Calculation of NoLoad Losses in an Induction Motor using an Inverse Vector Preisach Model and an Eddy Current Loss Model

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Abstract—This paper deals with the direct inclusion of vector hysteresis and eddy current losses in a 2D finite element (FE) time stepping analysis. It is shown that a vector Preisach model can be inverted in an efficient way by means of the Newton-Raphson method and that it thus can easily be included in the Newton-Raphson iterations of the FE equations. The eddy current losses are accounted for by considering an additional conductivity matrix in the FE equations. The method is applied to the no-load simulation of an induction motor. The calculation results are discussed and compared to measurements.

Index terms—hysteresis, eddy currents, magnetic losses, finite element methods, induction machines

I. INTRODUCTION

In 2D FE time stepping simulations of electrical machines, single-valued $BH$-curves are usually adopted for all materials comprised in the model. The irreversible material behaviour in the iron core is thus completely ignored in the time stepped magnetic field equations, but the ensuing iron losses may readily be estimated a posteriori [1]-[3].

The inclusion of a hysteresis model in the 2D FE simulation is hampered by the fact that most hysteresis models, and in particular the (vector) Preisach model, take the magnetic field $\mathbf{B}$ as input, while the magnetic vector potential $\mathbf{A}$ is based on the induction $\mathbf{B}$. As will be demonstrated, this problem can be overcome by explicitly inverting the Preisach model, which can be done efficiently by means of the Newton-Raphson (NR) method.

The eddy currents in the iron core, mainly flowing parallel to the plane of the laminations, cannot be included as such in the 2D model. However, the ensuing losses may simply be effected by considering an additional conductivity matrix in the FE equations, as will be shown.

II. CONVENTIONAL FE EQUATIONS

For 2D magnetic field computations (in the $xy$-plane) by means of the FE method, the magnetic vector potential (MVP) $\mathbf{A} = A_z(x, y, t) \mathbf{1}_z$ is commonly introduced. The derived induction $\mathbf{B} = \nabla \times \mathbf{A} = \frac{\partial A_y}{\partial x} \mathbf{1}_x - \frac{\partial A_x}{\partial y} \mathbf{1}_y$ automatically satisfies $\nabla \cdot \mathbf{B} = 0$. A FE discretisation of the 2D domain with $n_\Omega$ nodes leads to a set of $n_\Omega$ algebraic equations:

$$S(A) A = I_p,$$

with $S$ the stiffness matrix, $A$ the column matrix of the MVP nodal values and $I_p$ the column matrix of equivalent nodal currents. It can easily been shown that the $i$-th equation of (1) is equivalent to

$$(SA)_{i} = \oint_{C_i} \mathbf{H} \cdot d\ell \equiv (I_p)_{i} = \int_{S_{i}} J_z dx dy,$$

with $C_i$ an elementary contour around the $i$-th node [4].

In the dynamic case distinction is commonly made between stranded ($s$) and massive conductors ($m$), which are current and voltage driven respectively. Equation (1) can be rewritten as a system of differential equations:

$$SA + T_m \frac{dA}{dt} = F(I_s, V_d),$$

with $T_m$ the conventional conductivity matrix. Coupling with an electrical circuit is easily accounted for, but for sake of brevity not considered in the discussion hereafter.

Time discretisation using e.g. the 'β-scheme' ($0.5 \leq \beta \leq 1$), leads to a system of nonlinear algebraic equations for each time step from $t^* \rightarrow t^{*} + \Delta t$:

$$S(A^{*}) + \frac{T_m}{\beta \Delta t} \Delta A^{*} = F^{*}(I_{s}^{*}, V_{d}^{*}, I_{m}^{*}, V_{d}^{*}),$$

which is resolved by means of the NR method. Starting from the initial estimate $A_{0}^{*} = A^{*}$, subsequent increments $\Delta A_{i}^{*} (i = 1, 2, \ldots)$ and approximations $A_{i}^{*} = A_{(i-1)}^{*} + \Delta A_{i}^{*}$, are defined by the following linearised system:

$$S^{*}(A_{(i-1)}^{*}) + \frac{T_m}{\beta \Delta t} \Delta A_{i}^{*} = F^{*} - S^{*}(A_{(i-1)}^{*}) - \frac{T_m}{\beta \Delta t} A_{(i-1)}^{*},$$

where $S^{*}$ is the Jacobian of the stiffness matrix.

III. INCLUSION OF THE EDDY CURRENT LOSSES

We consider a wide lamination of thickness $d$ and conductivity $\sigma$ which carries a time varying flux parallel to the plane of the lamination. For low frequencies the induction $B$ is approximately constant over the thickness
(no skin effect). The following relation between the magnetic field \( \vec{H} \) at the surface of the laminator, the average magnetic field \( \bar{H}_a \) and the average induction \( \bar{B}_a \) is readily derived:

\[
\bar{B}_a(t) = \bar{H}_a(t) + \frac{\sigma \partial^2 \bar{B}_a}{12} \frac{d\bar{B}_a}{dt}. \tag{6}
\]

For a sinusoidal induction with amplitude \( \bar{B}_a \) and (sufficiently low) frequency \( f \), the (time averaged) classical eddy current losses \( p_{cl} \) [W/m²] are given by the following well known expression:

\[
p_{cl} = \frac{1}{6} \sigma \pi^2 \frac{d^2}{f^2} \bar{B}_a^2. \tag{7}
\]

We now consider a laminated iron core in the 2D FE model and a node \( i \) situated in its cross-section. Equation (2) only holds for a contour \( C_i \) situated on the surface of the laminator, i.e., for \( \vec{H} = \bar{H}_a \). The integral along \( C_i \) of the first term \( \bar{B}_a \) in (6) gives rise to the stiffness matrix \( S \), with \( \bar{B}_a \) and \( \bar{H}_a \) related through the average (reversible or hysteretic) material behaviour. The integral of the second term in (6) can be rewritten as follows:

\[
\frac{\int_{C_i} \sigma \partial^2 \bar{B}_a}{12} \frac{d\bar{B}_a}{dt} \cdot dI = \left( T_{en} \frac{dA}{dt} \right)_i, \tag{8}
\]

with the \( n_p \times n_p \) conductivity matrix \( T_{en} \) given by:

\[
(T_{en})_{ij} = \int_{\Omega_{en}} \frac{\sigma \partial^2 \bar{B}_a}{12} \nabla \alpha_i \cdot \nabla \alpha_j \, dx \, dy,
\]

where \( \alpha_i(x,y) \) is the \( i \)-th interpolation function.

Equation (3) thus becomes:

\[
SA + (T_{en} + T_{m}) \frac{dA}{dt} = F(I, V, \delta). \tag{10}
\]

IV. INCLUSION OF THE VECTOR PREISACH MODEL

A. Scalar Preisach Model

The classical Preisach model, which is scalar and rate-independent, has been described and employed in numerous publications, see e.g., [5]. In this work an analytical distribution function containing six parameters is adopted [6][7]. The differential permeability \( \bar{\mu}(H, H_{\text{ext}}) = \frac{\partial \bar{B}}{\partial H} \) can be evaluated analytically as a function of the actual magnetic field \( \bar{H} \) and its past, denoted \( H_{\text{ext}} \), which consists of a number of extreme \( H \)-values. The induction \( B(H, H_{\text{ext}}) \) is obtained by Gaussian integration of \( \bar{\mu} \).

B. Isotropic Vector Extension of the Scalar Model

In the vector Preisach model [3][2][7][4], the projection \( H_\theta = \bar{H} \cdot \hat{e}_\theta = H_\theta \cos \theta + H_y \sin \theta \) of the magnetic field \( \bar{H} \) on an axis which encloses an angle \( \theta \) (\( 0 \leq \theta \leq \pi \)) with e.g., the \( x \)-axis, is input to a scalar Preisach model. The induction \( \bar{B} \) and the differential permeability tensor \( \bar{\mu} = \frac{\partial \bar{B}}{\partial H} \) are given by:

\[
\bar{B}(H, H_{\text{ext}}) = \frac{1}{\pi} \int_0^\pi d\theta \, B_\theta(H_\theta, H_{\theta\text{ext}}) \hat{e}_\theta, \tag{11}
\]

\[
\bar{\mu}(\bar{H}, H_{\text{ext}}) = \frac{1}{\pi} \int_0^\pi d\theta \, B_\theta(H_\theta, H_{\theta\text{ext}}) \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}, \tag{12}
\]

where \( B_\theta \) and \( \mu_\theta = \frac{\partial B_\theta}{\partial H_\theta} \) are supplied by the scalar Preisach model. In practical applications a finite number of angles have to be considered, e.g., \( \theta_i = (i-1)\pi/n\theta \), \( i = 1, \ldots, n\theta \).

C. Inversion of the Vector Model

The inverse vector Preisach model, which supplies the magnetic field \( \bar{H} \) as a function of its past \( \bar{H}_{\text{ext}} \) and the actual induction \( \bar{B} \), is required for the FE incorporation of the Preisach model presented in this paper. The direct vector Preisach model can be inverted as follows.

For a known state \( (\bar{H}^-, \bar{H}^-_{\text{ext}} \bar{B}^-) \) at \( t^- \), and a given \( \bar{B}^+ \), the sought \( \bar{H}^+ \) is the (a) solution of the following nonlinear vector equation:

\[
\bar{B}^+(\bar{H}^+, \bar{H}^-_{\text{ext}}) = \bar{B}^+. \tag{13}
\]

Starting from the initial estimate \( \bar{H}^{+1} = \bar{H}^- \), subsequent approximations \( \bar{H}^{+i+1} = \bar{H}^{+i} + \Delta \bar{H}^{+i} \) \( i = 1, 2, \ldots \), are obtained by means of the NR method:

\[
\Delta \bar{H}^{+i} = \left( \bar{\mu}^+(\bar{H}^{+i-1}, \bar{H}^-_{\text{ext}}) \right)^{-1} \left( \bar{B}^- - \bar{B}(\bar{H}^{+i-1}, \bar{H}^-_{\text{ext}}) \right), \tag{14}
\]

where \( \bar{B}(\bar{H}, \bar{H}_{\text{ext}}) \) and \( \bar{\mu}^+(\bar{H}, \bar{H}_{\text{ext}}) \) are given by the direct model (11-12).

In practice fast convergence is usually observed, even when \( \bar{\mu}^+ \) displays discontinuities in the considered \( \bar{H} \)-interval. Relaxation of the NR scheme is rarely required.

D. Inclusion in the FE Equations

The vector Preisach model may easily be included in the FE equations (4, 5) in a natural way as follows.

The contribution of each hysteretic element to the Jacobian matrix \( S \) at the \( i \)-th NR iteration (5) can readily be computed from \( \bar{\mu}(\bar{H}^{+i}, \bar{H}^-_{\text{ext}}) \), which is supplied by the (direct) vector Preisach model (12). The contribution of each hysteretic element to the column matrices \( S(A_-^i) A_-^i \) in (5), and \( S(A_{\text{ext}}^{(i-1)}) A_{\text{ext}}^{(i-1)} \) in (5), can be calculated from \( \bar{H}^- \) and \( \bar{H}_{\text{ext}}^{(i-1)} \), respectively, considering (2).

From the \( i \)-th approximation \( A_{\text{ext}}^{(i-1)} \), the induction \( \bar{H}_{\text{ext}}^{+i} \) follows in each hysteretic element. At this point the inverse vector Preisach model is required to obtain the corresponding \( \bar{H}^{+i} \), after which the next NR iteration can be started. When the NR process has converged sufficiently, the material history is updated in each hysteretic element (yielding \( \bar{H}_{\text{ext}}^{+i} \)).

In [4] a simpler but less accurate method, which does not require inversion of the Preisach model, is proposed. For the mild application example considered in [4] (a three phase transformer) it gives satisfactory results. However, for the simulation of an induction motor (featuring high local saturation and elevated slot harmonics) it was found to be inadequate, even with a very small time step.

In [7] the inclusion of a vector Preisach model in the FE equations is handled by means of the fixed point technique. A mild application example is considered.
V. NOLOAD SIMULATION OF AN INDUCTION MOTOR

No load measurements and simulations have been carried out on a 3kW 4-pole squirrel-cage induction motor [2][3] with 32 unskewed and open rotor slots. The stator phases are delta connected to the 50 Hz 220 V mains. Only one pole is modelled. A FE mesh having 3000 nodes and 6000 first order elements (including three layers in the airgap) is used. The stator and rotor iron core, comprising 2566 elements in the FE mesh, are modelled using the vector Preisach model (with \(n_p=20\)) and the conductivity matrix \(T_{km}\). From a \(BH\)-curve (Fig. 1) and a 50 Hz iron loss curve of the electrical steel (VH500-65D, \(d=0.65\) mm, \(\sigma=6 \times 10^6\) S/m, mass density 7850 kg/m\(^3\)), the six parameters of the vector Preisach model are fitted. The quasi-static hysteresis loss curve is obtained by subtracting the eddy current losses (7) from the total iron losses, as shown in Fig. 2. This implies that the excess losses (proportional to \(f^{3/2}\) in case of sinusoidal \(B\) [1][6]) are disregarded. As both the eddy current losses and the quasi-static hysteresis losses are directly included in the FE model, the correct iron loss dependence on \(B\) will be effected in case of sinusoidal 50 Hz induction, regardless of the value of conductivity \(\sigma\) in (7). In case of harmonic distortion, the implemented iron loss law is possibly inaccurate due to nonnegligible excess losses.

Fig. 1. Measured and fitted \(BH\)-curve

![Fig. 1. Measured and fitted BH-curve](image)

Fig. 2. Measured and fitted hysteresis and iron losses at 50 Hz

![Fig. 2. Measured and fitted hysteresis and iron losses at 50 Hz](image)

Fig. 3 shows the measured and calculated stator phase currents, which agree reasonably well. Fig. 4 shows the calculated current in the first rotor bar. By subtracting the measured stator Joule losses (112 W), the calculated rotor Joule losses (30.3 W) and the mechanical power losses (estimated 7 W) from the electrical power input (261 W), the iron losses are estimated at 112 W.

Fig. 5 shows the \(B\)-locus in four points in the stator core, the position of which is indicated in Fig. 5. Fig. 6 shows the corresponding \(B_{\alpha x}\), \(H_{\alpha x}\), \(B_{\alpha y}\), \(H_{\alpha y}\), \(B_{\alpha z}\), and \(H_{\alpha z}\)-loops. Fig. 7 shows the \(B_{\alpha x}\) and \(H_{\alpha y}\)-locus in point 4 in more detail. The area of a \(B_{\alpha z}\)-loop \((x+y)\) equals the quasi-static hysteresis loss density per cycle \(\text{[W/m}^2\text{]}\) produced by the Preisach model. The area of a \(B_{\alpha x}\)-loop equals the total loss density per cycle. The difference between the two areas is the eddy current loss density and is effected by the matrix \(T_{km}\).

![Fig. 3. Measured (left) and calculated (right) stator phase currents](image)

![Fig. 4. Calculated current in the first rotor bar](image)

Table I lists the calculated losses and average loss densities (eddy current losses, quasi-static hysteresis losses and total losses) in six regions in the stator and rotor core (defined in Fig. 5) and in the whole motor. The total iron losses are underestimated by about 10%. Some possible causes for the common underestimation of the iron losses are discussed in [1].

![Fig. 5. Points and regions in stator and rotor core](image)

![Fig. 6. B-locus in points 1, 2, 3 and 4 in the stator](image)

In the stator yoke (region I, point 1) the induction is slightly rotational and harmonically distorted. Near the stator tooth losses (region II, point 2) the induction is considerably rotational. In the stator teeth (region III, point 3) the induction is quasi-unidirectional. Near the airgap (regions IV and V, point 4) the induction is very distorted, resulting in a high hysteresis loss density and
Fig. 7. $B_{azH_{az}}$- and $B_{ayH_{ay}}$-loops (full line) and $B_{azH_{az}}$- and $B_{ayH_{ay}}$-loops (dotted line) in points 1, 2, 3 and 4.

![Diagram](image)

Fig. 8. $B_{azH_{az}}$- and $B_{ayH_{ay}}$-loops in point 4.

![Diagram](image)

Table I. Loss distribution in the motor

<table>
<thead>
<tr>
<th>Region</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{Aa}$ [W]</td>
<td>17.6</td>
<td>3.56</td>
<td>12.9</td>
<td>12.9</td>
<td>20.9</td>
<td>2.44</td>
<td>70.3</td>
</tr>
<tr>
<td>$P_{Bb}$ [W]</td>
<td>16.2</td>
<td>2.31</td>
<td>9.11</td>
<td>1.36</td>
<td>1.44</td>
<td>0.42</td>
<td>30.8</td>
</tr>
<tr>
<td>$P_{tot}$ [W]</td>
<td>33.8</td>
<td>5.87</td>
<td>22.0</td>
<td>14.2</td>
<td>22.4</td>
<td>2.86</td>
<td>101.</td>
</tr>
<tr>
<td>$p_{IR}$ [W/kg]</td>
<td>3.63</td>
<td>4.60</td>
<td>6.72</td>
<td>7.59</td>
<td>9.28</td>
<td>0.62</td>
<td>5.93</td>
</tr>
<tr>
<td>$p_{Bb}$ [W/kg]</td>
<td>2.33</td>
<td>2.59</td>
<td>4.75</td>
<td>8.03</td>
<td>6.96</td>
<td>0.11</td>
<td>2.60</td>
</tr>
<tr>
<td>$p_{tot}$ [W/kg]</td>
<td>6.96</td>
<td>7.59</td>
<td>11.5</td>
<td>83.9</td>
<td>107.</td>
<td>0.73</td>
<td>8.54</td>
</tr>
</tbody>
</table>

and compared to measurements.

VI. Conclusion

The eddy current losses in the laminated core are easily included in the 2D FE simulation by means of an additional conductivity matrix. The skin effect in the laminations is hereby neglected. The direct inclusion of a vector Preisach model in the NR iterations of the FE equations, as presented in this paper, requires the inversion of the Preisach model, for which the NR method can be utilized as well.

A no-load simulation of an induction motor has been carried out. The calculation results have been discussed.

VII. References


