On the Computation of Prices Indices*

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1 Introduction

While there are no significant investment characteristics that inhibit art from being considered as an asset, a major hurdle has long been the lack of a systematic measure of its financial performance. Due to its heterogeneity (each piece is different) and its infrequency of trading (the exact same piece does not come to the market very often), the determination of changes in market value is difficult to ascertain.

Anderson (1974) and Stein (1977) were among the very first to study returns in art markets. But it is undoubtedly Baumol's (1986) surprising result about the very small long-term return of art (0.55 percent between 1600 and 1950), coupled with the price boom in the late 1980s, shortly after his paper was published, that triggered most of the literature. It became so large, that it is hard to find, quote and do justice to all studies.

Many different art or antique markets have been looked at: African art, books, vintage cars, ancient coins, Old Master drawings, furniture, crafted ivory objects, jewellery, photographs, prints, sculpture, silverware, stamps, and probably others. Of course, the largest effort was devoted to portfolios of paintings, often Impressionists and their followers, but also American, Belgian, Canadian, Italian, Latin-American, Pre-Raphaelite, and other groups of painters. Some papers look at individuals painters, some study individual collections.

There are several important uses for a well-constructed art market index:

(a) outline general market trends, much like the Dow Jones Industrial Average describing the general direction of the US stock market. This would help us measure art market returns and compare it to other asset class so that we can ascertain whether art is a viable investment asset class.

(b) provide a concise measure of art market volatility as well as its correlation with other financial instruments, such as stocks and bonds, making it possible to address the question whether investing in art would diversify risk in a long-term investment portfolio.

(c) allowing to examine what are the major social and economic factors affecting art market price movements. It would be easy to examine, for example, how inflation affects art market prices.

(d) provide a simple way of appraising the value of art works. By assuming artworks appreciate at the art market rate, just as assuming a US stock price moves at the rate of the S&P 500, one can mark-to-market and derive a simple valuation for artworks. Recent studies, such as Mei, Moses, and Xiong (2004) have discovered that this simple valuation estimate

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1 As well as wines, which can also, in some sense, be considered as art.
2 See Appendix 1 for details and references.
can provide some anchor to art prices, much like book-value is a useful valuation tool for stocks.

There are several important properties that art indices must have in order to serve as investment benchmarks:

(a) they must be based on reliable and publicly available price information (catalogues or computer databases) so that they can be independently replicated;
(b) methods used to construct indices must address the heterogeneity issue raised above and should be based on well-established economic models and estimation procedures that make the results transparent and easy to verify;
(c) the data should avoid sample selection biases;
(d) indices should distinguish many different collecting categories, since returns may vary dramatically;
(e) the data must be regularly updated to provide a real time investment guide; it may also allow an art portfolio to be "marked to market" on a timely basis.

We now briefly turn to each of these aspects. Most papers use auction prices, since these are easily available, very few are able to get hold of other data. Gallery prices are analyzed by Candela and Scorcu (2001), while the analysis of returns for stamps (Feuillolay, 1996) or coins (Verbert, 1991) is sometimes carried out on catalogues.

Two estimation methods are commonly used to construct indices. Repeat-sales regression (RSR) uses prices of individual objects traded at two distinct moments in time. If the characteristics of an object do not change (which is usually so for collectibles), the heterogeneity issue is bypassed. This approach, extensively used in real estate studies, is applied to art markets by Anderson (1974), Baumol (1986), Goetzmann (1990, 1993, 1996), Goetzmann and Spiegel (2003), Locatelli Biey and Zanola (1999), Mei and Moses (2002a, 2002b), Pesando (1993), and Pesando and Shum (1996, 1999). The basic idea of the hedonic regression (HR) method is to regress prices on various attributes of objects (dimensions, artist, subject matter, etc.) and to use the residuals of the regression which can be considered as "characteristic-free prices" to compute the price index. The method was used by Frey and Pommerehne (1989a, 1989b), Buelens and Ginsburgh (1993), Chanel et al. (1993) and became very popular in the mid-1990s. The benefit of HR is that the index is constructed from all sales, not from an (often small) subset of the available transactions. One of its drawbacks is that it depends on the characteristics used to describe the objects, and on the functional form of the equation.

Auction prices may give a biased view, since auctions seem to represent a mere 10 to 20 percent of the art market. Other biases are problematic with auction prices. The first is

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3 Note that Baumol does not construct an index. See below.
especially relevant for studies that use repeat-sales sales, since some transactions do not go through salesrooms. This point was raised by Guerzoni (1994), who shows that Reitlinger's (1961, 1971) data, on which Baumol (1986), Frey and Pommerehne (1989a, 1989b), Buelens and Ginsburgh (1993), and others base their calculations miss some important transactions, and this may seriously alter rates of return. The other problem is that works that are bought-in at auction are generally not included, since there is no hammer price. Ashenfelter, Graddy and Stevens (2001), Ekelund, Ressler, and Watson (1998), Goetzmann and Peng (2003), Goetzmann and Spiegel (2003), and Mc Andrew and Thompson (2003, 2004) analyze the consequences, and suggest some solutions, such as estimating the secret reserve prices, or using some percentage of the lower bound of the pre-sale estimate published nowadays by most salesrooms.5

As mentioned above, many categories of collectibles have been studied: African art, books, vintage cars, coins, Old Master drawings, furniture, ivory objects, jewellery, photographs, prints, sculpture, musical instruments, silver, stamps, etc. Some are reasonably easy to describe (drawings), some are less so (furniture, sculpture). Some are sold frequently, since there are many "copies" available (prints), some are seen very rarely at auction (paintings by Vermeer). This points to which type of method (RSR v. HR) can be used with some success, given the shortcomings of each one, and in particular, their dependence on the number of observations, and the possibility to describe objects by characteristics.

Timeliness is another important issue. There are fortunately more and more databases that are updated on a regularly, if not daily, basis, such as Artprice.com (http://web.artprice.com), Hislop's Art Sales Index (http://www.art-sales-index.com/system/index.html), Gabrius (http://gabrius.com/), Gordon's On Line Art Mall (http://www.onlineartmall.com/limited/lgordon/) and probably some other. Still, for many types of collectibles (furniture, violins) this is not so. In some cases, prices are made available in annual catalogues (such as Mayer for paintings and drawings, Gordon's for prints), but often it is necessary to go to individual sales catalogues of auctioneers to get hold of the data. This again has an influence on the estimation method used to construct price indices.

The paper is organized as follows. Section 2 deals with the basics of hedonic and repeat-sales estimators, and tries to interpret in economic terms what both are trying to achieve. Section 3 goes into some more technical details which may be useful for researchers who want to construct such indices. Section 4 briefly discusses on how to go about collecting data, and the choice between RSR and HR that this induces. Section 5 compares both methods using simulated returns, and tries to point to which method should be used given the data at hand.

4 This may of course affect all resale data, not only those collected by Reitlinger.
5 Mei and Moses (2004) show that the publication of pre-sale estimates since the mid-1970s has an influence on hammer prices.
2 Hedonic and Repeat-sales Estimators: A First Approach

In this section, we describe possible estimators of price indices, obtained from observing a set of $2N$ transactions related to $i = 1, 2, ..., N$ different objects, that are also described in terms of some attributes or characteristics. To simplify exposition, we assume that each object is transacted twice. The set of dates (say, years) is $t = 0, 1, ..., T$ and defines possible periods (period $t$ goes from date $t-1$ to date $t$) or market runs. There exist data on prices for each object during some (here, 2) periods, but not for all objects in every period. A transaction of object $i$ in period $t$ is indexed by subscripts $(i, t)$.

The estimators considered are the hedonic and the repeat-sales estimators, and include some others, that can be obtained as special cases. In particular, we show that the repeat-sales estimator is a special case of the hedonic estimator.

We illustrate our discussion using an example in which there are $2N = 12$ sales of $N = 6$ objects at three possible dates ($T = 2$). We denote by $P_{it}$ be the (log of the) price of object $i$, sold at date $t$, and assume objects 1 and 4 were sold in $t = 0$ and 1, objects 3 and 5 were sold in $t = 1$ and 2 and finally, objects 2 and 6 were sold in $t = 0$ and 2. The logged price (column) vector is denoted by $p = (p_{10}, p_{20}, p_{40}, p_{60}, p_{11}, p_{31}, p_{41}, p_{51}, p_{22}, p_{32}, p_{52}, p_{62})$. For convenience, we rank the observations for $t = 0$ first, then those for $t = 1$, etc., without taking into account that some of the prices concern resales of the same object. We also define a vector $y = (p_{11} - p_{10}, p_{22} - p_{20}, p_{32} - p_{31}, p_{41} - p_{40}, p_{52} - p_{51}, p_{62} - p_{60})$ with elements $y_i$ (the logged differences of prices obtained at two dates for object $i$).

The hedonic estimator

Hedonic regression is typically used to control for the changing quality of goods transacted. It seems to have been introduced by Court (1939), and subsequently used by Griliches (1961), Triplett (1969), and Griliches (1971a, 1971b) to construct price indices for automobiles. It is widely used by statistical offices to estimate consumer price indices, and in academic work to establish price indices for real estate, computers, automobiles, dishwashers, the arts, and more generally for cases in which quality varies over space and/or time.

We start, however, with the simple case that ignores characteristics. Let $C$ be a matrix consisting of $2N$ rows and $T+1$ columns, denoted $c_0, c_1, ..., c_T$. Element $c_{it}$ is equal to one if a transaction on commodity $i$ occurs in year $t$, and is zero otherwise. For the example at hand in

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6 See for example Abraham, Greenless and Moulton (1998), Boskin (1996), Boskin et al. (1998) and of course the recent book by Triplett (2004) that is entirely devoted to this issue.
which the (column) vector of prices is \((p_{10}, p_{20}, p_{40}, p_{60}, p_{11}, p_{31}, p_{41}, p_{51}, p_{22}, p_{32}, p_{52}, p_{62})\), the first column of \(C\) contains four ones, followed by 8 zeros, the second contains four zeros, followed by four ones and four zeros, etc.

We estimate the parameters \(\delta_t\) of the linear model:

\[
(p_{it} = \sum_{\tau=0}^{T} \delta_{\tau} c_{i\tau} + \epsilon_{it},
\]

where \(\epsilon_{it}\) is a random disturbance. The OLS estimator is:

\[
\hat{\delta} = (C'C)^{-1}C'p.
\]

It is straightforward to check that the estimator for the price in year \(\tau\) is simply the average of the (log of) prices of the \(n_{\tau}\) objects sold during that year, that is a geometric average of prices:

\[
\hat{\delta} = \frac{1}{n_{\tau}} \sum_{i} p_{i\tau}, \quad \tau = 0, 1, ..., T.
\]

The index, normalized to 1 in \(\tau = 0\), is obtained as the sequence of \(\exp \left( \hat{\delta}_\tau / \exp \hat{\delta}_0 \right)\). Obviously, this is a sound approach as long as the mix of objects sold in each year has the same characteristics, or is of the same quality. This is often not the case, and then the hedonic approach is useful since it homogenizes sales mixes over time.

Consider therefore the set of objects sold in a specific year \(t\) and assume that the price of an object \(i\) sold in \(t\) can be considered as a function of \(m\) time-invariant characteristics \(v_{ik}\), \(k = 1, 2, ..., m\) (e.g. the dimensions of a painting) and of \(n\) time-varying characteristics \(w_{ij}\), \(j = 1, 2, ..., n\). If this assumption holds, we can write that \(p_{it} = f(v_{i1}, ..., v_{im}, w_{i10}, ..., w_{i1t}, w_{i20}, ..., )\). We specialize the functional form to:

\[
\begin{align*}
\text{(3)} & \quad \hat{\delta} = \frac{1}{n_{\tau}} \sum_{i} p_{i\tau}, \quad \tau = 0, 1, ..., T. \\
\text{(4)} & \quad \hat{\delta}_0 = \frac{1}{n_{\tau}} \sum_{i} p_{i\tau}, \quad \tau = 0, 1, ..., T.
\end{align*}
\]

\[\text{It is obvious that one can also derive an estimator for arithmetic averages of prices, using } P_{it} \text{ instead of } p_{it}.\]

\[\text{Alternatively, one can set } c_0 \text{ as a vector of ones, and estimate equation (1) with an intercept. Then the sequence of } \exp \hat{\delta}_\tau \text{ gives directly the index.}\]

\[\text{Note that the antilog of the OLS estimates of the } \delta \text{ are not unbiased, and that a correction has to be made, by adding one half of the coefficient's squared standard error to the estimated coefficient. Since such standard errors are usually small, this makes little difference. See Triplett (2004, chapter 3, footnote 12).}\]

\[\text{See more on the choice of functional forms e.g. in Halvorsen and Pollakowski (1981).}\]
\( p_{it} = \sum_{k=1}^{m} \alpha_k v_{ik} + \sum_{\tau=0}^{T} \sum_{j=1}^{n} \theta_{j\tau} w_{ij\tau} + \epsilon_{it}. \)

The parameters \( \alpha_k \) and \( \theta_{j\tau} \) appearing in (4) are, often abusively, interpreted as implicit prices of the various characteristics describing the commodity, and \( \epsilon_{it} \) is a random error term. These implicit prices are thus obtained by a regression of prices on observable characteristics (also often in logarithmic form); once they are known, it is possible to compute, like in (3), the average price \( \hat{\delta}^\tau \) of a characteristic-free, or quality-adjusted commodity in year \( \tau \) as:

\[
\hat{\delta}_\tau = \frac{1}{n_\tau} \sum_{i=1}^{n_\tau} (p_{it} - \sum_{k=1}^{m} \alpha_k v_{ik} - \sum_{\tau=0}^{T} \sum_{j=1}^{n} \theta_{j\tau} w_{ij\tau}).
\]

The sequence of \( \hat{\delta}_\tau, \tau = 0, \ldots, T \), would then describe the price of an (artificial) characteristic-free commodity over time, and can obviously be obtained by a hedonic regression pooling the sales over time, by combining (1) and (4):

\[
p_u = \sum_{k=1}^{m} \alpha_k v_{ik} + \sum_{\tau=0}^{T} \sum_{j=1}^{n} \theta_{j\tau} w_{ij\tau} + \sum_{\tau=0}^{T} \delta_{c\tau} c_{\tau} + \epsilon_{u}. \]

The method allows for interactions between time and characteristics, if one believes that the implicit prices of some characteristics vary over time. For this, one merely has to introduce new variables \( \omega_{k\tau} = v_{k\tau} c_{\tau} \), which pick regression coefficients that describe the time path of the implicit price of characteristic \( k \). The two previous estimators can also provide such information, by computing the parameters on sub-samples (e.g. a specific painter). However, given that the number of observations will be small compared with the total number of sales, the estimated coefficients will be estimated with little precision.

Obviously, there are many other ways to specify the way in which prices depend on time. The \( \Sigma_{\tau} \delta_{c\tau} \) formulation makes it possible to construct a price index, in a reasonably flexible way. Alternatively, one can use a variable \( \tau \) which takes the values 0, 1, 2, ..., \( T \) and specify (6) with a term \( \phi_\tau \), where \( \phi \) would be an estimate of the price trend. One can also estimate time trends for sub-periods.

11 See the discussion below.

12 Note that, following the usual specification, one can introduce an intercept \( \alpha_0 \), and estimate only \( T-1 \) parameters \( \delta_\tau \), and normalize \( \delta_0 \) to be equal to 1.
The repeat-sales estimator

The repeat-sales estimator was developed to derive price indexes for real estate. The method was formalized by Bailey, Muth and Nourse (1963), though they refer to previous suggestions by Wenzlick (1952) and Wyngarden (1927). The Bailey, Muth and Nourse paper was followed by a large number of theoretical approaches as well as applications, in most of the field journals. This estimator is now also used for other markets, including artworks.

The usual approach to derive the underpinnings of the estimator is as follows. Assume that \( r_{it} \), the continuously compounded return for a certain art asset \( i \) in period \( t \) may be represented by \( \delta_t \), the continuously compounded return of "art," and an error term \( \eta_{it} \):

\[
r_{it} = \delta_t + \eta_{it},
\]

where \( \delta_t \) may be thought of as the average return in period \( t \) of artworks in the portfolio. We will use sales data about individual works to estimate the index \( \delta \) (a \( T \)-dimensional vector whose individual elements are \( \delta_t \)) over some interval \( t = 1, ..., T \). The observed data consist of purchase and sales (logged) price pairs \( (p_{ib}, p_{is}) \) of individual objects \( i \), as well as the dates of purchase and sale, designated by \( b_i \) and \( s_i \). Then, the logged relative price for object \( i \), held between its purchase date \( b_i \) and its sales date \( s_i \), may be expressed as:

\[
r_i = p_{is} - p_{ib} = \sum_{t=b_i+1}^{s_i} r_{it}.
\]

There are many ways to derive the repeat-sales estimator to understand how the regression should be run. An easy way, which also happens to provide a link between the RSR and the HR estimators is to start with the hedonic equation, in which characteristics are constant over time:

\[
p_{it} = \sum_{k=1}^{m} \alpha_k v_{ik} + \sum_{t=0}^{T} \delta_t c_{it} + \varepsilon_{it}.
\]

For notational purposes, it is convenient to define, as above, \( p_{ib} \) as the first sale, and \( p_{is} \) as the second one, and to redefine accordingly the columns of the matrix \( C \) and the \( \delta_t \) parameters. Then,

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14 We derive the so-called geometric repeat-sales estimator. See Shiller (1991) and Goetzmann and Peng (2001) who discuss an arithmetic estimator.
\( p_{is} - p_{ib} = \delta_s c_{is} - \delta_b c_{ib} + \eta_i, \)

where the \( \Sigma_k \alpha_k v_{ik} \) terms vanish, since the characteristics are identical over time, and \( \eta_i = \epsilon_{is} - \epsilon_{ib} \). It is easy to check that for our example with six repeat-sales, this leads to the following system of equations:

\[
\begin{bmatrix}
  p_{11} - p_{10} \\
  p_{22} - p_{20} \\
  p_{32} - p_{31} \\
  p_{41} - p_{40} \\
  p_{52} - p_{51} \\
  p_{62} - p_{60}
\end{bmatrix} =
\begin{bmatrix}
  -1 & 1 & 0 \\
  -1 & 0 & 1 \\
  0 & -1 & 1 \\
  -1 & 1 & 0 \\
  0 & -1 & 1 \\
  -1 & 0 & 1
\end{bmatrix}\begin{bmatrix}
  \delta_0 \\
  \delta_1 \\
  \delta_2 \\
  \eta_0 \\
  \eta_1 \\
  \eta_2
\end{bmatrix} + \eta,
\]

where \( \eta \) is a vector of the six error terms \( \eta_i \).

The three columns of the matrix that appears in the right-hand side are linearly dependent, so that one column, say the first, should be discarded. This will set the normalization \( \delta_0 = 0 \), and leave us with an \( N \times T (= 6 \times 2) \) matrix \( Z \) that will be interpreted later on. The \( T \) (here 2) regression coefficients can be estimated by OLS:

\[
\hat{\eta} = (Z'Z)^{-1}Z'y.
\]

An alternative way to derive the estimator—which leads to an easy interpretation of the coefficients—is to construct an \( N \times T \) matrix \( X \), the columns of which represent periods (not dates); observation \( i \) is in row \( i \), which contains \( \text{ones} \) for periods during which the object was held and \( \text{zeros} \) otherwise. For our example, this matrix is:

\[
X = \begin{bmatrix}
  1 & 0 \\
  1 & 1 \\
  0 & 1 \\
  1 & 0 \\
  0 & 1 \\
  1 & 1
\end{bmatrix}
\]

The OLS estimator of the two coefficients, say \( \beta_1 \) and \( \beta_2 \), is given by:

\[
\hat{\beta} = (XX)^{-1}X'y.
\]

The two normal equations are:
\[ 4 \hat{\beta}_1 + 2 \hat{\beta}_2 = (p_{11} - p_{10}) + (p_{22} - p_{20}) + (p_{41} - p_{40}) + (p_{62} - p_{60}). \]
\[ 2 \hat{\beta}_1 + 4 \hat{\beta}_2 = (p_{22} - p_{20}) + (p_{32} - p_{31}) + (p_{52} - p_{51}) + (p_{62} - p_{60}). \]

They can also be written:

\[ \hat{\beta}_1 = \frac{1}{4} \left[ (p_{11} - p_{10}) + ((p_{22} - \hat{\beta}_2) - p_{20}) + (p_{41} - p_{40}) + ((p_{62} - \hat{\beta}_2) - p_{60}) \right], \]
\[ \hat{\beta}_2 = \frac{1}{4} \left[ (p_{22} - (p_{20} + \hat{\beta}_1)) + (p_{32} - p_{31}) + (p_{52} - p_{51}) + (p_{62} - (p_{60} + \hat{\beta}_1)) \right]. \]

If we now interpret \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) as being estimates of the mean rates of return in periods 1 and 2 respectively, \( (p_{22} - \hat{\beta}_2) \) and \( (p_{62} - \hat{\beta}_2) \) are estimates of the prices of objects 2 and 6, had they been resold in year 1 instead of year 2, while \( (p_{20} + \hat{\beta}_1) \) and \( (p_{60} + \hat{\beta}_1) \) are estimates of the prices of the same objects, had they been sold for the first time in year 1 instead of year 0.

Once this interpretation is accepted, one immediately sees that \( \hat{\beta}_1 \) is the average return of the objects sold in \( t = 0 \) and in \( t = 1 \), while \( \hat{\beta}_2 \) is the average return of the objects sold in \( t = 1 \) and in \( t = 2 \).

It is straightforward to link formulations (9) and (10). Let \( B \) be a \( T \times T \) matrix constructed as follows: row \( t \) starts with \( t \) ones, while the other elements of the row are zeros.

For our example, this matrix is:

\[ B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}. \]

We then construct a matrix \( XB^{-1} \) of explanatory variables, which is exactly the matrix \( Z \) used in (9). Some straightforward matrix algebra shows that (9) can also be written

\[ \hat{\delta} = B \hat{\beta}, \]

which relates estimators (9) and (10), implying that

\[ \hat{\delta}_t = \sum_{i=1}^{t} \hat{\beta}_i, \quad t = 1, 2, ..., T. \]

For our example, this means that \( \hat{\delta}_1 = \hat{\beta}_1 \) and \( \hat{\delta}_2 = \hat{\beta}_1 + \hat{\beta}_2 \). Since we can set \( \hat{\delta}_0 = \hat{\beta}_0 = 0 \), the sequence \( \exp(\hat{\delta}_0), \exp(\hat{\delta}_1), \exp(\hat{\delta}_2) \) is the price index over the three years.
A special case of the geometric repeat-sales estimator is the geometric mean estimator. Start with the following linear model:

\[ \frac{y_i}{\tau_i} = \gamma + \varepsilon_i, \]

where \( \gamma \) is a parameter to be estimated and \( \tau_i \) is a variable which takes as value the number of periods during which an object was held by an owner (i.e. not sold); in the example, \( \tau_i \) is thus equal to 1 for \( i = 1, 3, 4 \) and 5 and equal to 2 for \( i = 2 \) and 6; \( \varepsilon_i \) is a random disturbance with the usual properties. The variable \( \frac{y_i}{\tau_i} \) is the annualized rate of return on commodity \( i \). The parameter \( \gamma \) can be estimated by running a regression of \( \frac{y}{\tau} \) on a variable which takes the value one for each observation. It is trivial to check that the OLS estimate for \( \gamma \) is the average of annualized returns:

(12) \[ \hat{\gamma} = \frac{1}{N} \sum \frac{y_i}{\tau_i}. \]

For our example, (12) leads to:

\[ \hat{\beta} = \frac{1}{6} \left[ (p_{11}-p_{10}) + (p_{22}-p_{20})/2 + (p_{32}-p_{31}) + (p_{41}-p_{40}) + (p_{52}-p_{51}) + (p_{62}-p_{60})/2 \right]. \]

This is the estimator used by Baumol (1986) and Frey and Pommerehne (1989b).\(^{15} \) It is obviously very easy to compute, but does not provide an index over time. Moreover, it puts equal weights on all annualized rates, irrespective of the length of time during which the object was held.

3 Hedonic, Repeat-sales and Other Estimators: Further Issues

*Rosen's interpretation of hedonic models*

Lancaster (1966) was at the root of giving to the purely econometric technique used in hedonic regression, its theoretical foundations, based on the idea that commodities were not consumed *per se*, but for their combination of characteristics say, in the case of automobiles, speed, mileage, length, engine capacity, number of doors, etc., each of which carrying an implicit price. Rosen (1974) suggested that the simple estimation of such implicit prices by

\(^{15} \) Actually, Baumol and Frey and Pommerehne use the exact formula \( (P_{it}/P_{it'})^{1/(t-t')} \) to compute the annual return of a work sold in \( t' \) and subsequently in \( t \). We use the approximation instead. The two lead to comparable results if \( P_{it} \) is close to \( P_{it'} \). \( (P_{it}/P_{it'})^{1/(t-t')} \)
regressing observed prices on characteristics, was flawed, in the sense that, in general, the
function does not allow "to recover the underlying utility and cost functions from such data
alone." (Griliches, 1990, p. 189). Implicit prices emerge from the equilibrium between
demand and supply on markets for characteristics. Rosen outlines a two step procedure in
which the first step is to estimate a hedonic function $p = f(v)$, to evaluate its derivatives at
points corresponding to the observed values of $v$, and use these derivatives as prices in a
system containing supply and demand functions for characteristics, paying attention to the
usual identification problems in estimating simultaneous equations. Brown and Rosen
(1982), Epple (1987), Bartik (1987) showed that the problem was even more complicated
than what Rosen had thought, and suggest alternatives to Rosen' estimation procedure.
Empirical estimation was carried out by Wittke, Sumka and Erekson (1979), Brown and

Rosen's (1974) approach deals with perfect competition. The analysis was carried to
imperfect competition (or at least to cases in which the number of producers is small) by

In many cases, and art markets are one of these, the purpose of hedonics is merely to
determine a price index over time, and not, as in the case of hedonics in consumer price
analysis, to derive consumer welfare, which is possible only if utility parameters can be
inferred from the estimated model. Therefore, in some sense, it is superfluous to appeal to
Rosen's theory in the case of art price indices.

**Issues with hedonic indexing**

Still, reliable and unbiased estimates have to be obtained, and the problems that are
present in single-equation model estimation, such as heteroskedasticity, multicollinearity,
choice of functional form, choice of characteristics (independent variables), etc. have also to
be dealt with.

In order for the index to be as little contaminated as possible by the heterogeneity of
the sales mix over time, and account correctly for quality adjusted price changes, the choice
of hedonic characteristics is important. This is dealt with in some detail below.

A further issue is whether one can assume that the coefficients of the hedonic
variables (other than time dummies) are constant over time. Though it is doubtful that the
hedonic equation really captures changes in consumers' tastes, these can vary over time.

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16 On this issue, see also Palmquist (1992), Feenstra (1995) and Diewert (2001).
17 One feels somewhat uncomfortable to discuss in one double-spaced page, a topic for
18 Pakes (2003) observes that characteristics are often highly correlated for industrial
products (in his case, computers). This is unlikely to be the case for artworks.
Characteristics whose implicit prices are assumed to vary can be made variable, by introducing interaction terms between characteristics and time, or by running adjacent-period regressions, and testing whether the null hypothesis of constancy has to be rejected. Triplett (2004, chapter 3, section 2c) argues that constraining the hedonic coefficients to be equal over time does make a difference on the time index, and that, as long as the number of observations is sufficient, running a sequence of adjacent-period regressions produces better results.

A problem that is not encountered in art indexes, but in many other cases, such as computers, cars, and most other consumer durables, is that the ordinary least squares estimator does not take into account the size of the market for each brand or make. The same weight is given to every observation. The issue is dealt with in Silver and Heravi (2004) and Triplett (2004, chapter 6, section D).

A related problem is that brands often disappear from the market, and new brands with other characteristics are introduced. This is what makes for most of the difficulty in the construction of consumer price indices. See Boskin (1996) and Boskin et al. (1998), as well as Triplett (2004, chapter 2).

Finally, we mention a problem analyzed by Melser (2004), who shows that the time-dummy method suggested above may fail to satisfy the monotonicity axiom, which requires the following properties to hold: a price index which compares two periods must increase (decline) if the second (first) period's prices rise, holding other factors fixed. Melser implements a suggestion made by Diewert (2001) which satisfies the axiom. Diewert (2003) also discusses many other unresolved issues in hedonic price indexing.

**Functional form of the hedonic function**

The most widely used functional form is the double-log function in which both prices and characteristics (as long as these are captured by continuous variables) are transformed into natural logarithms, though, Rosen (1974) shows that, in general, theory has nothing to say on the functional form. Therefore, as mentioned by Triplett (2004, chapter 6, p. 10), "the form of the hedonic function is entirely an empirical matter ... accordingly, one should choose the functional form that best fits the data, empirically." The empirical question is analyzed (for houses) by Cropper, Deck and McConnel (1988) who run some Monte Carlo experiments to determine which form should be used to determine implicit prices as correctly as possible.

19 Note that the space of characteristics is usually not dense. One can hardly find a car whose engine capacity is 1,527.3 cc, or a computer whose memory cannot be expressed by an exponential of 2. Therefore, there is no need for the functional form to be smooth.

They show that the linear Box-Cox functional form may be a reasonable compromise.\(^{21}\) Wallace (1996) suggests non parametric estimation which avoids the problem of choosing or imposing a functional form.

**Hedonic characteristics for artworks**

A good starting point is de Piles *balance des peintres*, added as an appendix to his *Cours de peinture par principes* (1708). The balance is a table in which de Piles decomposes painting into four fundamental characteristics: composition, drawing, colour, expression,\(^{22}\) and rates each of these on a scale between zero and twenty for 56 painters from his and previous times. Rembrandt, for example, is very low on drawing and obtains 15, 6, 17 and 12 on the characteristics just mentioned, while Michelangelo is very high on drawing, with scores of 8, 17, 4 and 8 respectively. De Piles himself looked at this as a game, but his contemporaries considered it as a "clever way to characterize genius" (Thuillier, 1989, p. xxvii). Later on, this view changed, and many art historians hate this idea, describing it as a "notorious aberration" (Gombrich, 1966, p. 76) or thinking that de Piles was "at his worst when he tried to be most systematic" (Puttfarken, 1985, p. 42). The originality of the *balance* is that it introduces a view of aesthetics that breaks up beauty into its parts. This is of course the very same idea as the one expressed by Lancaster (1966), and used in the early econometric work on hedonics.

Short of having been pursued by many other art critics,\(^{23}\) such characterizations do hardly exist, and we must rely on alternative descriptions, that are surrogates for aesthetic characteristics. Mandeville, a physician, "attracted attention mainly as the author of provocative essays on economic and social subjects" (De Marchi and Van Miegroet, 1994). In one of his essays (1728), he lists four factors which explain value: the name of the master, his age (in Mandeville's words "the time of his age"), the scarcity of his work, and the rank of those owning them.\(^{24}\) Characteristics clearly depend on the type of collectible that is studied.\(^{25}\) For paintings, which have been studied most, Sagot-Duvaouroux (2003) singles out as characteristics signature (therefore, name of the artist), provenance (former owners, including exhibitions and literature describing the work), technique (oil, mixed media, etc.), subject matter (landscape, portrait, etc.), support (canvas, paper, etc.), size, place of sale. Nothing very new

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\(^{21}\) See also Halvorsen and Pollakowski (1981) and the discussion in Triplett (2004, chapter 6, section C).

\(^{22}\) Note that the decomposition itself goes back to Plinus.

\(^{23}\) See Ginsburgh and Weyers (2004) who used de Piles' characteristics to price out their values in terms of today's prices and aesthetic values.

\(^{24}\) See De Marchi and Van Miegroet (1994, p. 454).

\(^{25}\) For details, see the papers cited in Appendix 1.
with respect to Mandeville. Note that, in the past, prices also depended on colors used, subject, number of figures represented.

Since most empirical studies are based on auction data, they use the characteristics made available in sales catalogues, which include name of the artist, size, technique, elements of provenance (though names of previous owners rarely appear). Photographs of the paintings are also often included, so that subject matter, colors, etc. can be coded as characteristics as well.\textsuperscript{26} Art prices may also be influenced by the rarity, as suggested by Mandeville. It is however not clear whether rarity (for example based on the number of works of an artist that are still in private hands) can be introduced as a hedonic characteristic since it is not a characteristic of the work (or of the artist), but of the market.

Table 1 shows the results for some of the studies that included characteristics in a reasonably uniform manner. Height and width always carry a positive sign, but the negative sign of the coefficient picked by the area variable shows that dimensions cannot become too large: price is a concave function of dimensions. Oil paintings and canvasses are generally more expensive than other media or supports. There is a positive influence of signature and date, when these appear on a painting.

The most controversial characteristic is probably the name of the master (dummy variables that are introduced when portfolios of art are studied), since this is no explanation for why his works are valued. Sociologists, art philosophers and economists seem however to agree that names do change values. Becker (1982), Moulin (1992) and Bonus and Ronte (1997) document in great detail how artists' reputations are constructed, and are responsible for the sometimes high prices that some works command. Art philosophers also hold the opinion that names make for aesthetic value. It is interesting to bring in what Danto, one of the great American analytic art philosophers, writes in one of his essays on contemporary art (Danto, 1986, pp. 13-14 and 45): "Duchamp's Fountain is, as everyone knows, to all outward appearances a urinal--it was a urinal until it became a work of art and acquired such further properties as works of art possess in excess of those possessed by mere real things like urinals..." or "The interpretation is not something outside the work: work and interpretation arise together in aesthetic consciousness. As interpretation is inseparable from work, it is inseparable from the artist if it is the artist's work." As economist, Grampp (1989, p. 131) purports that the name of the artist belongs to his work: the aesthetic object consists of the painting, the artist and the title. "Imagine," he writes, "how a dealer would fare if he alone in the market and none of his competitors did not provide information about the painting he offered for sale: no name, no title, no provenance...nothing but the price." Attribution matters. There are hundreds of such examples, one of the latest being Raphael's \textit{Madonna of the Pinks}, worth some £8,000 until its attribution to Raphael in 1991, and paid £22 million by

\textsuperscript{26} Czujack (1997) and Lazzaro (2003) who study respectively paintings by Picasso paintings and prints by Rembrandt are probably the two papers that make use of the largest set of hedonic characteristics.
the National Gallery in 2004. All these are good reasons for not being afraid of introducing artist dummies as hedonic characteristics.

The main purpose of the work is to estimate a price index—though this may be biased if the choice of characteristics is poor, or if some important characteristics are omitted. Other hedonic coefficients should be interpreted with some care, given Rosen's (1974) warnings. In concluding this section, however, it is hard resisting to quote Triplett (2004, chapter 5, p. 3) who suggests that "the first principle for conducting a hedonic study is [to] know your product."

**Other repeat-sales estimators**

The OLS approach in (9) àr (10) simply sets the weighting matrix as the identity matrix. While it is the easiest to use, it nonetheless makes the frequently unrealistic assumption that the error term in equation (8) is homoskedastic. Generalized least square (GLS) set the weighting matrix as a diagonal matrix where the elements are the ex post squared estimation errors of a first step OLS estimation.

Palmquist (1982) shows that the use of OLS is problematic once there are more than two sales for a specific object and that GLS are needed to derive minimum variance estimates. Goetzmann (1992) shows that GLS provide maximum likelihood estimates of the parameters.

Case and Shiller (1987, 1989) propose an alternative weighting matrix for $\Omega$. They assume that the variance of the error term in equation (8) is a simple linear function of the holding period plus a constant, and suggest a three-stage least square approach to estimate $\delta$. In the first stage, OLS are used. Then one computes the error terms for equation (8). In the second stage, one regresses the squared error terms against a constant and the holding period. Then, based on the regression, one computes an estimated variance for each object using the holding period. Finally, one sets the diagonal term using the estimated variance and then runs a third stage regression using GLS.

The repeat-sales regressions discussed above are known to introduce certain biases. One of the most serious is a spurious negative autocorrelation. This bias is potentially severe at the beginning of the estimated series. Goetzmann (1992) proposes a two-stage Bayesian regression to mitigate the problem over the early periods. The Bayesian formulation imposes an additional restriction that the $\delta$ are normally, independently and identically-distributed. The effect on the estimate is dramatic for the early period when data are scarce, and minimal for the period during which data are plentiful. Goetzmann (1992) also suggests a couple of other Bayes estimators.

The repeat-sales estimator is usually presented under its geometric form, since this gives returns in a straightforward way. Shiller (1991), and Goetzmann and Peng (2001) also
propose arithmetic repeat-sales estimators that are unbiased and based on arithmetic averages of returns.

*Combining repeat-sales and hedonic estimators*

Case and Quigley (1991) try to use all the information and combine sales and resales (of houses) in a system of equations.\(^{27}\) They use a hedonic equation for sales and a repeat-sales equation for resales. They also distinguish resales for which characteristics have changed from other resales. Quigley (1995) complements Case and Quigley (1991) by introducing a procedure that is based on an explicit structure of the error term.

Though the results are extremely interesting—indeed, the suggestion is hard to apply to paintings, since characteristics are mainly described by qualitative variables, while Case and Quigley deal with (a small number of) continuous variables only. Since in most cases, time is represented by annual dummies, one would need to introduce a very large number of variables.

*Other estimators*

Many alternative estimators have been suggested and used. A very simple one is discussed in the beginning of the section concerning hedonic regression, the geometric mean in each period. Clearly, one can also think of simple estimators such as arithmetic means or median prices in each period,\(^ {28}\) trimmed or not for outliers. During the 1980s, Sotheby's was carrying an index based on representative objects, the prices of which were reassessed by experts at regular time intervals.\(^ {29}\)

Mark and Goldberg (1984) add a long list of alternative methods, often based on hedonics:\(^ {30}\)

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\(^{27}\) See also Schiller (1991) for more references and a discussion of combining both estimators.

\(^{28}\) Median prices are used by the U.S. National Association of Realtors. An index based on average prices is computed by Art Market Research in the U.K. and is regularly published by the *Daily Telegraph*, and the *Art Newspaper*.

\(^{29}\) See also Clapp and Giacotto (1992) who use price assessments by tax assessors for residential properties, and show that these are strongly correlated with observed prices, so that assessments can play the role of hedonic characteristics. This is very close to using pre-sale estimates by salesrooms as describing artworks, and thus avoiding hedonic characteristics. See e.g. Beggs and Graddy (1997).

\(^{30}\) See also Triplett (2004, chapter 3) for such methods, that he calls "characteristic price index methods," in which implicit prices are used to compute an index, without any need for time dummies.
- the "transactions weighted by base period weights" index, in which base period implicit prices (obtained from a hedonic regression run on sales in the first year) are used to compute predicted prices for all units sold in each other year, and the unweighted arithmetic mean of the predicted values is used as the basic price series. This reflects changes in the distribution of the characteristics of units actually sold through time;
- the "transactions weighted by final period weights" index; the idea is the same as in the previous method, except that final period implicit prices are used;
- the "changing annual weights in every period" index; in this method, the weights change every year; predicted prices are calculated and serve as basis for the index;
- a "Laspeyre's index analog"; the Laspeyre index uses base period quantities as weights; the analog suggested uses as weights the average bundle of characteristics from the base year sample, while the changing prices are the hedonic coefficients obtained for each year;
- a "Paasche's index analog"; the Paasche index uses final period quantities as weights; otherwise, calculations are identical to the Laspeyre's index analog.


4 Developing Hedonic and Repeat-sales Models: Data Acquisition

We will develop this section assuming a researcher is starting at a moment in time and deciding on how to go about collecting data so that he can develop an index that portrays the movement of art prices or returns over some historical period ending with the current period. We assume that the only valid data to use for this type of analysis is auction data either from the direct source catalogs and/or an online computer database.

The RSR-researcher is only interested in the information contained in the provenance of the piece. This gives all the prior owners and any prior auction transaction, to the best of the auction houses ability. Prior auction prices are normally not provided and to find these requires finding the prior auction catalog and price lists. The prior transaction price can sometimes be found on computer databases such as the Hislop Art Sales Index or Art Net but care must be taken to insure that one has found the correct pairing (Still Life by Cezanne hardly gives a clue). Moreover titles can change and sizes can be miss-recorded. The shortcoming of the RSR technique is that the number of objects with prior sale results is usually much smaller than the total number of lots offered.

The HR-researcher can use the information given in sales catalogs to serve as a set of independent variables in a hedonic regression. This would allow him to forgo the search for the prior price data. With the help of on line databases, which provide a subset of the data
available in catalogs, the task can be made even simpler. The major shortcoming of this
technique is that two works can have very similar characteristics for the same year but widely
different prices leaving the year dummy alone to try to explain the difference. But since the
data collection is easier all the works that sell in each period can be used.

The following variables are usually available: artist, title, size, lot number, date,
auction house, medium and support, whether the object is signed or dated and the date, and
since 1973 the high and low price estimates. For important works, catalogs also include
illustrations, ownership and exhibition history as well as the number of literature citations,
and their dates. The ownership history is essential for those hoping to use repeat-sales
techniques since it will mention any prior auction sale but rarely the hammer price. This is
found by using a good library with an extensive auction catalog collection.

5 Comparing Hedonic and Repeat-sales Estimation

The pros and the cons of both types of indices

Number of observations. The number of cases in which one can retrieve repeat-sales is
usually small, and certainly much smaller than the total number of sales. Ashenfelter and
Graddy (2004) compare a hedonic index using 8,792 observations with a repeat-sales index
with 474 observations. Mei and Moses (2002) have 4,900 pairs (9,800 observations) for the
period 1875-2000 (of which 2,300 are for Old Masters). By comparison, the database used by
de la Barre, Docclo and Ginsburgh (1994) on Impressionists and followers for paintings sold
between 1962 and 1991 contains 24,500 observations. In the Mei Moses All Art Index that
will be used in Section 5, the proportion of repeat-sales with respect to all sales is roughly
7% within a ten year time span (1971-1980), 13% within a 20 year span (1971-1990) and
15% within a 30 year span (1971-2000).

In some cases, such as prints which exist in several copies, the number of repeat-sales
is much larger. Pesando (1993) collected almost 28,000 repeat-sales, though prints with the
same title are not necessarily interchangeable: some may be in better condition than others,
some have a good provenance, etc.

Repeat-sales estimation also needs much longer periods on which the index is
estimated, since the one before the last sale may have happened long before the last one. The
method hardly allows fine disaggregation into submarkets, not to mention constructing
indices for individual artists, because the number of observations is usually too small.
Goetzmann (1992) explores the issue of sparse data, and suggests some solutions. Sparse data
are prone to generate spurious negative autocorrelation and too much volatility. Finally, there
is the problem that most data (those for which no resales can be retrieved) are not used, and information is wasted.\textsuperscript{31}

Sample biases in repeat-sales estimators. Gatzlaff and Haurin (1997) show that because only a small percentage of objects (in their case, houses) sell each year, the sample of those that sell may have non-random statistical properties, due for example to changing economic conditions which influence reserve prices of sellers. They suggest using Heckman's (1979) sample selection correction model. More generally, repeat-sales estimation may suffer from several other sample biases, such as transactions which did not go through auction and whose prices are missing, and outliers, which have more importance than for hedonic indexes, given that data are more sparse.

Specification biases. Choosing the functional form and the variables that represent quality are pervasive in hedonic indexing, and can lead to all the problems linked to mis-specification. This is of course avoided in repeat-sales estimation. However, the repeat-sales estimator will not capture the effects of changes in the characteristics of an object between two sales. This happens very often in the case of houses, less so in the case of art objects, though these may get damaged, restored or even reattributed over time, and this may change their hedonic characteristics.

Revision volatility. Wang and Zorn (1997) among others note that "revision volatility" affects both types of indices. As new data become available, previous estimates of the index change, though this may be thought of as good since they result from increased efficiency in the estimators.

Price inflation and exchange rates. Inflation can be corrected easily, either by deflating prices themselves, or by deflating the index obtained using either method, since deflating is a log-linear operation. Exchange rates are hardly more problematic, except that here individual prices have to be corrected before going to the estimation work. Think however of a Japanese collector who buys in London in pounds, pays in yen, and sells a few years later in New York in dollars that he changes into yen. Obviously, the only thing that can be done is to compute the rate of return based on the first sale in pounds, translate the pounds into dollars at the going exchange rate, and compute the return in dollars with respect to the second sale. This computed return may be positive, though it may be negative for the collector. At least part of this will not happen with stocks, since most of these are bought and sold in the same place.

\textsuperscript{31} See however the discussion on hybrid methods.
The literature on comparing indices

Many papers deal with the comparison of indices, but there is very little done in a systematic way. In most cases, real data are used, different indices are constructed, but since the "true" underlying return is not known, the indices are simply compared with each other. Most comparisons deal with real estate prices.

Mark and Goldberg (1984) find that among the indices that are enumerated under "Other estimators" in Section 3 (to which they add a hedonic index constructed on the basis of time dummies for every year, a hedonic index run on adjacent years, a repeat-sales index, an arithmetic (unweighted) mean of sale prices in each period index, a median sale price in each period index), the arithmetic mean, surprisingly, performs well as do several of the hedonic price indices. Case (1986) and Case and Shiller (1987) compare RSR with median prices, and find that the latter are not a good measure of appreciation.

Goetzmann (1992) runs simulations on seven different repeat-sales estimators, including those that are discussed above, using NYSE data from which he draws random samples. He shows that all estimators perform well when the number of repeat-sales is large relative to the number of time intervals. Otherwise, Bayesian-type estimators, such as those discussed in Goetzmann (1990, 1992) are the only ones to perform well.

Chanel, Gérard-Varet and Ginsburgh (1996) run simulations by bootstrapping from the real set of Reitlinger's (1961, 1971) art data, but compute only a mean return and not an index. They compare the hedonic, the repeat-sales and the geometric mean estimators and find that none of them is biased, but the hedonic method provides much smaller standard errors for the return coefficient (which is obviously the consequence of a larger number of observations).

Meese and Wallace (1997) compare hedonic, repeat-sales and hybrid methods. They find that hedonic techniques are better suited. Repeat-sales techniques are subject to sample selection bias, they violate the assumption of constant implicit prices over time, they are too sensitive to small samples and to influential observations, and the usual method to correct for heteroskedasticity is inappropriate.

The results are thus far from clearcut, since most studies (with exception of Goetzmann, 1992 and Chanel, Gérard-Varet and Ginsburgh, 1996) cannot really tell which method is better, since they do not control for the underlying "true" returns. Goetzmann (1992) only compares various repeat-sales estimators (and does not compare these with hedonic estimators), and the experiments run by Chanel, Gérard-Varet and Ginsburgh (1996) are not comprehensive enough to draw solid conclusions.32

32 See also Case, Pollakowski and Wachter (1991), and Crone and Voith (1992) for more comparisons.
This is the reason for which we decided to run some Monte-Carlo experiments, which are described and discussed now.

Comparing hedonic and repeat-sales estimation: Some Monte-Carlo experimentation

The major concern that faces researchers on finding index values for art have to do with the amount of information available and the time frame of reference. If we knew with certainty and transparency the purchase and the sale price of every object sold for a long period of time there would be no question that RSR would give the best measure of the financial return to holding the set of objects included. When the time frame is short or the ratio of repeat-sales data to total sales data is small, questions arise as to whether a hedonic regression with large amounts of data would give a better reading of market returns.

To study this question we utilized the data collected on the Impressionist and Modern market from 1950 to 2002 and used in the Mei Moses All Art Index. For any sale that originated in New York during that period we used the existing catalogue description of ownership to search out any prior auction sale anywhere in the world in any time period, as well as for the prior hammer price. If prices were available for both sales they were included in the database. This research produced over 2,000 repeat-sale pairs, of which 1,671 were usable (at least three observations per artist). In addition to collecting the prior prices we also collected hedonic variables for these pairs. These include dummies for artist, salesroom, shape, medium, aspect (height/width), as well as height, width, and area. Other characteristics (signature, date, number of previous owners, number of times auctioned, number of exhibitions, number of citations and whether pictured in the sales catalogue, shape, and subject) were also collected but not used in the Monte-Carlo experiments.

The steps taken next are as follows, and ensure that both the repeat-sales and the hedonic regressions should generate returns that can be compared with some "true" reference returns.

Step 1. Generate a "reference series" of returns, that will be used to compare the results obtained by repeat-sales and hedonic regression over varying time frames. This reference series is obtained from a repeat-sales regression using the 1,671 pairs (3,342 observations) over the 31 year period. Store the estimated returns, as well as the residual variance of the regression.

Step 2. Construct artificial repeat-sales prices using the estimated returns in Step 1.

Step 3. For each time frame consisting in years \( t = 1, 2, ..., T \), to be discussed later, construct 100 samples \( i = 1, 2, ..., 100 \) of prices obtained by adding to the artificial repeat-sales prices

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33 Using the resources of the Watson Library at the Metropolitan Museum of Art and the New York Public library, 42nd Street branch.
generated in Step 2, a normal random disturbance with the usual properties and whose variance is equal to the residual variance of the regression in Step 1. Run a repeat-sales regression on each of the 100 samples, compute the mean return, and the mean tracking error $\text{TE}_R$ obtained as the square root of the sum of squares of the differences between the "true" and the mean returns over the whole period, that is, $\text{TE}_R = \left[ \frac{1}{T} \sum_t (\hat{\gamma}_{R,t} - \gamma_R,t) \right]^{1/2}$, where $\gamma_R,t$ is the mean over the 100 samples of the RSR return in year $t$, $\gamma_R,t$ is the "true" return in year $t$.

Compute also the mean standard error of the estimated returns, obtained as $\text{SD}_R = \frac{1}{T} \sum_t \left[ \frac{1}{100} \sum_i (\hat{\gamma}_{R,i,t} - \gamma_R,t) \right]^{1/2}$, where $\gamma_R,i,t$ is the RSR return in sample $i$, for year $t$.

**Step 4.** Compute a hedonic regression including the characteristics described above, as well as time dummies, using the 3,342 observations. Store the estimated implicit prices for characteristics, as well as the residual variance of the regression.

**Step 5.** Construct artificial hedonic prices using the estimated implicit prices obtained in Step 2, and the "true" returns obtained in Step 1.

**Step 6.** For different times frames to be discussed later, pick 14, 8 and 6 times each observation\(^{34}\) and construct 100 samples of prices obtained by adding to the artificial hedonic prices generated in Step 5, a normal random disturbance with the usual properties and whose variance is equal to the residual variance of the regression in Step 4.

**Step 6.** Run a hedonic regression on each of the 100 samples, compute the mean return, and the mean tracking error $\text{TE}_H$, obtained as the square root of the sum of squares of the differences between the "true" and the mean returns over the whole period, that is $\text{TE}_H = \left[ \frac{1}{T} \sum_t (\hat{\gamma}_{H,t} - \gamma_H,t) \right]^{1/2}$, where $\gamma_H,t$ is the mean over the 100 samples of the HR return in year $t$, $\gamma_H,t$ is the "true" return in year $t$. Compute also the mean standard error of the estimated returns, obtained as $\text{SD}_H = \frac{1}{T} \sum_t \left[ \frac{1}{100} \sum_i (\hat{\gamma}_{H,i,t} - \gamma_H,t) \right]^{1/2}$, where $\gamma_H,i,t$ is the HR return in sample $i$, for year $t$.

Observations cover the period 1950-2001. The time frames chosen are 10 years (1972-1981), 20 years (1972-1991) and 30 years (1972-2001), so that one can compare the performance of both estimators over ten, twenty and thirty year periods. There is one problem due to repeat-sales. In order to estimate a repeat-sales regression over the period 1972-1981 say, one may need observations for earlier years during which the first sale occurred. For this reason, all the regressions (repeat-sales and hedonic) are started in 1950, but the tracking errors and the mean standard errors are computed with 1972 as starting year.

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\(^{34}\) Each sale (or resale) is taken 14, 8 and 6 times in order to "simulte" the fact that there are 14, 8 and 6 times more sales than resales in the 10, 20 and 30 year database.
Three estimators are compared: the OLS repeat-sales estimators, the OLS hedonic estimator using all of the characteristics described above, and OLS hedonics that use only a subset of characteristics, to check how much ignoring some of the characteristics (in this case, dimensions: height, width, area and shape, that is width/height) matters. The three estimators are compared in Table 2 in terms of tracking errors and mean square errors.

The tracking errors $TE_R$ and $TE_H$ are much lower for hedonics than for repeat-sales for small samples (156 and 612 pairs), but, as expected, both methods tend to produce comparable results once the sample gets larger (1,671 pairs). Ignoring characteristics does not seem to matter: both tracking errors and mean standard errors are of the same order of magnitude than before. Figures 1, 2, and 3 compare both the RSR and the HR returns and indices with the "true" ones. Simulation results also show that the mean standard errors are lower for hedonics than for repeat-sales.

As expected, HR performs much better than RSR, even if some of the characteristics are ignored, when the sample size is small. It seems clear that RSR methods should not be used for time frames that include less than 20 years, unless the number of pairs is large.

Comparing three GLS estimators for repeat-sales estimation

Using the same dataset (1950-2001), we now compare the OLS estimator and three generalized least-squares estimators to compute repeat-sales regressions: the standard generalized least-squares estimator, with a diagonal weighting matrix in which the weights are equal to the squared estimation errors of a first step OLS estimation, the Case and Shiller (1987) three-stage least squares estimator and Goetzmann's (1992) two-stage Bayesian estimator. Table 3 reproduces the main results in terms of correlation between indices. Results show that though the mean return resulting from all four methods are almost identical, the correlation of the returns between the OLS and GLS estimators is not as high as the correlation of one GLS estimator with another. It also shows that all three GLS estimators give almost identical results.

35 Since in our artificial samples, errors are normal IID, a GLS estimator would lead to the same results as an OLS estimator. Therefore, the OLS estimator has all the properties requested.
36 For some reason that we do not understand, they are even smaller in some cases.
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Appendix 1 Studies on returns


Many more studies are compiled and surveyed in Ashenfelter and Graddy (2003, 2004), Burton and Jacobsen (1999), and Frey and Eichenberger (1995a, 1995b), who provide convenient tables with comparative results.

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37 As well as wines, which can also, in some sense, be considered as art.
38 See e.g. The Toronto Globe and Mail, July 5, 1996 (quoted by Pesando and Shum, 1999), or Forbes, December 5, 1994 (quoted by Burton and Jacobsen, 1999).
### Table 1
Examples of Results for Hedonic Equations for Painters

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height (in cm)</td>
<td>0.0111 (0.0002)</td>
<td>0.0080 (0.0010)</td>
<td>0.0090 (0.0002)</td>
<td>0.0102 (0.0004)</td>
<td>0.0572 (0.0040)</td>
</tr>
<tr>
<td>Width (in cm)</td>
<td>0.0077 (0.0002)</td>
<td>0.0050 (0.0010)</td>
<td>0.0081 (0.0003)</td>
<td>0.0132 (0.0004)</td>
<td>0.0804 (0.0051)</td>
</tr>
<tr>
<td>Area (in 1,000 sq. cm)</td>
<td>-0.1898 (0.0053)</td>
<td>-0.2400 (0.0080)</td>
<td>0.0269 (0.0013)</td>
<td>-0.0622 (0.0036)</td>
<td>-1.3594 (0.1000)</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.7705 (0.0783)</td>
</tr>
<tr>
<td>Collage</td>
<td>-0.5306 (0.0928)</td>
<td>-0.1830 (0.0840)</td>
<td>0.2690 (0.0721)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pastel</td>
<td>-0.6159 (0.0132)</td>
<td>-</td>
<td>-</td>
<td>0.0605 (0.1207)</td>
<td></td>
</tr>
<tr>
<td>Drawing</td>
<td>-0.6817 (0.0928)</td>
<td>-</td>
<td>-</td>
<td>-0.6817 (0.1003)</td>
<td></td>
</tr>
<tr>
<td>Mixed media</td>
<td>-0.1132 (0.0460)</td>
<td>-</td>
<td>0.1330</td>
<td>-0.2163 (0.0569)</td>
<td>0.0000</td>
</tr>
<tr>
<td>Other</td>
<td>-0.3829 (0.0652)</td>
<td>-</td>
<td>-</td>
<td>-0.3829 (0.0652)</td>
<td></td>
</tr>
<tr>
<td>Support</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canvas</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-</td>
</tr>
<tr>
<td>Wood panel</td>
<td>-0.0790 (0.0550)</td>
<td>-</td>
<td>-0.009 (0.0201)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cardboard</td>
<td>-0.1300 (0.0710)</td>
<td>-</td>
<td>-0.2229 (0.0438)</td>
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<tr>
<td>Paper</td>
<td>-0.2163 (0.0569)</td>
<td>-</td>
<td>-0.2163 (0.0569)</td>
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<tr>
<td>Not signed</td>
<td>-0.3829 (0.0652)</td>
<td>-</td>
<td>-0.3829 (0.0652)</td>
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</tr>
<tr>
<td>Not dated</td>
<td>-0.1440 (0.0481)</td>
<td>-</td>
<td>-0.1440 (0.0481)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>No. of observations</td>
<td>24,540</td>
<td>6,410</td>
<td>6,224</td>
<td>12,118</td>
<td>3,342</td>
</tr>
</tbody>
</table>

Sources. Impressionists and Modern, and Other Europeans: de la Barre, Docclo and Ginsburgh (1994); American painters: Demortier (1992); Belgian painters: Ginsburgh and Mertens (1994); Mei and Moses database. All the regressions contain dummies for painters, years and auction houses, but the detailed results are not reported in the table.
Table 2
Comparing OLS hedonic and repeat-sales estimators
(tracking errors and standard deviations over various time frames)

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<tr>
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</thead>
<tbody>
<tr>
<td>No. of RSR observations</td>
<td>312</td>
<td>1,224</td>
<td>3,342</td>
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<td>No. of HR observations</td>
<td>4,368</td>
<td>9,792</td>
<td>20,052</td>
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<td>Tracking errors</td>
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<tr>
<td>RSR</td>
<td>0.2723</td>
<td>0.1566</td>
<td>0.0306</td>
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<tr>
<td>HR full</td>
<td>0.0333</td>
<td>0.0110</td>
<td>0.0070</td>
</tr>
<tr>
<td>HR without dimensions</td>
<td>0.0307</td>
<td>0.0012</td>
<td>0.0026</td>
</tr>
<tr>
<td>Standard errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RSR</td>
<td>0.6580</td>
<td>0.4903</td>
<td>0.3775</td>
</tr>
<tr>
<td>HR full</td>
<td>0.3485</td>
<td>0.2983</td>
<td>0.2925</td>
</tr>
<tr>
<td>HR without dimensions</td>
<td>0.3914</td>
<td>0.3201</td>
<td>0.2665</td>
</tr>
</tbody>
</table>

HR without dimensions contains the same regressors as HR full, except Height, Width and Area of the work.
Table 3
Comparison of OLS and GLS repeat-sales estimators

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>GLS</th>
<th>Case-Shiller</th>
<th>Bayes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return</td>
<td>0.118</td>
<td>0.119</td>
<td>0.118</td>
<td>0.119</td>
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<tr>
<td>Standard deviation of return</td>
<td>0.223</td>
<td>0.203</td>
<td>0.194</td>
<td>0.202</td>
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</tbody>
</table>

Correlations between annual returns

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>GLS</th>
<th>Case and Shiller</th>
<th>Bayes</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GLS</td>
<td>0.74</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case and Shiller</td>
<td>0.87</td>
<td>0.96</td>
<td>1.00</td>
<td>-</td>
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<tr>
<td>Bayes</td>
<td>0.74</td>
<td>0.99</td>
<td>0.96</td>
<td>1.00</td>
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</tbody>
</table>
Figure 1 Repeat-sales and Hedonic Returns and Indices 1972-1981
Figure 2 Repeat-sales and Hedonic Returns and Indices 1972-1991
Figure 3 Repeat-sales and Hedonic Returns and Indices 1972-2000