# Vibration based damage detection using large array sensors and spatial filters

Arnaud Deraemaeker* and André Preumont

*ULB, Active Structures Laboratory*

*50 av. Franklin Roosevelt, CP 165/42, B-1050 Brussels, Belgium*

*Tel: 32 - 2.650.26.89 Fax: 32 - 2.650.46.60*

*email : aderaema@ulb.ac.be, apreumon@ulb.ac.be*

## Abstract

In this paper, a novel approach for vibration based damage detection is proposed. The approach relies on the use of a large network of sensors (possibly hundreds of them) to which a programmable linear combiner is attached. The linear combiner is programmed to work as a modal filter. The frequency content of the output of the modal filter is proposed as feature for damage detection. It is shown that if a local damage is present, spurious peaks appear in the FRF of the modal filter whereas if temperature changes are considered, the FRF of the modal filter is shifted but its shape remains unchanged. The approach is interesting because of the ability to differentiate between local damage and global environmental changes to a structure.

**Keywords :** Health monitoring, sensor array, damage detection, spatial filtering

# Introduction

Health monitoring problems have received increased interest from many different fields of engineering for the last two decades. The problem is : to be able to detect, locate and assess the extent of damage in a structure so that its remaining life can be known and possibly extended. If the location of the damage is known, local techniques such as ultrasonic methods, radiography, eddy-current methods or thermal field methods are of common use. The need for global damage detection (when the vicinity of damage is not known) has led the researchers into developing vibration based methods. The literature on the subject is very large [1]. The key point for a successful damage detection is to extract from the measurements features which are sensitive to damage. The most common features found in the literature are modal properties (eigenfrequencies [2] and eigenmodes [3]). Recent developments in "modal based" damage detection techniques aim at acquiring modal properties of large structures by using ambient sources of vibrations (wind, traffic ...) which are in principle not known so that one has to use so-called output-only modal identification methods [4, 5, 6, 7].

In the frequency domain, other methods exists based on either frequency response functions [8] or transmittance (also called transmissibility) functions [9, 10, 11, 12]. For frequency response functions the input must be accurately known.

This is a drawback for the design of a health monitoring system under operating conditions since, as mentioned earlier, the input is generally not known, unless the excitation is imposed by the monitoring system. In the case of transmittance functions, frequencies much higher than those used for modal identification are used because transmittance functions are sensitive to changes in vibration characteristics of two neighboring points which occur only at high frequencies. Such an approach will most probably fail for large and heavy structures for which such high frequencies are not sufficiently excited by ambient sources.

More recently, researchers have turned towards features extracted from the time domain data in order to detect damage. Working on time domain data allows to be able to detect non-linearities [13] which arise for certain types of damage such as crack propagation, or unusual events in the signals [14]. This information is lost when the time domain data is projected in the frequency domain. These methods rely on signal processing techniques such as the wavelet transform.

It is also worth noting that there is an increased interest for the monitoring of composite materials. Traditional modal based methods can be used, but also more advanced techniques are under development. Features of interest are : Lamb Waves (high frequency traction/compression waves [15, 16, 17]), random decrement (RANDEC) signatures [18], identified NARMAX (nonlinear autoregressive moving average with exogenous input) model coefficients [19], specific damping capacity (SDC, [20]).

Although many researchers have been working on the subject for quite a few years, there are still problems that have not found satisfactory solutions. A major one is the fact that environmental changes (temperature, humidity) are responsible for changes in features of interest of the same order of magnitude (or more) as damage, making very difficult to determine whether the structure is damaged. A few recent studies can be found in which by monitoring the features over a long period of time, it is possible to eliminate the effect of environmental conditions of the data [21, 22]. On the other hand, it is important to point out that experimental equipment evolves very rapidly and allows one to envision the use of large networks of wireless sensors for health monitoring in a not too distant future. These technological advances might be the key to successful vibration-based automated damage detection techniques if intelligent methods are produced in order to take advantage of the enormous amount of information provided by these large networks.

The aim of this paper is to study the damage detection problem in the framework of large networks of sensors and address the issue of differentiating the environmental (global) changes from the damage (local) changes to a structure dynamic response. For this purpose, a new feature based on the concept of spatial filters is proposed. Spatial filters can be seen as a single sensor built from a large network of sensors. The individual outputs are combined into a single output by means of a linear combiner. This single output can be designed such that it mimics the behavior of a single degree of freedom system in which case the filter is called a ”modal filter”. The filter can be tuned to any of the modes of the structure in the frequency band of interest. There is in addition a possibility of multiplexing by changing in real time the linear combiner coefficients.

One application of the use of modal filters for damage detection can be found in [23], where it is proposed to use modal norms (extracted using modal filters) as a damage indicator. The philosophy developed in this paper is quite different. It is based on the fact that if a global change is applied to the structure (i.e. temperature), the shape of the output of the modal filter does not change whereas if a local stiffness change is applied, spurious peaks appear in the filter at the resonant frequencies of the structure. The appearance of peaks in the modal filter is therefore an attractive feature in order to differentiate between environmental and damage effects.

The paper is organized as follows : in the first part, the concept of spatial and modal filtering is presented. In the second part, the impact of damage on modal filters is studied with a numerical example. In the last part, the ability of modal filters to differentiate between global and local changes is demonstrated on a simply supported beam example. Comparisons are made with the usual modal features for damage detection (MAC and frequency deviation).

## Spatial filtering and modal filters

[Figure 1 about here.]

Let us consider a structure equipped with an array of $n$ sensors (Fig.1). Spatial filtering consists in combining linearly the outputs of the network of sensors into one single output according to $y = \sum \alpha_k y_k$. Upon proper selection of $\alpha_i$, various meaningful outputs may be constructed, as, for example, modal filters. The idea behind modal filtering is to configure the linear combiner in such a way that it is orthogonal to all the modes of a structure in a frequency band of interest, except mode $l$. The modal filter is then said to be tuned to mode $l$ and all the contributions from the other modes will be removed from the signal. This is illustrated in Fig. 2 where the FRF of such a modal filter is represented. Because of spatial aliasing, there are some restrictions on the frequency band where modal filters can be built, for a given size of the sensor array.

[Figure 2 about here.]

The coefficients of the linear combiner can be either computed from a known model of the structure, or directly computed from experimental measurements (FRFs). For more details on the determination of the modal filter coefficients, the reader should refer to [24, 25, 26]. Note that the coefficients of the modal filter are independent of the excitation type and location.

## Modal filtering with an array of sensors

Let us assume that the structure equipped with the linear combiner is discretized using the finite element method. Let us note $[\mathbf{K}]$, $[\mathbf{M}]$ and $[\mathbf{C}]$ its stiffness, mass and damping matrix. The structure is subject to an external excitation $\{f\}$. The finite element method leads to a system of equations of the type

$$[\mathbf{K}]\{u\} + [\mathbf{M}]\{\ddot{u}\} + [\mathbf{C}]\{\dot{u}\} = \{f\} \tag{1}$$

where $\{u\}$ is a vector containing the nodal values of the displacement of the structure. If we assume that the excitation is harmonic, the response will also be harmonic and we can write :

$$\{u(t)\} = Real\left(\{U\}e^{j\omega t}\right) \tag{2}$$

$$\{f(t)\} = Real\left(\{F\}e^{j\omega t}\right) \tag{3}$$

$\{U\}$ and $\{F\}$ are complex valued vectors containing the information about the amplitude and phase of the signals. $\{U\}^*$ denotes the complex conjugate of $\{U\}$. Equation (1) becomes :

$$\left([\mathbf{K}] - \omega^2[\mathbf{M}] + j\omega[\mathbf{C}]\right)\{U\} = \{F\} \tag{4}$$

For a system discretized with $N$ degrees of freedom, there exists $N$ couples of eigenmodes and eigenfrequencies $(\{\Phi_i\}, \omega_i)$. The eigenmodes (column vectors) can be put in a matrix $[\Phi]$ and the eigenvalues on the diagonal of a matrix $[\Lambda]$ :

$$[\Phi] = [\Phi_1 \Phi_2 ... \Phi_n] \tag{5}$$

$$[\Lambda] = \left[diag(\omega_i^2)\right] \tag{6}$$

If the eigenmodes are mass normalized, the following orthogonality properties hold :

$$[\Phi]^T [\mathbf{K}] [\Phi] = [\Lambda] \tag{7}$$

$$[\Phi]^T [\mathbf{M}] [\Phi] = [\mathbf{I}] \tag{8}$$

In modal coordinates, vector $\{U\}$ can be written

$$\{U\} = [\Phi] \{Z\} \tag{9}$$

where $\{Z\}$ is the vector of modal amplitudes. Replacing and premultiplying Equation (4) by $[\Phi]^T$, we get :

$$\left([\Lambda] - \omega^2[\mathbf{I}] + j\omega [\Phi]^T [\mathbf{C}] [\Phi]\right) \{Z\} = \{b\} \tag{10}$$

with :

$$\{b\} = [\Phi]^T \{F\} \tag{11}$$

$b_i$ is called the modal input gain. Assuming that $[\Phi]^T [\mathbf{C}] [\Phi]$ is diagonal, this leads to a system of $N$ decoupled equations of the type

$$\left(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega\right) z_i = b_i \tag{12}$$

The modal amplitudes $z_i$ are therefore given by :

$$z_i = \frac{b_i}{(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)} \tag{13}$$

and the response of the structure is :

$$\{U\} = \sum_{i=1}^{N} \frac{b_i}{(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)}\{\Phi_i\} \tag{14}$$

We assume that the structure is equipped with an array of $n$ sensors whose measured values are put in a vector $\{Y\}$. The output of the sensors is a linear combination of the response of the structure :

$$\{Y\} = [c]\{U\} \tag{15}$$

which leads to

$$\{Y\} = \sum_{i=1}^{N} \frac{b_i}{(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)}\left([c]\{\Phi_i\}\right) \tag{16}$$

The output on sensor $k$ of the network as a function of $\omega$ therefore reads :

$$Y_k(\omega) = \sum_{i=1}^{N} \frac{c_{ki} b_i}{(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)} \tag{17}$$

where $c_{ki}$ is the result of the product of line $k$ of $[c]$ with $\{\Phi_i\}$ and is usually called the modal output gain.

If the $n$ sensors in the array are connected to a linear combiner with gain $\alpha_k$ for sensor $k$ (Fig.1), the output of the linear combiner is $y = \sum_{k=1}^{n} \alpha_k y_k$ and the global frequency response is :

$$G(\omega) = \sum_{k=1}^{n} \alpha_k Y_k(\omega) = \sum_{i=1}^{N} \frac{\{\sum_{k=1}^{n} \alpha_k c_{ki}\} b_i}{(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)} \tag{18}$$

A modal filter which isolates mode $l$ can be constructed by selecting the weighing coefficients $\alpha_k$ of the linear combiner in such a way that

$$\sum_{k=1}^{n} \alpha_k c_{ki}(\omega) = \delta_{li} \tag{19}$$

For more details on the subject, we refer to [26].

# Impact of damage on modal filters

If we now assume that the structure is damaged, equation (4) becomes :

$$\left([\mathbf{K}+\mathbf{\Delta K}]-\omega^2[\mathbf{M}]+j\omega[\mathbf{C}]\right)\{\tilde{U}\}=\{F\} \tag{20}$$

where $[\mathbf{\Delta K}]$ represents a change in stiffness du to damage on the structure. The quantities related to the damaged structure will be noted by a ˜ . The output of the linear combiner is now :

$$G(\omega)=\sum_{k=1}^{n}\alpha_k\tilde{Y}_k(\omega)=\sum_{i=1}^{N}\frac{\{\sum_{k=1}^{n}\alpha_k\tilde{c_{ki}}\}\tilde{b_i}}{(\tilde{\omega_i}^2-\omega^2+2j\tilde{\xi_i}\tilde{\omega_i}\omega)} \tag{21}$$

The impact of damage can be decomposed into three effects :

- $\tilde{b_i}$ : the change in the modeshapes of the structure will affect the modal input gain which changes the amplitude of the modal filter ;
- $(\tilde{\omega_i},\tilde{\xi_i})$ : the change in the eigenfrequencies and modal damping will affect respectively the position and peak amplitude of the modal filters ;
- $\sum_{k=1}^{n}\alpha_k\tilde{c_{ki}}$ : because of the change in the modeshapes, equation(19) may not be satisfied. In this case, the output of the modal filter does not isolate mode $l$ perfectly and the other modes may appear in the response.

The third effect is interesting because it is a clear indicator that the shape of the eigenmodes has changed. Indeed, changes in the peak position and amplitude of the modal filter can be caused by shape changes as well as damping or frequency changes. One particular case worth nothing is when the stiffness change is global ($[\mathbf{\Delta K}]=\beta[\mathbf{K}]$). In this case, the modeshapes are not altered and Equation (19) still holds. In all other cases, it is expected that when damage occurs, peaks will appear at all the resonant frequencies of the damaged system. This is the feature which is proposed for damage detection.

# Numerical example

In order to illustrate the impact of a structural modification on the output of modal filters, we have built a numerical demonstrator with the following features :

- Generation of random inputs with prescribed power spectral densities ;
- Computation of structural response in the time domain based on modal decomposition ;
- Generation of noise on the outputs based on RMS value of the signals ;
- Estimation of FRF and PSD of outputs based on Welch's average periodogram method [27] ;
- Computation of linear combiner coefficients for modal filtering.

## Simply supported beam

The demonstrator has been applied to an example of a simply supported beam represented in Fig. 3. It is made of aluminum (Young's modulus = 68.8 $GPa$, Poisson's ratio = 0.3, density = 2718 $kg/m^3$), the length of the beam is $L = 326mm$ and the cross section is rectangular ($b = 50mm$, $h = 4mm$). The modal damping is 1% for all the modes. The beam is equipped with an array of $49$ equally spaced velocity sensors and a linear combiner as described on the figure. In this study, we assume perfect measurements (noiseless and perfectly synchronized).

[Figure 3 about here.]

A numerical model of the structure is built using 100 Euler-Bernoulli beam finite elements. The beam is excited through pseudo-random band-limited noise (repeated sequences) in the frequency band [0,10000] Hz (containing the first 10 modes of the beam) at the location indicated on the figure. The time response of the structure is computed and the linear combiner coefficients $\alpha_k$ are determined based on the eigenmodes of the structure (for more details, see [26]). The individual time domain data of each sensor are combined to form the modal filter output in the time domain. The Welch's average periodogram method is then used in order to estimate the FRF of the modal filter.

## Output of modal filters

The estimated FRFs of the output of the linear combiner tuned to modes 1 to 6 for the initial (undamaged) structure are shown on Fig. 4. We then investigate the effect of damage on the output of these modal filters. In particular, we will see if the effect of a local damage can be distinguished from a global stiffness change (due essentially to temperature changes). We therefore consider two scenarios :

- Scenario 1, local damage : a stiffness reduction of 10% in the section between $0.1L$ and $0.2L$ is applied to the initial structure.

- Scenario 2, environmental effects : a stiffness reduction of 2% on the whole beam is considered. This is a simplified model of a stiffness change due to temperature dependant properties of the materials of the structure.

These structural changes affect the output of the modal filters. We represent the FRF of the output of the modal filters for the first scenario on Fig. 5 and for the second scenario on Fig. 6.

[Figure 4 about here.]

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

As expected from the theoretical developments, it can be seen that an interesting feature of the output of the modal filters is that when the damage is local, new peaks appear in the modal filter (this effect is more pronounced for modal filters tuned to modes 4 to 6), whereas when the damage is global, the modal filter is shifted in frequency but the general shape is not altered. It is also interesting to look at the frequency deviation defined by :

$$\Delta f_i = \frac{f_i - \tilde{f}_i}{\tilde{f}_i} * 100(\%) \tag{22}$$

where $f_i$ is the i$^{th}$ eigenfrequency of the damaged structure and $\tilde{f}_i$ is the i$^{th}$ eigenfrequency of the initial reference structure, and the $MAC$ defined by :

$$MAC(\phi_i, \tilde{\phi}_j) = \sqrt{\frac{(\phi_i^T \tilde{\phi}_j)^2}{(\phi_i^T \phi_i)(\tilde{\phi}_j^{\ T} \tilde{\phi}_j)}} \qquad (23)$$

where $\phi_i$ is a column vector containing the i$^{th}$ eigenmode of the initial structure and $\tilde{\phi}_j$ is a column vector containing the j$^{th}$ eigenmode of the damaged structure. These two classical indicators are shown in Fig. 7 and 8 where one can see that the average frequency deviation is equivalent for both damage scenarios and that the MAC is not affected by any of the scenarios considered.

In real conditions, the temperature field may not be perfectly uniform. Therefore, we now consider the following scenario where the temperature field is different at the two ends of the beam :

- A stiffness reduction of 1.5 % between elements $0$ and $.1L$ and between $.9L$ and $L$, as well as a stiffness reduction of 2 % between $.1L$ and $.9L$

The FRFs output of the linear combiner tuned to modes 1 to 6 are represented on Fig 9. One can see that small peaks appear on modal filter 6 only. The amplitudes of the peaks are much smaller than for the local damage scenario, which shows that it is still possible to differentiate between global and local damage. The frequency deviation for this damage case is represented in Fig 10. It is equivalent to the two damage scenarios considered before.

[Figure 9 about here.]

[Figure 10 about here.]

# Conclusion

In this paper, we have studied the vibration based damage detection problem using a large network of sensors. Our aim was to find a feature that could differentiate between local and global (environmental) stiffness changes. The novel idea presented consists in using the frequency domain output of so-called modal filters. A combined theoretical and numerical study showed that an interesting feature of the output of modal filters is the appearance of spurious peaks for local stiffness changes which does not occur for global changes. The appearance of peaks in the output of the modal filters was therefore identified as a good indicator in order to differentiate between damage and environmental effects.

A comparison with the classical MAC and $\Delta f$ features has shown that these estimators were not able to detect damage in the example considered. Indeed, on one hand, the frequency deviation due to environmental effects was of the same order of magnitude as the one due to damage, and on the other hand, we have shown that the MAC values were not altered by the local damage scenario. On the contrary, the modal filters showed the clear appearance of peaks for the local damage scenario and their shape was not altered by temperature changes. In the case were the stiffness change is not perfectly uniform, small peaks may appear but their amplitudes are much smaller than in the case of a local stiffness change.

It is therefore still possible to differentiate the global effect of environment and the local effect of damage even when the stiffness change is not perfectly uniform.

There remains however a large amount of work in order to see if such a feature can be used in practice. One further step consists in studying the behavior of modal filters for output-only measurements which are much more practical for real structures where the input consisting of environmental and traffic excitations is generally not known. A first study on the subject can be found in [28]. Further research should also focus on issues related to the robustness of the damage detection method such as (i) sensor placement, (ii) influence of noise and delays on the measurements, (iii) influence of sensor failure.

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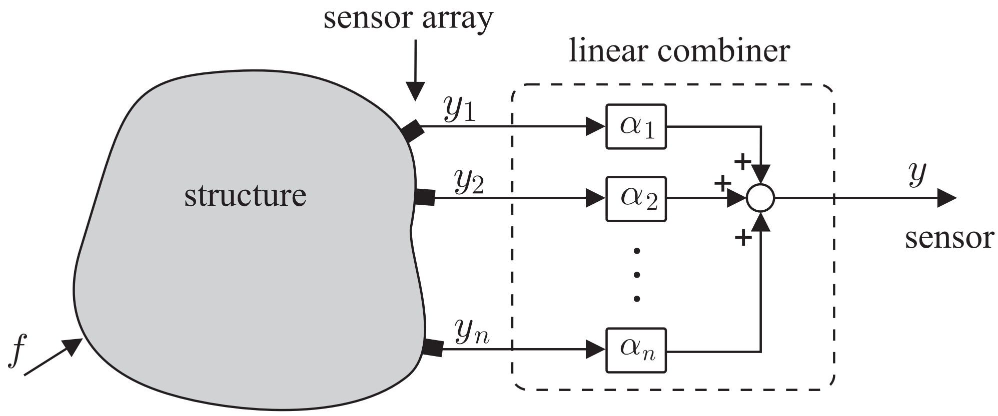


Figure 1: Representation of the spatial filter using $n$ discrete sensors and a linear combiner.

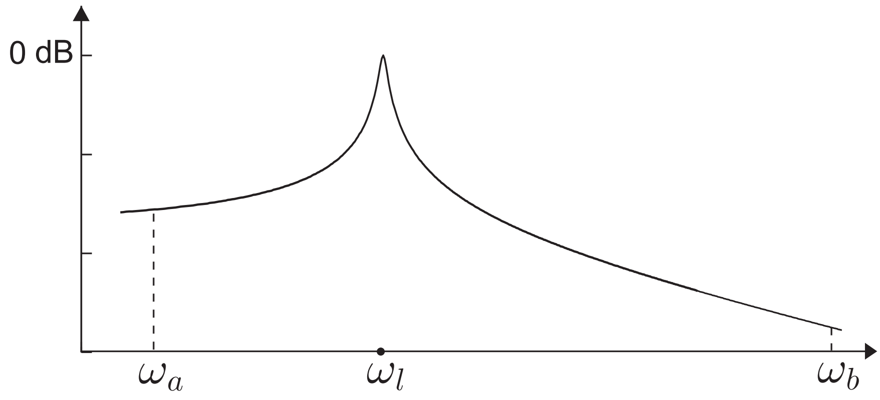


Figure 2: Perfect modal filter within the bandwidth $[\omega_a, \omega_b]$.

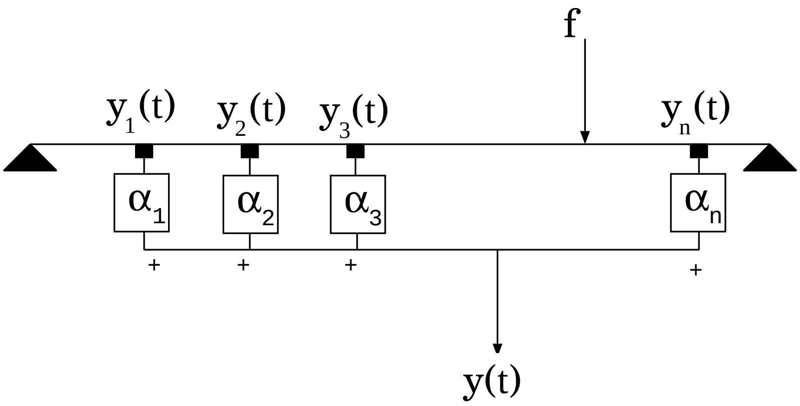


Figure 3: Simply supported beam with array of sensors and linear combiner

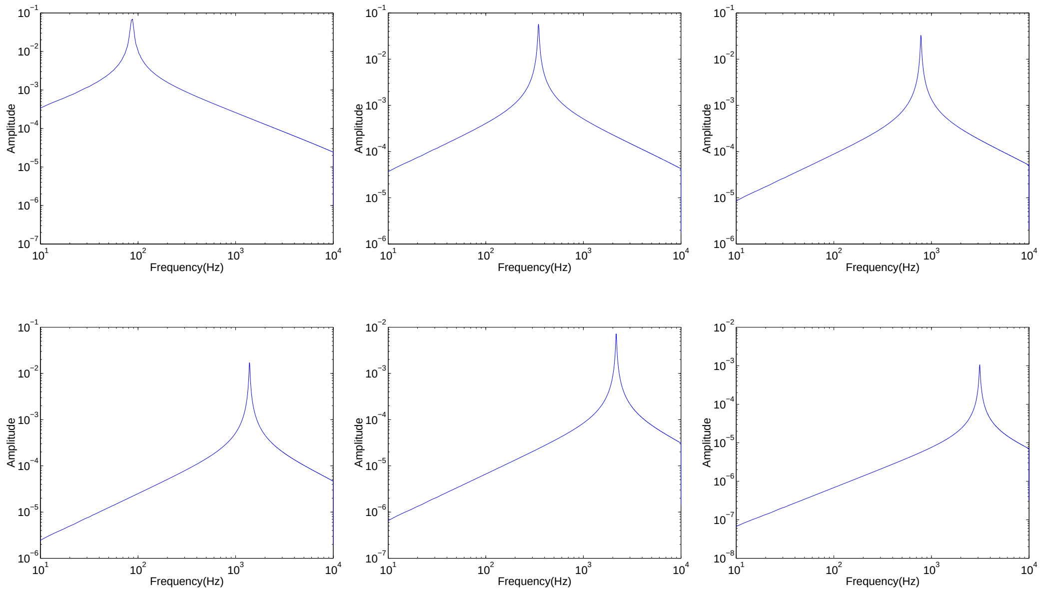


Figure 4: FRF of output of linear combiner tuned to modes 1 to 6, initial undamaged structure

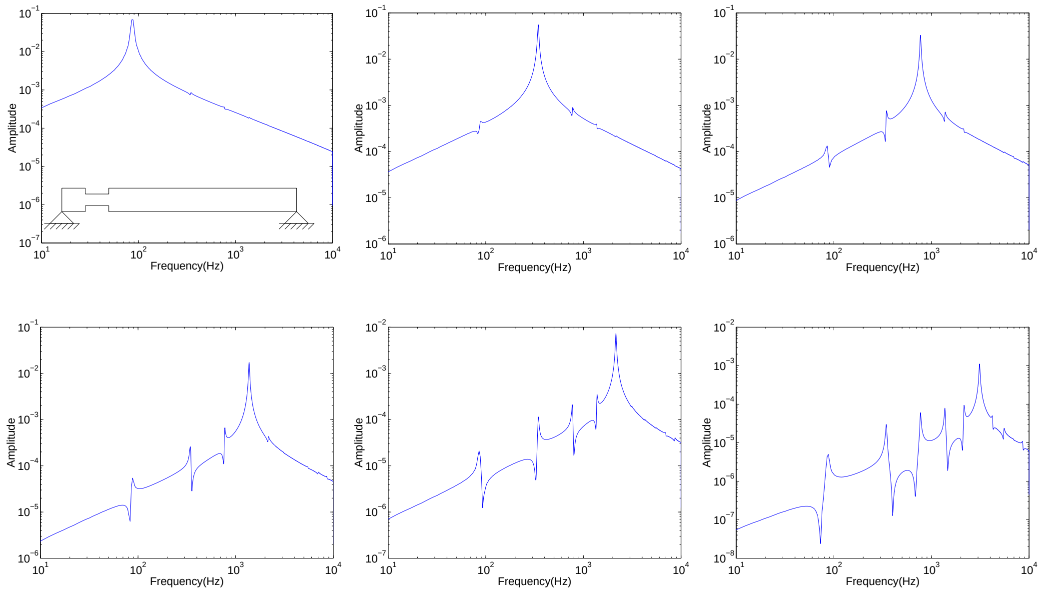


Figure 5: FRF of output of linear combiner tuned to modes 1 to 6, local damage : 10% stiffness reduction between $0.1L$ and $0.2L$

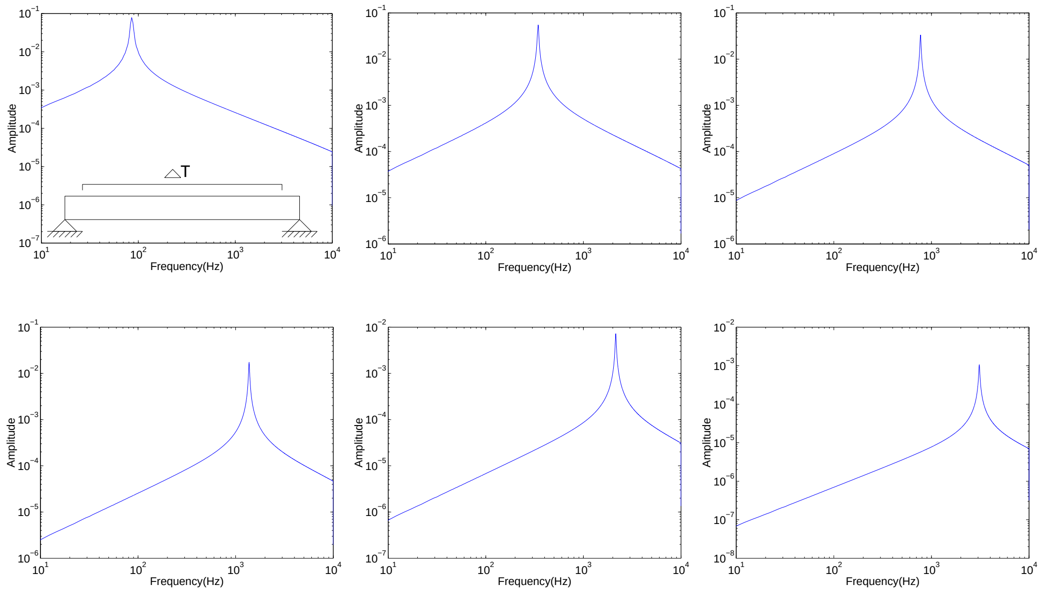


Figure 6: FRF of output of linear combiner tuned to modes 1 to 6, environmental effects : 2% stiffness reduction on the whole beam

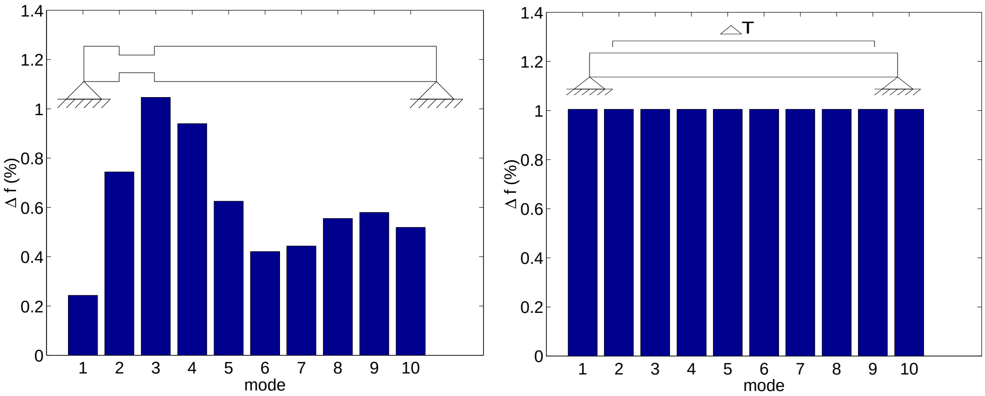


Figure 7: Frequency deviation for the two scenarios, local damage (left : 10% stiffness reduction between $0.1L$ and $0.2L$) and environmental effects (right : 2% stiffness reduction on the whole beam)

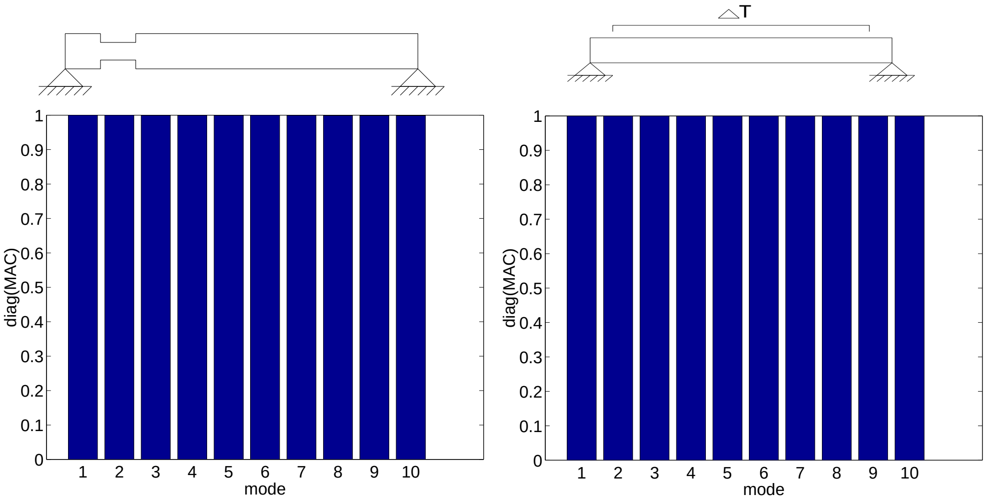


Figure 8: Diagonal of MAC values with reference to initial state for the two scenarios, local damage (left : 10% stiffness reduction between $0.1L$ and $0.2L$) and environmental effects (right : 2% stiffness reduction on the whole beam)

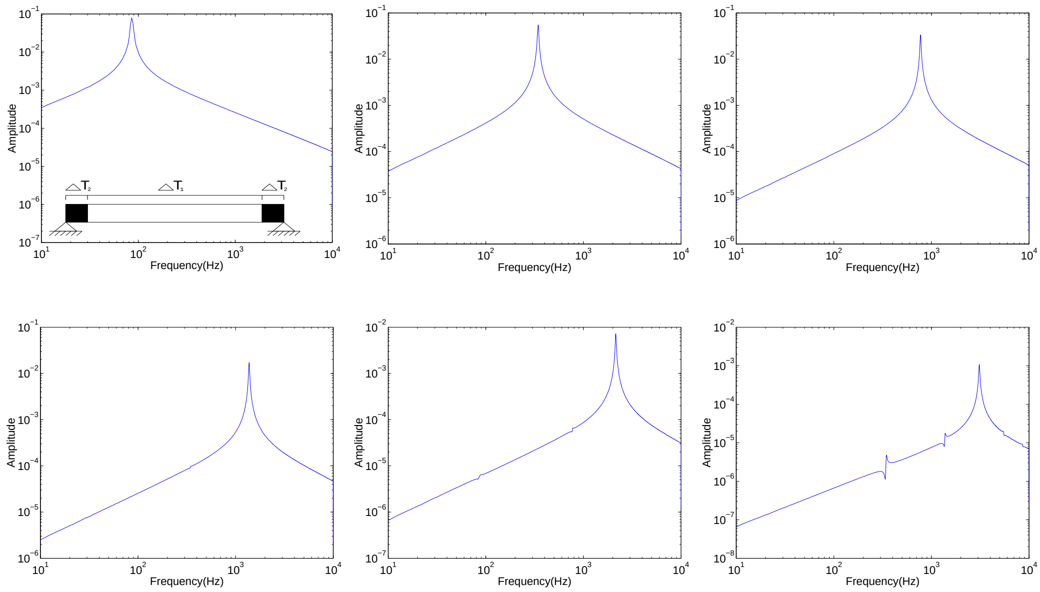


Figure 9: FRF of output of linear combiner tuned to modes 1 to 6, nearly uniform stiffness change on the whole beam : stiffness reduction of 1.5% between $0$ and $.1L$, $.9L$ and $L$, stiffness reduction of 2% between $.1L$ and $.9L$

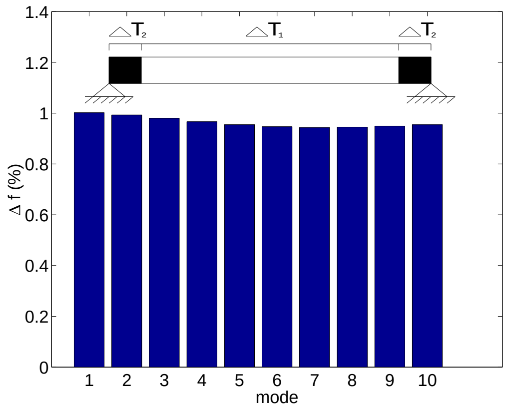


Figure 10: Frequency deviation for a nearly uniform change of the stiffness distribution : stiffness reduction of 1.5% between $0$ and $.1L$, $.9L$ and $L$, stiffness reduction of 2% between $.1L$ and $.9L$