THE EXCHANGE RATE IN A DYNAMIC-OPTIMIZING
BUSINESS CYCLE MODEL WITH NOMINAL RIGIDITIES:
A QUANTITATIVE INVESTIGATION

Robert Kollmann (*)
University of Paris XII
December 4, 2000

This paper studies a quantitative dynamic-optimizing business cycle model of a small open economy with staggered price and wage setting. The model exhibits exchange rate overshooting in response to money supply shocks. The predicted variability of the nominal and, especially, of the real exchange rate is noticeably higher than in standard Real Business Cycle models with flexible prices and wages. A positive domestic money supply shock is predicted to lower the domestic interest rate, raise GDP, and trigger a depreciation of both the nominal and real exchange rate. Increases in domestic productivity and in the world interest rate are also predicted to induce a nominal and real exchange rate depreciation.

JEL classification: F31, F32, E32.

Key words: exchange rates; nominal rigidities; business cycles.

(*) Department of Economics; University of Paris XII
61, Av. du Général de Gaulle; F-94010 Créteil Cedex; France
e-mail: robert_kollmann@yahoo.com

I thank three anonymous referees for constructive comments. I am indebted to C. Erceg for advice at the initial stage of this project and thank P. Beaudry, V.V. Chari, M. Devereux, H. Hau, O. Jeanne, P. Kehoe, M. Kiley, as well as workshop participants at several Universities and conferences for useful discussions. This project was supported by grants from SSHRC (Canada), the HCM program (EU Comission), and INQUIRE.
1. Introduction

The nominal and real exchange rates of major currencies against the U.S. dollar are highly volatile. Also, nominal and real exchange rates are strongly positively correlated. For example, the standard deviations of Hodrick-Prescott (HP) filtered (logged) quarterly nominal and real exchange rates of Japan, Germany, and the U.K. (G3, henceforth) vis-à-vis the U.S. were roughly 9% during the post-Bretton Woods era, compared to standard deviations of GDP of about 1.5%. The correlation between HP filtered nominal and real U.S. dollar exchange rates was about 0.97 for these countries.

In an attempt to explain key features of international macroeconomic data, much effort has recently been devoted to developing quantitative open economy business cycle models with explicit microfoundations. Following the Real Business Cycle (RBC) approach, this work has generally considered models without money or models in which money is (basically) neutral, since prices and wages are assumed to be fully flexible. In these models, productivity shocks are the main source of economic fluctuations. (See Backus et al. (1995) for a survey of that literature.) One striking limitation of these models is that they tend to generate a predicted variability of the nominal and, particularly, the real exchange rate that is much too small when compared to actual data for periods with floating exchange rates. For example, in Schlagenhauf and Wrase's (1995) monetary model with flexible prices and wages, the predicted standard deviations of exchange rates are five to ten times smaller than historical standard deviations of G3/U.S. exchange rates. Non-monetary models generate standard deviations of (real) exchange rates that are even smaller (see, for
example, Backus et al. (1995)).

The present paper studies a quantitative dynamic-optimizing business cycle model of a small open economy in which nominal prices and wages are sticky. Overlapping price and wage contracts, à la Calvo (1983), are assumed. The average interval between price and wage changes, at the micro-economic level, is set at four quarters, consistent with empirical evidence on price and wage adjustment. A flexible exchange rate and four types of exogenous shocks are assumed: shocks to the domestic money supply, domestic productivity, the foreign price level, and the foreign interest rate. The model is calibrated to post-Bretton Woods data for the G3 countries.

Predicted standard deviations of the nominal and, particularly, of the real exchange rate are noticeably higher—and, hence, closer to the data—in the nominal rigidities structure considered here, compared to a structure with flexible prices and wages. The nominal rigidities structure captures 40 to 50 percent of the historical standard deviations of nominal and real G3/U.S. exchange rates during the post-Bretton Woods era. It also generates improved predictions for other business cycle statistics: the predicted correlation between the nominal and the real exchange rate is markedly higher (and closer to the data) than when flexible prices and wages are assumed. In addition, the structure captures more closely the historical variability of GDP, consumption, and the nominal interest rate.

The nominal rigidities model predicts that an increase in the domestic money supply induces a sizable rise in domestic GDP, a depreciation of the country's currency, and a decline in the domestic interest rate. On impact, the nominal exchange rate overshoots its long-run
response. Owing to the sluggishness of the price level, the nominal exchange rate depreciation produces, on impact, an almost equi-proportional real depreciation. By contrast, in a version of the model without nominal rigidities, money supply shocks have a negligible effect on the real exchange rate (and other real variables), and there is no overshooting of the nominal exchange rate. The nominal rigidities model also predicts that increases in domestic productivity and the foreign interest rate induce a nominal and real currency depreciation.

By assuming nominal rigidities, the model builds on Keynesian open economy models developed during the 1960s and 1970s (for example, Mundell (1968) and Dornbusch (1976)). However, these models lack the explicit micro-foundations for the private sector’s consumption, investment, and production decisions that characterize the dynamic-optimizing approach adopted here.

The present paper is also related to Obstfeld and Rogoff’s (1995) widely discussed dynamic-optimizing open economy model, in which nominal prices are fixed in the short run, as firms are assumed to set their prices one period in advance. However, these authors’ analysis is entirely qualitative and their model is highly stylized—they consider an economy without physical capital and without uncertainty (except for one-time unanticipated shocks), in which the Law of One Price holds and the real exchange rate is constant. That model also seems unable to generate sufficient nominal exchange rate volatility.\(^1\) Owing to one-period price stickiness, it generates very simple dynamics: for example, after a permanent money supply shock, the economy is predicted to adjust to its new long-run equilibrium in a single period.

In contrast, the present paper develops a quantitative (calibrated)
stochastic business cycle model with physical capital, multi-period price and wage setting, and deviations from the Law of One Price. It predicts a gradual adjustment of prices to a money supply increase—which empirically seems more realistic—and, hence, a persistent increase in real balances and a persistent reduction in the nominal interest rate, nominal exchange rate overshooting, and a sizable real exchange rate response.

More recently, other papers have also studied dynamic-optimizing open economy models with nominal rigidities—see Lane (1999) for a detailed survey. Most of that research builds rather closely on the basic Obstfeld-Rogoff (1995) framework (prices set one period in advance, no capital), and offers only limited quantitative results. A contribution by Betts and Devereux (2000) shows that pricing to market (PTM) behavior by firms (limited pass-through of exchange rate movements into prices due to local currency price setting) increases nominal and real exchange rate volatility, compared to a setting where the Law of One Price holds. Given the empirical rejection of the Law of One Price (for example, Knetter (1993)), the present paper also assumes PTM. Stochastic extensions of the Obstfeld-Rogoff (1995) analysis are considered by Obstfeld and Rogoff (2000) and Engel and Devereux (2000). Based on a highly stylized structure, these authors derive exact closed-form model solutions that are used to study the welfare effects of alternative monetary policy regimes. In contrast, I here consider a richer business cycle model that is solved numerically.

The methodology used here builds on recent quantitative dynamic general equilibrium models of closed economies with sticky prices or wages. See, for example, Rotemberg and Woodford (1997) and Erceg et al. (2000). Quantitative two-country models with multi-period price stickiness have
recently been considered by Betts and Devereux (1998) and Chari et al. (1998). The present paper differs from these studies, inter alia by using a model of a small open economy with price and wage stickiness.\(^2\)

Section 2 discusses the model. Sect. 3 reports macroeconomic stylized facts for the G3. Sect. 4 presents simulation results. Sect. 5 concludes.

2. The model

I consider a small open economy with a representative household, firms, and a government that issues a national currency. The country produces a single non-tradable final good and a continuum of tradable intermediate goods indexed by \( s \in [0,1] \): it imports a continuum of foreign intermediate goods, also indexed by \( s \in [0,1] \). Domestic and foreign intermediate goods are used by perfectly competitive firms to produce the final good; the latter is consumed and used for investment. There is monopolistic competition in intermediate goods markets--each intermediate good is produced or imported by a single firm. Intermediate goods producers use domestic capital and labor as inputs--capital and labor are immobile internationally. The household owns all domestic firms and the capital stock, which it rents to firms. The rental market for capital is competitive. Capital can be moved across firms without cost. The household supplies a continuum of differentiated labor services, indexed by \( h \in [0,1] \); it acts as a wage setter.

2.1. Final good production

The final good is produced using the aggregate technology

\[
Z_t = \left( \alpha^d \right)^{1/\varphi} \left( Q^d_t \right)^{(\theta-1)/\varphi} + \left( \alpha^m \right)^{1/\varphi} \left( Q^m_t \right)^{(\theta-1)/\varphi} \left( Q^m_t \right)^{1/\varphi},
\]

with \( \alpha^d > 0, \alpha^d + \alpha^m = 1, \varphi > 0 \). \( Z_t \) is final good output at date \( t \); \( Q^d_t, Q^m_t \) are
quantity indices of domestic and imported intermediate goods, respectively:

$$Q_t^i = (\int_0^1 q_t^i(s)^{\nu-1}/\nu ds)^{\nu/(\nu-1)}$$

with $\nu > 1$, for $i = d, m$, where $q_t^d(s)$ and $q_t^m(s)$ are quantities of the domestic and imported type 's' intermediate goods. Let $\rho_t^d(s)$ and $\rho_t^m(s)$ be the prices of these goods, in domestic currency.

Cost minimization in final good production implies:

$$q_t^i = (\rho_t^i)^{-\nu} Q_t^i, \quad Q_t^i = \alpha_1 (P_t^d / P_t^m)^{-\theta} Z_t$$

for $i = d, m$, (2)

with $P_t^{d} = \int_0^1 (\rho_t^d(s))^{1-\nu} ds$ and $P_t^{m} = \int_0^1 (\rho_t^m(s))^{1-\nu} ds$.

$P_t^{d}$ and $P_t^{m}$ are price indexes for domestic [imported] intermediate goods.

Perfect competition in the final good market implies that the good's price is $P_t$ (its marginal cost is $\alpha_1 (P_t^{d1-\theta} + \alpha_m (P_t^{m1-\theta})^{1/(1-\theta)})$).

2.2. Intermediate goods firms

The technology of the firm that produces domestic intermediate good 's' is:

$$y_t(s) = \theta_t(K_t(s))^{\psi}(L_t(s))^{1-\psi}, \quad 0 < \psi < 1.$$ (4)

$y_t(s)$ is the firm's output at date $t$; $\theta_t$ is an exogenous productivity parameter that is identical for all domestic intermediate goods producers; $K_t(s)$ and $L_t(s)$ are the capital stock and an index of the labor types used by the firm: $L_t(s) = (\int_0^1 \ell_t(h; s)^{(\gamma-1)/\gamma} dh)^{\gamma/(\gamma-1)}$, with $\gamma > 1$, where $\ell_t(h; s)$ is the quantity of type $h$ labor.

Let $R_t$ and $w_t(h)$ be the rental rate of capital and the wage rate for type $h$ labor. Cost minimization conditions for the firm can be written as:

$$\ell_t(h; s) = (1-\psi)\psi^{-1} w_t(h)^{-\gamma}(W_t)^{-1} R_t K_t(s), \quad \text{with} \quad W_t = \int_0^1 (w_t(h))^{1-\gamma} dh)^{1/(1-\gamma)}.$$ (5)

The firm's marginal cost is: $\xi_t = (1/\theta_t)(R_t)^{\psi} (W_t)^{-\psi} (1-\psi)^{(1-\psi)}$. (6)

The firm's good is sold in the domestic market and exported: $y_t(s) = q_t^d(s) + q_t^X(s)$, where $q_t^d(s)$ [$q_t^X(s)$] is domestic [export] demand. The export demand function is assumed to resemble the domestic demand function (2):
\[ q_t^X = (\rho_t^X(s)/\bar{p}_t^X)^{-\nu} Q_t^X, \quad \text{with} \quad Q_t^X = (p_t^*/\bar{p}_t^*)^{-\eta}, \quad \eta > 0, \]  
(7)

where \( \rho_t^X(s) \) is the firm's export price, in foreign currency, while

\[ Q_t^X = \left( \int_0^1 (q_t^X(s))^{(\nu-1)/\nu} ds \right)^{1/(\nu-1)}, \quad P_t^* = (\int_0^1 (\rho_t^X(s))^{1-\nu} ds)^{1/(1-\nu)} \]  
(8)

are a quantity index and a price index for the country's exports. \( P_t^* \) is the foreign price level and also represents the purchase price of foreign intermediate goods paid by domestic importers; \( P_t^* \) is exogenous.

The profits of a domestic intermediate good producer, \( \pi_t^{dx} \), and of an intermediate good importer, \( \pi_t^{m} \), are:

\[ \pi_t^{dx}(\rho_t^d(s), \rho_t^X(s)) = (\rho_t^d(s)/\bar{p}_t^d)^{-\nu} Q_t^d + (e_t \rho_t^X(s)/\bar{p}_t^X)^{-\nu} Q_t^X, \]
\[ \pi_t^{m}(\rho_t^m(s)) = (\rho_t^m(s) - e_t P_t^*) (\rho_t^m(s)/\bar{p}_t^m)^{-\nu} Q_t^m, \]

where \( e_t \) is the nominal exchange rate, expressed as the domestic currency price of foreign currency.

Motivated by the empirical failure of the Law of One Price, and in particular by widespread pricing-to-market behavior (e.g., Knetter (1993)), it is assumed that intermediate goods firms can price discriminate between domestic and foreign markets (\( \rho_t^d(s) \neq \rho_t^X(s) \) is possible), and that they set prices in terms of the currencys of their customers.

There is staggered price setting, à la Calvo (1983): domestic intermediate goods firms cannot change prices, in buyer currency, unless they receive a random "price-change signal". The probability that the price (in buyer currency) of a given intermediate good can be changed in any particular period is \( 1-\delta \), a constant. Thus, the mean price-change-interval is \( 1/(1-\delta) \). Firms are assumed to meet the demand for their good, at the posted price, until a new "price-change signal" is received.

Consider an intermediate good producer that, at \( t \), sets a new price in the domestic market, \( \rho_{t,t}^d \). With probability \( \delta \), \( \rho_{t,t}^d \) is still in force
at \( t+\tau \). The firm sets
\[
\rho_{t,t}^d = \text{Arg Max}_{\rho} \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \rho_{t,t+\tau} \pi_{t+\tau} (s) \} / P_{t+\tau},
\]
where \( \rho_{t,t+\tau} \) is a pricing kernel (for valuing date \( t+\tau \) pay-offs) that is assumed to equal the household's intertemporal marginal rate of substitution in consumption: \( \rho_{t,t+\tau} = \beta^U_{t+\tau} / U_{t+\tau}^C \), where \( U_{t+\tau}^C \) is the household's marginal utility of consumption at \( t+\tau \) (household preferences are described in Section 2.3). Let \( E_{t,t+\tau}^{i} = \rho_{t,t+\tau} Q_{t+\tau}^i (\nu P_{t+\tau})^{\nu} / P_{t+\tau} \), for \( i=d,x,m \). The solution of the firm's maximization problem regarding \( \rho_{t,t}^d \) is:
\[
\rho_{t,t}^d = (s/(s-1)) \left( \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} / \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} \right).
\]

Analogously, an intermediate good producer and an intermediate good importer that get to choose a new export price/new sales price in the domestic market set these prices at, respectively:
\[
\rho_{t,t}^d = (s/(s-1)) \left( \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} / \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} \right),
\]
\[
\rho_{t,t}^m = (s/(s-1)) \left( \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} / \sum_{\tau=0}^{\tau=0} \mathbb{E}_t \{ \tau_{t,t+\tau} \} \right).
\]

(9)-(11) imply that, up to a certainty-equivalent approximation, prices set at \( t \) equal a weighted average of current and expected future marginal costs (or foreign purchase prices), multiplied by the markup factor \( s/(s-1) > 1 \).

The price indices \( p_{t,t}^{d}, p_{t,t}^{m}, p_{t,t}^{x} \) (see (3), (8)) evolve according to:
\[
(p_{t,t}^{i})^{1-s} = \delta (p_{t-1,t}^{i})^{1-s} + (1-\delta) (\rho_{t,t}^{i})^{1-s}, \quad i=d,m,x.
\]

2.3. The representative household

The preferences of the representative household are described by:
\[
E_0 \sum_{t=0}^{\tau=0} \beta^t U(C_t, M_t / P_t, S_t^1(h) dh).
\]

\( E_0 \) denotes the mathematical expectation conditional on date \( t=0 \) information. \( 0 < \beta < 1 \) is a subjective discount factor, and \( U \) is a utility function. \( C_t \) is period \( t \) consumption, \( M_t \) is the household's stock of domestic money at the beginning of \( t \), and \( l_t(h) \) is the amount of type \( h \) labor provided by the
household \( (h \in [0,1]) \). The utility function \( U \) takes the following form:

\[
U(C,M/P,\int f(h) dh) = (1-\psi)^{-1} \{ [C^\sigma + \kappa (M/P)^\Gamma]^{1/\sigma} \}^{1-\psi} - \int f(h) dh.
\]

As indicated earlier, the household owns all domestic firms and accumulates physical capital. The law of motion of the capital stock is:

\[
K_{t+1} + \phi(K_{t+1}, K_t) = K_t (1-\delta) + I_t,
\]

where \( I_t \) is gross investment, \( 0 < \delta < 1 \) is the depreciation rate of capital, and \( \phi \) is an adjustment cost function: \( \phi(K_{t+1}, K_t) = 0.5 \Phi (K_{t+1} - K_t)^2 / K_t, \Phi > 0 \).

The household also holds domestic money and nominal one-period domestic and foreign currency bonds. Its period \( t \) budget constraint is:

\[
M_{t+1} + A_{t+1} + e_t B_{t+1} + P_t (C_t + I_t) = M_t + T_t + A_t (1+1_{t-1}^*) + e_t B_t (1+1_{t-1}^*) + \nonumber
\]

\[
R_t K_t + \int_0^1 (\pi_t^d(s) + \pi_t^e(s)) ds + \int_0^{1} \int_0^1 w_t(h) e_t(h;s) dh ds.
\]

\( A_t \) and \( B_t \) are net stocks of domestic and foreign currency bonds that mature in period \( t \). \( 1_{t-1}^* \) and \( 1_{t-1}^* \) are the nominal interest rates on these bonds.

The foreign rate, \( i^* \), is exogenous. \( T_t \) is a government cash transfer. The last two terms in (15) are the household's dividend and labor income.

There is staggered wage setting by the household, subject to the constraint that the wage rate for labor of a given type can be changed only when a random "wage-change signal" is received for that type; at any given date, the probability of receiving this signal is \( 1-\Omega \), a constant. Let \( w_{t,t} \) be the wage set at \( t \). The date \( t \) wage for type \( h \) labor, \( w_t(h) \), equals the wage set the last time (up to \( t \)), for that type:

\[
w_t(h) = w_{\tau(h,t), \tau(h,t)}, \quad \text{with } \tau(h,t) = \text{Max } \{ \tau: s_{\tau}(h) = 1, \tau \leq t \},
\]

where \( s_{\tau}(h) \in \{0,1\} \) is an i.i.d. random variable, with \( \text{Prob}(s_{\tau}(h) = 1) = 1-\Omega \) (a wage change for type \( h \) labor can occur at date \( \tau \) iff \( s_{\tau}(h) = 1 \)).

The household is assumed to take the average wage \( \bar{w} \) as given when setting \( w_{t,t} \) and to always meet the demand for each labor type, at the
prevailing wage:
\[ l_t(h) = \int_0^1 \ell_t(h,s) ds, \]
where \( l_t(h) \) is the amount of type \( h \) labor provided by the household (see (13)), while \( \int_0^1 \ell_t(h,s) ds \) is total demand for type \( h \) labor by firms (see (5)).

The household chooses a strategy \( \{A_{t+1}, B_{t+1}, M_{t+1}, K_{t+1}, C_t, W_{t, t'} \}^{t=\omega}_{t=0} \) to maximize its expected lifetime utility (13), subject to constraints (14)-(17) and to initial values \( A_0, B_0, M_0, K_0, \{W_{t, t'} \}^{t=\omega}_{t=0} \). Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problem:

\[ 1 = (1+i_t) \beta E_t\{(U_{C, t+1}/U_{C, t}) (P_{t}/P_{t+1}) \}, \]
\[ 1 = (1+i_t) \beta E_t\{(U_{C, t+1}/U_{C, t}) (P_{t e_t+1}/P_{t+1} e_t) \}, \]
\[ 1 = \beta E_t\{(U_{C, t+1}/U_{C, t}) (K_{t+1}/P_{t+1} + 1-\delta - \phi_{2, t+1})/(1 + \phi_{1, t}) \}, \]
\[ E_t\{(M_{t+1}/P_{t+1})^{1-\sigma} (U_{C, t+1}/P_{t+1}) \} \gamma \Gamma/\sigma = 1 \]
\[ E_t\{(C_{t+1})^{1-\sigma} (U_{C, t+1}/P_{t+1}) \} \Gamma - 1 \]
\[ \frac{\kappa \Gamma/\sigma}{E_t\{(U_{C, t+1}/P_{t+1}) \}} = 1 \]
\[ E_t\{U_{C, t+1}/P_{t+1} \}, \]
\[ w_{t, t'} = (\gamma/(\gamma-1)) \sum_{t=0}^{\infty} (\beta^t) \chi_{t+\tau, t+\tau} = (\gamma/(\gamma-1)) \sum_{t=0}^{\infty} (\beta^t) \chi_{t+\tau, t+\tau} \]
\[ \chi_{t+\tau, t+\tau} \equiv (1-\psi) \psi^{-1}(W_{t, t'})(\gamma-1) K_{t+\tau, t+\tau} \int_0^1 K_{s, t+\tau} ds, \]
\[ \phi_{1, t} \equiv \phi(K_{t+1}, K_t)/\partial K_{t+1} \]
\[ \phi_{2, t} \equiv \phi(K_{t+2}, K_{t+1})/\partial K_{t+1}. \]

(18)-(20) are Euler conditions, (21) can be viewed as a money demand condition, and (22) determines the contract wage, \( w_{t, t'} \). The wage index \( W_t \) (see (5)) evolves according to:

\[ (W_t)^{1-\gamma} = D (W_{t-1})^{1-\gamma} + (1-D) (W_{t, t'})^{1-\gamma}. \]

2.4. Government

The government prints the local currency. Let \( M_t \) be the money stock at beginning of date \( t \). \( M \) is exogenous. Increases in \( M \) are paid to the household, as a transfer (T):

\[ M_{t+1} = M_t + T_t. \]
2.5. Market clearing conditions

Supply equals demand in markets for labor and intermediate goods as, by assumption, the household and intermediate goods firms always meet the demand for labor/their goods. Market clearing for the final good and rental capital requires:
\[ Z_t = C_t + I_t \quad \text{and} \quad K_t = \int_0^1 K_t(s)ds, \]  
(25)

where \( Z_t \) is final good output, \( K_t \) is the aggregate capital stock, and \( \int_0^1 K_t(s)ds \) is total demand for rental capital by intermediate goods firms.

It is assumed that foreigners do not hold the country's currency or bonds denominated in that currency. Thus, money market equilibrium requires:
\[ M_t = M_t', \]  
(26)

where \( M_t \) and \( M_t' \) are the domestic money stock and the household's desired money balances, respectively; market clearing for domestic currency bonds requires that the household's (net) stock of bonds of this type is zero:
\[ A_t = 0. \]  
(27)

2.6. Solution method

An approximate model solution is obtained by taking a linear approximation of equations (1)-(12), (14)-(27) around the deterministic steady state in which the country's net foreign asset position is zero. (Log-)linear stochastic processes are specified for the shocks (see (28)). The resulting linear dynamic model is solved using Blanchard and Kahn's (1980) formulae.

2.7. Parameter values

The household's coefficient of relative risk aversion is set at \( \psi = 2 \). I focus on business cycles in Japan, Germany and the U.K. (G3). \( \psi = 2 \) is consistent with estimates of \( \psi \) for these countries (Barrionuevo (1991)) and is also in the range of values typically used in macro models. As is usual in models
calibrated to quarterly data, the steady state real interest rate, \( r \), is set at \( r = 0.01 \) (which corresponds roughly to the long-run average return on capital), while the subjective discount factor is set at \( \beta = 1/1.01 \) (the existence of a deterministic steady state requires that \((1+r)\beta = 1\) holds).

Up to a certainty-equivalent approximation, (21) can be written as 
\[
(M_{t+1}/P_{t+1})^{\Gamma-1} \kappa \Gamma / \sigma = (c_{t+1})^{\sigma-1} + \eta_{t+1},
\]
where \( \eta_{t+1} \) is a forecast error \( \mathbb{E}_{t} \eta_{t+1} = 0 \). Thus, the elasticities of money demand with respect to the interest rate and to consumption are \( \varepsilon_{i} = 1/(\Gamma - 1) \) and \( \varepsilon_{c} = (\sigma - 1)/(\Gamma - 1) \), respectively. Based on Fair's (1987) estimates of \( \varepsilon_{i} \) for the G3 countries, and Faig's (1989) estimates of \( \varepsilon_{c} \) (for Germany and the U.K.), I set \( \varepsilon_{i} = -0.04 \) and \( \varepsilon_{c} = 0.33 \). The preference parameter \( \kappa \) is set so that the steady state consumption velocity of money, \( PC/M \), is 0.5 (which corresponds roughly to average post-Bretton Woods M1 velocity in the G3).

The price elasticities of the country's aggregate imports and exports (see (2), (7)) are set at \( \phi = \eta = 0.6 \); this is the median value of the estimates of \( \phi \) and \( \eta \) for the G3 countries reported by Hooper and Marquez (1995). \( \alpha^{m} \) (see (1)) is set so that the steady state imports/GDP ratio is 25\%, consistent with U.K. and German data. (The imports/GDP ratio is \( \approx 10\% \) for Japan; using a 10\% steady state ratio does not change the key results.)

The steady state markup of price over marginal cost for intermediate goods is set at \( 1/(\nu - 1) = 0.2 \), consistent with the findings of Martins et al. (1996) for the G3 countries. The technology parameter \( \psi \) (see (4)) is set at \( \psi = 0.24 \), which entails a 60\% steady state labor income/GDP ratio, consistent with G3 data. Aggregate data suggest a quarterly capital depreciation rate of about 2.5\%; thus, \( \delta = 0.025 \) is used. The capital adjustment cost parameter \( \Phi \) is set at \( \Phi = 15 \), in order to match the fact that the standard deviation of
investment is three to four times larger than that of GDP in G3 countries. 3

In G3 countries, wages are generally changed once a year (Bruno and Sachs (1985)). Thus, the average wage-change-interval is set at 4 periods, i.e. $D=0.75$ is used, as the model is calibrated to quarterly data. I am not aware of estimates of the frequency of price changes in the G3. In the U.S., the average price-change-interval is about 1 year, for many goods (Romer (1996)). Thus, the mean price-change-interval is set at 4 periods: $\delta=0.75$.

Domestic money and productivity, and the foreign price level and expected real interest rate ($r^*_t=(1+i_t^*)E_t^*(P_t^*/P_{t+1}^*E_t^*)^{-1}$) follow these processes:

$$
\ln(M_{t+1}/M_t) = \rho^m \ln(M_{t-1}/M_{t-1}) + \epsilon^m_t, \quad \ln(\theta_t) = \rho^\theta \ln(\theta_{t-1}) + \epsilon^\theta_t,
$$

$$
\ln(P_t^*/P_{t-1}^*) = \rho^P \ln(P_{t-2}^*/P_{t-2}^*) + \epsilon^P_t, \quad r_t^* = (1-r^r)r^r + \rho^r r_{t-1}^* + \epsilon^r_t,
$$

where $\epsilon^m_t$, $\epsilon^\theta_t$, $\epsilon^P_t$, and $\epsilon^r_t$ are independent white noises with standard deviations $\sigma^m$, $\sigma^\theta$, $\sigma^P$, and $\sigma^r$. The parameters of the money and productivity processes are set at $\rho^m=0.15$, $\sigma^m=0.017$, $\rho^\theta=0.82$, $\sigma^\theta=0.011$. These values are parameter estimates (for quarterly post-Bretton Woods data) that were averaged across the G3 countries. 4 G3/U.S. exchange rates are considered below. Using the U.S. CPI and the U.S. 3-month CD rate (Citibase series FYUSCLU) as measures of the foreign price level $P_t^*$ and nominal interest rate $i_t^*$, the following parameter estimates are obtained for 1973Q1-94Q4: $\rho^P=0.80$, $\sigma^P=0.005$, $\rho^r=0.76$, $\sigma^r=0.005$. These values are used in the simulations. 5

3. Stylized facts about economic fluctuations (Post-Bretton Woods era)

Table 1 reports statistics on the cyclical behavior of key G3 quarterly macroeconomic time series since 1973 (all series have been logged, with the exception of interest rates. and Hodrick-Prescott (HP) filtered).

Most statistics are similar across the G3 countries. The standard
deviation of GDP is roughly 1.5%; consumption is about as volatile as GDP; physical investment is more volatile. With standard deviations of about 9%, the nominal and real exchange rates of the G3 countries (vis-à-vis the U.S.) are more volatile than the other variables in Table 1. The correlation between nominal and real exchange rates is high (about 0.97).

Consumption, investment and the money stock are procyclical (positive correlation with domestic GDP), while net exports and the price level are countercyclical. The variables in Table 1 are highly autocorrelated.

4. Simulation results

Simulation results are reported in Table 2. Columns (6)-(10) pertain to the nominal rigidities model; Cols. (1)-(5) consider a structure without nominal rigidities (in which the price/wage adjustment parameters δ, θ are set at δ=θ=0). The country’s output and price level are measured by its real GDP and the price of the final good, P_t; the real exchange rate is defined as \( RER_t = \frac{P^*_t}{P_t} \). Variants are considered in which each of the four types of shocks occurs separately, as well as variants with the four simultaneous shocks. Model statistics pertain to variables that have been logged (with the exception of the interest rate) and HP filtered.

4.1. Money supply shocks

Cols. (1) and (6) of Table 2 show results for the case with only money shocks. When prices and wages are flexible (δ=θ=0), then these shocks have little effect on GDP, consumption, investment, net exports and the real exchange rate: the predicted standard deviations of these variables do not exceed 0.10% (Col. 1); in contrast, the standard deviation of the price