Strategic Asset Allocation with Heterogeneous Beliefs

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Abstract

We study how the presence of long term investors using different return forecasting strategies and switching them based on their past performance generates the price trends observed in financial markets. In the empirical section, we assume that investors choose how to allocate their portfolios among four major stock indices: Dow Jones, FTSE, Nikkei and Hang Seng. The exercise shows that a decrease in the proportion of fundamentalists is related to movements in prices that are subsequentialy reverted. In this paper, we bridge the literatures on intertemporal asset allocation and on heterogeneous beliefs. The interaction between two switching types of agents, e.g. fundamentalists and chartists, is responsible for endogenously generating the observed price trends.

Key words: asset pricing, intertemporal asset allocation, heterogeneous beliefs, adaptative learning.

JEL classification numbers: G11, G12, D83, D84

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1 Introduction

Traditional asset pricing models in Finance usually present problematic empirical features, the so called "puzzles" documented by researchers. Some stilyzed facts in financial series, such as deviations from fundamentals, excess price volatility, abnormal and predicable returns, and others are hard to justify using standard models. [1]

In an attempt to build a framework capable of reproducing real market prices behaviour, many alternatives have been proposed. A large literature attempts this relaxing some of the assumptions made in standard models. In the Behavioural Finance literature, for example, it is usually assumed that investors behave irrationally to some extent. [2] Other alternatives include assumptions about preferences, like habit formation, to explain the irrealistic high risk aversion required by these models to replicate real financial data or some ad hoc measure of risk as in Fama and French (1992).

These assumptions were found capable of (to some extent) conciliating real

[1] See, for instance, the "volatility puzzle" in Campbell (1999), the related "equity premium puzzle" in Mehra and Prescott (1985), the poor empirical performance of the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) in Fama and French (1988, 1992, 1996), the limited application of the Arbitrage Pricing Theory of Ross (1976) and the debate regarding its classification as an equilibrium asset pricing model, as in Shanken (1982, 1992).

[2] See, e.g., Barberis, Shleifer and Vishny (1998) who create a model where they assume that agents show psychological biases when forming expectations or Daniel, Hirshleifer and Subrahmanyam (1998) who show how agents can misinterpret private information leading to deviations from fundamentals.

asset market data and theoretical formulation. We believe, however, that such assumptions may be hard to justify, so in this paper we propose an alternative model.

We assume that the market is populated by many different agents, and that those agents, aware of this fact, know that some forecasts may even be theoretically supported, but unless the majority of the other agents agree with them, they are not going to be the most accurate. These agents, therefore, adjust their forecasts taking into account what they believe is the dominant forecasting strategy in the market. Later we show that these assumptions alone are capable of generating the price trends we observe. This happens in the presence of very general assumptions about risk and time preferences, investment horizon and number of assets.

We start to build our model assuming long term investors, selecting portfolios among n risky assets. The justification is that this case nests as parcitular ones all other possible investment horizons and number of assets. It is even possible, for instance, to obtain the widely used myopic horizon of Markowitz (1952) with one or more risky assets by making the appropriate restrictions to the parameters.

Next, we describe investors preferences as belonging to the (general) class of preferences introduced by Epstein and Zin (1989, 1991). We, thus, assume that investors derive utility from consumption, and that they discount future consumption by a given intertemporal discount factor. Using this class of preferences allows to separate risk aversion (that is meaningful even in a single period formulation) from intertemporal substitution effects (that are meaningful even in the absense of risk).

For simplicity, for most of the analisys agents will be split into only two switching types: Fundamentalists, that are traders using fundamentalist strategies to forecast future prices; and chartists, which are traders using trend following forecasting strategies. Although controversial, it is hard to classify investors of the second type (chartists) as irrational with such strong empirical evidence showing the profitability of momentum (trend following) strategies in so many different markets as in Jagadeesh and Titman (1993) for instance. Also, Friedman's argument, that only rational investors survive in the market because those are the profitable investors, is based on the profitability of a given strategy. This implies that as long as a strategy is profitable, investors using it will not be "eliminated" from the market.

Investors are concerned with forecasting returns as accuratelly as possible. We expect, therefore, that they switch types (forecasting strategy) according to what they believe is the dominant one in the market. This evolution of types is endogenous in the model and we assume that it is based on the previous period's performance (profit) of each strategy. So, if a given strategy was successful in the previous period, we expect an increase in the number of traders of that given type.[3]

The strategic features of the model can be interpreted as a coordination game, where agents are always better off if they manage to coordinate with the market. Previous strategy perfomance in this case can be seen as a sign that helps agents to coordinate.

[3] The difference from the representative agent approach is that at any given time, some agents will be using fundamentalist strategies and others will be using chartist strategies. The interaction between these two is what generates the endogenous equilibrium price.

Some may argue that using only last period's performance to infer the dominant type in the market is a form of bounded rationality since it doesn't take into account what happened in the previous periods. However, we justify the rationality of this choice assuming that agents *believe that others,* bounded rational ones, choose their type in this way. The assumption of some degree of inefficiency from other agents is in the heart of the Eficient Market Hipothesys: markets can only be efficient if agents believe that they are not, and activelly search for profits. In our model, agents also believe that the market is inefficient and search for profits. The difference is about how they do that, implying that under our assumptions innefficiencies don't have any reason to disapear.

To compare performances, investors need to know what is the optimal portfolio given by each strategy. However, there are many different forecasting models within each class of strategies. For instance within the fundamentalist class, there are lots of possible return forecasting variables: Current/forecasted price-earnings ratio, or the current/forecasted dividend-price ratio, or present value discounted cash flow model, or many others. The same happens among momentum strategies, where agents may use different time horizons to compute the autocorrelation in prices. To incorporate these facts, therefore, we represent the type by one of these strategies and assume that performances are observed with noise.

Noise in the observed performance is what prevents all agents in our model from switching abruptely from the loosing strategy to the winning one as they observe any performance differences. In that extreme case the entire market would be either fundamentalist or chartist at a given time, depending on which strategy was the winner in the previous period.

We model heterogeneity as Brock and Hommes (1998), henceforth BH98, where the interaction between two switching agent types (e.g., fundamentalists and chartists) is responsible for endogenously generating the observed price trends. The intertemporal asset allocation problem has analitical solutions only for some particular cases and to obtain it for the other ones we use the aproximation in Campbell et al (2001).

Our paper bridges the literature on intertemporal asset allocation and on heterogeneous beliefs. Therefore, its contribution is twofold: From the intertemporal asset allocation perspective, we add the possibility of heterogeneity of agents, and therefore the representative agent approach (implicitly assuming homogeneous agents) is not valid anymore. The implications are that the resulting equilibrium price and allocation vectors are completely different since they will now be determined by the interaction of agents with different expectations. Moreover, as the proportion of types evolves in time, different equilibria are obtained.

From the heterogeneous agents perspective, we add to the existing literature by allowing agents to choose between a larger number of risky assets, n, in a more general consumption based intertemporal otimization formulation. This contrasts with the previous literature of mean variance investors with only one risky asset.

In the empirical section we estimate the model using data on the US, UK, Japan and Hong Kong's stock markets. We start by estimating, for each of these four markets, a simple dividend-price return forecasting model and a simple momentum model. Next, we use these models respectivelly as the fundamentalist and chartist strategies and examine the resulting dynamics. The results are consistent with our predictions. We observe periods when prices deviate from fundamentals followed by a later reversal. Consistenly with our model, the fraction of fundamentalists declines during the first trend when prices move away from fundamentals, and increases during the second when they start to move back.

The paper is organized in four sections apart from this introduction. Section two contains the motivations for our model. In the third section, we present the asset pricing model together with its theoretical results. The fourth is the empirical section and we conclude in the fifth section.

2 Motivation

2.1 The intertemporal framework

There is a long tradition in Finance to cast investment problems into a mean-variance framework since the seminal work of Markowitz (1952). He showed how investors should allocate their portfolio if all they cared about was the mean (with a positive weight) and the variance (with a negative weight) of their portfolio return between two periods. For its simplicity and intuitive closed form solution, it has the appealing features that seduce both researchers and practitioners.

However, this model relies on very strong assumptions. One of those relates to the investment horizon itself. Markowitz (1952) supposes that agents have a myopic planning horizon, in the sense that their choices are made solely based on their implications to next period's returns. Since at least Merton (1969) and Samuelson (1969), however, it is known the (restrictive) situations under which this solution is optimal for long term investors. [4] An implication of the investment horizon is the definition of risk. In the mean-variance framework, cash is regarded as the risk free asset, because its next period's value is known. Long term bonds are considered risky, as their short term value varies with the interest rate. For a long term investor, however, this does not necessarily hold in the same way. Cash is risky in the long run, since it needs to be reinvested at unknown rates in the future while inflation-indexed bonds provide a fixed stream of real payments (consumption) and may be classified as risk free, even if their value is not known in the short run. [5]

The mean-variance formulation also does not address consumption and therefore agents are supposed to be interested in high mean and low variance of returns for its own sake. In fact, the maximization of a quadratic utility function over wealth generates similar results in the sense that agents trade mean and variance in a linear fashion. However, among other problems and against empirical evidence, this utility function shows increasing relative and absolute risk aversion over wealth and, even more importantly, it is not

[4] If investors are able to rebalance their portfolios each period and if they can hedge against changing investment opportunities, they only behave myopically if asset returns are iid over time (what means, for instance, that the mean return of the risky and risk free asset are constant over time) or if they have utilities of the log form (which is the limit of a power utility with elasticity of intertemporal substitution and risk aversion coefficient equal to one).

[5] Discussions about the relationship between risk and investment horizon are already present in Modigliani and Sutch (1966), who argue that long-term bonds are safe for long-term investors, and later in Stiglitz (1970) and Rubinstein (1976) who theoretically confirmed those predictions.

monotonically increasing in wealth.

In our model, therefore, we assume that investors intertemporaly maximize a consumption based utility function, avoiding all these inconsistencies.[6]

2.2 Heterogeneous beliefs models

It is widely documented, in many independent markets, that some variables such as fundamentalist ratios or past returns are good forecasters for future return. For instance, the forecasting power of scaled price ratios has been known for a long time.[7] Those results suggest that a winning strategy is selling overvalued assets and buying undervalued ones from a fundamentalist perpective. On the other hand, there is also evidence of momentum in the short run, as in Jegadeesh and Titman (1993), and the subsequent long-term reversal, as in De Bondt and Thaler (1985). While long term reversal may be consistent with fundamentalists forecasts, driving prices back after some deviation, the presence of momentum in returns suggests another optimal strategy, based solely on trend following.

From both theorecial and empirical sides, there is evidence of heterogeneity in investors' behaviour. Several no trade theorems (as in Milgrom and Stokey (1982) for instance) have been developed showing that in an economy where all agents are rational there will be no trade based on private information. This contrasts with the high observed trading volumes in many asset markets

[6] And we explain in details how we do that in section 3.1.

[7] See, e.g., Graham (1949), Dreman (1977), Banz (1981), Basu (1983), Rosenberg, Reid and Lanstein (1985), Fama and French (1988, 1992), and Lakonishok, Shleifer and Vishny (1994):

and also suggests the existence of heterogeneous beliefs, since it is necessary difference in opinions (or heterogeneous beliefs) as a condition to generate trade.

In fact, Table I, from Frankel and Froot (1990), is a good example of heterogeneity. It shows not only that companies apply different forecasting strategies but also that the fraction of companies using one or other strategy changes in time.

Year	*Total*	*Chartist*	*Fundamentalist*	*Both*
1978	23	3	19	0
1981	13	1	11	0
1983	11	8	1	1
1984	13	9	0	2
1985	24	15	5	3
1988	31	18	7	6

Table I: Survey of strategies used by forecasting companies[8]

Further evidence is given by surveys in Frankel and Froot (1987), Ito (1990), Taylor and Allen (1992) and, more recently, Mentkhoff (1997) surveying ex-

[8] Note that some companies did not complete the survey.

change rate expectations of financial specialists. They show that investors tend to use different trading strategies depending on the investment horizon they are trying to forecast. Basically, chartists/trend following strategies are used in the short run while fundamentalists/mean reverting strategies are kept for long horizons.

Considering stocks, Kandel and Pearson (1995) and Bamber, Barron and Stober (1999) provide evidence of heterogeneity of analyst expectations about earnings around announcements. Shiller (2000), analyzing bubbles, also provides evidence of heterogeneity in expectations about the market's future performance.

All the facts presented here are hard to conciliate with rational homogeneous agents models. In the model that we describe next, we show how rational heteronegous agents interacting with each other can generate the price trends observed in financial markets.

3 The model

There is an infinite number of long-term investors that can be classified into H different types. [9] Agent type h is determined by the trading strategy he is currently using to forecast returns. In this paper, we restrict the analysis to

[9] Those long-term investors face time varying investment opportunities, meaning that, as pointed by Campbel et al (2001) the productivity of wealth is varying over time. We, thus, expect that they may be willing to hedge against those shocks, and this is what gives rise to their intertemporal hedging demand.

$H = 2$ (i.e. fundamentalist or chartist types).[10] Agents extract information from prices. They switch between trading strategies (changing their types) responding to their previous performances. Investors do not receive perfect information about strategies's performances.

Merton's intertemporal model only has a closed form analytical solution in some situations. One of them is when the elasticity of intertemporal substitution (ψ) equals to 1. Campbell et al (2001) provide an approximate analytical solution to Merton's model, based on perturbations of this known exact solution. Since this is an approximate solution, it is expected to be valid only in a neighborhood of the true analytical solution, i.e. provided that the intertemporal elasticity of substitution is not too far from one.

3.1 The investor's maximization problem

Time is discrete and infinitely-lived investors maximize Epstein-Zin (1989, 1991) recursive preferences defined over an stream of consumption.[11] There are n risky assets in the economy and investors allocate their wealth among these assets and consumption. Conditioned on his type, the investor's problem

[10] Our framework, however, can be easily applied to any number of agent types since our theoretical results correspond to this general case.

[11] It may be interesting to note that the power utility is a special case of the Epstein-Zin function. We can obtain it by letting $\gamma = \psi^{-1}$ (and hence $\theta = 1$). Besides, as the log utility is a special case of the power utility, it is easy to obtain it just by adding the restriction $\gamma = 1 = \psi^{-1}$. With time varying investment opportunities, this is a condition to generate the myopic portfolio allocation. But as Giovannini and Weil (1989) show, $\gamma = 1$ or $\psi^{-1} = 1$ alone are not sufficient for this result.

is to choose at every time t, the portfolio allocation $\alpha^*_{h,t}$ and consumption $C^*_{h,t}$ that maximize his utility restricted by his budget constraint and portfolio return:[12]

$$
\begin{aligned}
(\alpha^*_{h,t}, C_{h,t}) = \arg \max_{\alpha_{h,t}\in\mathbb{R}^n, C_{h,t}\in\mathbb{R}} \quad & U(C_t, E_t[U_{t+1}]) = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\delta}} \\
\text{s.t.} \quad & W_{t+1} = (W_t - C_t)(1 + R_{p,t+1}), \\
& R_{p,t+1} = \textstyle\sum_{i=2}^{n} \alpha_{h,i,t}(R_{i,t+1} - R_{1,t+1}) + R_{1,t+1}.
\end{aligned}
\tag{1}
$$

where C_t is the agent's consumption and $E_t(.)$ is his conditional expectation operator in time t. The agent's relative risk aversion coefficient is $\gamma > 0$, $\psi > 0$ is his elasticity of intertemporal substitution coefficient, $0 < \delta < 1$ is his time discount factor and $\theta \equiv (1-\gamma)/(1-\psi^{-1})$. In the consumption based budget constraint, W_t is wealth at time t, and $R_{p,t+1}$ is the portfolio return. Finally, $\alpha_{h,i,t}$ is the portfolio weight on asset i in time t and $R_{i,t+1}$ is its next period's return. The first asset ($i = 1$) is a short-term instrument whose real return is $R_{1,t+1}$. This asset is a proxy for a risk-free one.

Epstein and Zin (1989, 1991) find that solving the problem in (1) results in the Euler equation

$$
E_t\left[\left\{\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right\}^{\theta}(1+R_{p,t+1})^{-(1-\theta)}(1+R_{i,t+1})\right] = 1 \tag{2}
$$

[12] To save on notation and because the maximization problem is the same for every agent type, we don't write the subscripts here.

that must hold for any asset i, (including the portfolio p) along the optimum consumption path. The equation shows the relationship between portfolio allocation (and consumption) and expectations. It is possible to rewrite it to explicetely show the parameters that will change for agent type h:

$$E_{h,t}\left[\left\{\delta\left(\frac{C_{h,t+1}}{C_{h,t}}\right)^{-\frac{1}{\psi}}\right\}^{\theta}(1+R_{h,p,t+1})^{-(1-\theta)}(1+R_{i,t+1})\right]=1, \tag{3}$$

where $C_{h,t}$ is consumption chosen by agent type h in time t, $E_{h,t}(.)$ is his conditional expectation operator in time t and $R_{h,p,t+1}$ is his portfolio return in $t+1$ (that depends on his asset allocation vector $\alpha_{h,t}$).The problem with this formulation is that analytical solutions can only be derived for some particular cases. Therefore, we apply the same approximate solution of Campbell et al (2001) described in the next subsections.

3.1.1 Return's dynamics

We start by postulating that agents describe the dynamics of the relevant state variables as a first-order vector autoregressive process $VAR(1)$.

Formally we define

$$x_{t+1}=\begin{bmatrix} r_{2,t+1}-r_{1,t+1} \\ r_{3,t+1}-r_{1,t+1} \\ \dots \\ r_{n,t+1}-r_{1,t+1}\end{bmatrix}, \tag{4}$$

where $r_{i,t+1} = \ln(1 + R_{i,t+1}) \; \forall \; i$, and x_{t+1} is a vector of (log)excess returns. We also include other state variables s_{t+1}, such as the price-earnings ratio, realized returns or other return forecasters, stacking them all: $r_{1,t+1}$, x_{t+1} and s_{t+1} into an $m \times 1$ vector z_{t+1}:

$$z_{t+1} = \begin{bmatrix} r_{1,t+1} \\ x_{t+1} \\ s_{t+1} \end{bmatrix}. \tag{5}$$

Conditioned on the strategy that the investor is actually using to forecast returns (his type), there will be different ways of modeling the market dynamics. A fundamentalist agent will model it considering fundamentalist forecasters, while chartists will decide based on a different collection of return forecasters. The difference between them, therefore, is on the coeficients of the VAR, as in equation (6):

$$z_{h,t+1} = \phi_{h,0} + \phi_{h,1} z_t + v_{h,t+1}. \tag{6}$$

The coefficients $\phi_{h,0}$, the $m \times 1$ vector of intercepts, and $\phi_{h,1}$, the $m \times m$ matrix of slope coefficients, are determined by the trading strategy that agent h is actually using with shocks $v_{h,t+1}$ that satisfy:

$$v_{h,t+1} \sim i.i.d. \; N(0, \Sigma_{h,v}), \tag{7}$$

$$\Sigma_{h,v} \equiv Var_t(v_{h,t+1}) = \begin{bmatrix} \sigma^2_{h,1} & \sigma'_{h,1x} & \sigma'_{h,1s} \\ \sigma_{h,1x} & \Sigma_{h,xx} & \Sigma'_{h,xs} \\ \sigma_{h,1s} & \Sigma_{h,xs} & \Sigma_{h,ss} \end{bmatrix}. \tag{8}$$

Those distributional assumptions mean that shocks can be cross-sectionally correlated, but they are homoskedastic and iid over time.[13] Given the homoskedastic $VAR(1)$ formulation, the unconditional distribution of z_{t+1} is easily derived because it inherits the normality of the shocks v_{t+1}. Diferently from BH98, we assume that agents may also disagree about the estimated variances and covariances in shocks.

3.1.2 Approximate solution

Epstein and Zin (1989, 1991) show that the value function obtained from the maximization in (1) per unit of wealth can be written as a power function of the optimal consumption-wealth ratio:

[13] The assumption of homoskedasticity is quite restrictive because it rules out the possibility that state variables predict changes in risk. So they can only affect portfolio choice by predicting changes in expected returns. However, there are many previous work showing the limited effect of those risk changes over portfolio choice. Campbell (1987), Harvey (1989, 1991), Glosten, Jagannathan, and Runkle (1993) have found only modest effects that are dominated by the effects of the state variables on expected returns. Also, Chacko and Viceira (1999) find that changes in risk are not persistent enough to have large effects on the intertemporal hedging demand.

$$V_t \equiv \frac{U_t}{W_t} = (1-\delta)^{-\frac{\psi}{1-\psi}} \left(\frac{C_t}{W_t}\right)^{\frac{1}{1-\psi}}. \tag{9}$$

Campbell and Viceira (1999) note that:

$$\lim_{\psi \to 1} \frac{C_t}{W_t} = (1-\delta), \tag{10}$$

and this guarantees that the value function (9) has a finite limit as ψ tends to 1.This result is important because it allows for an approximation close to this limit, where there is an analytical solution to the model.

Following Campbell et al (2001), the return on the portfolio in problem (1) can be approximated, exactly in continuous time and very close to the true value at short time intervals, by

$$r_{p,t+1} = r_{1,t+1} + \alpha_t' x_{t+1} + \frac{1}{2}\alpha_t'(\sigma_x^2 - \Sigma_{xx}\alpha_t), \tag{11}$$

where lower cases indicate log variables, $\sigma_x^2 \equiv diag(\Sigma_{xx})$ is the diagonal elements of Σ_{xx}, i.e. the variances of (log) excess returns.

As in Campbell (1993, 1996), we can also log-linearize the budget constraint in the same problem around the unconditional mean of the log consumption-wealth ratio. This results in

$$\Delta w_{t+1} \approx r_{p,t+1} + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) + k, \tag{12}$$

where Δ is the difference operator; $\rho \equiv 1 - \exp(E[c_t - w_t])$; and $k = \ln(\rho) + (1-\rho)\ln(1-\rho)/\rho$ is endogenous because it depends on the optimal level of

c_t relative to w_t. When $\psi = 1$, $c_t - w_t$ is constant and $\rho = \delta$.[14] In this case, the budget constraint approximation is exact.

Applying a second-order Taylor expansion to the Euler equation (**??**) around the conditional means of Δc_{t+1}, $r_{p,t+1}$, $r_{i,t+1}$ results in:

$$
\begin{aligned}
0 = {} & \theta ln\delta - \frac{\theta}{\psi} E_t \Delta c_{t+1} - (1-\theta) E_t r_{p,t+1} + E_t r_{i,t+1} \\
& + \frac{1}{2} Var_t \left[-\frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta) r_{p,t+1} + r_{i,t+1} \right].
\end{aligned} \tag{13}
$$

This log-linearized Euler equation is exact if consumption and asset returns are jointly lognormally distributed and this is the case when the elasticity of intertemporal substitution equals one ($\psi = 1$).

Considering $i = 1$ in equation (13), subtracting it from its general form, and noting that $\Delta c_{t+1} = \Delta(c_{t+1} - w_{t+1}) + \Delta w_{t+1}$, we obtain, for the other $n-1$ assets:

$$
\begin{aligned}
E_t(r_{i,t+1} - r_{1,t+1}) + \frac{1}{2} Var_t(r_{i,t+1} - r_{1,t+1}) = {} & \frac{\theta}{\psi}(\sigma_{i,c-w,t} - \sigma_{1,c-w,t}) \\
& + \gamma(\sigma_{i,p,t} - \sigma_{1,p,t}) - (\sigma_{i,1,t} - \sigma_{1,1,t}),
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
\sigma_{i,c-w,t} &= Cov_t(r_{i,t+1}, c_{t+1} - w_{t+1}), && (15)\\
\sigma_{1,c-w,t} &= Cov_t(r_{1,t+1}, c_{t+1} - w_{t+1}), && (16)\\
\sigma_{i,p,t} &= Cov_t(r_{i,t+1}, r_{p,t+1}), && (17)\\
\sigma_{1,p,t} &= Cov_t(r_{1,t+1}, r_{p,t+1}), && (18)\\
\sigma_{i,1,t} &= Cov_t(r_{i,t+1}, r_{1,t+1}), && (19)\\
\sigma_{1,1,t} &= Var_t(r_{1,t+1}). && (20)
\end{aligned}
$$

[14] This gives ρ the interpretation of a discount factor as well.

On the left hand side of equation (14), we have the risk premium on asset i over asset 1 required by each agent, adding one-half the variance of the excess return because we are considering log returns.[15]

On the right hand side, we have the factors that determine the required excess return on each asset. Factors that contribute to raise the risk premium are the excess covariance with consumption growth and excess covariance with the portfolio return. The last term, that cancels when asset 1 is risk-free, relates the covariance of the asset's excess return with the benchmark return to the required risk premium.

Since consumption growth and portfolio return are endogenous, this is only a first-order condition describing the optimal solution. So, to solve the model, it is necessary to determine those values.

Guessing that the optimal portfolio rule is linear in the VAR state vector but with a quadratic optimal consumption rule gives equations (21) and (22):

$$\alpha_t = A_0 + A_1 z_{t,} \tag{21}$$

$$c_t - w_t = b_0 + B_1' z_t + z_t' B_2 z_t. \tag{22}$$

Here A_0, A_1, b_0, B_1, and B_2 are constant coefficient matrices to be determined, with dimensions $(n-1) \times 1$, $(n-1) \times m$, 1×1, $m \times 1$, and $m \times m$,

[15] Note that the left hand side of equation (14) is determined by the dynamics of z_t, that also determines the variances and covariances on the right hand side. However, the second term $(\gamma(\sigma_{i,p,t} - \sigma_{1,p,t}))$ is a function of portfolio choice, α_t, that is made in order to equal both sides.

respectively.

Now, we simply write the conditional moments that appear in equation (14) as functions of the VAR and the unknown parameters in equations (21) and (22), solving for the parameters that finally satisfy equation (14).

For agent type h, we can write the conditional expectation on the left hand side of (14) as:

$$E_{h,t}(x_{t+1}) + \frac{1}{2}Var_{h,t}(x_{t+1}) = H_x\phi_{h,0} + H_x\phi_{h,1}z_t + \frac{1}{2}\sigma^2_{h,x}, \tag{23}$$

where H_x is a selection matrix that selects the vector of excess returns from the full state vector and $Var_{h,t}$ is the conditional volatility estimated by agent h in time t.

Campbell et al (2001) also show that the right hand side of equation (14), can be written as linear functions of the state variables:

$$\sigma_{h,c-w,t} - \sigma_{h,1,c-w,t}\iota \equiv [\sigma_{h,i,c-w,t} - \sigma_{h,1,c-w,t}]_{i=2,3,\dots,n} = \Lambda_{h,0} + \Lambda_{h,1}z_t, \tag{24}$$

$$\sigma_{h,p,t} - \sigma_{h,1,p,t}\iota \equiv [\sigma_{h,i,p,t} - \sigma_{h,1,p,t}]_{i=2,3,\dots,n} = \Sigma_{h,xx}\alpha_{h,t} + \sigma_{h,1x}, \tag{25}$$

$$\sigma_{h,1,t} - \sigma_{h,1,1,t}\iota \equiv [\sigma_{h,i,1,t} - \sigma_{h,1,1,t}]_{i=2,3,\dots,n} = \sigma_{h,1x}, \tag{26}$$

where ι is a vector of ones.

3.1.3 *Agent h's approximate demand for assets*

Finally, substituting equations (24) (25) and (26), and also (23) into the Euler equation (14) and solving for the portfolio rule we get the optimal asset demand for each investor type h:

$$\alpha^*_{h,t} = \overbrace{\frac{1}{\gamma}\Sigma^{-1}_{h,xx}\left[E_{h,t}(x_{t+1}) + \frac{1}{2}Var_{h,t}(x_{t+1}) + (1-\gamma)\sigma_{h,1x}\right]}^{\text{Myopic Demand}} \\ \underbrace{+\frac{1}{\gamma}\Sigma^{-1}_{h,xx}\left[-\frac{\theta}{\psi}\left(\sigma_{h,c-w,t} - \sigma_{h,1,c-w,t}\iota\right)\right]}_{\text{Intertemporal hedging demand}}. \tag{27}$$

Equation (27) is the multiple-asset demand generalization of Restoy (1992) and Campbell and Viceira (1999) for agent type h. This equation characterizes the optimal portfolio choice as the sum of two components: a myopic demand term and an intertemporal hedging demand term.

The first one is exactly the myopic demand with many risky assets and log-normal returns. Since this is a myopic component, it does not depend on the elasticity of intertemporal substitution. The second term is the intertemporal hedging demand. As we assumed time varying investments opportunities, Merton (1969, 1971) already predicted that an investor more risk averse than a logarithmic one would want to hedge against those shocks.[16] This can be verified by noting that the second term depends, indeed, on the excess co-variance between the risky asset's return and consumption growth. It may

[16] A logarithmic investor has coeficient of risk aversion $\gamma = 1$, and hence $\theta = 0$. Therefore, his portfolio rule is simply myopic, as we would expect. Note that $\theta = 0$ sets the intertemporal hedging demand term to zero, and the only term left (that does not even depend on θ) is the myopic one.

be interesting to note that, as the investor is willing to smooth consumption, he demands more of assets whose returns are negatively correlated with his consumption growth. This is what gives the negative sign to the intertemporal hedging demand term.

The differences in our model with respect to myopic ones are a consequence of this equation. As we introduce an intertemporal hedging demand term, the optimal allocation changes and so does the equilibrium obtained.

In order to proceed, it is worth defining the Intertemporal Hedging Demand in time t for agent type h ($IHD_{h,t}$) as:

$$IHD_{h,t} \equiv \frac{1}{\gamma}\Sigma_{h,xx}^{-1}\left[-\frac{\theta}{\psi}\left(\sigma_{h,c-w,t} - \sigma_{h,1,c-w,t}\iota\right)\right], \tag{28}$$

stressing that it depends on h and that it can also be time variant.

Equation (27) can then be rewritten as:

$$\alpha_{ht}^{*} = \frac{1}{\gamma}\Sigma_{h,xx}^{-1}\left[E_{h,t}(x_{t+1}) + \frac{1}{2}Var_{h,t}(x_{t+1}) + (1-\gamma)\sigma_{h,1x}\right] + IHD_{h,t}. \tag{29}$$

3.2 Evolution of trader types

So far, we have derived the demand for assets of a given agent type but said very little about how they choose their types in the first place. In this section we model the evolution of η_{ht}, the fraction of agent type h in time t. This is the so called "evolutionary part" of the model, that describes how beliefs about the best strategy are updated over time.

Following Brock and Hommes (1997, 1998) we state that agents choose their strategies based on their observed performance. Agents have access to fitness measures, but subject to noise due to measurement errors or non-observable characteristics as we explained before. Observed fitness of strategy h, $\tilde{U}_{h,t}$, is given by:

$$\tilde{U}_{h,t} = U_{h,t} + \varepsilon_{h,t}, \tag{30}$$

where $U_{h,t}$ is the deterministic part of the measure, and $\varepsilon_{h,t}$ represents the noise in its observation. We assume that $\varepsilon_{h,t}$ is *iid* across types, drown from a double exponential distribution. In this case, as the number of agents tends to infinity, the probability that a given agent chooses strategy h is given by the multinomial logit probabilities of a discrete choice. We describe the fractions n_{ht} of trader types as:

$$n_{ht} = \frac{\exp(\beta U_{h,t-1})}{\chi_{t-1}}, \tag{31}$$

$$\chi_{t-1} = \sum_{h=1}^{H} \exp(\beta U_{h,t-1}), \tag{32}$$

where $U_{h,t-1}$ is the fitness measure of strategy h evaluated in period $t-1, \chi_{t-1}$ is a normalization term and the parameter β is the intensity of choice, that is inversely proportional to the variance of the noise $\varepsilon_{h,t}$.[17]

[17] When the noise's variance is infinity, $\beta = 0$. In this case, agents cannot observe differences in fitness and are not sensible to differences in strategies performance. The other extreme situation is when there is perfect observation of strategies performances, or $\beta = \infty$. In this case, all agents switch strategies when they see differences

We assume that a measure of evolutionary fitness of strategy h is realized profits over a given period, that is given by

$$U_{ht} = (x_t) \bullet \alpha_{h,t-1} + \omega U_{h,t-1}, \tag{33}$$

where ω is a memory parameter that reflects how slowly agents discount past strategy success for selecting their trading rules. We consider the most simple case, with no memory, i.e. $\omega = 0$. In this case, equation (33) becomes

$$U_{ht} = (x_t) \bullet \alpha_{h,t-1}. \tag{34}$$

3.3 The equilibrium price

Considering that the market is in equilibrium, i.e. total demand, α_t^d, equals total supply, α_t^s, for each asset, equation

$$\sum_{h=1}^{H} \eta_{ht} \bullet \alpha_{ht} = \alpha_t^d = \alpha_t^s \tag{35}$$

holds, where the vector η_{ht} denotes the (possibly different) fraction of trader type h at date t in each of the asset markets, considering H different trader types. And where $\bullet$ is the direct product operator.

Substituting equation (29) into equation (35) for the case of zero outside supply shares, i.e. $\alpha_t^s = 0$, equation (36) represents the market clearing condition that closes the model:

in performances.

$$\sum_{h=1}^{H} \eta_{ht} \bullet \left\{ \begin{array}{c} \frac{1}{\gamma}\Sigma_{h,xx}^{-1} \left[E_{h,t}(x_{t+1}) + \frac{1}{2} Var_{h,t}(x_{t+1}) + (1-\gamma)\sigma_{h,1x} \right] \\ + IHD_{h,t} \end{array} \right\} = 0. \qquad (36)$$

4 Empirical application

In this section, we assume the perspective of a global investor who can allocate funds between four major stock markets: USA (Dow Jones Industrials), UK (FTSE all share), Japan (Nikkei 500) and Hong Kong (Hang Seng).

There are two main questions we want to answer in our exercise: The first one is if traders tend to use the same strategies (fundamentalist or graphist) across markets. The second is how the use of strategies relate to the observed price trends in each market.

We find that the proportions of fundamentalists (and graphists) tend to be positively correlated between the FTSE, Hang Seng and Nikkei indices, especially in the first two. But, on the other hand, fundamentalist's proportions in the Dow Jones index tend to be negatively correlated to all other indices.

This may suggest that the use of strategies is related to the trader's preferred strategy and not so much to each market condition, i.e. a fundamentalist trader uses fundamentalist strategies regardless the market (or asset) he is evaluating. [18]

[18] We still need to know more about the fraction of global traders in each of these markets to make it possible to check this hypothesis. A large (relative) presence of investors who trade both in the UK and in Hong Kong would provide further signif-

We also note that the proportion of different trader types fluctuate according to the market conditions. These fluctuations are more proeminent in the Nikkei and Hang Seng and less clear in the Dow Jones and FTSE indices.

One possible explanation is the fact that the first two markets show stronger tendencies during the observed period, while in the last ones the tendency is not so easy to determinate.

4.1 Data Description

Our database is made of quarterly data for the US, UK, Japan and Hong Kong's stock market that is described on Table 2. All relevant returns and consumption data are given in dollars and relative to the American economy. The adjusted quarterly data goes from 1993:1 until 2007:1. Index values and dividend-price ratios are obtained from DataStream. Quarterly data on the American consumption-wealth ratio is obtained from the updated dataset in Lettau and Ludvigson (2004). The CPI series is obtained from the U.S. Department of Labor: Bureau of Labor Statistics.

We construct the real log stock return in dollars as the difference between the log return on the stock index of each country and the US inflation in the period, using the Consumer Price Index. We report the results for $\Psi = 0.5$, $\beta = 0.75$ and $\gamma = 2$. However, we have estimated the model for $\beta = \{0.25, 0.75, 0.5\ , 1\}$ and $\gamma = \{1, 2, 5, 20\}$ with the same general results.

icance to these findings. For the same reason, given the overall negative correlation between the chosen strategies (or types) between the Dow Jones and the rest of the indices, we would not expect a significant proportion of global investors in the USA.

Returns	*Dow*	*FTSE*	*Nikkei*	*Hang Seng*
Mean	0.0085	0.0070	−0.0004	0.0178
Median	0.0172	0.0150	−0.0001	0.0437
Maximum	0.1966	0.5763	0.2427	0.5158
Minimum	−0.3211	−0.3493	−0.3737	−0.7288
Std.Dev.	0.08	0.11	0.13	0.19
Skewness	−0.84	0.34	−0.46	−0.83
Kurtosis	5.1	7.2	3.2	5.6
Observations	226	180	85	170

Table 2: Descriptive statistics for the series of Dow Jones Industrials, FTSE all shares, Nikkei 500 and Hang Seng real quarterly returns in US$.

4.2 Estimation

We estimate a slightly less complex version of our model, assuming only $H = 2$ different agent types and $n = 4$ assets. We, then, follow two steps: Finding the demands of fundamentalists and graphists in equation (27) and then the evolution of types given by equation (31). Variances and covariances are estimated using the constant conditional correlation multi-variate GARCH specification proposed by Bollerslev (1990).

4.2.1 Estimated trader's models

Fundamentalists agents use a model to predict asset's i real return in time t ($x_{i,t}$) that considers the dividend-price ratio in time t ($DP_{i,t}$) as a forecaster for return in time $t+1$ ($r_{i,t+1}$). This model is given by

$$x_{i,t} = \mu_i + \rho_o x_{i,t-1} + \rho_1 DP_{i,t} + \rho_2 DP_{i,t-1} + e_{i,t}. \tag{37}$$

Past real return ($x_{i,t-1}$) is included to eliminate endogeneity in the equation; $e_{i,t}$ is an error term.

Graphist traders use only past returns to forecast future return for asset i, given by equation

$$x_{i,t} = \mu_i + \rho x_{i,t-1} + e_{i,t}. \tag{38}$$

These models are estimated for each one of the $n = 4$ assets. They give the inputs for the (restricted) VAR that agents use to describe the market. The parameters in equations (37) and (38) are estimated recursively based on the information available to the traders. This means, for instance, that to estimate ρ in 1999:4, traders use the information available up to 1999:3.

4.2.2 Estimated evolution of trader types

The second step is to find the evolution of types in our model. We consider different values for β, $\beta \in \{0.25, 0.5, 0.75, 1\}$, obtaining the same qualitative results. As explained before, we assume that investors measure fitness of a given strategy by its present observed profit, i.e. last period's observed profits are forgotten so there is no memory in the fitness function. Fitness is, thus, given by the observed real returns in time t, x_t, and the estimated allocation by agent type h, in the previous period, $\alpha_{h,t-1}$:

$$U_{h,t} = (x_t) \bullet \alpha_{h,t-1}. \tag{39}$$

4.3 Results

Our first empirical exercise is to examine the effect of changing β and γ in the estimated proportions of fundamentalists and graphists evolution. We see from figure (1) that changing these values affect how agents respond to differences in the fitness of a given strategy.

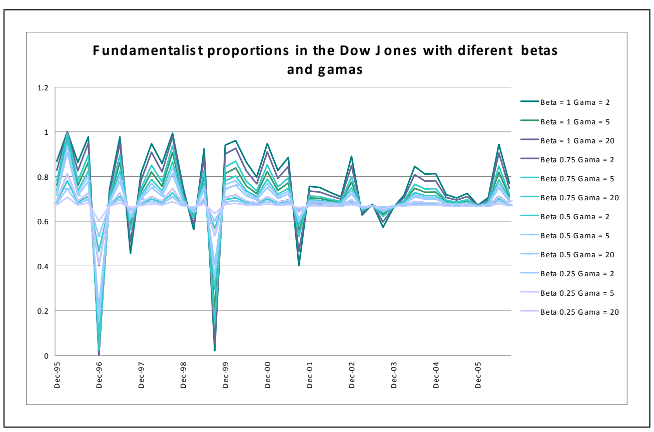

Fig. 1. **Fundamentalist proportions in the Dow Jones for diferent values of β and γ**

This result was already expected: β is the intensity of choice and is negatively correlated to the noise in the strategy's observed performance. In other words, we expect that a high value of β corresponds to the situation where traders better observe differences in performance between the two strategies and this increases the likelihood of changing types, resulting in a higher variance in the type's proportions through time.

Equation (29) shows the effect of the risk aversion coefficient, γ, over asset's demand in our model. We note that increases in γ make the intertemporal hedging demand term more important relatively to the myopic term when everything else remains constant.

The increase in γ, however, is associated with decreases in the variance of trader's proportions though time. This happens because traders respond to difference in performance (profits). If the asset allocation is similar, there is less difference between the two strategies and therefore traders don't switch types so often.

The difference in the myopic term between the two strategies tend to be higher than the difference between their intertemporal hedging demand term. So, when we consider more weight on the second one, asset demands tend to be less different.

We then examine how trader's proportions in each market evolve through time. In table 2 we see the correlation between fundamentalist's proportions. In the FTSE, Hang Seng and Nikkei they are positively correlated, especially between the first two. On the other hand, this correlation is negative between the Dow Jones and these other three markets. As we noted before, one possible explanation is that investors choose their types regardless of the assets they are trading. In this case, we would expect a large proportion of (the same) global investors trading in the FTSE and Hang Seng markets[19].

Correlation	*Dow*	*FTSE*	*Hang Seng*	*Nikkei*
Dow	1	−0.51	−0.60	−0.04
FTSE		1	0.50	0.13
Hang Seng			1	0.06
Nikkei				1

Table 2: Descriptive statistics for the series of Dow Jones Industrials, FTSE all shares, Nikkei 500 and Hang Seng real future quarterly returns in US$.

Finally, we explore the variation in fundamentalist traders proportions in the 4 different markets. We have plot a "normalized" version of the indices to make it possible to observe the variations in trader's proportions, (between 0 and 1), in the same graph as the variations in prices.[20]

Starting with the Nikkei index, we see in figure (2) that the fraction of fundamentalists significantly drops in the second half of 1998 in a trend that persists until the end of the first quarter of 1999 when the market reaches

[19] We still need more information about investors in each of those markets to fully understand this relationship.

[20] We normalize the index dividing it by its value in 1994.

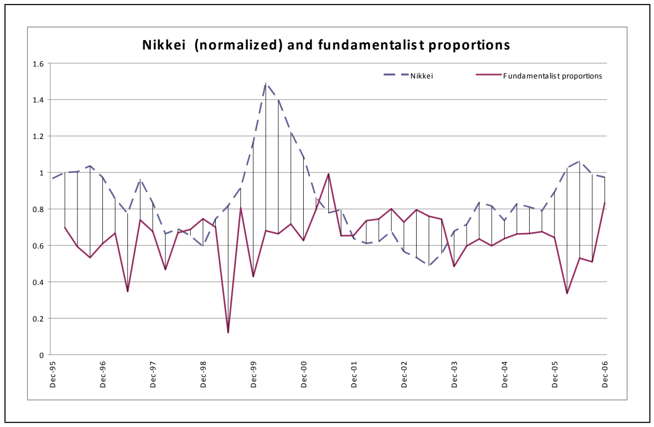

Fig. 2. **The normalized Nikkei index and fundamentalist proportions**

its peak. At this stage, the proportion of fundamentalists starts to increase consistently while the index value goes back to the level where it was before. If we consider that a bubble occurs when prices deviate from fundamentals and resumes when they start to reflect them again, this period could be a good example of how it happens in our model.

The fraction of fundamentalist traders decreases when prices go up until the end of 1999:1. This happens because since prices are not following fundamentals, this strategy is not successful to forecast returns. When prices start to revert back to fundamentals, traders start to believe that fundamentalist strategies are correct and their proportion increases consistently until 2000:2, where the market prices are back to the same level as before, reflecting fundamentals again.

In the Hang Seng index we observe the same pattern. Fundamentalists leave the market (or become graphists) when their strategies are not successful in predicting the market's movement. From figure (3) we see that the last increase in prices starting in 2002:2 seem to be consistent with fundamentals as given by our model. We also note that from the end of 1999 to the end of 2001, the same drop in fundamentalist's proportion happened in this market, only

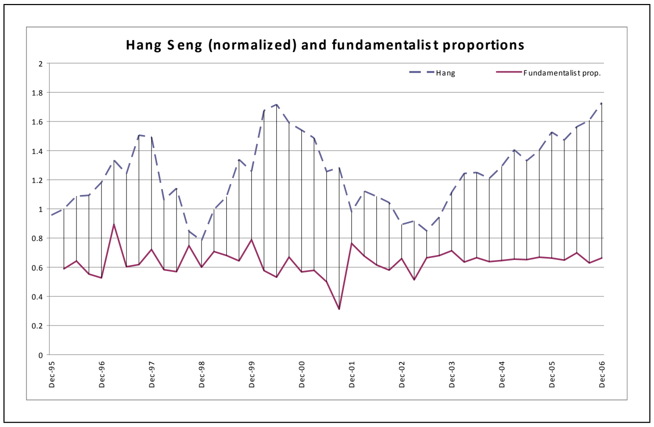

Fig. 3. **Normalized Hang Seng index and fundamentalist proportions**

reaching its normal values after the decrease in prices observed in the last quarter of 2001.

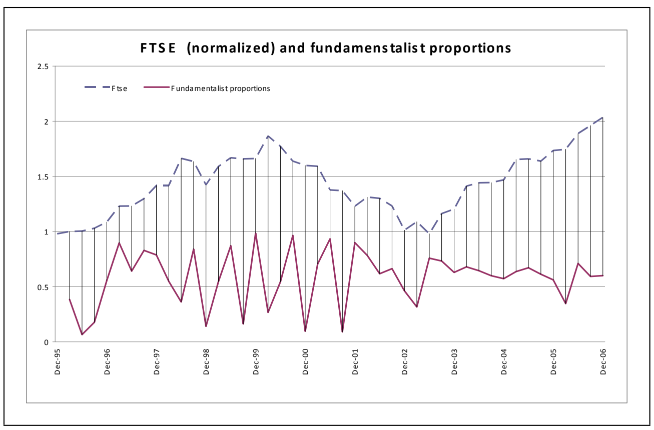

Fig. 4. **Normalized FTSE index and fundamentalist proportions**

Regarding the FTSE index, in figure (4), one important observation is about the last price trend since 2002:2. The fraction of fundamentalists shows a slight decrease during this time, indicating that this movement is not completely driven by fundamentals. In this case, we expect prices to eventually revert back in the future.

Finally, when we examine the Dow Jones index in figure (5), its value appears to be consistent with fundamentals in most of the time. There are some drops in fundamentalist proportions but nothing that can be characterized as a trend or periods of consistently low participation of fundamentalist traders in the market. In general, there are no extreme variations or observed trends in the trader's proportions.

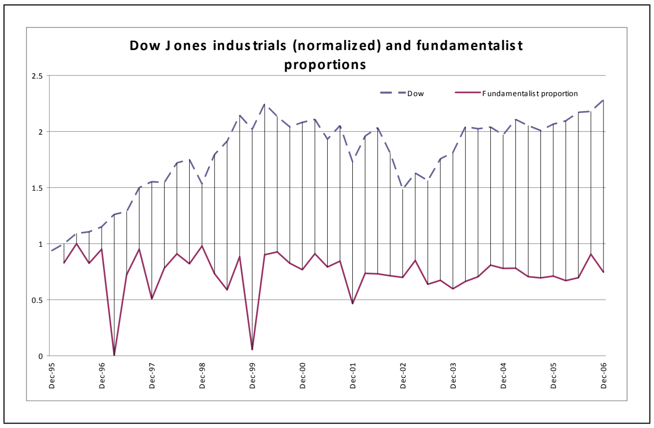

Fig. 5. **Normalized Dow Jones index and fundamentalist proportions**

5 Concluding remarks

We have proposed a new asset pricing model, where rational agents, with long investment horizons, and maximizing a recursive utility function choose the strategy used to forecast returns based on its previous profitability. We wanted to check if this formulation was enough to generate the observed price trends in the market. We were specially concerned about movemens in prices that are not driven by fundamentals, because of its relative conflict with rational agents models.

We concluded that theoretically our model would be able to generate price trends that were not related to fundamentals as long as a large fraction of investors were (rationally) using chartist strategies. In the previous section, we also concluded that empirically our model is also capable of generating these price trends.

The model adds to the literature in a few ways: Comparing it with the original strategic asset allocation homogeneous model, it is possible to see that the implied equilibrium asset prices are completelly different. In particular, one needs to know the proportions of each agent type to compute the resulting equilibrium price. These proportions are not known beforehand since they depend on the market conditions and, thus, are varying over time.

We have also found results that are different from the ones in BH98, since the asset allocation has changed given the long term intertemporal asset demand, in contrast with the myopic one in their original work. The main difference is that in our model there is a higher demand for assets with intertemporal hedging properties as explained before. This result is specially important when we consider, again differently from BH98, n risky assets in the economy and differences in relative prices become more important.

Finally, we show that this problem is separable, meaning that the adaptive part of the model (i.e. modeling the evolution of trader types fractions) is independent from the prevailing fundamental price. Since the strategic asset allocation demand is also separable itself (into a myopic and intertemporal hedging demand), we end up having a two step solution. The first one is to determine the demand for each asset, and the second is to determine how agents are going to interact, choosing their types.

The empirical part of our exercise shows one of many applications for our model. An important result in this section is the ability of the model to reproduce the apparently "irrational" market's behaviour during periods of deviations from fundamentals.

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