CA-CDMA: Channel Adapted CDMA for MAI/ISI-free Burst Transmission

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Abstract—
This paper proposes a precoding technique for the complete elimination of both MAI and ISI in the uplink of a CDMA system based on burst transmission, with reduction of the MMSE joint detector to a simple filter matched to the total impulse responses. If the received power is fixed, this system minimizes the variance of the symbol estimation error. An infinity of solutions exists to orthogonalize the system. Two objectives are added: a possible progression in the number of users and a minimization of the average emitted power. In an ideal progressive system, users can enter or leave the system easily without recomputing the codes of the other users. The minimization of the emitted power is a complex optimization problem. An approximate solution is proposed. It is shown that the system introduced in this paper outperforms burst systems using a conventional set of codes.

Index terms: code division multiple access, code design, multiuser channels.

I. Introduction

Orthogonality is the underlying concept in the design of many communication systems. Namely, there is orthogonality in the frequency domain (no interference between different carriers) and orthogonality in the time domain (no interference between the different symbols transmitted on the same carrier at different time slots). Multiuser communication systems can be described by means of synthesis/analysis operators (transmultiplexers) as it is shown in [3]. In frequency division multiple access (FDMA) based systems, the communication channel is divided into separate frequency subchannels. Hence, the synthesis and analysis filters are nonoverlapping sinusoïds. For time division multiple access (TDMA), the impulse responses of the analysis and synthesis filters are rectangular windows at different locations in time. With direct sequence code division multiple access (DS-CDMA), these filters are related to the codes allocated to the different users.

The code families presented above do not take into account the time dispersion of the current channels. The interesting orthogonality properties of the transmitted signal are often lost at the receiver due to the high multipath component. It is interesting to investigate new code design methods taking into account the channel effects. Hence, the purpose of this paper is to investigate design methods assuming that the channel impulse responses (CIRs) of the active users are known.

Salz has first reported on the problem of joint transmitter and receiver optimization for coupled multi-input multi-output (MIMO) systems in additive noise [4]. A solution is obtained only for the restrictive case of a system bandlimited to the first Nyquist zone ($[-1/2T, 1/2T]$). A significant feature is that the optimization of the cross-coupled system leads to an equivalent decoupled system consisting of parallel subchannels. In their paper [5], Yang and Roy extend the result to MIMO systems with arbitrary bandwidth and unequal numbers of inputs and outputs. However, a close form solution is not given. As shown in [7], the redundancy introduced by the nonmaximally decimated filterbank can overcome the difficulties encountered in the standard MIMO systems for channels with spectral nulls. A close form solution is presented for the joint transmitter-receiver optimization under the average transmission power constraint. A unifying framework based on a multirate filterbank model is introduced in [8] by Scaglione et al. An optimum solution is presented for the design of transmitter-receiver filterbank pairs able to miminize the symbol estimation mean square error (MSE) under a constraint of interference suppression at the output of the receiver. It turns out that this problem is equivalent to the minimization of the symbol estimation MSE under a received power constraint: the optimal solution is to eliminate all the interference in order to reach the matched filter bound (MFB) given by the constraint. The symbol estimation MSE is also minimized under a transmitted power constraint. The two problems give rise to a singular value decomposition (SVD) of the channel matrix.

The optimization of the precoding scheme in case of synchronous multiuser downlink transmission over multipath channels is considered in [9]. The purpose is to find linear transformations applied on the symbols at the transmitter and at the receiver that minimize the symbol estimation MSE under average and peak emitted power constraints. In the following paper [10], a precoding scheme is introduced that eliminates completely the multiple-access interference (MAI) and the inter-symbol interference (ISI). A filter matched to the CIRs is used at each receiver. The precoding matrix is the inverse of the matrix that represents the impulse responses including the matched filters at the receivers. The multiuser detection problem reduces to independent parallel single user detection problems. In the last two schemes [9, 10], the precoding is applied on the data symbols, rather than on the transmitted waveforms, so that the higher resolution inherent to a CDMA system is not exploited. In [11], the transmit waveforms themselves are modified. The receivers are made of simple pre-designed code correlators. Knowing the structure of the receiver, the spreading sequences used at the transmitter are designed in such a way that the interference be totally eliminated by the receivers. In this scheme, the receivers are not composed of filters matched to the total impulse responses.

In [12], low complexity CDMA transceivers are introduced that are capable of eliminating MAI deterministically in the presence of unknown and even rapidly varying multipath, provided that the users asynchronism is limited. The recent so-called Lagrange-Vandermonde CDMA transceivers generalize the orthogonal frequency division multiple access (OFDMA) technique.

In this paper, the design of spreading sequences which orthogonalize the channels of all users under a received power constraint, is considered for the uplink of a system based on burst transmission. We assume that the CIRs are known at the transmitter. The issue of channel estimation is crucial but beyond the scope of the present paper. The effects of MAI and ISI due to multipath fading are mitigated by the use of a simple filter matched to the total impulse responses at the receiver. The feasibility is first demonstrated based on the subspace theory. Since there is an infinite number of ways to orthogonalize the user channels, two specific strategies for the sequence design are considered. In Section IV, an algorithm which is progressive in the number of users is presented and, in Section V, the transmit power is approximately minimized.

II. System model

The lowpass equivalent uplink of a multiuser system is considered. The base station receives the contributions of K different users, modified by the user specific channels and the additive noise. A detailed model of the transmission system is given in Figure 1. Each active user k transmits a sequence of symbols $d_k(n)$ at the rate $1/T$. The symbols are spread by the user specific codes $s_k(n)$ and shaped by means of the chip shaping filter with impulse response $u_a(t)$ (it is assumed to be a half-root Nyquist). T_c is the chip duration and N_c is the spreading factor. Each user signal is transmitted over a user specific channel with lowpass equivalent impulse response $c_{a,k}(t)$. At the receiver, $p_a(t)$ represents an ideal lowpass presampling filter. $w_a(t)$ is the additive noise and $v_a(t)$ is the noise filtered by $p_a(t)$. The received signal is denoted by $r_a(t)$. We define the impulse responses $h_{a,k}(t) \stackrel{def}{=} u_a(t) \otimes c_{a,k}(t) \otimes p_a(t)$. The operator $\otimes$ denotes a convolution. A sampling of the received and noise signals at the rate $1/T_s = M/T_c = MN_c/T$ gives the received sequence $r(n) = r_a(t = nT_s)$ and the noise sequence $v(n) = v_a(t = nT_s)$. To get a discrete time model, we define the discrete time impulse responses $h_k(n) = h_{a,k}(t = nT_s)$. In our computations, we further

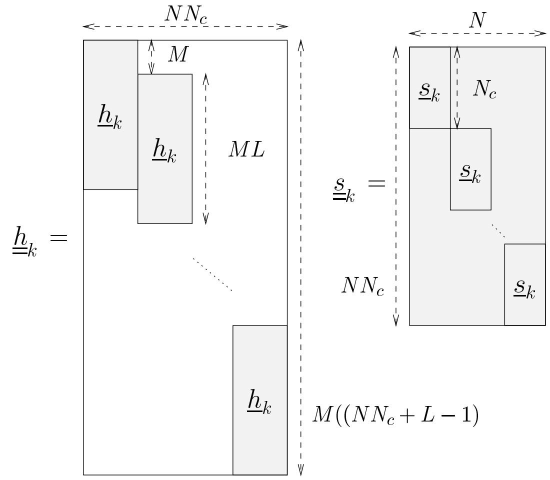

Fig. 2. User k specific matrices

assume that the power control is such that the received energy be equal for all user symbols.

In case of burst transmission, each user is assumed to send a packet of N symbols. In our model, we want to separate the spreading codes from the other elements of the composite channels. A matrix $\underline{\underline{s}}_b$ represents the transmitter precoding operation and a matrix $\underline{\underline{h}}_b$ represents the impulse responses of the different users. The observation model becomes [1,2]

$$\underline{r}_b = \underline{\underline{g}}_b \, \underline{d}_b + \underline{v}_b \tag{1}$$

where $\underline{\underline{g}}_b = \underline{\underline{h}}_b \underline{\underline{s}}_b$. The following definitions have been introduced:

$$\underline{d}_b \stackrel{def}{=} \left[\begin{array}{ccc} \underline{d}_1^T & \cdots & \underline{d}_K^T \end{array} \right]^T \tag{2}$$

$$\underline{\underline{s}}_b \stackrel{def}{=} \left[\begin{array}{ccc} \underline{s}_1 & \cdots & \underline{\underline{0}}_{NN_c \times N} \\ \vdots & \ddots & \vdots \\ \underline{\underline{0}}_{NN_c \times N} & \cdots & \underline{s}_K \end{array} \right] \tag{3}$$

where $\underline{d}_k \stackrel{def}{=} \left[\begin{array}{ccc} d_k(0) & \cdots & d_k(N-1) \end{array} \right]^T$. Furthermore, we have

$$\underline{\underline{h}}_b \stackrel{def}{=} \left[\begin{array}{cccc} \underline{\underline{h}}_1 & \underline{\underline{h}}_2 & \cdots & \underline{\underline{h}}_K \end{array} \right] \tag{4}$$

$$\underline{r}_b \stackrel{def}{=} \left[\begin{array}{ccc} r(0) & \cdots & r(M(NN_c+L-1)-1) \end{array} \right]^T \tag{5}$$

$$\underline{v}_b \stackrel{def}{=} \left[\begin{array}{ccc} v(0) & \cdots & v(M(NN_c+L-1)-1) \end{array} \right]^T \tag{6}$$

L is the size of the impulse responses $h_{a,k}(t)$ evaluated in numbers of T_c.
A description of the matrix $\underline{\underline{h}}_k$ of size $M(NN_c + L - 1) \times NN_c$ is given in the left part of Figure 2 in which the vector $\underline{h}_k$ is the impulse response corresponding to channel k. We have $\underline{h}_k \stackrel{def}{=} \left[\begin{array}{ccc} h_k(0) & \cdots & h_k(ML-1) \end{array} \right]^T$.
A description of the precoding matrix $\underline{\underline{s}}_k$ of size $NN_c \times N$ is given in the right part of Figure 2 in which the vector $\underline{s}_k$ is the code allocated to user k in a conventional precoding scheme. We have $\underline{s}_k \stackrel{def}{=} \left[\begin{array}{ccc} s_k(0) & \cdots & s_k(N_c-1) \end{array} \right]^T$.

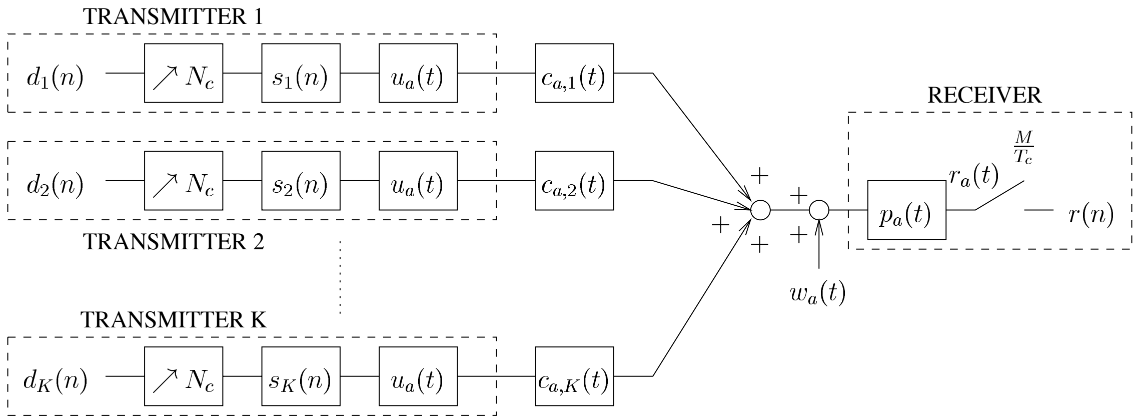

Fig. 1. Uplink transmission model

After multiplication of the symbol vector $\underline{d}_b$ with the matrix $\underline{\underline{s}}_b$, we get a sequence at chip rate corresponding to the symbols spread by the codes. If the length of the codes does not exceed the symbol period, each symbol is spread by a particular code with no a priori interference with the symbols around. However such a constraint can be relaxed in practice: the length of the codes can be larger than a symbol period, different codes can be assigned to the successive symbols of a user, and finally, the codes can be constituted with complex elements. *As a key element to our precoding scheme, we assume that the matrices $\underline{\underline{s}}_k$ shown in Figure 2 can be totally filled in with complex elements.* This actually amounts to having codes overlapping with each other and being different from one symbol to another (even for the same user). The choice of this precoding matrix which is an extension of the classical spreading, is a key element for rendering the system MAI/ISI free.

III. Optimum system under a received power constraint

The joint transmitter and receiver optimization is performed for the minimum mean square error (MMSE) criterion. At the output of the MMSE linear joint detector [2]

$$\underline{\underline{f}}_b = \left[\frac{1}{\sigma_d^2}\underline{\underline{I}}_{KN} + \frac{1}{\sigma_v^2}\underline{\underline{g}}_b^H\underline{\underline{g}}_b\right]^{-1}\left(\frac{1}{\sigma_v^2}\underline{\underline{g}}_b^H\right) \tag{7}$$

applied on the received vector (1), the symbol estimation error auto-correlation matrix is equal to

$$\underline{\underline{R}}_{b,\varepsilon\varepsilon} = \left[\frac{1}{\sigma_d^2}\underline{\underline{I}}_{KN} + \frac{1}{\sigma_v^2}\underline{\underline{g}}_b^H\underline{\underline{g}}_b\right]^{-1} \tag{8}$$

where ε denotes the symbol estimation error vector and $\underline{\underline{I}}_{KN}$ is the identity matrix of size KN. Cross-correlation between two vectors in a burst transmission scheme is defined as $\underline{\underline{R}}_{b,xy} = E\left[\underline{x}_b\,\underline{y}_b^H\right]$ where $E[]$ denotes the expectation operator. Independent symbols of variance equal to σ_d^2 and additive white Gaussian noise (AWGN) of one-sided power spectral density equal to N_0 are assumed. σ_v^2 denotes the noise variance at the output of the presampling filter. The error variance of any symbol estimate can be found at the appropriate location on the diagonal of the matrix $\underline{\underline{R}}_{b,\varepsilon\varepsilon}$.

The optimization of the transmitters is performed under a constraint of constant received symbol energy E_s. In this case, the MFB is equal to the constant $\gamma \stackrel{def}{=} E_s/N_0$ for each user. As the symbols are independent, we minimize the sum of the symbol estimation error variances. The problem becomes

$$\min_{\underline{\underline{s}}_b} \quad \mathrm{tr}\left[\underline{\underline{R}}_{b,\varepsilon\varepsilon}\right] \tag{9}$$

subject to

$$\frac{\sigma_d^2}{\sigma_v^2}\left(\underline{\underline{g}}_b^H\,\underline{\underline{g}}_b\right)_{ii} = \gamma \tag{10}$$

for $i = 1, \cdots, KN$. The operator $\mathrm{tr}\left[\underline{\underline{x}}\right]$ denotes the trace of the matrix $\underline{\underline{x}}$. An interesting result due to Witsenhausen [13], restated in [7] and introduced here as a proposition, allows the optimization.

Proposition *Let $\underline{\underline{q}}$ be a square non-negative definite hermitian matrix and $\underline{\underline{q}}_d$ be the diagonal matrix obtained from $\underline{\underline{q}}$ by setting all off-diagonal elements to zero. It can be proven, for a square non negative diagonal matrix $\underline{\underline{\Lambda}}$, that*

$$tr\left[\left(\underline{\underline{I}} + \underline{\underline{q}}_d\underline{\underline{\Lambda}}\right)^{-1}\right] \quad \leq \quad tr\left[\left(\underline{\underline{I}} + \underline{\underline{q}}\,\underline{\underline{\Lambda}}\right)^{-1}\right]. \tag{11}$$

The diagonal of the matrix $\underline{\underline{g}}_b^H\,\underline{\underline{g}}_b$ is constant due to the received symbol energy constraint. As a result of the proposition, a diagonalization of the matrix $\underline{\underline{g}}_b^H\,\underline{\underline{g}}_b$ results in the minimization of the trace of the error auto-correlation matrix $\underline{\underline{R}}_{b,\varepsilon\varepsilon}$. Hence, the precoding matrix $\underline{\underline{s}}_b$ is ideally chosen such that $\underline{\underline{s}}_b^H\,\underline{\underline{h}}_b^H\,\underline{\underline{h}}_b\,\underline{\underline{s}}_b$ be diagonal with the elements equal to $\gamma\,(\sigma_v^2/\sigma_d^2)$.

In this case, the conventional MMSE linear joint detector (7) reduces to the whitening matched filter multiplied by a factor $\sigma_d^2/(1+\gamma)$. Thanks to this channel adapted precoding, the interference is eliminated by the use of a filter matched to the total impulse responses. Consequently, the MFB is reached for each user.

From a matrix analysis point of view, we have to find a matrix $\underline{\underline{s}}_b$ such that the columns of the matrix $\underline{\underline{g}}_b$ be orthogonal. The orthogonality feasibility is proven with the aid of the vectorial subspace theory. Each part $\underline{\underline{h}}_k$ of the matrix $\underline{\underline{h}}_b$ corresponding to the specific user k constitutes a vectorial subspace of dimension NN_c in the space of dimension $M\,(NN_c + L - 1)$. Since the columns of $\underline{\underline{h}}_k$ are delayed versions of the same vector $\underline{h}_k$, they are linearly independent and form a base of the subspace associated with user k. The product $\underline{\underline{h}}_b\,\underline{\underline{s}}_b$ is a juxtaposition of the respective products $\underline{\underline{h}}_k\,\underline{\underline{s}}_k$. These products are equivalent to choosing N vectors in the subspace associated with each user k. They are linear combinations (achieved by the coefficients contained in the N columns of the matrices $\underline{\underline{s}}_k$) of the base vectors of a particular subspace. It is shown hereafter that all those vectors can be chosen orthogonal.

- N orthogonal vectors are chosen in the first subspace $\underline{\underline{h}}_1$.
- The N vectors assigned to user 1 are projected onto the subspace $\underline{\underline{h}}_2$ of user 2. A subspace of dimension $N(N_c-1)$ orthogonal to the N projections and included in subspace $\underline{\underline{h}}_2$ exists. It is possible to choose N orthogonal vectors in this subspace. Of course those vectors orthogonal to the projections are also orthogonal to the initial N vectors. They are assigned to user 2.
- The same operations can be performed with the subspace $\underline{\underline{h}}_3$ of user 3. We look for the subspace of dimension $N(N_c-2)$ orthogonal to the projections of the N vectors assigned to user 1 and to the projections of the N vectors assigned to user 2. N orthogonal vectors are chosen in this subspace and assigned to user 3.
- This procedure is followed up to user K.

Two remarks are important:

- This iterative procedure is only possible if the spreading factor N_c is at least equal to the number of users K. The size of the initial subspaces is NN_c. At each step, we have to find a subspace orthogonal to the projections of the N vectors assigned to the previous users. For the last user, we need a subspace of size N included in the subspace K of size NN_c and orthogonal to $N(K-1)$ projections. Thus N_c must be at least equal to K.
- At each step, an infinite number of solutions exists.

The last remark requires a criterion for optimization. The maximization of the signal-to-noise-ratio (or the minimization of the MSE) cannot be considered since it is a constant equal to the MFB. In the following sections, some objectives are introduced:

- The precoding matrices can be chosen in order to get some progression in the number of users. A flexible system should be such that any user can enter or leave the system without the need to recompute the codes of the other users.
- The average emitted power depends on the precoding matrices. It should be as low as possible.

We define the channel auto and cross-correlation matrices $\underline{\underline{m}}_{k_1k_2} = \underline{\underline{h}}_{k_1}^H\,\underline{\underline{h}}_{k_2}$ for $k_1, k_2 = 1, \cdots, K$ in order to simplify the notations in the following sections.

IV. A system progressive in the number of users

The number of users that can be accommodated in the system is limited by the spreading factor N_c as explained in Section III (at least in the context of this paper where the target is a perfect orthogonalization of the system). Since the bandwidth is constant, N_c must be as low as possible to increase the bit rates. There is thus a serious advantage in keeping N_c equal to the number of users. It will be the case in the procedure described below in which the precoding matrices $\underline{\underline{s}}_k$ are adapted when the number of users varies. The purpose of the method explained in this section is to get some progression in the number of users. Ideally users can be added and removed easily without recomputing the codes of the other users.

In the initialization period, the base station should have an estimation of the durations the users stay active in the system (for instance the users are expected to declare the time they will be active in the system). A user active for a long period will receive a high priority: his symbol codes will be computed first. The code allocation is pursued until the user with the lowest priority (short presence in the system) be processed. After the initialization period, any user can enter (N_c is increased by one) or leave (N_c is decreased by one) the system. The current system does not have to be completely recomputed.

In the proposed procedure, only a limited number of vectors in the initial vectorial subspaces $\underline{\underline{h}}_k$ is used.

- The codes are first computed for user k_1 with the highest priority. An orthogonal base of dimension N is selected in the subspace generated by the first N columns of the matrix $\underline{\underline{h}}_{k_1}$. The last $N(N_c-1)$ column vectors of $\underline{\underline{h}}_{k_1}$ are not used. This means that the $N(N_c-1)$ last elements of all columns of the matrix $\underline{\underline{s}}_{k_1}$ are equal to 0.
- The codes are computed for user k_2 which is the second user with the highest priority. An orthogonal base of dimension N is selected in the subspace of dimension N orthogonal to the N vectors allocated to user k_1 and included in the subspace generated by the first $2N$ columns of the matrix $\underline{\underline{h}}_{k_2}$. The other $N(N_c-2)$ column vectors of $\underline{\underline{h}}_{k_2}$ are not used. This means that the $N(N_c-2)$ last elements of all columns of the matrix $\underline{\underline{s}}_{k_2}$ are equal to 0.
- The procedure is followed up to the last user k_K.

The Gram-Schmidt orthogonalization procedure is used at each step to get an orthogonal base of a particular subspace. Some vectors in the initial subspaces are not used. The associated code elements are equal to "0". This makes possible the progression in the number of users. The process allows the addition of an orthogonal user in the system without having to recompute the codes of the current users. The codes of the already active users are simply extended with zeros. As the bandwidth is constant, this means of course a reduced symbol rate for these users. Only the codes of the new user have to be explicitly computed. When a particular user k is removed, the spreading factor can be decreased by one in order to increase the bit rates of the other users. The last N rows of the code matrices corresponding to the users who have entered the system before user k are made of zeros. They are removed without loss of orthogonality. Only the codes corresponding to the users who have entered the system after user k have to be recomputed. The system introduces some kind of TDMA component since most of the code elements are equal to "0" (this will be confirmed by the numerical results). It should be mentioned that if burst error times are in the order of TDMA slots, interleaving will not be able to distribute errors wide enough. Then coding will not be able to correct the burst errors. The number of operations required to compute the entire system is $O(K^4N^3)$ (as compared to the complexity required to compute the MMSE joint detectors which is $O(K^3N^3)$). If a new user is added in the system, the number of operations required to compute his codes is only $O(K^3N^3)$.

V. Emitted power minimization

We would like to investigate the issue of the transmission power minimization for a system where the MFB is reached. This is a complex problem of non-linear optimization. A lower bound on the emitted power at the output of each transmitter is first computed and an approximate solution is provided next.

A. A lower bound

In this section, we are only interested in the computation of a lower bound (LB) on the emitted power. The detection of the data symbols is not considered. It is thus not necessary to take ISI and MAI into account and the minimum emitted power is obtained by assuming that all symbols of each user k are spread by the same code $\underline{s}_k$ that minimizes the emitted power under a constraint of constant received symbol energy. The precoding matrix $\underline{\underline{s}}_k$ is the repetition N times of the column vector $\underline{s}_k$ of dimension NN_c. The different data symbols can not be distinguished after spreading. The optimal code vector $\underline{s}_k$ is the solution of

$$\min_{\underline{s}_k} \quad \mathrm{tr}\left[\underline{s}_k\,\underline{s}_k^H\right] \tag{12}$$

subject to

$$\frac{\sigma_d^2}{\sigma_v^2}\left(\underline{s}_k^H\,\underline{\underline{m}}_{kk}\,\underline{s}_k\right) = \gamma\,. \tag{13}$$

This is a classical problem of constrained optimization. A modified code vector $\underline{\phi}_k$ is defined such that $\underline{s}_k \stackrel{def}{=} \underline{\underline{v}}_k\,\underline{\phi}_k$ where $\underline{\underline{v}}_k$ is the matrix of eigen-vectors of $\underline{\underline{m}}_{kk}$. The solution requires that only one element of the vector $\underline{\phi}_k$ be different from zero. The element associated with the largest eigen-value of the matrix $\underline{\underline{m}}_{kk}$ is chosen in order to minimize the emitted power.

In the special case of flat fading channels, the columns of each matrix $\underline{\underline{h}}_k$ are already orthogonal. Any vector $\underline{s}_k$ which satisfies the received power constraint can be chosen for each symbol. It is possible to achieve a system completely orthogonal where each symbol is sent at a minimum emitted power, for instance by using the classical TDMA, synchronous CDMA or digital multi-tone (DMT) codes.

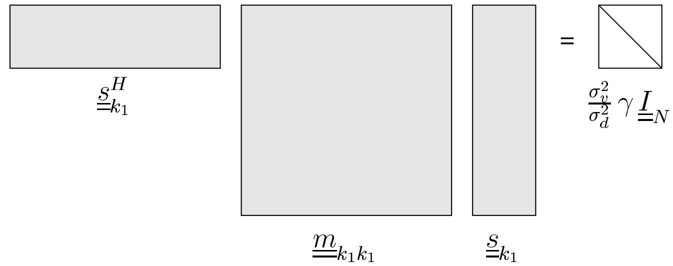

Fig. 3. Constraint of orthogonality for user 1

B. The problem and an attractive approximate solution

This section studies the minimization of the emitted power of each transmitter for a system where the MFB is reached. The total interference suppression by the use of a matched filter is assumed. The problem is formulated as

$$\min_{\underline{\underline{s}}_k} \quad \sum_{k=1}^{K} \mathrm{tr}\left[\underline{\underline{s}}_k\,\underline{\underline{s}}_k^H\right] \tag{14}$$

subject to

$$\frac{\sigma_d^2}{\sigma_v^2}\left(\underline{\underline{s}}_k^H\,\underline{\underline{m}}_{kl}\,\underline{\underline{s}}_l\right) = \gamma\,\delta_{kl}\,\underline{\underline{I}}_N \tag{15}$$

where $k,l = 1\cdots K$. δ_{kl} denotes the Kronecker symbol. It is a complex problem of non linear optimization. An easily realizable solution close to the optimal system is achieved.

A LB on the emitted power is available for each user. It allows a classification of the users according to the attenuation of their channels. The codes are first computed for the user requiring the largest emitted power in order to decrease his power as much as possible. The algorithm is pursued until the user requiring the lowest emitted power. Orthogonality is kept at each step.

B.1 User k_1

The codes are first computed for user k_1. The problem is to choose an orthogonal base of dimension N in a subspace of dimension NN_c that minimizes the emitted power of the user. The problem is thus formulated as (see Figure 3)

$$\min_{\underline{\underline{s}}_{k_1}} \quad \mathrm{tr}\left[\underline{\underline{s}}_{k_1}\underline{\underline{s}}_{k_1}^H\right] \tag{16}$$

subject to

$$\frac{\sigma_d^2}{\sigma_v^2}\left(\underline{\underline{s}}_{k_1}^H\,\underline{\underline{m}}_{k_1k_1}\,\underline{\underline{s}}_{k_1}\right) = \gamma\,\underline{\underline{I}}_N\,. \tag{17}$$

To get a simple solution, each code assigned to a particular symbol will be computed regardless of the following symbols. The first symbol receives the LB code. The eigen-value decomposition of the matrix $\underline{\underline{m}}_{k_1k_1}$ is computed. The N codes allocated to user k_1 are the eigen-vectors associated with the N largest eigen-values. We have $\underline{\underline{s}}_{k_1} = \left(\frac{\sigma_v^2}{\sigma_d^2}\gamma\right)^{1/2} \underline{\underline{\tilde{v}}}_{k_1}\,\underline{\underline{\tilde{\Lambda}}}_{k_1}^{-1/2}$ where $\underline{\underline{\tilde{\Lambda}}}_{k_1}$ denotes a diagonal matrix of dimension N with the largest eigen-values

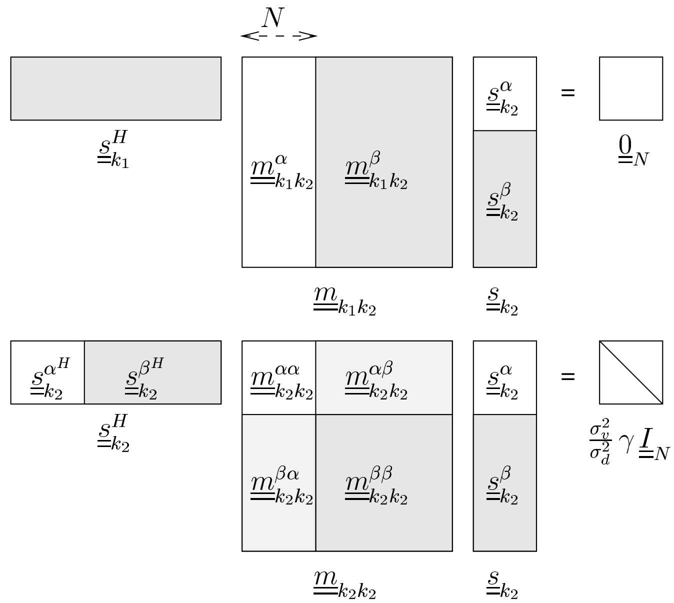

Fig. 4. Constraints of orthogonality for user 2

in the decreasing order on the diagonal and $\tilde{\underline{\underline{v}}}_{k_1}$ denotes the associated eigen-vectors in columns.

B.2 User k_2

The second user is denoted by k_2. The solution must be orthogonal with the first user k_1. The N vectors allocated to user k_1 are projected onto the vectorial subspace of user k_2. The problem is to choose an orthogonal base of dimension N in a subspace of dimension $N(N_c-1)$ (included in the vectorial subspace of user k_2 and orthogonal to the projections of the vectors of user k_1) that minimizes the emitted power of user k_2. The problem is written

$$\min_{\underline{\underline{s}}_{k_2}} \quad \mathrm{tr}\left[\underline{\underline{s}}_{k_2}\underline{\underline{s}}_{k_2}^H\right] \tag{18}$$

subject to

$$\left\{\begin{array}{l} \underline{\underline{s}}_{k_1}^H\,\underline{\underline{m}}_{k_1k_2}\,\underline{\underline{s}}_{k_2} = \underline{\underline{0}}_N \\ \frac{\sigma_d^2}{\sigma_v^2}\left(\underline{\underline{s}}_{k_2}^H\,\underline{\underline{m}}_{k_2k_2}\,\underline{\underline{s}}_{k_2}\right) = \gamma\,\underline{\underline{I}}_N\,. \end{array}\right. \tag{19}$$

To get a favorable solution, two different steps are followed.
- The solution must be orthogonal with the first user. The first constraint becomes (see the definitions introduced in the upper part of Figure 4) $\underline{\underline{s}}_{k_2}^{\alpha} = -\underline{\underline{t}}_{k_2}\,\underline{\underline{s}}_{k_2}^{\beta}$ where $\underline{\underline{t}}_{k_2} \stackrel{def}{=} \left(\underline{\underline{s}}_{k_1}^H\,\underline{\underline{m}}_{k_1k_2}^{\alpha}\right)^{-1}\underline{\underline{s}}_{k_1}^H\,\underline{\underline{m}}_{k_1k_2}^{\beta}$ is a matrix of dimension $N\times N(N_c-1)$.
- The problem is rewritten taking the last result into account. It becomes (see the definitions introduced in the lower part of Figure 4)

$$\min_{\underline{\underline{s}}_{k_2}^{\beta}} \quad \mathrm{tr}\left[(\underline{\underline{s}}_{k_2}^{\beta})^H\left(\underline{\underline{I}}_{N(N_c-1)} + \underline{\underline{t}}_{k_2}^H\,\underline{\underline{t}}_{k_2}\right)\underline{\underline{s}}_{k_2}^{\beta}\right] \tag{20}$$

subject to

$$\frac{\sigma_d^2}{\sigma_v^2}\left((\underline{\underline{s}}_{k_2}^{\beta})^H\,\tilde{\underline{\underline{m}}}_{k_2k_2}^{\beta\beta}\,\underline{\underline{s}}_{k_2}^{\beta}\right) = \gamma\,\underline{\underline{I}}_N \tag{21}$$

where $\tilde{\underline{\underline{m}}}_{k_2k_2}^{\beta\beta} \stackrel{def}{=} \underline{\underline{t}}_{k_2}^H\,\underline{\underline{m}}_{k_2k_2}^{\alpha\alpha}\,\underline{\underline{t}}_{k_2} + \underline{\underline{t}}_{k_2}^H\,\underline{\underline{m}}_{k_2k_2}^{\alpha\beta} + \underline{\underline{m}}_{k_2k_2}^{\beta\alpha}\,\underline{\underline{t}}_{k_2} + \underline{\underline{m}}_{k_2k_2}^{\beta\beta}$. The eigen-value decomposition of the matrix $\underline{\underline{I}}_{N(N_c-1)} + \underline{\underline{t}}_{k_2}^H\,\underline{\underline{t}}_{k_2}$ is computed and a modified precoding matrix is defined in order to get the same formulation of the problem that has been approximately solved for user k_1.
The same kind of decomposition is extended until the last user k_K.

The solution achieved with this procedure is not the optimum solution of the initial problem. Each code allocated to a particular symbol of a user is computed in order to minimize the emitted power while avoiding interference with the previous symbols and users. The next symbols and users are not taken into account to compute the current symbol. This explains the degradation with respect to the optimum solution. It will be seen later that this system is however quite effective. The number of operations required to compute the entire system is also $O(K^4N^3)$.

VI. Results

The main purpose of this section is to compare the precoding schemes providing orthogonality for all users at the receiver with a classical set of codes. Different criteria of performance are studied: the average emitted power, the instantaneous received power, the received power spectral density, the achievable signal-to-interference-and-noise ratios (SINRs) and the corresponding bit rates. The average emitted power of each user is compared to the LB which is the solution of (12). The statistical property of stationarity, required for the computation of the power spectral density, is lost in a burst system. Nevertheless, it can be recovered if we suppose that an infinite number of bursts are transmitted. A guard time is introduced between two successive bursts in order to avoid inter-burst interference at the receiver. The user bit rates are computed with the assumption that the symbol error probability is equal to 10^{-7}. Firstly, Hadamard codes are used to spread the symbols of the different users. Then, a precoded system which is progressive in the number of users is studied. Finally, the performance of the precoded system that approximately minimizes the emitted power is studied.

A. A small size system

A system composed of 2 users communicating with the base station has first been assumed. For each user, the emitted burst contains 10 symbols. The spreading factor is equal to the number of users. The bandwidth is equal to 10 [MHz]. The chip shaping filter (halfroot Nyquist) has a rolloff factor equal to 0.2 so that the chip duration is equal to 0.12 [μs]. Noise is added which is white and Gaussian. The ratio E_s/N_0 is fixed at 20 [dB] for each symbol. The single sided noise power spectral density N_0 is equal to -100 [dBm/Hz]. The user specific channels are characterized by multipath propagation. The taps are complex random variables. Four-paths lowpass equivalent complex CIRs are assumed. They are characterized by the following parameters:

- attenuation [dB]:

user 1	user 2
40	45

- relative amplitudes:

	user 1	user 2
path 1	0.5832	0.4303
path 2	0.5249	0.5164
path 3	0.4666	0.6025
path 4	0.4082	0.4303

- delays [μs]:

	user 1	user 2
path 1	0.00	0.00
path 2	0.15	0.10
path 3	0.25	0.20
path 4	0.35	0.25

- phases:

	user 1	user 2
path 1	0	0
path 2	$14\pi/8$	$2\pi/8$
path 3	$4\pi/8$	$10\pi/8$
path 4	$2\pi/8$	$14\pi/8$

A.1 Hadamard codes

	user 1	user 2
Emitted power LB [mW]	130.79	320.09
Emitted power [mW]	267.97	633.00
Matched filter [dB]	2.65	2.80
MMSE linear [dB]	14.61	16.91
MMSE linear [Mbits/s]	5.00	5.55
MMSE DF [dB]	17.52	18.86
MMSE DF [Mbits/s]	5.83	8.33

TABLE I
EMITTED POWER, SINRS AND BIT RATES; HADAMARD CODES, $K = 2$, $N = 10$.

In case of Hadamard codes, the interference (ISI and MAI) can not be completely eliminated. The results are given in Table I. The SINRs at the output of the linear and decision feedback (DF) MMSE joint detectors are smaller than the MFB equal to 20 dB. The matched filter performs poorly. The DF joint detector outperforms the linear one but requires an increased working complexity and suffers from error propagation. The average emitted power of each user is larger than the emitted power LB.

A.2 Progression in the users

In case of a system progressive in the users, the codes allocated to user 1 are computed before the codes allocated to user 2. The results are given in Table II. This system requires a large emitted power (as compared to the LB). The MFB is reached for the symbols of each user since there is no interference left at the output of the matched filter. Figure 5 shows the instantaneous received power.

	user 1	user 2
Emitted power LB [mW]	130.79	320.09
Emitted power [mW]	901.20	1287.20
Matched filter [dB]	20.00	20.00
Matched filter [Mbits/s]	8.33	8.33

TABLE II
EMITTED POWER, SINRS AND BIT RATES; PROGRESSIVE SYSTEM, $K = 2$, $N = 10$.

The contribution of each user is illustrated separately. It is shown that the signal energy of user 1 is received before the signal energy of user 2. The users are separated in the time domain. So it appears that the solution is close to a TDMA operating mode.

A.3 Emitted power close to optimum system

	user 1	user 2
Emitted power LB [mW]	130.79	320.09
Emitted power [mW]	358.57	554.97
Matched filter [dB]	20.00	20.00
Matched filter [Mbits/s]	8.33	8.33

TABLE III
EMITTED POWER, SINRS AND BIT RATES; CLOSE TO OPTIMUM SYSTEM, $K = 2$, $N = 10$.

Since the emitted power LB of user 2 is larger (user 2 suffers a larger attenuation than user 1), the codes allocated to user 2 are computed before the codes allocated to user 1. The results are given in Table III. In this case, the emitted power of user 2 is relatively close to the LB. However, user 1 needs an increased emitted power. The difference between the emitted power and the LB of each user is increasing according to the order of user code computation. Figure 6 shows that the received signals of the different users are located in different frequency bands. The users are separated in the frequency domain. The algorithm allocates the frequency bands favorable to user 2 and the remaining frequency bands to user 1.

B. A larger size system

The performance of a 16 user system has also been investigated. Each user sends a burst of 10 symbols. The spreading factor is equal to the number of users. Four-path lowpass equivalent complex CIRs are assumed. The conclusions are the same as for the case of a 2 user system. Table IV compares the total emitted power between the small size system and the large size one. It is shown that the systems providing orthogonality at the receiver require a total emitted power closer to the LB for a larger size system (especially for the close to optimum system). The system using a set of Hadamard codes needs an increased emitted power. Table V illustrates the total bit rates for the small

	2 users	16 users
MMSE (linear and DF)	1.99	2.67
Progressive system	4.85	3.78
Close to optimum system	2.02	1.24

TABLE IV

Ratio between the total emitted power and the associated LB for a 2 user system and a 16 user system.

size system and the large size one. When the number of users becomes larger, the symbol duration is increased (the bandwidth is constant and the spreading factor is equal to the number of users). The inter-burst separation becomes negligible as compared to the burst duration so that the bit rates are improved. Figure 7 illustrates the instantaneous

	2 users	16 users
MMSE linear [Mbits/s]	10.55	12.07
MMSE DF [Mbits/s]	14.16	17.89
Orthogonal systems [Mbits/s]	16.66	22.72

TABLE V

Total bit rate for a 2 user system and for a 16 user system.

received power contributions of users 3 and 11 for a 16 user system which is progressive in the users. The users are separated in the time domain. Figure 8 illustrates the received power spectral density contributions of users 3 and 11 for an emitted power close to optimum system. The users are separated in the frequency domain.

VII. Conclusion

In this paper, a joint transmitter and detector optimization has been investigated for a multiple access system. A burst transmission system with constant average received power has been assumed. The proposed solution leads to a total orthogonalization of the system at the receiver: a simple matched filter eliminates completely the interference. In this case, the MFB is reached. An infinite number of solutions exists to make the system orthogonal. Two objectives have been added: a possible progression in the number of users and a minimization of the average emitted power. The solution of the last objective is a complex problem of non linear optimization. An approximate solution has been introduced based on the eigen-value decomposition of the channels auto and cross-correlation matrices.

It is shown that the system introduced in this paper outperforms the burst system using a conventional set of codes. Nevertheless, an initial information about the impulse responses is needed at the transmitters. The base station should compute the precoding matrix of the whole system and send the particular precoding signatures to each user. The channels are assumed to be constant within the burst and also slowly variant from burst to burst. The asynchronism between the users has to be reduced in order to decrease the time separation between two following bursts.

References

[1] A. Klein and P.W. Baier, "Linear Unbiased Data Estimation in Mobile Radio Systems Applying CDMA", *IEEE Journal on Selected Areas in Communications*, vol. 11, No. 7, September 1993, pp. 1058-1066.

[2] A. Klein, G.K. Kaleh and P.W. Baier, "Zero Forcing and Minimum Mean Square Error Equalization for Multiuser Detection in Code Division Multiple Access Channels", *IEEE Transactions on Vehicular Technology*, vol. 45, No. 2, May 1996, pp. 276-287.

[3] Ali N. Akansu, Pierre Duhamel, Xueming Lin, Marc de Courville, "Orthogonal Transmultiplexers in Communication: A Review", *IEEE Transactions on Signal Processing*, vol. 46, No. 4, April 1998, pp. 979-995.

[4] J. Salz, "Digital transmission over cross-coupled linear channels", *AT&T Bell Lab. Tech. J.*, July-Augustus 1985, pp. 1147-1159.

[5] Jiang Yang, Sumit Roy, "On Joint Transmitter and Receiver Optimization for Multiple-Input-Multiple-Output (MIMO) Transmission Systems", *IEEE Transactions on Communications*, vol. 42, No. 12, December 1994, pp. 3221-3231.

[6] Xiang-Gen Xia, "New Precoding for Intersymbol Interference Cancellation Using Nonmaximally Decimated Multirate Filterbanks with Ideal FIR Equalizers", *IEEE Transactions on Signal Processing*, vol. 45, No. 10, October 1997, pp. 2431-2441.

[7] Tongtong Li, Zhi Ding, "Joint Transmitter-Receiver Optimization for Partial Response Channels Based on Nonmaximally Decimated Filterbank Precoding Technique", *IEEE Transactions on Signal Processing*, vol. 47, No. 9, September 1999, pp. 2407-2414.

[8] Anna Scaglione, Georgios B. Giannakis, Sergio Barbarossa, "Redundant Filterbank Precoders and Equalizers
Part1: Unification and Optimal Designs", *IEEE Transactions on Signal Processing*, vol. 47, No. 7, July 1999, pp. 1988-2006.

[9] Won Mee Jang, Branimir R. Vodčič, Raymond L. Pickholtz, "Joint Transmitter-Receiver Optimization in Synchronous Multiuser Communications over Multipath Channels", *IEEE Transactions on Communications*, vol. 46, No. 2, February 1998, pp. 269-278.

[10] Branimir R. Vodčič, Won Mee Jang, "Transmitter Precoding in Synchronous Multiuser Communications", *IEEE Transactions on Communications*, vol. 46, No. 10, October 1998, pp. 1346-1355.

[11] Maïté Brandt-Pearce and Anupama Dharap, "Transmitter-Based Multiuser Interference Rejection for the Down-link of a Wireless CDMA System in a Multipath Environment", *IEEE Journal on Selected Areas of Communication*, vol. 18, No. 3, March 2000, pp. 407-417.

[12] Anna Scaglione, Georgios B. Giannakis, and S. Barbarossa, "Lagrange/Vandermonde MUI eliminating User Codes for Quasi-Synchronous CDMA in Unknown Multipath", *IEEE Transactions on Signal Processing*, vol. 48, No. 7, July 2000, pp. 2057-2073.

[13] H.S. Witsenhausen, "A determinant maximization problem occurring in the theory of data communications" *SIAM J.Appl.Math.*, vol. 29, No. 3, November 1975.

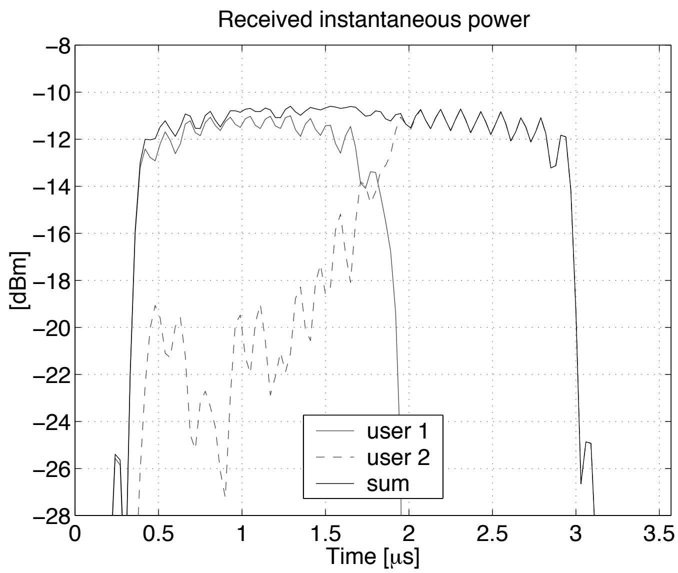

Fig. 5. Instantaneous received power during a burst; progressive system, $K = 2$, $N = 10$.

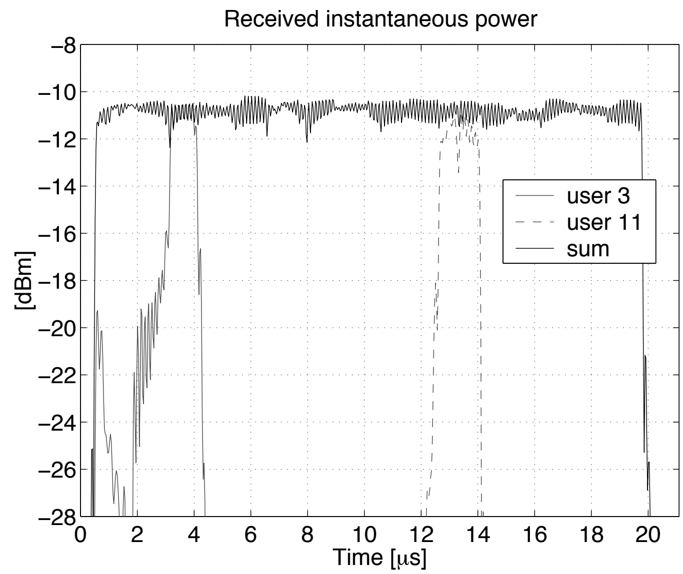

Fig. 7. Instantaneous received power during a burst; progressive system, $K = 16$, $N = 10$.

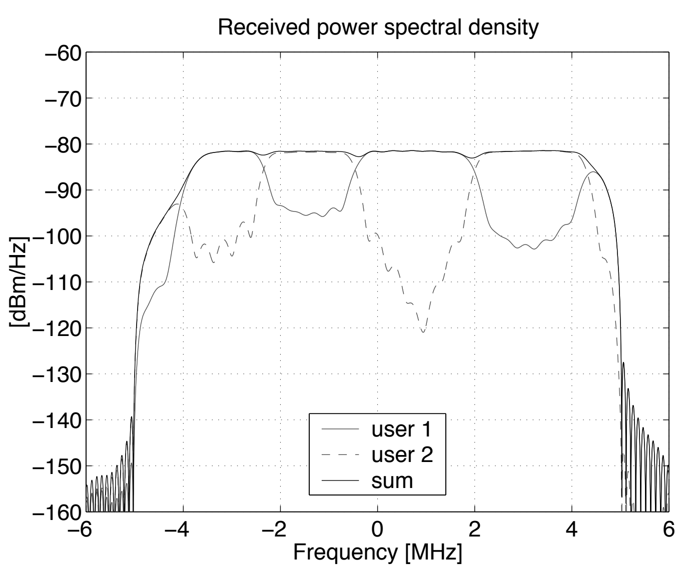

Fig. 6. Received power spectral density; close to optimum system, $K = 2$, $N = 10$.

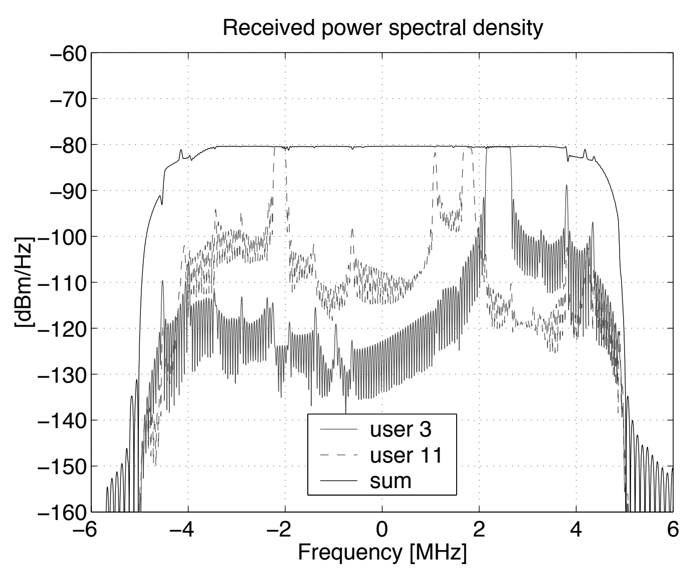

Fig. 8. Received power spectral density; close to optimum system, $K = 16$, $N = 10$.