On the RANS simulation of neutral ABL flows

A. Parente\textsuperscript{a} C. Benocci\textsuperscript{b}

\textsuperscript{a} Von Karman Institute for Fluid Dynamics, Belgium, parente@vki.ac.be
\textsuperscript{b} Von Karman Institute for Fluid Dynamics, Belgium, benocci@vki.ac.be

ABSTRACT:
The present paper reports a novel implementation of a general purpose wall function for the numerical modeling of ABL flows under neutral stratification. Moreover, a modification of the standard $k$-$\varepsilon$ model consisting in the introduction of source terms in the transport equation of turbulent kinetic energy and dissipation rate is proposed, to allow using different sets of fully developed inlet profiles. The methodology is demonstrated through implementation in the commercial code FLUENT 6.3 and in OpenFoam 1.6. Results are presented for a neutral boundary layer and indicate the strong potentials of the proposed methodology in ensuring the homogeneity of velocity and turbulent quantities throughout the computational domain.

1 INTRODUCTION

The limitations related to the RANS simulation of neutral atmospheric boundary layer (ABL) with commercial CFD codes are well known and documented in the literature (Franke et al., 2007; Blocken et al., 2007a,b; Riddle et al., 2004; Hargreaves and Wright, 2007). The cause of such unsatisfactory behavior is directly related to the inconsistencies between the formulation of the law of the wall for rough surfaces and the inlet conditions for ABL simulations. Remedial measures have been proposed in the literature (Blocken et al. 2007b); however, these are generally code dependent and do not provide a general solution to the problem.

As far as inlet profiles for ABL simulations are concerned, profiles by Richard and Hoxey (1993) are usually used as inlet conditions. However, the assumption of constant kinetic energy, $k$, contrasts to wind-tunnel and full scale measurements, where a decrease of $k$ with height is observed. Following these considerations, Yang et al. (2009), proposed a new set of inlet conditions, with a $k$ profile decreasing with height. However, it remains still unclear how this new set of inlet conditions affects the formulation of the $k$-$\varepsilon$ model. In a recent work, Gorlé et al. (2009) proposed a modification of the $\sigma_{\varepsilon}$ constant to ensure streamwise homogeneity when using the $k$ profile by Yang et al. (2009).

The present paper addresses both the aforementioned aspects. In particular, a novel implementation of a general purpose wall function for the numerical modeling of ABL flows under neutral stratification is presented. Moreover, a modification of the standard $k$-$\varepsilon$ model is proposed. The latter is based on the introduction of two source terms in the transport equations of $k$ and $\varepsilon$, respectively, to allow using different sets of fully developed inlet conditions for homogeneous ABL flows.

2 THEORY

The numerical simulation of the homogeneous atmospheric boundary layer (ABL) is usually carried out using the standard $k$-$\varepsilon$ model. Under the hypothesis of no vertical velocity, constant pres-
sure in vertical and streamwise directions and constant shear stress throughout the boundary layer, the transport equation for turbulent kinetic energy, $k$, and turbulent dissipation rate, $\varepsilon$, reduce to:

$$\frac{\partial}{\partial z} \left( \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial z} \right) + G_k \frac{\varepsilon}{k} - \rho \varepsilon = 0$$ (1)

$$\frac{\partial}{\partial z} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) + C_{\varepsilon 1} G_k \frac{\varepsilon}{k} - C_{\varepsilon 2} \rho D \frac{\varepsilon^2}{k} = 0$$ (2)

where the production of turbulent kinetic energy, $G_k$, and the turbulent viscosity, $\mu_t$, are given by

$$G_k = \mu_t \left( \frac{\partial u}{\partial z} \right)^2$$

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$$ (3)

and $\sigma_k$, $\sigma_\varepsilon$, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ and $C_\mu$ are constants of the k-\(\varepsilon\) turbulence model.

2.1 **Inlet conditions for homogeneous ABL flows**

Richard and Hoxey (1993) proposed fully developed inlet profiles of mean velocity, turbulent kinetic energy and dissipation rate for neutral stratification conditions:

$$u = \frac{u^*}{\kappa} \ln \left( \frac{z + z_0}{z_0} \right)$$ (4)

$$k = \frac{u^*^2}{\sqrt{C_\mu}}$$ (5)

$$\varepsilon = \frac{u^*^3}{\kappa (z + z_0)}$$ (6)

It can be showed that Equations 4-6 are analytical solutions of the standard k-\(\varepsilon\) model when the turbulent dissipation number Prandtl number is given by

$$\sigma_\varepsilon = \frac{\kappa^2}{(C_{\varepsilon 2} - C_{\varepsilon 1}) \sqrt{C_\mu}}$$ (7)

The condition expressed by Equation 7 is obtained by substituting the profiles of turbulent kinetic energy and dissipation rate in the transport equation for $\varepsilon$. In alternative, the constant value of $\sigma_\varepsilon$ can be kept and a source term added to the dissipation rate equation (Pontiggia et al., 2009):

$$S_\varepsilon(z) = \frac{\rho u^*^4}{(z + z_0)^2} \left( \frac{(C_{\varepsilon 2} - C_{\varepsilon 1}) \sqrt{C_\mu}}{\kappa^2} - \frac{1}{\sigma_\varepsilon} \right)$$ (8)

The latter approach may appear less straightforward than that expressed by Equation 7. However, it is very convenient when adopting different profiles for the turbulent quantities, as it will be shown in the following.
The constant turbulent kinetic energy, $k$, profile in Equation 5 lacks of physical sense, since experimental evidence shows a decreasing trend of $k$ with height. Recently, Yang et al. (2009) proposed the following alternative inlet condition for $k$:

$$k(z) = \sqrt{C_1 \ln(z + z_0) + C_2} \quad (9)$$

where $C_1$ and $C_2$ are constants determined by fitting the equations to the measured profile of $k$. Consequently, the turbulent dissipation rate profile can be expressed using the equilibrium assumption as:

$$\varepsilon(z) = \sqrt{C_\mu k} \left( \frac{dU}{dz} \right) \quad (10)$$

The profile expressed by Equation 9 satisfies identically the transport equation for $k$ (Yang et al., 2009). However, assuming a non constant profile for $k$ complicates the formulation of the $k$-$\varepsilon$ model constants. In particular, Equation 9 implies a non constant profile for $C_\mu$, to satisfy the continuity equation:

$$\mu_T \frac{\partial \mu_\tau}{\partial z} = \tau_w = \rho \mu^2 \quad (11)$$

Therefore, a general expression for $C_\mu$ as a function of the height can be deduced:

$$C_\mu = \frac{u^4}{k^2} \quad (12)$$

However, if the formulation of Equation 12 is used for $C_\mu$, Equation 7 is no longer valid and a rather complex differential equation should be solved for $\sigma_\varepsilon$. Recently, Gorlé et al. (2009) provided a solution to this problem for the case of a constant $C_\mu$, obtained from the solution of Equation 12 at the first cell centroid. However, no closed form for $\sigma_\varepsilon$ was found for the general case of a variable $C_\mu$. On the other hand, if the source term approach is used, Equation 8 can be adopted. The reason for that lies in the equilibrium assumption and in the generalization of $C_\mu$, which make the first term of the $\varepsilon$ transport equation independent of the form of the specified inlet profile. In particular:

$$\frac{\partial}{\partial z} \left( \frac{\mu_\tau}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) = - \frac{f \mu^4}{\sigma_\varepsilon (z + z_0)^2} \quad (13)$$

The non uniformity of $C_\mu$ imposes the definition of a source term for the turbulent kinetic energy equation. Differently from the source term for the turbulent dissipation rate, the one for $k$ is dependent on the form of the inlet profile, as the latter influences directly the definition of $C_\mu$. A general source term, valid for any turbulent kinetic energy profile, can be expressed as:

$$S_{tke}(z) = \frac{f \mu^*}{\sigma_k} \left( \frac{\partial}{\partial z} \left( \frac{\partial k}{\partial z} \right) \right) \quad (14)$$

If Equation 9 is used for $k$, then Equation 14 becomes:
\[ S_{
u e}(z) = \frac{1}{2} C_1 \frac{\kappa}{\sigma_k} \frac{\partial \sqrt{C \mu}}{\partial z} \]  

(15)

2.2 Wall treatment

The standard approach for rough surfaces in CFD software consists in a modification of the standard smooth law of the wall:

\[ \frac{U}{u^*} = \frac{1}{\kappa} \ln(Ez^+) - \Delta B(k_s^+) \]  

(16)

where \( u^* \) is the friction velocity, \( \kappa \) is the Von Karman constant, \( E \) is an integration constant (\( E=9-9.793 \)) and \( z^+ \) is the non-dimensional distance from the wall, defined as \( z^+ = z u^* / \nu \). The function \( \Delta B(k_s) \) depends on the dimensionless roughness height \( k_s^+ = k_s u^* / \nu \) and measures the departure of the wall velocity from smooth conditions. It can assume different forms, depending on the equivalent sand grain roughness values (Cebeci and Bradshaw, 1977; Nikuradse, 1933). In particular, when \( k_s^+ > 90 \), Equation 16 can be reformulated as:

\[ \frac{U}{u^*} = \frac{1}{\kappa} \ln(Ez^+) - \frac{1}{\kappa} \ln(1 + C_s k_s^+) = \frac{1}{\kappa} \ln \left( \frac{Ey^+}{1 + C_s k_s^+} \right) \approx \frac{1}{\kappa} \ln \left( \frac{Ey^+}{C_s k_s^+} \right) \]  

(17)

where \( C_s \) is a roughness constant which should be set to ensure first order matching between the law of the wall and the inlet profile (Blocken et al., 2007b)

\[ C_s = \frac{Ez_0}{k_s} \approx \frac{Ez_0}{z_c} \]  

(18)

Different implementations of this law can be found in commercial CFD codes. Currently, the value of \( C_s \) can be freely set in the commercial software StarCCM+ (CD-Adapco, 2008), which provides a roughness function similar to that expressed by Equation 16, but with two parameters, \( B \) and \( C \), e.g. \( \Delta B(k_s^+) = 1/\kappa \cdot \ln(B + C k_s^+) \). In CFX, a fixed value for the roughness constant is adopted, \( C_s=0.3 \) (Ansys Ltd., 2005), whereas FLUENT limits \( C_s \) in the interval \([0, 1]\) (Fluent, 2006); however such constraint can be overcome by defining custom profiles for \( C_s \) through user defined functions (UDF).

However, even when Equation 18 is satisfied, the standard rough wall function suffers from two main drawbacks. First, it poses strong limitations on the maximum size of the first inner cell, being the maximum allowable value for \( C_s \) limited by the wall function constant \( E \). Moreover, the standard wall function does not imply a direct effect of the roughness properties on the turbulence quantities at the wall. Following the above considerations, a new approach has been developed to overcome the aforementioned limitation of the standard rough wall function. The proposed wall function conditions are taken from Richard and Hoxey (1993), with a slight difference regarding the calculation of the production of \( k \) in the wall adjacent region:
\[ u = \frac{u^*}{\kappa} \ln \left( \frac{z+z_0}{z_0} \right) \]  
(19)

\[ \varepsilon_w = \frac{C_{\mu}^{0.75} k^{1.5}}{\kappa(z_p+z_0)} \]  
(20)

\[ k_w \leftarrow G_k = \tau_w \left( \frac{\partial u}{\partial z} \right) = \frac{\tau_w^2}{\rho k C_{\mu}^{0.25} k^{0.5} (z_p+z_0)} \]  
(21)

Differently to Richard and Hoxey (1993), the production of turbulent kinetic energy at the wall (Equation 21) is not integrated over the first cell height but computed in \( z_p+z_0 \). This formulation is consistent with the equilibrium assumption, \( \varepsilon = \sqrt{C_{\mu} k dU/dz} \), and it does not result the peak of turbulent kinetic energy at the wall observed in Hargreaves and Wright (2007).

As far as the numerical implementation is concerned, the wall function has been implemented into FLUENT 6.3 by means of a User Defined Function (UDF) and in OpenFOAM 1.6, by direct modification of the source code. The implementation procedure in the two cases is analogous. To allow a specification of the wall boundary condition in terms of non-dimensional parameters, the law of the wall is expressed as:

\[ u = \frac{u^*}{\kappa} \ln (E' z^+) \]  
(22)

where a new wall function constant and non-dimensional wall distance are defined as:

\[ E' = \frac{v}{z_0 U' } \quad z^+ = \frac{(z_p+z_0) u^*}{v} \]  
(23)

The \( z^+ \) in Equation 23 is simply a \( z^+ \) shifted by the aerodynamic roughness whereas the new wall function constant depends on the roughness characteristics of the surface. In Equation 23 the friction velocity, \( u^* \), is not kept constant but calculated as \( u^* = C_{\mu}^{0.25} k^{0.5} \).

The described approach removes the drawbacks of the standard wall function, without reducing its flexibility. In fact, it can be easily extended to mixed rough and smooth configurations through a redefinition of the law of the wall constants. In particular, the roughness properties for each boundary are retrieved and, then, the values constants are properly determined for both smooth and rough surfaces. Finally, it should be observed that, for FLUENT, the definition of \( z^+ \) automatically ensures a modification of the turbulent dissipation rate and production of turbulent kinetic energy at the wall adjacent cell according to Equations 20-21, as these terms are directly computed from the velocity derivative. As far as OpenFoam is concerned, Equations 20-21 have been directly implemented in the dedicated wall function files.

3 TEST CASES

Results are presented for a 2D domain of 4 m length and 1 m height (wind tunnel scale), with a grid of 400x71 cells. For FLUENT, the generated grid is uniform in the longitudinal direction and stretched in the vertical direction to have the centre point of the wall adjacent cell at a height of 0.0025 m. In OpenFoam, the same grid is extruded in the third direction by one layer, as OpenFoam can only deal with 3D domains.

As far as boundary conditions are concerned, inlet profiles of velocity, turbulent kinetic energy and dissipation rate are specified at the domain inlet with \( u^* = 0.374 \text{m/s} \), \( z_0 = 0.00075 \text{m} \), \( C_1 = \)
0.025 and \( C_2 = 0.41 \). A pressure outlet condition is used for the outlet section. A wall function (Table 1) with both standard and modified formulations (Section 2.2), is applied at the lower boundary whereas a constant strain is applied to the upper boundary, following the recommendation of Richard and Hoxey, 1993. This is accomplished by specifying a velocity at the upper boundary equal to:

\[
u = \frac{u^*}{k} \ln \left( \frac{z_h + z_0}{z_0} \right)\]

(24)

where \( z_h \) is the height of the computational domain.

The objective of the study is to evaluate the overall proposed methodology in sustaining two different sets of inlet conditions, the classic ones (Richard and Hoxey, 1993) defined by Equations 4-6, and the ones based on a non constant profile of turbulent kinetic energy (Equations 4, 9-10). The investigated conditions and model settings are summarized in Table 1.

### Table 1 – Summary of the investigated runs and model settings.

<table>
<thead>
<tr>
<th>Run</th>
<th>( k )</th>
<th>Wall function</th>
<th>( C_{\mu} )</th>
<th>( S_k )</th>
<th>( S_{\varepsilon} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( k = \frac{u'^2}{\sqrt{C_{\mu}}} )</td>
<td>STD - ( C_s = E\varepsilon_0/z_c )</td>
<td>0.09</td>
<td>-</td>
<td>( S_{\varepsilon}(z) = \frac{C_{\mu}^4}{(2+c_0)^3} \left( \frac{(C_{\mu}^2-C_{\mu})\sqrt{C_{\mu}}}{k^4} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( \sqrt{C_1 \ln(z+z_0)} + C_2 )</td>
<td>MOD</td>
<td>( C_{\mu} = \frac{u'^4}{k^2} )</td>
<td>( S_{\varepsilon}(z) = \frac{C_{\mu}^4}{(2+c_0)^3} \left( \frac{(C_{\mu}^2-C_{\mu})\sqrt{C_{\mu}}}{k^4} \right) )</td>
<td></td>
</tr>
</tbody>
</table>

4 RESULTS

Figure 1 shows the profiles of velocity, turbulent kinetic energy and dissipation rate at the inlet and outlet section of the computational domain, for the inlet conditions specified by Equations 4-6 (Figure 1a-c) and by Equations 4, 9-10 (Figure 1a'-c'). Due to length limitations, only the results obtained with FLUENT are presented.

For the case of a constant profile for turbulent kinetic energy, the results obtained with the standard wall function with a constant \( C_s \) defined via a UDF according to Equations 18 are compared to those provided by the modified law of the wall. Results (Figure 1a-c) indicate that the two approaches are consistent; however, the modified wall approach results in a more homogeneity of the velocity field and turbulence quantities throughout the domain. This already represent a significant outcome, as the alternative well function approach provides results which are better of the best achievable with a standard wall function approach. In particular, Figure 1b-c confirms that the modification of \( C_s \) only affects the velocity profile, whereas the turbulent dissipation rate at the wall is overestimated, being the aerodynamic roughness omitted in the denominator of Equation 20. Consequently, the turbulent kinetic energy at the wall adjacent cell results slightly underestimated. For the case of a non constant \( k \) profile, the comparison between the profiles of velocity, turbulent kinetic energy and dissipation rate at the inlet and outlet sections of the domain indicate that the proposed source terms (Section 2) and the implemented wall function ensure the longitudinal homogeneity. The highest differences are observed for the \( k \) profile; however, these are safely below 4% in all cases.
Figure 2 shows the evolution of the shear stress along the axial coordinate, when using the constant and varying inlet $k$ profiles. It can be concluded that the present approach ensures homogeneity of velocity and turbulence quantities, as it guarantees the uniformity of the wall shear stress. Moreover, when the profile in Equation 9 and the $k$-ε turbulence model constants are modified, the recovered shear stress is collapsing onto the theoretical one, whereas small differences (below 6%) are observed for the constant $k$ case.

5 CONCLUSIONS

The present paper reports a novel implementation of a general purpose wall function for the numerical modeling of ABL flows. The wall function is based on the formulation proposed by Richard and Hoxey (1993), but with a modification on the turbulent kinetic energy production term. Moreover, a modification of the standard $k$-ε model consisting in the introduction of source terms for turbulent kinetic energy and dissipation rate is proposed, to allow introducing arbitrary profiles of turbulent kinetic energy and dissipation rate. Results of the numerical simulations are presented for a neutral boundary layer and indicate the strong potentials of the proposed methodology, able to provide homogeneity of velocity and turbulent quantities throughout the computational domain.
Figure 1. Profiles of velocity, turbulent kinetic energy and turbulent dissipation rate at inlet and outlet section of the computational domain, obtained when applying inlet conditions given by Equations 4-6 (Figure 1a-c) and Equations 4, 9-10 (Figure 1a’-c’). STD WF = Standard Wall Function; MOD WF = Modified Wall Function.
Figure 2 – Wall shear stress as a function of the axial coordinate. MOD WF = Modified Wall Function.

6 REFERENCES