Observer-based friction compensation system for teleoperation

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Abstract—In this paper, an observer-based friction compensation (OBFC) suited for teleoperation is analyzed. Because the observer is based on a model of the dynamics of the system rather than a model of the friction, it is well suited when friction presents time varying properties. This is precisely the case in teleoperated minimally invasive surgery. In this paper, a stability analysis of this observer w.r.t. errors in the model parameters is performed. The impact of indirect speed measurement is also investigated and finally, simulations are performed to illustrate the above mentioned issues.

I. INTRODUCTION

Friction is a nonlinear phenomenon affecting most motion systems. Besides making the use of classical controller design methods delicate, it leads to unwanted behaviors like larger position errors in robot motion control for example. In order to avoid these drawbacks, friction compensation systems have been widely studied in the literature (see [2] for an overview).

When considering the application of teleoperated minimally invasive surgery, the arm of the slave robot enters the body of the patient through a seal called trocar. The trocar opposes friction forces to the motion of the slave arm which are of the same order of magnitude as the interaction forces between the slave and its environment. Consequently, in order to be able to safely use linear methods to design the controller (see [4], [3] and [8]) and to avoid transparency losses when implementing it, friction in the trocar must be compensated. Unfortunately, the properties of the friction of the trocar are time varying. In [7], the friction of six trocars commonly used in laparoscopy was investigated. It was found that friction properties vary dramatically from one type of trocar to the other. Moreover, moistening the shaft with water reduced friction up to 45 %.

Online estimation of the friction in order to compensate it can therefore not be based on a classical fixed friction model. The observer presented in [1] could be used instead. As it is based on a model of the dynamics of the mechanical system rather than on a model of the friction, it can be adapted to estimate friction with time varying properties.

In this paper the behavior of a mechanical device equipped with a friction compensation system based on this observer is analyzed from a stability and performance point of view. The remaining of the paper is structured as follows. In section II, the equations of the friction observer are given. Next, the effect of two types of imperfections of the observer are investigated: errors on the estimated parameters in the dynamical model of the mechanical system and indirect (via software sensor) speed measurements. They are respectively discussed in sections III and IV. In order to validate these results, simulations are presented in section V and finally, section VI provides concluding remarks.

II. OBSERVER BASED FRICTION COMPENSATION (OBFC)

The friction observer implemented in this paper is taken from [1]. It will be presented for a one degree of freedom (dof) device but the results of this section hold for multiple dofs. Let us consider the linear device depicted in figure 1 which corresponds to the slave instrument sliding into a trocar.

Assume that the force applied by the DC motor, \( \tau_{actu} \), is available (by current measurement for example) and that a force sensor is placed at the end effector to measure the interaction force between the tool and the organ, denoted \( \tau_{ext} \). The different components of the friction, namely the friction of the motor, the friction of the linear guide and of course the friction of the trocar are reduced to a single friction component denoted \( \tau_{fric} \) applied to the instrument. The forces are considered to be positive when they result in a positive steady-state velocity of the instrument \( (\dot{x} > 0) \). With the above mentioned forces acting on it, the dynamics of the instrument are described by equation (1) where \( m \) is the inertia, \( b \) is the damping and \( x \) is the position of the instrument.

\[
m \ddot{x} + b \dot{x} = \tau_{actu} + \tau_{ext} + \tau_{fric} \tag{1}
\]

The aim of the observer, whose block diagram is depicted in figure 2, is to estimate the friction torque \( \tau_{fric} \). To do so, a residual \( r \), which turns out to be equal to the filtered friction...
The latter shows that the output of the observer, namely to maximize the bandwidth of the observer while avoiding by a first order low pass filter. The gain $K$ should be adjusted so that

$$r(t^*) = K\{p(t^*) - \int_0^{t^*} (r(t) - b\dot{x}(t) + \tau_{actu}(t) + \tau_{ext}(t))dt + p(0)\}$$

(2)

The relation between $\tau_{fric}$ and the residual is obtained below. First, the derivative of equation (2) with respect to time is computed:

$$\dot{r} = K\{\dot{p} - [r - b\dot{x} + \tau_{actu} + \tau_{ext}]\}$$

(3)

$$= K\{m\ddot{x} + b\dot{x} - \tau_{actu} - \tau_{ext} - r\}$$

(4)

Then, substituting (1) for the first two terms in (4) yields:

$$\dot{r} = K\{\tau_{actu} + \tau_{ext} + \tau_{fric} - \tau_{actu} - \tau_{ext} - r\}$$

(5)

$$= K\{\tau_{fric} - r\}$$

(6)

In the frequency domain, equation (6) yields equation (7). The latter shows that the output of the observer, namely residual $R(s)$, corresponds to the friction torque processed by a first order low pass filter. The gain $K$ should be adjusted to maximize the bandwidth of the observer while avoiding significant distortions due to measurement noise.

$$R(s) = \frac{K}{s + K}\tau_{fric}(s)$$

(7)

The estimate of the friction $r(t)$ is used in this paper to perform feedforward friction compensation as illustrated in figure 3. In the remaining of the paper, we will refer to the system equipped with the OBFC as the compensated system.

III. COMPENSATION OF THE DYNAMICS OF THE ROBOT

So far, exact knowledge of $m$ and $b$ has been assumed. In this section, the impact of a difference between the estimated parameters $\hat{m}$ and $\hat{b}$ and the actual values $m$ and $b$ on the behavior of the compensated system is investigated. First, it will be shown how this difference affects the dynamics of the compensated system. Then, fundamental stability issues will be discussed.

A. Modification of the dynamics of the robot

If the estimated values are used instead of the exact values, the derivative of the residual (see equation (3)) is equal to:

$$\dot{r} = K\{\hat{m}\ddot{x} + \hat{b}\dot{x} - \tau_{actu} - \tau_{ext} - r\}$$

$$= K\{(m\ddot{x} + b\dot{x}) + (\hat{m} - m)\ddot{x} + (\hat{b} - b)\dot{x} - (\tau_{actu} + \tau_{ext}) - r\}$$

(8)

Again, substituting (1) for the first two terms in (8) yields:

$$\dot{r} = K\{(\tau_{fric} + \tau_{actu} + \tau_{ext}) + (\hat{m} - m)\ddot{x} + (\hat{b} - b)\dot{x} - (\tau_{actu} + \tau_{ext}) - r\}$$

(9)

Using equation (9), the residual can be computed in the frequency domain, which yields:

$$R = \frac{K}{s + K}\tau_{fric} + \frac{K}{s + K}\{(\hat{m} - m)s^2 + (\hat{b} - b)s\}$$

(10)

If the compensation of figure 3 is implemented, the dry friction component $\tau_{fric}$ is compensated as before. However the mechanical impedance of the compensated system, i.e. the Laplace transform of the ratio between the applied force and the resulting position is modified due to the additional terms. The mechanical impedance (denoted $Z(s)$) of the compensated system is written as follows:

$$Z(s) = (m + \Delta m)\frac{K}{s + K}s^2 + (b + \Delta b)\frac{K}{s + K}s$$

(11)

where $\Delta m = \hat{m} - m$ and $\Delta b = \hat{b} - b$. Based on the latter equation, the following conclusions can be drawn.

1) At low frequencies with respect to the bandwidth of the observer, i.e. $\omega \ll K$, the dynamics are approximated by:

$$Z_{LF} = (m + \Delta m)s^2 + (b + \Delta b)s$$

$$= \hat{m}s^2 + \hat{b}s$$

(12)

- if $0 < \hat{b} < b$, the damping (i.e. the viscous friction) of the compensated system is reduced w.r.t. the nominal system,
- if $\hat{b} = 0$, the viscous friction is completely compensated,
- if $\hat{b} > b$, the viscous friction is increased w.r.t. the nominal system,
* if $0 < \hat{m} < m$, the inertia of the compensated system is reduced w.r.t. the nominal system,
* if $\hat{m} = 0$, the inertia is completely compensated,
* if $\hat{m} > m$, the inertia is increased w.r.t. the nominal system.

2) At high frequencies with respect to the bandwidth of the observer, i.e. $\omega \gg K$, the dynamics are approximated by

$$Z_{HF} = ms^2 + bs$$

so, no modification of the dynamics takes place.

In this section, it has been shown that a difference between the estimated values of inertia and damping and their true values has no impact on the compensation of dry friction. However, the dynamics of the system are modified. By carefully selecting the parameters $\hat{m}$ and $\hat{b}$, it is possible to compensate the inertia of the system as well as the viscous friction in the frequency band of the observer. However, this compensation has an impact on stability which must be evaluated.

**B. Stability issues**

Stability depends on the choice of control architecture and control parameters. Here, the stability of the compensated instrument interacting with a pure stiffness environment (denoted $S$) is addressed without making assumptions about the control structure. This can be conservative as the tele-operation controller can be designed to stabilize an unstable compensated instrument. Note that this is equivalent to the stability analysis of a haptic interface interacting with a virtual environment of stiffness $S$ without assumptions about the dynamics of the user. The dynamics of the compensated system connected to the spring are given by equation (13).

$$Z^{-1}(s) = \frac{s + K}{ms^3 + (K\hat{m} + b)s^2 + (K\hat{b} + S)s + SK}$$

Using the stability criterion of Routh, the following necessary and sufficient conditions for stability are obtained:

$$K > 0 \quad (14)$$

The latter shows that if the inertia is overestimated ($\Delta m > 0$) and if $\hat{b}$ takes any arbitrary positive value, their is no limit induced by stability on the achievable bandwidth of the observer. Necessary and sufficient conditions for stability are given below for two particular cases of practical interest.

1) Total compensation ($\hat{m} = 0$ and $\hat{b} = 0$): the stability conditions (expressions 14) become:

$$0 < K < \frac{b}{m} \quad (15)$$

The evolution of the poles of the system can be visualized as follows. Before connecting it to the spring, the dynamics of the compensated system are given by

$$Z^{-1}(s) = \frac{s + K}{(ms^2 + bs)s}$$

If the compensated system is connected to a spring of stiffness $S$, the latter will act as a proportional position controller of gain $S$. The evolution of the poles of the system for different values of $S$ can therefore be investigated by analyzing the root locus of the compensated system in closed loop with a proportional position controller of gain $S$. Three cases can be distinguished as depicted in figure 4 below. The arrows on the root loci show the evolution of the poles for increasing values of the stiffness of the environment.

1) $0 < K < \frac{b}{m}$: for low values of the filter bandwidth, the system remains stable for every stiffness even though the system becomes less damped when $S$ is increased (part (a) of figure 4).
2) $K = \frac{b}{m}$: the system is marginally stable. This is the highest gain achievable for the observer if complete compensation is performed (part (b) of figure 4).
3) $K > \frac{b}{m}$: unstable system for any value of the environment stiffness (part (c) of figure 4).

![Fig. 4. Evolution of the poles of the system with respect to parameters $S$ and $K$.](image)

2) Partial compensation of the inertia ($\hat{m} \neq 0$) and total compensation of the viscous friction ($\hat{b} = 0$): in this section, we will see that the highest achievable bandwidth can be extended w.r.t. the previous case if only a partial compensation of the inertia is performed (i.e. $\hat{m} \neq 0$). According to equation (11), the dynamics of the compensated system before being connected to the spring are now:

$$Z^{-1}(s) = \frac{s + K}{(ms^2 + (b + K\hat{m})s)s}$$

The poles and zero of the system are in the configuration of figure 4. As before, there are two poles at the origin and a zero located at $s = -K$. The pole previously located at $s = -b/m$ has now moved to $s = -\frac{b + K\hat{m}}{m}$. The stability condition is now written

$$0 < K < \frac{b}{m - \hat{m}}$$

Viscous friction can be completely compensated (i.e. $\hat{b} = 0$) with no limit on $K$ if the inertia is perfectly known ($\hat{m} =$
m). If inertia compensation is wanted or if an error is made such that \( \hat{m} < m \), the bandwidth of the observer is limited by (18) to guarantee stability if \( \hat{b} = 0 \) is wanted.

IV. IMPACT OF A SPEED OBSERVER

Most often, direct speed measurement is not available and speed must be estimated based on quantized position measurement. Rather than computing speed using finite differences, which leads to a noisy estimate, a filter like a low pass or a Kalman filter is often used instead. In this section, we investigate the impact of the use of such a filter on the achieved compensation. The estimated speed will be denoted \( \hat{v}(t) \) and the filter will be defined by the following relation in the frequency domain:

\[
\hat{V}(s) = F(s)X(s)
\]

Let us consider equation (8) in the frequency domain where \( V(s) = sX(s) \) has been replaced by its estimated value and where the force measurements have been processed by a filter denoted \( A(s) \).

\[
R = \frac{K}{s + K} \left[ (ms + b) + (\hat{m} - m)s \right] + \{\hat{b} - b\}V - A(s)(\tau_{actu} + \tau_{ext})
\]

\[
= \frac{K}{s + K} \left[ \frac{F(s)}{s} \left( (ms^2 + bs) + (\hat{m} - m)s^2 \right) + \{\hat{b} - b\}X - A(s)(\tau_{actu} + \tau_{ext}) \right]
\]

\[
= \frac{K}{s + K} \left[ \frac{F(s)}{s} \left( (s\tau_{actu} + \tau_{ext} + \tau_{fric}) + (\hat{m} - m)s^2X \right) + \{\hat{b} - b\}X - A(s)(\tau_{actu} + \tau_{ext}) \right]
\]

\[
= \frac{K}{s + K} \left[ \frac{F(s)}{s} - A(s) \right] \left( (s\tau_{actu} + \tau_{fric}) + (\hat{m} - m)s^2X + (\hat{b} - b)sX \right)
\]

\[
+ \frac{K}{s + K} \left[ \frac{F(s)}{s} - A(s) \right] (\tau_{actu} + \tau_{ext})
\]

It can be observed that the filter modifies the bandwidth of the observer and adds a component which is a function of the measured torques. The latter component can be responsible for unwanted transients in the estimate of the friction. In order to cancel it, \( A(s) \) should be adjusted according to \( A(s) = \frac{F(s)}{s} \).

V. SIMULATION RESULTS

In this section, the different points addressed previously in this paper will be validated and illustrated through simulations. At first, the parameters of the simulated system are described. Then, the response of the friction observer using indirect speed measurements (see section IV) is computed for different choices of \( A(s) \) and compared to the response when direct speed measurements are available. Next, validation of the stability condition of section III-B is performed for a given choice of dynamical model parameters and finally the behavior of the compensated system for a variation of the properties of the friction is presented. The simulated systems are discretized and sampled at 10 kHz.

A. Simulated system

1) Mechanical part: the mechanical system of figure 1 is considered with the following parameters: \( m = 0.2 \) kg and \( b = 10 \) Ns/m. The organ is modeled by a spring of 500 N/m. The trocar opposes a friction force to the motion of this system which is approximated by a Karnopp model (see [6]) given by equation (21) below where \( \tau_{tot} = \tau_{ext} + \tau_{actu} \), \( \epsilon \) is a constant parameter and \( \tau_C \) is the Coulomb friction parameter. In this paper, \( \tau_C \) will vary with time between an upper bound \( \tau_{C, upper} = 2.2 \) N corresponding to a dry instrument sliding into the trocar and a lower bound \( \tau_{C, lower} = 1.2 \) N corresponding to a moistened instrument. These values are consistent with those that can be found in [7].

\[
\tau_{fric} = \begin{cases} 
-\text{sign}(\tau_{tot}) \min(|\tau_{tot}|, \tau_C) & \text{if } |\dot{x}| \leq \epsilon \\
-\text{sign}(\dot{x})\tau_C & \text{if } |\dot{x}| > \epsilon 
\end{cases}
\]

Parameter \( \epsilon \) defines a region of small velocity \(( -\epsilon < \dot{x} < \epsilon )\) where the speed is considered to be zero (stiction zone). It is adjusted to avoid numerical computation instabilities.

In our case, \( \epsilon = 0.5 \) mm/s was chosen. To avoid nonzero velocities in the stiction zone, the speed was set to zero when the momentum \( p = m\dot{x} \) calculated by \( \dot{p} = \tau_{tot} - r \) was inside the region defined by \(-\frac{p}{m} < p < \frac{p}{m}\).

2) Simulated OBFC: the implementation of the OBFC is based on the assumption that direct force measurements are available. Direct speed measurement however is not available. A digital Kalman filter described in [5] was implemented to estimate the speed based on position measurements. It provides an estimate of the state vector defined by \( x_S(k) = [x(k) \dot{x}(k) \dot{z}(k)]^T \). It is based on the following discrete time model where \( T \) denotes the sampling period, \( y(k) \) represents the measurement and \( v(k) \) and \( w(k) \) stand for the measurement and process noise respectively.

\[
x_S(k+1) = \begin{pmatrix} \frac{1}{T} & \frac{T^2}{2} & 0 \\ 0 & 1 & \frac{T}{2} \\ 0 & 0 & 1 \end{pmatrix} x_S(k) + \begin{pmatrix} 0 \\ 0 \\ w(k) \end{pmatrix}
\]

\[
y(k) = x_S(k) + v(k)
\]

The covariance of the measurement noise is assumed to be equal to \( \frac{p^2}{4} \) where \( d \) stands for the quantization step of the position measurement (equal to 10 \( \mu \)m). The covariance of the process noise, denoted by \( q \) must be adjusted. Increasing \( q \) will increase the bandwidth of the filter but also the impact of the measurement noise. In our case, the choice of \( q \) resulted in a phase lag of \( 10^\circ \) at 50 Hz between the actual value of the speed and its estimate. Note that even if the filter is designed for quantized position measurements, position was not quantized during the simulations.
B. Impact of indirect speed measurement

During this simulation, \( \dot{m} = m, \dot{b} = b, \tau_C = \tau_C,_{upper} \) and the system is not compensated. It is loaded with an external force which is increased from time \( t = 2 \) s at the rate of \( 2 \) N/s. When the force reaches \( 20 \) N, it is released and the responses of the observer are recorded for different configurations. They are plotted in figure 5 for \( K = 100 \) and in figure 6 for \( K = 400 \). Three cases are considered w.r.t. the discussion of section IV: direct speed measurements available and \( A(s) = 1 \) (ideal case, thin line), indirect speed measurements with \( A(s) = 1 \) (thick line) and indirect speed measurements with \( A(s) = \frac{F(s)}{s} \) (dashed line) where \( F(s) \) is the Kalman filter described in section V-A.2. The red step corresponds to the actual friction in the system.

![Fig. 5. Responses of the observer with \( K = 100 \).](image)

![Fig. 6. Responses of the observer with \( K = 400 \).](image)

The use of a Kalman filter to compute an estimate of the speed clearly modifies the response of the observer w.r.t. the ideal case. For low values of \( K \), the impact of \( F(s) \) is low, i.e. the response of the observer is close to the ideal one. This is because the bandwidth of \( \frac{K}{s+K} \) is low with respect to that of \( F(s) \). When \( K \) is increased, the impact of the Kalman filter becomes more significant. If indirect measurements are used with \( A(s) = 1 \), the transient due to \( \tau_{ext} + \tau_{actu} \) (see section IV) is clearly noticeable on both figures and it becomes more important when \( K \) is increased. This transient is avoided by choosing \( A(s) = \frac{F(s)}{s} \) as calculated in section IV.

C. Validation of the stability criterion

In order to validate the stability criterion of section III-B, we now consider the compensated system. The following parameters are chosen: \( \dot{b} = 0, \dot{m} = 0.1 \) and \( \tau_C = \tau_C,_{upper} \). Indirect speed measurements with \( A(s) = \frac{F(s)}{s} \) are also considered even if the discussion of section III-B considered direct measurements. However, as showed above, for this low value of \( K \) and with \( A(s) = \frac{F(s)}{s} \), the impact on the behavior of the observer is low. The system is loaded as in section V-B. According to equation (18), the ultimate gain is equal to 100. The evolution of the position of the system after release of the force for \( K = 100 \) is depicted in figure 7. Although, large oscillations are noticed due to the compensation of both viscous and dry friction in the bandwidth of the OBFC, the system remains stable. The gain \( K \) must be increased up to \( K = 102 \) to notice instabilities (see figure 8). A possible
D. Tracking of changing friction parameters

In this last simulation, the ability of the OBFC to handle changing friction properties is illustrated. The following parameters are used: \( b = h, \); \( \dot{m} = m \) and \( K = 100 \). Parameter \( b \) is reduced to 1 Ns/m. The system is compensated and indirect speed measurement with \( A(s) = \frac{F(s)}{s} \) is used. The system is loaded as in section V-B. At instant \( t = 13s \), parameter \( \tau_c \) switches from \( \tau_{C, upper} \) to \( \tau_{C, lower} \). In figure 9 below, the output of the observer which corresponds to the thick line is compared to the actual friction represented by the thin line. It can be observed that the observer correctly tracks the new friction values.

VI. CONCLUSIONS

This paper addresses the compensation of the friction opposed by the trocar to the motion of the slave robot in robotized minimally invasive surgery. Because this friction vary with the type of trocar and moisture conditions, compensation based on fixed friction models cannot be implemented. The observer of [1] provides an estimate of the friction based on a model of the mechanical device. This estimate corresponds to the actual friction processed by a first order low pass filter of bandwidth \( K \). In this paper, it is used to perform feedforward friction compensation. The impact of an error on the parameters of the mechanical model of the system is analyzed. We show that it is possible, by carefully selecting these parameters to compensate viscous friction and inertia in the bandwidth defined by \( K \) without need for acceleration measurements. Stability boundaries for the gain \( K \) are provided and validated through simulations. The results show that the stability criterion is slightly more conservative than simulations maybe because the residual friction (frequency components of the friction beyond the bandwidth of the OBFC are not compensated) has not been taken into account in our linear stability analysis.

The impact of indirect speed measurements is also addressed. If a filter \( F(s) \) is used to estimate the speed based on position measurement, it is shown that the bandwidth of the observer is modified. Transients, which corresponds to the measured forces processed by \( \frac{K}{s + K} (F(s) - 1) \) appear and increase when \( K \) becomes closer to the bandwidth of \( F(s) \). It is also shown that these transients can be avoided if the measured forces are processed by \( F(s)/s \). These results are illustrated by simulations.

Future work will address the impact of the residual friction on stability. The role of measurement noise should also be investigated as it adds noise to the friction estimate. Reducing noise distortions requires reducing the bandwidth \( K \).

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REFERENCES