Damage localization in bridges using multi-scale filters and large strain sensor networks

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Abstract
An output-only vibration based method for very small damage localization is proposed and applied on a numerical example inspired from the Boirs viaduc near Liège in Belgium. The use of local modal filtering from a network of long-gage fibre-optic strain sensors allows to locate the area where the damage is situated without the need for a numerical model of the structure. The general methodology consists in extracting features from multiple filters and to use multivariate control charts in order to detect a deviation from normal condition in one of the filters, indicating the occurrence of damage in this area. The robustness of the new method for damage localization with respect to noise on sensors and environmental changes is assessed.

1 Introduction
For more than twenty years, many authors have been interested in vibration based damage detection techniques and focused mainly on modal parameters such as modal damping, eigen frequencies or mode shapes [1]. One of the major problems with this approach is the fact that small damage does not affect these global physical properties sufficiently to be detected, because its influence is very local. Nowadays, recent advances in sensors and instrumentation make it possible to put large sensor networks on structures such as fiber-optics strain sensors. On the contrary to global properties, strains are local properties and have been shown to be a very good candidate for non-model based damage localization ([2], [3]). The new method of modal filtering which has already been studied in [4] with a simply supported beam is applied in this paper on a 3D structure. In order to perform automated on-line structural health monitoring, we use multivariate control charts [5].

2 Damage localization using local modal filters
Consider a structure equipped with a large network of sensors. In order to proceed to a data reduction, one can combine sensors responses to form a single output response with a linear combiner as shown in Figure 1 and Equation (1):
Figure 1: Principle of spatial filtering on a network of \( n \) sensors.

\[
y(t) = \sum_{k=1}^{n} \alpha_{l,k} y_k(t)
\]

The idea behind modal filtering is to choose coefficients \( \alpha_{l,k} \) in Equation (1) in such a way that they are orthogonal to all the modes of the structure in a frequency band of interest, except mode \( l \).

These coefficients are the solution of Equation (2)

\[
[C]^T \{ \alpha_l \} = \{ e_l \}
\]

where \( \{ e_l \}^T = (00...10)^T \) is a vector with all entries equal to 0 except entry \( l \) (corresponding to the eigen frequency on which the modal filter is tuned) which is equal to 1 and \( [C] \) is the rectangular matrix of modal output gain. The pseudo-inverse of \( [C] \) can be computed using a SVD:

\[
[C]^T = [U][S][V]^T,
\]

with \( [S] = \text{diag}(\sigma_i) \).

The solution of Equation (2) is given by

\[
\{ \alpha_l \} = \{ \alpha_{l,1} \cdots \alpha_{l,n} \}^T = \left( \sum_{i=1}^{r} \frac{1}{\sigma_i} u_i v_i^T \right) \{ e_l \}
\]

The calculation of coefficients \( \alpha_l \) is governed by two parameters which are the number of modes \( nm \) taken into account in \( [C] \) (\( nm \) is limited to the number of sensors) and the number of singular values \( r \) to solve Equation (4) (\( r \leq nm \)).

Previous works pointed out that the method is strongly sensitive to the quality of the mode shapes (i.e. components of matrix \( [C] \)). It is therefore recommend to identify properly the mode shapes. Because the method is output-only oriented, the mode shapes are obtained using the MACEC Toolbox under Matlab which applies the stochastic subspace identification method [6]. Moreover, because only the modes shapes at the lowest frequencies are excited by the ambiance in real structures, we will only consider the low frequencies modeshapes.

When damage occurs, the orthogonality condition (2) is not satisfied anymore. When looking at the Fourier transform of \( y(t) \), or at its power spectral density (PSD), this results in spurious peaks near other natural frequencies than \( l \). This principle has already been studied in [7] and [8] for damage detection, using accelerations measurements. It also has the advantage to distinguish global effects from local effects (see Figure 2):
(a) Effect of damage.

(b) Effect of environment.

Figure 2: Modal filter tuned on mode \(l\).

The appearance of spurious peaks can be detected thanks to a peak indicator \(I_{\text{peak}}\) computed for a given bandwidth, which is equal to 1 if the function is constant and tends to zero when the peak grows. Figure 3 gives an example of \(I_{\text{peak}}\) values computed between \(\omega_a\) and \(\omega_b\) when a spurious peak grows around \(\omega_l\):

![Figure 3: Example of \(I_{\text{peak}}\) values for an increasing peak.](image)

An important aspect of the present work is to extend the previously developed damage detection method to damage localization. In order to do this without using a numerical model, features which are locally sensitive to damage should be monitored. Pandey et al. [2] showed that for beams, curvatures exhibit this local sensitivity. In beam structures, the kinematic assumptions are such that the curvatures are proportional to the longitudinal strain at a given height in a section. In general, for any type of structure, this local sensitivity can be generalized for strains, as shown in [3]. The basic idea in the present paper is to apply local rather than global modal filters to strain measurements in order to locate small damage. The principle is as follows: consider a network of \(n\) sensors divided in \(m\) local filters (Figure 4).
For each local filter, the modal coefficients $\alpha_{l,k}$ are computed such that they are orthogonal to the projection of all mode shapes on the $n$ sensors of the local filter, except mode $l$. From there, the data processing is carried out on each local filter independently, following the strategy presented in [8], which is summarized below:

1. Compute the PSD of the output of the local modal filter tuned on mode $l$;
2. Compute the peak indicator $I_{Peak}$ in a frequency band around the natural frequencies of the undamaged system (except around the frequency $\omega_l$ of the mode $l$);
3. Repeat steps 1 and 2 for each local modal filter, and for each mode $l$ on which the filter is tuned.

The process results in a number of peak indicators for each local filter which are used as features to be monitored. Because of the local sensitivity of strains with respect to damage, if damage occurs under local
filter \(i\), spurious peaks will mainly appear in this local filter for the low order modes, which allows to locate the damaged area. Note that the efficiency of the method is linked to the relative size of the damage compared to the size of the local filters, which means that the size of the local filters would need to be adapted, possibly in near real-time, leading to a multi-scale modal filters approach.

### 3 Statistical approach for an automated damage localization

As explained later in section 4.2, we will check the appearance of spurious peaks near the first and the second natural frequencies in order to locate damage. This results in two features which have to be monitored in order to detect when they are shifted from their in-control values (values of the features in the undamaged state). Control charts [5] are very efficient for these shifts detection. Control charts plot the feature as a function of the sample number. When operating conditions change, the feature will fall outside control limits (lower or upper) which are computed from samples when the conditions are assumed to be in control. Because the spurious peaks near the two first eigen frequencies will increase with the damage level, the two features are not independent and therefore, monitoring these two quantities independently by applying two univariate control charts can be very misleading as explained in [5]. Instead, multivariate control charts which allow the simultaneous monitoring of two or more related characteristics are used in this study: these control charts plot a new single feature which is computed from the multiple monitored features, and, as for the univariate control charts, when the new feature falls outside control limits, alarm occurs.

In this paper, we will apply the Hotelling \(T^2\) control chart. In addition, because we are interested in small changes of the features, we also apply a COT scheme ([9]) which is the individual control chart applied to the feature \(T^2\).

### 4 Application example on a simplified model of a bridge

#### 4.1 Case study description

In order to apply the damage localization with modal filters, we study the same structure that was investigated in [3]. The model is inspired from a real bridge in Boirs near Liège in Belgium (see Figure 5). It is made of concrete and modeled with hexa8 elements using the Structural Dynamics Toolbox [10] under Matlab.

![Finite element model inspired from a real bridge with damaged area and long-gage fibre-optic strain sensors network.](image)

The five first natural frequencies computed with the FE method are given in Table 1 and are considered as the reference eigen frequencies. The identified eigen frequencies with the stochastic subspace identification method were found to be close to the reference frequencies.
Table 1: Natural frequencies of the undamaged structure (Hz).

<table>
<thead>
<tr>
<th>Value</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8583</td>
<td>4.6250</td>
<td>7.2181</td>
<td>15.4024</td>
<td>16.5056</td>
<td></td>
</tr>
</tbody>
</table>

A unique damage between $z=17.1m$ and $z=17.3m$ is introduced under the form of a small local stiffness reduction. A simulator has been developed in order to compute time response of the sensors attached to the structure under a random excitation. The network of strain sensors is composed of $100 \times 0.25m$ long gage fibers installed along a straight line and equally spaced between points $A$ and $B$ and the bridge is excited with a vertical random force (white noise) at point $C$. We introduce a small level of white noise directly on sensors responses under the form of Equation (5), with $\beta = 0.01$:

$$y^n(t) = y^0(t) + [\beta \max(y^0(t))]N(0,1)(t),$$

where $N(0,1)(t)$ is a random Gaussian variable with zero-mean and unitary standard deviation, $y^n$ and $y^0$ the noisy and non-noisy measurements respectively. The five first identified mode shapes (projected on the strain sensors) for the undamaged case are represented in Figure 6:

![Figure 6: Modes shapes identified along the long-gage fibre-optic strain sensors network.](image)

The network is divided into 10 local filters, each composed with 10 long-gage fibre-optic strain sensors and the damage is located under local filter number 7 (Figure 7).

![Figure 7: Local filters along the long-gage fibre-optic strain sensors network.](image)

Each sample of measurements has been chosen to last for 100sec, after which the peak indicators can be computed for each filter. The following damage scenario is investigated:
1. No damage (samples 1 to 100): $U$

2. Unique damage with 2% of uniform stiffness reduction (samples 101 to 200): $D_1$

3. Unique damage with 4% of uniform stiffness reduction (samples 201 to 300): $D_2$

Note that the damage is very small: it is less than $1/100^{th}$ of the total bridge length, with only a few percents of stiffness reduction.

The eigen frequencies changes due to damage cases $D_1$ and $D_2$ are summarized in Table 2:

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>-0.0007%</td>
<td>-0.0044%</td>
<td>-0.0025%</td>
<td>-0.0028%</td>
<td>-0.0018%</td>
</tr>
<tr>
<td>$D_2$</td>
<td>-0.0014%</td>
<td>-0.0088%</td>
<td>-0.0051%</td>
<td>-0.0056%</td>
<td>-0.0036%</td>
</tr>
</tbody>
</table>

Table 2: Natural frequencies shifts due to damage.

Note that the eigen frequencies shifts are so small that they are lower than the accuracy of modal identification methods. Damage detection based on eigen frequencies is therefore not possible.

### 4.2 Choice of the features for the damage localization

The computation of the modal filters presents some difficulties which are outlined in the present section. Firstly, it has been observed that the method is strongly sensitive to the quality of the mode shapes which are taken into account to compute the modal filters (i.e. the modal coefficients $\alpha_k$). Secondly, when the size of the local filters decreases, the distinction between the modes projected on local filters becomes tricky to establish and thus, the frequency responses are not filtered in a sufficient way. This is illustrated in Figure 8, where the modal filters are computed with the five first mode shapes, that is, $r = nm = 5$ (only the filtered frequency responses for filter $F_1$ is shown):
Figure 8: Effect of the size of the local filters on the filtered frequency responses (filter $F_1$).

Fortunately, it is not necessary to filter all the mode shapes in a frequency band. Indeed, we can filter only one peak and observe the potential appearance of a spurious peak near the corresponding natural frequency. In fact, when we take less mode shapes into account, the modal filtering obtained for these mode shapes is much better. We can choose any mode shapes but as demonstrated in [3], the damage effect on the strains map is better concentrated close to the damage area for the lowest frequencies. It justifies why we have chosen to filter the first two mode shapes. The modal coefficients are thus computed with $[C] = [\Phi_1 \Phi_2]$ and $r = nm = 2$.

Figure 9 shows the filtered frequency responses which are obtained when applying this principle with 10 local filters of 10 sensors. One clearly observes that the peaks corresponding to the first and the second natural frequencies are correctly filtered.

Figure 9: Modal filtering applied on only the first two mode shapes (filter $F_1$ for 10 local filters of 10 sensors).
The effect of damage levels $D_1$ and $D_2$ on filtered PSDs of filter $F_7$ is illustrated on Figure 10. One can see the growth of spurious peaks.

![Figure 10: Growth of spurious peaks for interval 2(top) and 1(bottom).](image)

Because we have only two features, it is helpful to represent the second feature with respect to the first one as shown in Figure 11 (intervals for calculation of $I_{\text{Peak}}$ can be seen in Figure 9):

![Figure 11: 2D plot of features for local filter $F_7$.](image)

From the analysis of Figure 11, we clearly see that the clouds representing the peak indicators are shifted, and that the corresponding shift increases with the damage level. Note that $I_{\text{Peak}}$ computed with interval 2 is more sensitive to damage than $I_{\text{Peak}}$ computed with interval 1.

Two remarks should be mentioned concerning the calculation of the peak indicator:

1. In order to enhance the sensitivity of the peak indicator and to remove the background noise, we use gaussian second derivative matched filtering (see [8]) on the PSD before computing $I_{\text{Peak}}$.

2. Despite this filtering, the PSD is not always constant for the undamaged case, so that the initial value of $I_{\text{Peak}}$ can be different from 1. This is the case for interval 1.

For an automated damage localization, we could use control charts based on mean shifts or on variance shifts of the peak indicators. In our case, we found that control charts based on mean shifts gave better results.

In Figure 12, we plot the bivariate distribution of $I_{\text{Peak}}$ on interval 1 and 2 for local filter $F_7$ using 1000 samples of the undamaged structure.
We see that the two features are approximatively distributed according to a bivariate normal distribution. The same kind of distribution has been observed with the other filters and thus, the use of the Hotelling $T^2$ is justified because it is based on a binormal distribution of the features.

### 4.3 Automated damage localization

Figure 13 illustrates the application of Hotelling $T^2$ control chart and COT scheme for filter $F_7$ (which includes the damaged area) and filter $F_8$, just next to the damaged area.

![Figure 13: Application of multivariate control charts on filters $F_7$ and $F_8$.](image)

The damage alarms are summarized in Figure 14, showing that the damage localization succeeded for both levels of damage $D_1$ and $D_2$. 
The Hotelling $T^2$ control chart allows to locate the damage even if some true alarms are missing. On the other hand, the COT scheme is more sensitive to small mean shifts and no true alarm is missing. False alarms occur more often with the COT scheme than with the Hotelling $T^2$ scheme but can be clearly distinguished from true alarms which remain as long as the damage exists. Note that because the COT scheme is a time-weighted scheme of control charts, it accumulates the mean deviance from its in-control value. Therefore, if one wishes to locate an increase of the damage (from damage level $D_1$ to damage level $D_2$ for example), one will have to recompute the control limits based on $D_1$, when considering samples of the damaged structure.

### 4.4 Effect of environmental changes on the damage localization

We now introduce a linear variation of the Young’s modulus along the $z$ direction as an environmental change. The stiffness of the bridge increases from 35GPa to 35.5GPa as shown in Figure 15, which can be seen as a decrease of temperature of more or less ten degrees between the left side ($T \approx 0^\circ C$) and the right side ($T \approx -10^\circ C$), assuming a linear variation of $E$ with $T$.

![Figure 15: Linear change of stiffness.](image)

This environmental change shifts the eigen frequencies as shown in Table 3.
Table 3: Natural frequencies shifts due to environmental change.

<table>
<thead>
<tr>
<th>$\Delta f_i$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+0.1501%</td>
<td>+0.3527%</td>
<td>+0.3069%</td>
<td>+0.2857%</td>
<td>+0.2326%</td>
</tr>
</tbody>
</table>

We can see that the environmental changes have a bigger influence on the eigen frequencies than the damage itself. Choosing eigen frequencies as the feature to be monitored for damage detection will therefore lead to erroneous results.

For this second example, we only apply the Hotelling $T^2$ control chart to the following scenario:

1. Samples 1 to 300: same cases than previously ($U$, $D_1$ and $D_2$)
2. No damage, with linear stiffness change (samples 301 to 350): $U_{\Delta T}$
3. Unique damage with 2% of uniform stiffness reduction, with linear stiffness change (samples 351 to 400): $D_{\Delta T1}$
4. Unique damage with 4% of uniform stiffness reduction, with linear stiffness change (samples 401 to 450): $D_{\Delta T2}$

Figure 16: Summary of the damage alerts for all the filters in the case of environmental changes.

We summarize the damage alarms in Figure 16. We can observe that for case $U_{\Delta T}$, we have only 4/0 alarms and therefore, no damage is located. A damage of 2% of stiffness decrease is less clearly located with a linear change of stiffness (there are 20/50 correct alarms in damage case $D_{\Delta T1}$ while 77/100 correct alarms in damage case $D_1$), but the results are still satisfying. The damage of 4% of stiffness is correctly located (no alarm is missing).

We also tried to apply the COT scheme but it has been found that this scheme was too sensitive to environmental changes. Indeed, when the stiffness changes linearly, damage was wrongly located in filters $F_7$ and $F_8$ for cases $U_{\Delta T}$, $D_{\Delta T1}$ and $D_{\Delta T2}$.

5 Conclusion

In this paper, we have proposed a new methodology to detect and locate automatically very small damage in bridge-like structures based on ambient vibrations. The method is based on the use of local spatial filters and
very large strain sensor networks, as well as multivariate control charts and does not require to build a model of the structure. The application to a numerical example inspired from a real bridge showed that the method is effective to detect and locate very small damage: damage with a length of about 1/100 of the length of the bridge and consisting of a stiffness reduction of only 2% could be located even in the presence of noise on the sensors and environmental variability. The approach shows very promising results and the next step is to validate it on an experimental small-scale setup, before evaluating it on a real bridge.

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