Self-similar steady Marangoni boundary layers of spontaneous nature at a uniformly cooled free surface of a liquid

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Summary

An infinitely deep horizontal liquid layer with a constant and uniform heat loss at the free surface (here assumed flat), which can e.g. mimic the effect of evaporation, is considered, the gas phase being “passive”. If the temperature is initially constant everywhere, a transient horizontally-uniform temperature profile starts developing from the surface. It becomes Marangoni-unstable upon a certain (critical) time, the corresponding marginal condition for monotonic instability being here calculated analytically within the frozen-time approach. Other instability mechanisms, if any, are discarded.

On the other hand, as an apparently unrelated development, self-similar steady Marangoni-boundary-layer (hereafter MBL) solutions are obtained in this same setup. Interestingly enough, they represent stationary regimes where a purely conductive state can be nothing but transient. If put in the context of the above mentioned Marangoni instability of a developing transient conductive profile, they can be seen to be appropriate for times much greater than the critical one and associated with a high-Marangoni-number convection developing as a result of the instability. Even though these MBL regimes are most definitely unstable themselves, they are deemed to be nonetheless noteworthy points in the phase space of the problem.

Let us underscore that the MBLs considered here come due to a spontaneous breakdown of symmetry rather than an externally induced one. Unlike most of the self-similar MBL cases in the literature, where MBLs are analyzed for given temperature distributions along the surface, an essential element here is a feedback between the hydrodynamic and thermal boundary layers eventually resulting in the determination of the convection intensity as a nonlinear eigenvalue of the problem. Due to their spontaneous character, the initiation point of the MBLs can in principle be arbitrary. Both planar and axisymmetric cases are treated. Interpretation of the results in light of the expected cellular structure of the Marangoni-Bénard convection is provided. Namely, collisions of MBLs emanating from different initiation points must result in vertical downward jets convecting into the bulk of the liquid the negative heat generated at the surface, which can give rise to a kind of cells. In this scheme of things, the sizes of neighboring cells need not be anyhow correlated.

Keywords: Marangoni boundary layer, self-similar solution, Bénard instability, frozen-time approach

1. Background

The present study is carried out, on the one hand, in the context of Marangoni-Bénard instabilities of a horizontal liquid layer and convection patterns developing on their background (e.g. [1] and references therein), and on the other hand, in the context of MBLs (e.g. [2] and references therein). More specifically, the studied configuration may be related to an evaporating (into an inert gas) pure-liquid layer [3] at the initial stages of the process when the bottom is not yet reached by the course of events near/at the free surface. For an evaporating layer, the activeness of the gas phase, assumed passive here, becomes apparent first of all through an effective Biot number that is a function of the wavenumber of the perturbation [3]. The passiveness assumption corresponds then to a sufficiently small Biot number. In principle, a problem equivalent to the one considered here can also be formulated in terms of mass transfer and the solutal Marangoni effect. For instance, this can concern evaporation of a binary-liquid layer, where the solutal effects are likely to be predominant over the thermal ones, so that the latter can be neglected at least in a certain sufficiently large vicinity of the instability threshold [4]. Nonetheless, the present analysis formally assumes a Prandtl number of order unity, and no allowance is made...
for its possible large values as would be the case when it gets replaced by the Schmidt number (typically large in the liquid) in the solutal counterpart of the thermal problem.

2. Some details of the analysis

The configuration of the problem, already described in Summary, is schematically represented in figure 1, where note that \( q^* \) has the dimensions of energy per unit area and time. The presence or absence of an asterisk will distinguish between the dimensional and dimensionless quantities, respectively. A symbol in square brackets will refer to the dimensional scale of the corresponding quantity, i.e. \( f^* = [f] f \) for the typical quantity \( f \). Let \( l^* \) be the length scale of the problem (unspecified for the moment). Some other scales are then chosen as \( [x] = [z] = [k]^{-1} = l^* \), \([u] = [w] = \chi^* l^* \), \([t] = l^* t^2 / \chi^* \), \([T] = q^* l^* / \kappa^* \). Here \( t \) is the time, \( T \) is the temperature (counted from that of the bulk of the liquid), \( k \) will refer to the wavenumber of perturbations, \( \chi^* \) and \( \kappa^* \) are the thermal diffusivity and conductivity, other notations being clear from figure 1. In this way, the dimensionless formulation of the problem depends only on two dimensionless numbers, the Prandtl number \( \text{Pr} \) and the Marangoni number \( Ma = \sigma^*_T q^* l^* / \kappa^* \), where \( \mu^* \) and \( \nu^* \) are the dynamic and kinematic viscosities, and \( \sigma^*_T \) < 0 is the surface tension derivative with respect to temperature (so that \( Ma > 0 \) for \( q^* < 0 \) considered here). In fact, one can make \( Ma \equiv 1 \) by choosing \( l^* = l^*_M \equiv \left( \frac{\mu^* \chi^* \eta^*}{\kappa^* \nu^*} \right)^{1/2} \), which can be referred to as the Marangoni length scale. In general here, \( Ma = l^2 / l^2_M \). It is physically quite expected that the critical time mentioned in Summary is just related to the length scale \( l^*_M \), namely \( t^*_c \equiv 2 l^*_M / \chi^* \), the numerical coefficient resulting from calculation and the most unstable mode corresponding to \( k = 0 \). In fact, for the linear stability analysis carried out within the frozen-time approach here, it is advantageous to choose \( l^* = l^*_c \equiv (\chi^* t^*_c)^{1/2} \), for after all \( t^* \) is then just a parameter among others. In particular, with \( l^* = l^*_c \), the monotonic marginal stability curve is just given by \( Ma = \pi^{1/2 - 4k^2 + \pi^{1/2}(8k^2 - 1)} \exp(4k^2) \exp(2k)l^*_M \), from where one can e.g. recover the above result for \( t^*_c \). The MBL solution is sought in the form \( u = (Ma \text{Pr})^{1/2} U(\eta) \), \( T = (\text{Pr} / Ma)^{1/4} x^{1/2} \Theta(\eta) \) with \( \eta = (Ma / \text{Pr})^{1/4} z / x^{1/2} \). One can see that this can be valid only for \( l^* \gg l^*_M \) or \( Ma \gg 1 \), as noted in Summary. One obtains the following boundary-value problem in terms of \( \Psi(\eta) = \int_0^{\eta} U(\tilde{\eta}) d\tilde{\eta} \) and \( \Theta(\eta) \), the primes denoting the \( \eta \) derivatives: \((\frac{1}{2} + n) \Psi' + \Psi'' = 0 \) and \((\frac{1}{2} + n) \Psi' - \frac{1}{2} \Psi \Theta' + \text{Pr}^{-1} \Theta'' = 0 \) with \( \Psi = 0 \), \( \Psi'' + \frac{1}{2} \Theta' = 0 \) and \( \Theta' = -1 \) at \( \eta = 0 \), and \( \Psi' \to 0 \) and \( \Theta \to 0 \) as \( \eta \to +\infty \). Here \( n = 0, 1 \) correspond to the planar and axisymmetric cases, respectively. In particular, for \( \text{Pr} = 7 \), one thereby numerically obtains \( U(0) = 0.711 \) and \( \Theta(0) = 0.532 \) in the planar case, and \( U(0) = 0.470 \) and \( \Theta(0) = 0.495 \) in the axisymmetric case. Also represented in figure 1 is the MBL initiation/collision scenario discussed at the end of Summary.

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References