left(y-a\right)dF_Y\left(y\right)-\left(1-p\right)\int_{-\infty}^{a}\left(y-a\right)dF_Y\left(y\right).$$
(A2)

Taking derivatives with respect to a, yields:

$$F_Y\left(a\right)-p. \qquad \text{(A3)}$$

Putting this expression equal to zero,

$$F_Y\left(a_p^*\right)=p, \qquad \text{(A4)}$$

yields the minimum of the objective function in equation (A1), since the second derivative with respect to a of (A2) is the density of Y, $F_Y'\left(a\right)\equiv f_Y\left(a\right)$, and this is a non-negative function, and thus the problem is convex. Consequently:

$$a_p^*=F_Y^{-1}\left(p\right)=Q\left(p,F_Y\right). \qquad \text{(A5)}$$

Suppose now that a set of variables $\left(Y,X\right)$ is jointly distributed according to the distribution function $F_{Y,X}$. The associated conditional distributions of Y for given values x of X are defined as $F_{Y|X}\left(y|x\right)=\dfrac{\int_{-\infty}^{y}f_{Y,X}\left(u,x\right)du}{\int_{-\infty}^{\infty}f_{Y,X}\left(u,x\right)du}$, where $f_{Y,X}\left(y,x\right)\equiv\dfrac{\partial^2F_{Y,X}\left(y,x\right)}{\partial y\partial x}$, is the joint density of Y and X. Notice that the denominator of the conditional distribution, $\int_{-\infty}^{\infty} f_{Y,X}(u,x)\,du$ equals the unconditional density of X, denoted in the sequel as f_X. $F_{Y|X=x}$ will be used as a shorthand for the functional $F_{Y|X}$ evaluated at the point

$x : F_{Y|X=x}(y) = \dfrac{\int_{-\infty}^{y} f_{Y,X}(u,x)\,du}{f_X(x)}$.

By an analogous reasoning as for the unconditional quantile value, the conditional p-th quantile value, for a given value x of X can then be defined as:

$$\begin{aligned} Q\left(p,F_{Y|X=x}\right) &= \underset{a\in\mathbf{R}}{\operatorname{argmin}}\ \mathrm{E}_{F_{Y|X=x}}\left\{(1-p)|Y-a|\chi(Y\le a)+p|Y-a|\left(1-\chi(Y\le a)\right)\right\} \\ &= \underset{a\in\mathbf{R}}{\operatorname{argmin}}\left[p\int_a^{\infty}(y-a)\,dF_{Y|X=x}(y)-(1-p)\int_{-\infty}^{a}(y-a)\,dF_{Y|X=x}(y)\right], \end{aligned} \tag{A6}$$

where $dF_{Y|X=x}$ is the conditional density of Y for a given value

x of $X : dF_{Y|X=x}(y) = \dfrac{f_{Y,X}(y,x)}{f_X(x)}$.

For a set of observations, $(y_i, x_i)_{i=1}^{n}$, an empirical counterpart to formula (A6) can be formulated by using the Nadaraya-Watson estimator for conditional expectations:

$$\hat{Q}\left(p,F_{Y|X=x}\right) = \underset{a\in\mathbf{R}}{\operatorname{argmin}} \sum_{i=1}^{n}\left[(1-p)|y_i-a|\chi(y_i\le a)+p|y_i-a|\left(1-\chi(y_i\le a)\right)\right]\frac{K\left(\frac{x_i-x}{h_p}\right)}{\sum_{j=1}^{n}K\left(\frac{x_j-x}{h_p}\right)}, \tag{A7}$$

where K is a kernel and h_p is the bandwidth of the kernel.[11] We used the Gaussian

[11] A nonparametric estimator of the density f_X is $\hat{f}_X(x) = (nh)^{-1}\sum_{j=1}^{n} K\left(\frac{x_j - x}{h}\right)$ and thus the Nadaraya-Watson estimator implicitly uses $\hat{f}_{Y,X}(y_i,x_i) = (nh)^{-1} K\left(\frac{x_i-x}{h}\right)$ as an estimate of $f_{Y,X}(y_i,x_i)$.

kernel $K(u) = \frac{\exp\left(-\frac{u^2}{2}\right)}{\sqrt{2\pi}}$. For the bandwidth we used $h_p = \bar{h}\frac{p(1-p)}{\left(\varphi\left(\Phi^{-1}(p)\right)\right)^2}$, where φ and Φ are respectively the standard normal density and distribution function, and

$$\bar{h} = 1.06\min\left(\sigma_X, .75\left(Q\left(.75, F_X\right) - Q\left(.25, F_X\right)\right)\right)n^{-0.2},$$

where σ_X is the standard deviation of X.

A local linear approximation to the conditional quantile yields:

$$Q\left(p, F_{Y|X=x_i}\right) = Q\left(p, F_{Y|X=x}\right) + Q'\left(p, F_{Y|X=z}\right)(x_i - x) \equiv a + b(x_i - x)$$
$$\text{for some } z \in \left[\min(x_i, x), \max(x_i, x)\right].$$

(A8)

Adapting the nonparametric conditional quantile estimator (A7) for such a local linear approximation of the conditional quantile[12], we get (irrelevant constants are dropped):

$$\left(\hat{a}_p, \hat{b}_p\right) \equiv \underset{a,b\in\mathbf{R}}{\operatorname{argmin}} \tag{A9}$$
$$\sum_{i=1}^{n}\left[(1-p)\left|y_i - a - b(x_i - x)\right|\chi\left(y_i \le a + b(x_i - x)\right) + p\left|y_i - a - b(x_i - x)\right|\left(1 - \chi\left(y_i \le a + b(x_i - x)\right)\right)\right]K\left(\frac{x_i - x}{h_p}\right),$$

and

$$\hat{Q}\left(p, F_{Y|X=x}\right) = \hat{a}_p. \tag{A10}$$

The only novelty we introduce here, is the reformulation of the above problem such that a package for linear quantile regression can be used, to estimate the conditional quantiles.[13] More specifically, we estimated the following linear quantile

[12] In equation (A7) the p -th quantile is assumed to be locally constant. Hence, the proposal in (A8) (stemming from Yu and Jones, 1998) is more flexible.

[13] Given a set of observations $\left(y_i, \mathbf{x}_i\right) \in \mathbf{R}^{1\times k}$ for $i = 1, \ldots, n$, linear quantile regressions derive conditional quantile estimators from solving the following problem:

$$\hat{\mathbf{b}}_p \equiv \underset{\mathbf{b}\in\mathbf{R}^k}{\operatorname{argmin}}\sum_{i=1}^{n}\left[(1-p)\left|y_i - \mathbf{x}'_i\mathbf{b}\right|\chi\left(y_i \le \mathbf{x}'_i\mathbf{b}\right) + p\left|y_i - \mathbf{x}'_i\mathbf{b}\right|\left(1 - \chi\left(y_i \le \mathbf{x}'_i\mathbf{b}\right)\right)\right], \tag{A11}$$

and applying the rule:

$$\hat{Q}\left(p, \mathbf{x}_i\right) = \mathbf{x}'_i\hat{\mathbf{b}}_p. \tag{A12}$$

We used the experimental QUANTREG procedure running under sas 9.1. The R-function `qr` developed by Koenker (see Koenker, 2005, Appendix A) yielded similar coefficient estimates, but the inversion of rank-based confidence intervals for the estimated coefficients differed substantially at some instances in regressions:

$$\left(\hat{a}_p,\hat{b}_p\right) \equiv \underset{a,b\in\mathbf{R}}{\operatorname{argmin}} \tag{A13}$$

$$\sum_{i=1}^{n}\left[(1-p)\left|\tilde{y}_i - a\tilde{x}_{i,1} - b\tilde{x}_{i,2}\right|\chi\left(\tilde{y}_i \le a\tilde{x}_{i,1}+b\tilde{x}_{i,2}\right)+p\left|\tilde{y}_i - a\tilde{x}_{i,1} - b\tilde{x}_{i,2}\right|\left(1-\chi\left(\tilde{y}_i \le a\tilde{x}_{i,1}+b\tilde{x}_{i,2}\right)\right)\right],$$

where $\tilde{y}_i \equiv y_i K\left(\dfrac{x_i - x}{h_p}\right)$, $\tilde{x}_{i,1} \equiv K\left(\dfrac{x_i - x}{h_p}\right)$, and $\tilde{x}_{i,2} = \left(x_i - x\right)K\left(\dfrac{x_i - x}{h_p}\right)$.

The estimated coefficient associated with explanatory variable $\tilde{x}_{i,1}$ is to be interpreted as the conditional quantile estimator, i.e. $\hat{Q}\left(p, F_{Y|X=x}\right) = \hat{a}_p$ continues to hold. In our application, the explanatory variable x_i is the age of the person at the moment for which the dependent variables y_i (income, wealth, consumption and lifetime utility) are registered.

The regressions we performed by no means claim to test any behavioural response of welfare, consumption, income or wealth with respect to age. We only want to give a descriptive picture of the age-specific distributions of those variables, for reasons set out in the introductory section. Therefore we used a weighted version of regression (A13). This amounts to multiplying the triple $\left(\tilde{y}_i, \tilde{x}_{i,1}, \tilde{x}_{i,2}\right)$ with sample weight w_i provided by the data agency. Sample weights are designed to extrapolate the sample data to the population of people over 50 years old.

the two procedures.