MIMO-channel adapted precoding for MAI/ISI-free uplink burst transmission

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Abstract

This paper considers the uplink of a multi-user DS-CDMA communication system based on burst transmission over frequency selective channels. A new precoding method is presented which is able to take benefit of the space diversity introduced at the mobile transmitters and at the common receiver. The codes are designed so that filters matched to the total impulse responses are able to remove completely the interferences (ISI and MAI) and maximize the SINRs. It is analytically demonstrated that an infinity of solutions is available to orthogonalize the system. Three particular solutions are considered. A progressive solution is firstly proposed to provide orthogonality in a simple way. Then a close to optimum solution is proposed to orthogonalize the system while attempting to reduce the total emitted power. The minimization of the total emitted power is a complex problem of non linear optimization which can only be solved using an iterative algorithm. The third solution proposed is based on an iterative

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algorithm. Those three solutions are analyzed in term of their different performance, required complexity and flexibility properties when the number of users is varying. A comparison is also provided with a system using a conventional set of Hadamard codes and linear or DF MMSE joint detection. It is shown that the close to optimum and iterative solutions are the only ones that are able to fully benefit from the space diversity.

Index terms: MIMO channels, Code division multiple access, Adaptive codes, Multi-user communication, Space diversity.

1 Introduction

Communications over wireless channels are impaired by inter-symbol interferences (ISI) due to multi-path propagation and by multiple-access interferences (MAI). Diversity (redundancy in the time, frequency and spatial domains) can be used to provide the receiver with several replicas of the transmitted signal. While antenna diversity at the base station is used for reception today, antenna diversity at a mobile handset is more difficult to implement because of electromagnetic interaction of antenna elements on small platforms. Furthermore, the use of multiple antennas and radio frequency chains would make the remote units larger and more expensive. It is therefore more economical to add equipment to base stations rather than to remote units. These factors motivate the use of multiple antennas at the base station not only for the reception but also for the transmission.

Balaban and Salz [1, 2] propose to combine diversity reception, known to combat flat fading in radio transmission, with adaptive equalization, known to mitigate the effects of ISI, into a single robust receiver for a single user communication system. The overall structure of the combiner-equalizer is chosen to minimize the mean square error (MSE). Optimum combining and signal processing with multiple antennas are also envisaged for multi-user systems. In this case, only the signal processing at the receiver is modified whereas the transmission signaling scheme of the mobile units remains unchanged. In [3] Jung et al. evaluate the gain of performance after joint detection (JD) due to receiver antenna diversity in case of uplink burst transmission. Paper [4] derives and compares several space-time multi-user detection structures, including the maximum likelihood (ML) sequence detector, low complexity linear detectors based on iterative interference cancellation, and blind adaptive detectors for multi-path CDMA channels with receiver antenna arrays.

To avoid the use of antenna arrays at the mobile handsets, space diversity is also introduced at the base station for the transmission. Only recently has transmit diversity been studied extensively as a method of combating detrimental effects in wireless fading channels. The first schemes introduced in the literature require no feedback from the receivers to the transmitter. The first bandwidth efficient transmit diversity scheme was proposed by Wittneben [6, 7], and includes the delay diversity scheme of Seshadri and Winters [5] as a special case. Paper [8] generalizes the delay diversity scheme proposed by Wittneben by introducing a new class of codes referred to as the space-time codes. Unfortunately they have a complexity increasing exponentially as a function of the diversity order. In paper [9], a diversity scheme is proposed by simple processing across two transmit antennas. Thanks to this simple precoding scheme, ML decoding at the receiver is achieved in a simple linear way. The transmission scheme discovered by Alamouti is generalized in [10, 11] to an arbitrary number of transmit antennas. In the previous references, the space-time codes were constructed using design criteria derived for flat Rayleigh fading where ideal channel state information is available at the receivers. Recently, systems were proposed where the codes are designed according to the knowledge of the user channel impulse response (CIR). A spatio-temporal coding structure is suggested in [12] for a single user burst transmission over multi-path channels with space diversity at the transmitter and at the receiver. Paper [13] describes a spatial multiple-access system where the users are separated in the space domain. However, the delay spread due to the multi-path propagation is not taken into account in this last case. When the channel of each user has multi-path, the beamformer usually chooses one path, which has the strongest power, and performs the interference canceling process.

In the present paper, we consider the uplink of a burst direct sequence code division multiple access (DS-CDMA) communication system. We investigate the impact of space diversity on the overall performance. The joint transmitter and receiver optimization proposed in [15, 17] is performed for a system where space diversity is introduced both at the transmitters and at the receiver. The CIRs are assumed to be known to design the system. We allow the spreading waveforms of successive symbols to overlap and use symbol specific waveforms. The channel adapted waveforms designed here are such that they can be demodulated by means of a total-channel matched filter bank. This means that after this step ISI and MAI are perfectly removed. As degrees of freedom are still left at this stage additional objectives are introduced. It is further requested that the emitted power be minimized. The progressive solution and the close to optimum one introduced in [15, 17] are compared to the optimal solution obtained at the output of an iterative algorithm. A system using a conventional set of binary codes is also envisaged for the comparison.

Throughout the paper, the symbols $(.)^*$, $(.)^T$, $(.)^H$ will be used to denote the complex conjugate, the transpose and the complex conjugate transpose respectively of a matrix, or a vector, or a scalar. Notation $Tr\,[\,.\,]$ stands for the trace of a matrix.

2 System model

2.1 General model

We consider the uplink of a multi-user system with space diversity at the mobile terminals and at the base station. N_d^e denotes the number of antennas at the transmitters and N_d^r denotes the number of antennas at the receiver. Each antenna at the base station receives the contributions coming from each antenna of the K different users and a specific additive noise. A detailed model of the transmission system is given in Figure 1. Each active user transmits a sequence of symbols $d_k(n)$ at the rate $1/T$ $(k = 1, \cdots, K)$. The information is spread by different user codes $s_k^p(n)$ specific to each transmitter antenna p $(p = 1, \cdots, N_d^e)$ and shaped with the chip shaping filter $u_a(t)$. T_c is the chip duration and N_c is the spreading factor. Each signal is transmitted over N_d^r specific channels with low-pass equivalent impulse response $c_{a,k}^{p,q}(t)$ $(q = 1, \cdots, N_d^r)$. At the receiver $p_a(t)$ represents an ideal low-pass presampling filter. $w_a^q(t)$ is the additive noise corrupting the received signal at the antenna q and $v_a^q(t)$ is the noise filtered by $p_a(t)$. The received signal at the antenna q is denoted $r_a^q(t)$. We define the impulse responses $h_{a,k}^{p,q}(t) \stackrel{def}{=} u_a(t) \otimes c_{a,k}^{p,q}(t) \otimes p_a(t)$. The operator $\otimes$ represents the convolution. A sampling of the received and noise signals at a rate $1/T_s = M/T_c = MN_c/T$, gives the received sequences $r^q(n) = r_a^q(t = nT_s)$ and the noise sequences $v^q(n) = v_a^q(t = nT_s)$. To get a discrete time model, we define the discrete time impulse responses $h_k^{p,q}(n) = h_{a,k}^{p,q}(t = nT_s)$.

In our computations, we further assume that the power control is such that the total symbol received energies are equal for all users. Each symbol received energy is the sum of the corresponding symbol received energies at the N_d^r antennas.

2.2 Linear antenna arrays

Wireless communication channels are often characterized by severe multi-path. The transmitted signal propagates along L multiple paths created by reflection on physical objects. With each path is associated a complex gain (amplitude and phase information) and a delay. The user channel attenuation is denoted α_k. $\beta_{k,l}$ is the relative path amplitude and $\phi_{k,l}$ is the associated phase $(l = 1, \cdots, L)$. The delay is denoted $\tau_{k,l}$. Furthermore, since we consider a system using multiple antennas, the signal directions must be taken into account. Each path is thus also characterized

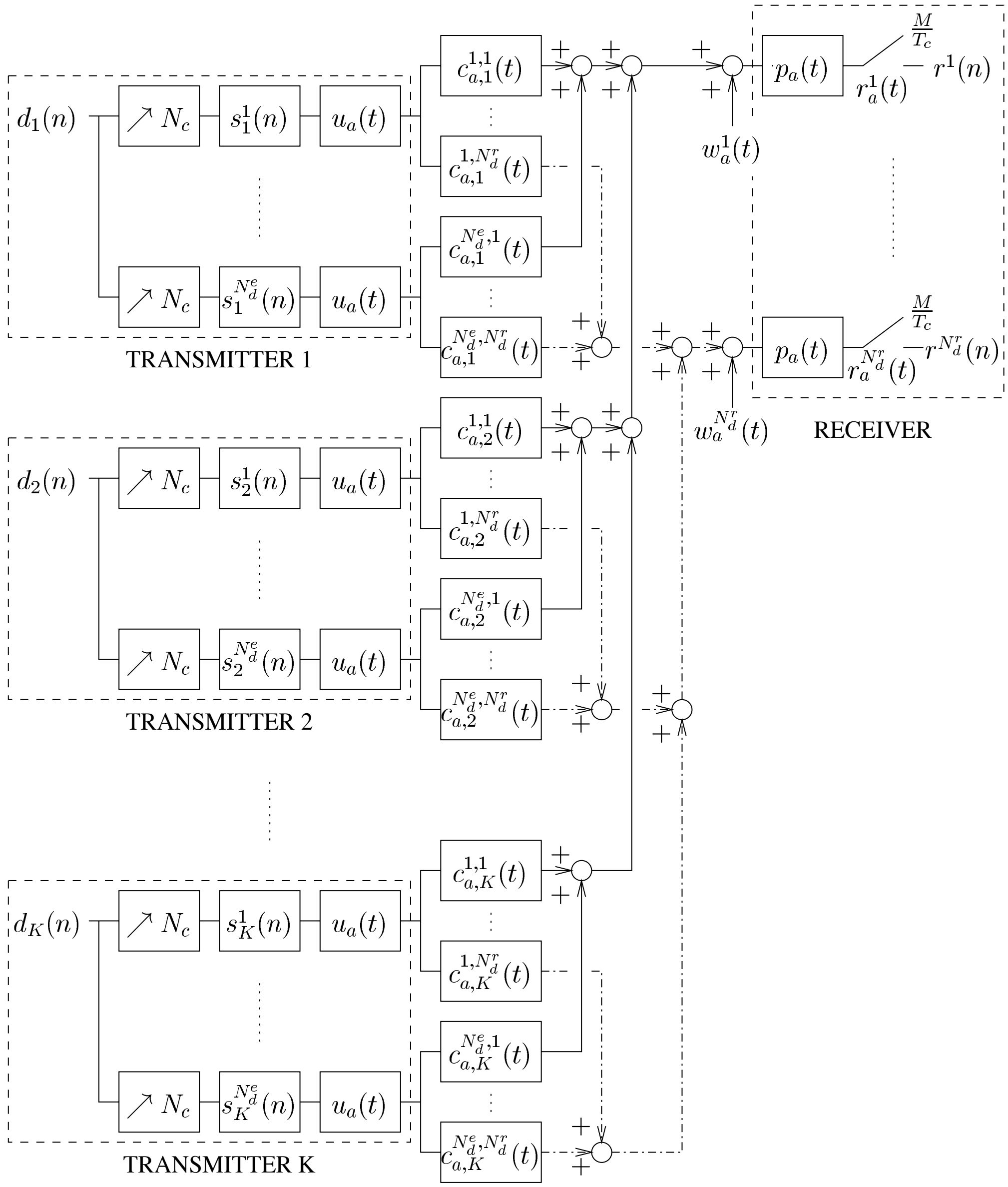

Figure 1: System model

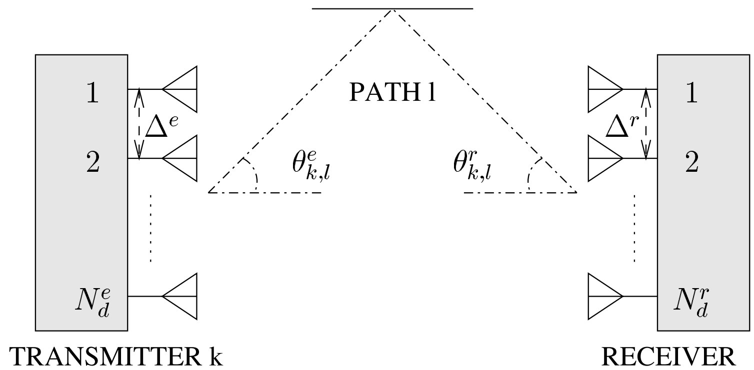

Figure 2: Antenna array

by an angle of departure $\theta^e_{k,l}$ from the antenna array at the terminals and by an angle of arrival $\theta^r_{k,l}$ on the antenna array at the base station.

Consider the situation represented in Figure 2. Linear arrays of antennas are assumed at the transmitters and at the receiver. Two successive antennas are separated by a distance Δ^e at the transmitters and Δ^r at the receiver. The signal propagation between the antenna p at the terminal k and the antenna q at the base station is modeled by the following channel impulse response

$$c^{p,q}_{a,k}(t) \;=\; \alpha_k \sum_{l=1}^{L} \beta_{k,l}\, e^{j\phi_{k,l}}\, a^e_p(\theta^e_{k,l})\, a^r_q(\theta^r_{k,l})\, \delta(t-\tau_{k,l}) \tag{1}$$

where $a^e_p(\theta^e_{k,l})$ denotes the gain relative to the antenna p at the emitter k and $a^r_q(\theta^r_{k,l})$ denotes the gain relative to the antenna q at the base station. The different antenna gains are due to the wave propagation phase delay between two successive antennas. We have that

$$a^e_p(\theta^e_{k,l}) \;\stackrel{def}{=}\; \exp\left(-j2\pi(p-1)\frac{\Delta^e}{\lambda}\sin\theta^e_{k,l}\right) \tag{2}$$

$$a^r_q(\theta^r_{k,l}) \;\stackrel{def}{=}\; \exp\left(-j2\pi(q-1)\frac{\Delta^r}{\lambda}\sin\theta^r_{k,l}\right) \tag{3}$$

where λ denotes the wavelength.

2.3 Burst transmission

In case of burst transmission, each user sends a packet of N symbols. In our model, we want to separate the spreading codes from the other elements of the composite channel. A matrix $\underline{\underline{s}}_b$

represents the transmitter precoding operation and a matrix $\underline{\underline{h}}_b$ represents the channel impulse responses of the different users. The observation model becomes [14]

$$\underline{r}_b = \underline{\underline{h}}_b \, \underline{\underline{s}}_b \, \underline{d}_b + \underline{v}_b \,. \tag{4}$$

The vector of symbols, denoted $\underline{d}_b$, is composed of different user parts

$$\underline{d}_b \overset{def}{=} \begin{bmatrix} (\underline{d}_1)^T & \cdots & (\underline{d}_K)^T \end{bmatrix}^T \tag{5}$$

where $\underline{d}_k \overset{def}{=} \begin{bmatrix} d_k(0) & \cdots & d_k(N-1) \end{bmatrix}^T$. The received and noise vectors are constituted by the juxtaposition of the different antenna received and noise vectors. We have

$$\underline{r}_b \overset{def}{=} \begin{bmatrix} (\underline{r}^1)^T & \cdots & (\underline{r}^{N_d^r})^T \end{bmatrix}^T \tag{6}$$

$$\underline{v}_b \overset{def}{=} \begin{bmatrix} (\underline{v}^1)^T & \cdots & (\underline{v}^{N_d^r})^T \end{bmatrix}^T \tag{7}$$

where $\underline{r}^q \overset{def}{=} \begin{bmatrix} r^q(0) & \cdots & r^q(M(NN_c+L_h-1)) \end{bmatrix}^T$ and $\underline{v}^q \overset{def}{=} \begin{bmatrix} v^q(0) & \cdots & v^q(M(NN_c+L_h-1)) \end{bmatrix}^T$. L_h is the size of the impulse responses $h_{a,k}^{p,q}(t)$ evaluated in numbers of T_c. Furthermore the received signal at each antenna is the sum of the signals coming from the different transmitter antennas and the noise. Hence the following matrices are defined

$$\underline{\underline{s}}_b \overset{def}{=} \begin{bmatrix} \underline{\underline{s}}_1 & \cdots & \underline{\underline{0}}_{NN_cN_d^e \times N} \\ \vdots & \ddots & \vdots \\ \underline{\underline{0}}_{NN_cN_d^e \times N} & \cdots & \underline{\underline{s}}_K \end{bmatrix} \tag{8}$$

$$\underline{\underline{h}}_b \overset{def}{=} \begin{bmatrix} \underline{\underline{h}}_1 & \cdots & \underline{\underline{h}}_K \end{bmatrix} \tag{9}$$

with

$$\underline{\underline{s}}_k \overset{def}{=} \begin{bmatrix} \underline{\underline{s}}_k^1 \\ \vdots \\ \underline{\underline{s}}_k^{N_d^e} \end{bmatrix} \tag{10}$$

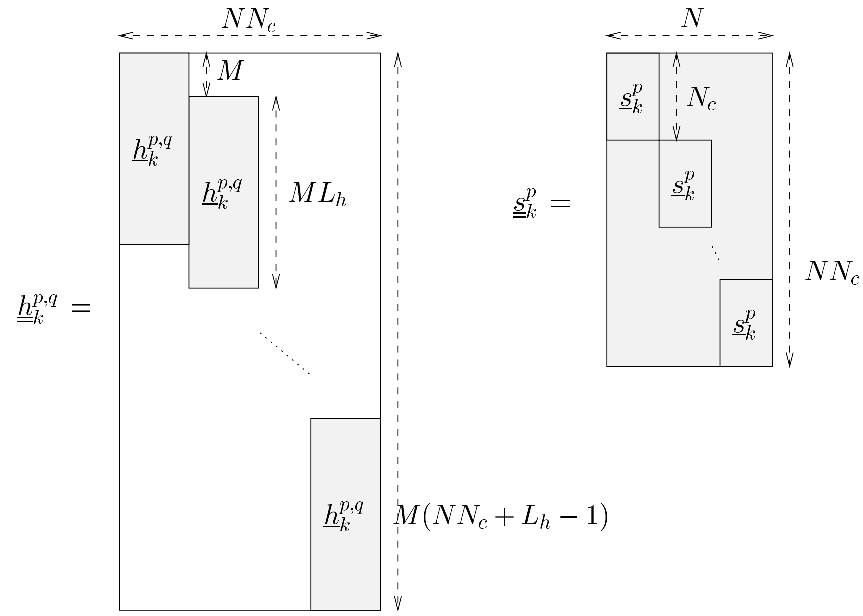

Figure 3: Uplink user k specific matrices

$$\underline{\underline{h}}_k \stackrel{def}{=} \begin{bmatrix} \underline{\underline{h}}_k^{1,1} & \cdots & \underline{\underline{h}}_k^{N_d^e,1} \\ \vdots & \ddots & \vdots \\ \underline{\underline{h}}_k^{1,N_d^r} & \cdots & \underline{\underline{h}}_k^{N_d^e,N_d^r} \end{bmatrix}. \tag{11}$$

A description of the precoding matrix $\underline{\underline{s}}_k^p$ and of the channel matrix $\underline{\underline{h}}_k^{p,q}$ is given in Figure 3 in which $\underline{s}_k^p$ is the code allocated to the antenna p of user k in a conventional precoding scheme and $\underline{h}_k^{p,q}$ is the impulse response vector. We have $\underline{s}_k^p \stackrel{def}{=} \begin{bmatrix} s_k^p(0) & \cdots & s_k^p(N_c-1) \end{bmatrix}^T$ and $\underline{h}_k^{p,q} \stackrel{def}{=} \begin{bmatrix} h_k^{p,q}(0) & \cdots & h_k^{p,q}(ML_h-1) \end{bmatrix}^T$.

3 Joint transmitter-receiver optimization

3.1 Design of the codes

Each product $\underline{\underline{s}}_k^p \underline{d}_k$ results in a sequence at the chip rate corresponding to the different symbols of user k spread by the codes specific to the antenna p. In a conventional precoding scheme, the length of the codes does not exceed the symbol period. In this case, each symbol is spread by a particular code $s_k^p(n)$ of length equal to the spreading factor, with no a priori interference with the symbols around. However such a constraint can be relaxed in practice: the length of the codes could be larger than a symbol period, different codes could be assigned to the successive symbols of a user (it is already the case in IS 95 or UMTS where a long code is superimposed on a short code), and finally, the codes could be constituted with complex elements. *As a key element to our precoding scheme, we assume that the composed matrices $\underline{\underline{s}}_k$ can be totally filled in with complex elements.* This actually amounts to having codes overlapping with each other and being different from one symbol to another (even for the same antenna).

3.2 Criterion

The joint transmitter and receiver optimization performed in papers [15, 17] is adapted to the system considered in this paper which includes space diversity at the transmitters and at the receiver. The different precoding matrices are designed in such a way that filters matched to the different total impulse responses are able to eliminate completely the interferences (ISI and MAI). In our computations, independent symbols of variance equal to σ_d^2 and an additive white Gaussian noise (AWGN) of one-sided power spectral density equal to N_0 are assumed. In this case, the matched filter bound (MFB) is equal to the constant $\gamma \overset{def}{=} E_s/N_0$ where E_s denotes the total received energy for each user symbol. According to [14], the matched filter receiver is the hermitian of the total impulse response matrix which includes the precoding and the channel impulse response matrices. Hence the precoding matrix $\underline{\underline{s}}_b$ should be chosen such that

$$(\frac{\sigma_d^2}{\sigma_v^2})\,(\underline{\underline{s}}_k)^H\,(\underline{\underline{h}}_k)^H\,(\underline{\underline{h}}_l)\,(\underline{\underline{s}}_l) \;=\; \delta_{kl}\,\gamma\,\underline{\underline{I}}_N \qquad (12)$$

for $k, l = 1, \cdots, K$. σ_v^2 is the noise variance at the output of the presampling filter. δ_{kl} denotes the Kronecker symbol and $\underline{\underline{I}}_N$ denotes the identity matrix of size N.

When the orthogonality conditions (12) are fulfilled, the signal to interference plus noise ratios

(SINRs) are maximized. A simple matched filter applied on the received signal is able to remove completely the interferences so that the MFB is reached for each user symbol (a detailed demonstration is provided in [15, 17]). In the following section, it is analytically demonstrated that an infinite number of solutions is available to orthogonalize the system.

3.3 Orthogonality realization

To analyze the possible orthogonalization of the system, we have to investigate the impact of the channel impulse responses on the rank of the channel matrices. Each channel is characterized by multi-path propagation. We have

$$h^{p,q}_{a,k}(t) \overset{def}{=} c^{p,q}_{a,k}(t) \otimes \tilde{u}_a(t) \tag{13}$$

$$= \alpha_k \sum_{l=1}^{L} \beta_{k,l}\, e^{j\phi_{k,l}}\, a^e_p(\theta^e_{k,l})\, a^r_q(\theta^r_{k,l})\, \tilde{u}_a(t-\tau_{k,l}) \tag{14}$$

where $\tilde{u}_a(t)$ is the cascade of the transmitter shaping filter and the receiver presampling filter ($\tilde{u}_a(t) \overset{def}{=} u_a(t) \otimes p_a(t)$). Each user channel matrix $\underline{\underline{h}}_k$ can be written as a sum of the contributions due to the different paths

$$\underline{\underline{h}}_k = \alpha_k \sum_{l=1}^{L} \beta_{k,l}\, e^{j\phi_{k,l}}\, \underline{\underline{a}}^r\, \underline{\underline{\tilde{u}}}_{k,l}\, \underline{\underline{a}}^e \tag{15}$$

where matrix $\underline{\underline{\tilde{u}}}_{k,l}$ is the chip shaping and presampling filter part. Matrix $\underline{\underline{\tilde{u}}}_{k,l}$ has exactly the same construction as a channel matrix $\underline{\underline{h}}^{p,q}_k$ in the initial model in which the vector $\underline{h}^{p,q}_k$ is replaced with a vector composed by the samples of the function $\tilde{u}_a(t-\tau_{k,l})$. Since the columns of the channel matrices $\underline{\underline{\tilde{u}}}_{k,l}$ are delayed versions of the same vector (see Figure 3), they are linearly independent. The matrices $\underline{\underline{a}}^e$ and $\underline{\underline{a}}^r$ are defined as

$$\underline{\underline{a}}^e \overset{def}{=} \begin{bmatrix} a^e_1(\theta^e_{k,l})\, \underline{\underline{I}}_{NN_c} & \cdots & a^e_{N^e_d}(\theta^e_{k,l})\, \underline{\underline{I}}_{NN_c} \end{bmatrix} \tag{16}$$

$$\underline{\underline{a}}^r \overset{def}{=} \begin{bmatrix} a^r_1(\theta^r_{k,l})\, \underline{\underline{I}}_{M(NN_c+L_h-1)} \\ \vdots \\ a^r_{N^r_d}(\theta^r_{k,l})\, \underline{\underline{I}}_{M(NN_c+L_h-1)} \end{bmatrix}. \tag{17}$$

Since the size of the matrices $\underline{\underline{\tilde{u}}}_{k,l}$ is $M(NN_c + L_h - 1) \times NN_c$, the rank of the composite matrix $\underline{\underline{h}}_k$ is the minimum between NN_cL (each term in the sum adds NN_c dimensions), $NN_cN_d^e$ and $M(NN_c+L_h-1)N_d^r$. In our computations, we assume that the channel impulse responses are long enough so that the rank is not limited by the number of rows $M(NN_c + L_h - 1)N_d^r$.

Using the vectorial subspace theory, it is possible to prove that the system can be made orthogonal under specific conditions on the spreading factor and on the space diversity orders. Each matrix $\underline{\underline{h}}_k$ can be viewed as a vectorial subspace of dimension equal to $\min[NN_cL, NN_cN_d^e]$ in the space of dimension $M\,(NN_c + L_h - 1)\,N_d^r$. The columns of matrix $\underline{\underline{h}}_k$ form a base of the subspace associated with user k. It is interesting to note that each product $\underline{\underline{h}}_k\,\underline{\underline{s}}_k$ is equivalent to choosing N vectors in the subspace associated with each user k. They are linear combinations (achieved by the coefficients contained in the N columns of the matrices $\underline{\underline{s}}_k$) of the base vectors. If the rank of matrix $\underline{\underline{h}}_k$ is at least equal to NK, all the vectors associated with the different users can be chosen orthogonal using classical results of the subspace theory so that all the orthogonality constraints (12) be verified. This is demonstrated hereafter.

- N orthogonal vectors can be chosen in the first subspace $\underline{\underline{h}}_1$.
- The N vectors assigned to user 1 are projected on the subspace $\underline{\underline{h}}_2$ of user 2. A subspace of dimension $N(K-1)$ orthogonal to the N projections and included in subspace $\underline{\underline{h}}_2$ exists. It is possible to choose N orthogonal vectors in this subspace. Of course those vectors orthogonal to the projections are also orthogonal to the initial N vectors. They are assigned to user 2.
- The same operations can be performed with the subspace $\underline{\underline{h}}_3$ of user 3. We look for the subspace of dimension $N(K-2)$ orthogonal to the projections of the N vectors assigned to user 1 and to the projections of the N vectors assigned to user 2. N orthogonal vectors are chosen in this subspace and assigned to user 3.
- This procedure is followed until user K.

To get a possible orthogonalization of the system, the minimum between the quantities NN_cL and $NN_cN_d^e$ must be at least equal to NK. If the spreading factor N_c is equal to the number of users K, the system can always be made orthogonal. However we can take benefit of the multi-path propagation jointly with the space diversity introduced at the transmitters in order to decrease the spreading factor while still allowing the system orthogonalization.

3.4 Solutions

At each step, an infinity of orthogonal vectors can be chosen. Hence infinities of solutions are available to orthogonalize the system. Two solutions proposed in [15, 17] and called the progressive and close to optimum solutions, are revisited briefly for a system where space diversity is introduced at the transmitters and at the receiver. In case of the progressive solution, the main concern is to provide orthogonality at the receiver. If the close to optimum solution is considered, the codes allocated to the different users are computed not only to provide orthogonality at the receiver but also to decrease the user emitted powers. However the minimization of the emitted power which is proportional to $\sigma_d^2 Tr\left[\underline{\underline{s}}_b \underline{\underline{s}}_b^H\right]$ is a complex problem of non linear optimization which can only be optimally solved using an iterative algorithm. This is the third solution considered in this section. The actual emitted power is compared to an unreachable lower bound (LB) computed in [15, 17] regardless of the different sources of interference.

The first solution of interest is the progressive solution. The codes are computed progressively following the procedure described in section 3.3. The user order of computation is not optimized. At each step, a vector orthogonal to the vectors allocated to the previous users and symbols is provided thanks to the Gram-Schmidt method of orthogonalization [18]. This vector belongs to the subspace spanned by the first columns of $\underline{\underline{h}}_k$ of size equal to the number of vectors which are already allocated plus one. Hence the number of code elements different from zero is minimized. The consequences are a small complexity needed to initialize the system (only $\frac{1}{4}K^4N^3$ complex operations are required to compute the codes [15, 17, 18]) and a poor performance. However, when the number of users is varying, this system offers interesting properties of flexibility. Two situations can be considered.

- We consider firstly that the spreading factor is constant. When a user enters the system, only the codes allocated to this user need to be optimized. It requires K^3N^3 operations. When a user leaves the system, the resulting system is still orthogonal. No additional computations are needed.

- We assume secondly that the spreading factor is kept as small as possible in order to increase the corresponding bit rates ($N_c = K/N_d^e$). Adding a user is easy. The initial users are still orthogonal if their code matrices are extended with N rows of zeros. The computation of the codes allocated to the new user requires K^3N^3 operations. When a user is removed, the spreading factor should be decreased by one in order to optimize the symbol rates. The last N rows of the code matrices corresponding to the users who entered the system before the user of interest are composed of zeros. They can be removed without loosing the orthogonality of those users. The codes of the users who entered the system after the user of interest need to be recomputed.

In case of the close to optimum solution, the users are classified according to the respective attenuation of their channels. Each code allocated to a particular user symbol is computed in order to minimize the corresponding emitted power while avoiding interferences with the previously computed users and symbols. The next symbols and users are not taken into account. The codes are firstly computed for the user suffering the largest channel attenuation in order to decrease his power as much as possible. The procedure is followed until the user suffering the lowest channel attenuation. The code matrices are completely filled in with complex elements. Hence the number of operations needed to initialize the system is larger ($11K^4N^3$ complex operations are needed to compute the codes if $N_c N_d^e = K$ [15, 17, 18]) and the performance is improved. Some flexibility properties are also lost. Only the first situation can be envisaged.

- We consider only a system where the spreading factor is constant. When a user enters the system, his codes are computed such that the user emitted power is minimized and such that orthogonality is provided with the initial users. It requires $2K^3N^3$ operations. When a user leaves the system, the resulting system is still orthogonal and no additional computations are needed. After a few changes in system configuration, the user order of computation does no longer correspond to a classification based on the attenuation of the channels. It can be shown that the emitted power increase is generally negligible. To conserve the classification based on the attenuations, the simple scheme proposed above has to be slightly modified. When a user enters the system, the codes of all the users suffering a lower attenuation than the user of interest are recomputed.

Finally we intend to minimize the emitted power while satisfying the constraints of orthogonality by means of an iterative algorithm. The problem introduced in this paper is generally referred to as a constrained non linear optimization problem. The iterative algorithm used here is based on a sequential quadratic programming method in which a quadratic subproblem is solved at each iteration. A constrained minimum of a non linear scalar function (in our case, the emitted power) of several variables (in our case, the real and imaginary parts of the different code elements) is found starting from an initial estimate. In our computations, the starting point is composed with the N eigen-vectors corresponding to the N largest eigen-values of the channel auto-correlation matrices. Hence no ISI is present at the beginning of the iterative process (the constraints of orthogonality corresponding to the ISI of each user are already fulfilled) which seems favorable to reach a local minimum corresponding to a small emitted power. The performance of this solution is still improved. However the complexity required at the initialization is high (the number of operations is $\mathcal{O}(K^8 N^6)$ to compute the codes) and this system does not offer any flexibility when the number of users is varying.

4 Performance comparison

In our computations, we consider the uplink of a 4 user system. Each user sends a burst of 5 symbols. A bandwidth equal to 10 MHz is assumed. The emitted signals are shaped with a half-root Nyquist. The roll-off factor is equal to 0.2 so that the chip duration is equal to 0.12 μs. A power control is applied such that the received symbol energies are equal for the symbols of each user. The ratio E_s/N_0 (E_s denotes the received symbol energy and N_0 denotes the one-sided noise power spectral density equal to -100 dBm/Hz) is fixed at 20 dB. At the receiver a chip fractional sampling is assumed ($M = 2$). The user bit rates are computed assuming that the interference at the output of the detector can be modeled as a white additive Gaussian noise of variance equal to its power. The symbol error probability is equal to 10^{-7}. For mobile communications, the channels can be modeled as multi-path channels. Four paths low-pass equivalent complex channel impulse responses are assumed. The coefficients are complex random variables which are chosen randomly. The delays are generally larger than one symbol. The separations between two successive antennas are given by $\Delta^e = 2\,\lambda$ and $\Delta^r = 5\,\lambda$ where λ denotes the wavelength.

4.1 Space diversity at the receiver

Figure 4 illustrates the required emitted power as a function of the number of antennas at the receiver. The spreading factor is equal to the number of users. There is only one antenna at each transmitter. The sum of all user emitted powers is considered for a system using a conventional set of Hadamard codes, for the progressive solution, for the close to optimum solution and finally for the iterative solution. The emitted power required for each system is compared to a lower bound which is the lowest emitted power required to get the received symbol energy corresponding to the fixed matched filter bound. In Figure 4, it is possible to see that the total emitted power lower bound decreases if the number of antennas at the base station becomes higher. It means that the total emitted power of a practical system is potentially lower. All systems are able to take benefit of this property. The emitted power of each system tends to the lower bound when the number of antennas becomes larger. The iterative solution and the close to optimum one seem to be rapidly effective as compared to the progressive solution. The advantage of the iterative solution as compared to the close to optimum one is negligible as soon as the number of antennas at the receiver is equal to 2. Figure 5 illustrates the emitted power of each user separately as a function of the number of antennas at the receiver for the close to optimum solution. The precoding matrices of the users who require potentially the largest emitted power are computed first. Their emitted powers are close to the lower bound. However the difference between the actual emitted power and the lower bound is large for the last computed users if there is only one antenna at the receiver. This is the cost paid to provide orthogonality to those users. When the number of antennas increases at the base station, the system is initially more orthogonal. The diagonal elements of the channel auto-correlation matrices become much larger than the non diagonal ones. Hence the emitted power required for the last computed users can be closer to their lower bound. It is also interesting to note that the order of user precoding matrix computation varies according to the number of antennas at the receiver.

Figure 6 illustrates the performance of each system. In case of an orthogonal system, the matched filter bound is reached for each user symbol since all the interferences are removed at the output of the matched filter (MF). The corresponding bit rates are constant. In case of a conventional set of binary codes, minimum mean square error (MMSE) linear and decision feedback (DF) joint detectors have to be considered to improve the performance. The performance of a system using a conventional set of Hadamard codes tends to be better when the number of antennas is larger at the receiver. However, as the emitted powers are generally lower when an antenna is added, the quantity of information available on the initial receiver antennas in order to take a decision on the emitted symbols is less important. In some cases this loss of information is not compensated by the information added on the new antenna in such a way that the SINRs can be lower.

4.2 Space diversity at the transmitters

Figure 7 illustrates the emitted power as a function of the number of antennas at the transmitters. The spreading factor is also equal to the number of users. The number of antennas at the receiver is only one. The power emitted at a specific terminal is the sum of the powers emitted at each antenna. The total emitted power is the sum of the different user powers. We consider a system using a conventional set of Hadamard codes, the progressive solution, the close to optimum solution and the iterative one. A lower bound on the emitted power is also illustrated. As the spreading factor is equal to the number of users, it seems difficult to share a simple set of Hadamard codes among the different antennas of each user. For the sake of simplicity, we consider that exactly the same information is sent by each antenna of a specific user. It means that the same code is used to spread the symbols of a particular user at all antennas. This is the optimal solution when the system suffers no multi-path propagation. It is shown in Figure 7 that the emitted power required for each user is potentially lower when the number of antennas at the transmitters increases (see the lower bound). The iterative solution and the close to optimum one tend to the lower bound. Furthermore it is possible to show that the maximal power level at each specific antenna becomes lower for those two solutions. The system using a conventional set of Hadamard codes has an emitted power which is strongly increased when space diversity is present at the transmitters. This is explained by the presence of destructive interferences due to multi-path propagation mechanisms. It is possible to obtain much lower values of the emitted power if different codes may be assigned to each antenna. Some kind of exhaustive search should be performed in order to find the best combination. However it can be stated that the conventional sets of codes are generally unable to exploit the space diversity introduced at the transmitters. As for optimized codes, the progressive solution does not take benefit of the space diversity at the transmitters. This is simply a consequence of the precoding matrix construction procedure. Figure 8 illustrates the evolution of the emitted power as a function of the number of antennas at the transmitters for each user separately in case of the close to optimum solution. When space diversity is present at the transmitters, the length of the codes is larger. However the number of orthogonality constraints remains the same. Hence degrees of freedom are added to orthogonalize the system so that the disadvantage for the last computed users becomes smaller. Their effective emitted power is close to the lower bound. Hence the emitted power required by the close to optimum solution becomes close to the emitted power required by the iterative solution when space diversity is present at the transmitters. The same order of user precoding matrix computation is kept for each number of antennas.

The performance of each system is shown in Figure 9. Of course the matched filter bound is reached for all orthogonal systems and the corresponding bit rates are constant. For a system using a conventional set of Hadamard codes the performance at the output of the MMSE linear and decision feedback joint detectors varies as a function of the number of antennas at the transmitters. Constructive and destructive interferences due to multi-path propagation mechanisms are observed so that the performance can be either better or lower when the diversity order is increased.

Figure 10 compares the influence of the number of antennas at the receiver and at the transmitters on the total emitted power. In the two cases, adding an antenna is mainly interesting when the initial number of antennas is small. The gain is principally due to the small space diversity orders. The emitted power required by the iterative and close to optimum solutions tends asymptotically to zero when the number of antennas goes to infinity. Space diversity at the receiver and at the transmitters has more or less the same effect on the required emitted power for the two systems considered. However the cost is much more important when space diversity is located at the transmitters since antenna arrays must be placed at all terminals.

4.3 Reduction of the spreading factor

It was demonstrated previously that the rank of each user specific matrix $\underline{\underline{h}}_k$ is the minimum between NN_cL, $NN_cN_d^e$ and $M(NN_c + L_h - 1)N_d^r$. We also know from the orthogonalization procedure that this rank must be at least equal to NK in order to be able to orthogonalize the system completely. If the number of paths is high enough and if the channel impulse responses are long enough, this rank is mainly limited by the quantity $NN_cN_d^e$. In that case, there exists a compromise between the spreading factor and the number of antennas at the transmitters. It seems possible to decrease the spreading factor by a factor equal to the transmit space diversity order in order to increase the corresponding bit rates. This is envisaged in this section for a system using a conventional set of codes, for the progressive solution, for the close to optimum solution and finally for the iterative solution.

In practice the number of antennas will be larger at the base station than at the mobile handsets in order to decrease the size and the cost of the last ones. We have considered that the numbers of antennas at the transmitters and at the receiver are equal to 2 and 4 respectively. The spreading factor ranges from 2 to 4. In case of a conventional set of codes, the same code is used to spread the symbols of a specific user at each antenna. We have considered random binary sequences in order to be able to generate a set of codes when the spreading factor is equal to 3.

In Figure 11, the influence of the spreading factor on the total emitted power required by each system is illustrated. A lower bound on the emitted powers is also given. Seemingly the emitted power lower bound depends weakly on the spreading factor. When the spreading factor is equal to 2, the number of degrees of freedom is the minimum accepted to provide orthogonality ($NK = NN_cN_d^e$). The emitted power should be large in that case. However the values are quite acceptable thanks to the large number of antennas at the receiver. When the spreading factor becomes higher, the obtained emitted powers can be close of the lower bound. Figure 12 shows the emitted power of each user as a function of the spreading factor in case of the close to optimum solution. For a spreading factor at least equal to 3, the number of degrees of freedom are sufficient to limit the disadvantage of the last computed users.

The performance of each system is given in Figure 13. For the range of spreading factors considered, the system can be configured so that a matched filter is able to eliminate all the interferences. In this case the matched filter bound is reached for each user symbol. The corresponding bit rates are significantly increased thanks to the possible reduction of the spreading factor. In case of a conventional set of the codes, the performance is still acceptable. However, when the spreading factor is equal to 2, the linear MMSE joint detector is unable to take benefit of the reduced spreading factor to increase the corresponding bit rates. If a decision feedback MMSE joint detector is considered, the SINRs are large enough to get improved bit rates. When the spreading factor is equal to 3 at least, the bits rates are the same as the bit rates obtained with an orthogonal system.

5 Conclusion

This paper studies the improvement of performance that can be achieved thanks to the presence of transmit and receive space diversity. The uplink of a DS-CDMA burst transmission is considered. New codes are designed which are able to provide orthogonality at the receiver. It is analytically demonstrated that an infinite number of solutions are available to solve this problem. We compare the performance of three specific solutions (a progressive solution, an emitted power close to optimum solution and the optimal solution obtained by the use of an iterative algorithm) with the one obtained by the use of a conventional set of binary codes. The potential of space diversity seems really attractive. Mainly, space diversity at the receiver allows a decrease of the required emitted power for all considered systems, as well as a performance improvement for the system using a conventional set of codes. The potentiality of space diversity at the transmitters can only be fully exploited if an optimized set of codes is considered (either the iterative solution or the close to optimum one). Furthermore the optimized orthogonal system can take benefit of space diversity at the transmitters to reduce the spreading factor in order to increase the corresponding bit rates. Thanks to the introduction of space diversity in the system, it is possible to get strongly improved bit rates for much lower user emitted powers.

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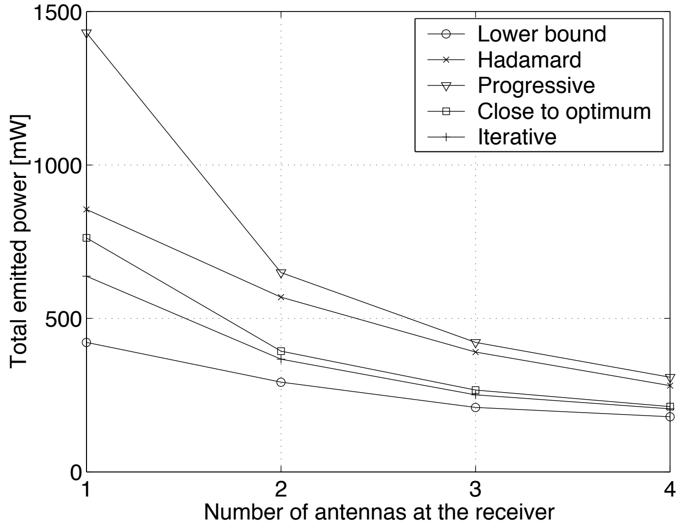

Figure 4: Total emitted power as a function of the number of antennas at the receiver, $K = 4$, $N_c = 4$, $N = 5$, $N_d^e = 1$.

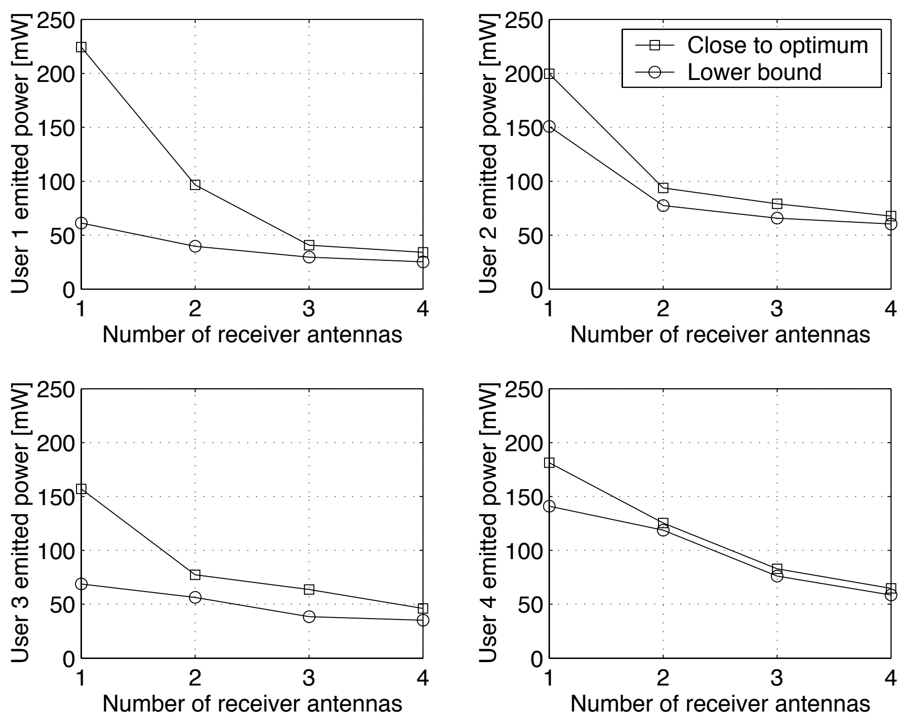

Figure 5: Emitted powers as a function of the number of antennas at the receiver in case of the close to optimum solution, $K = 4$, $N_c = 4$, $N = 5$, $N_d^e = 1$.

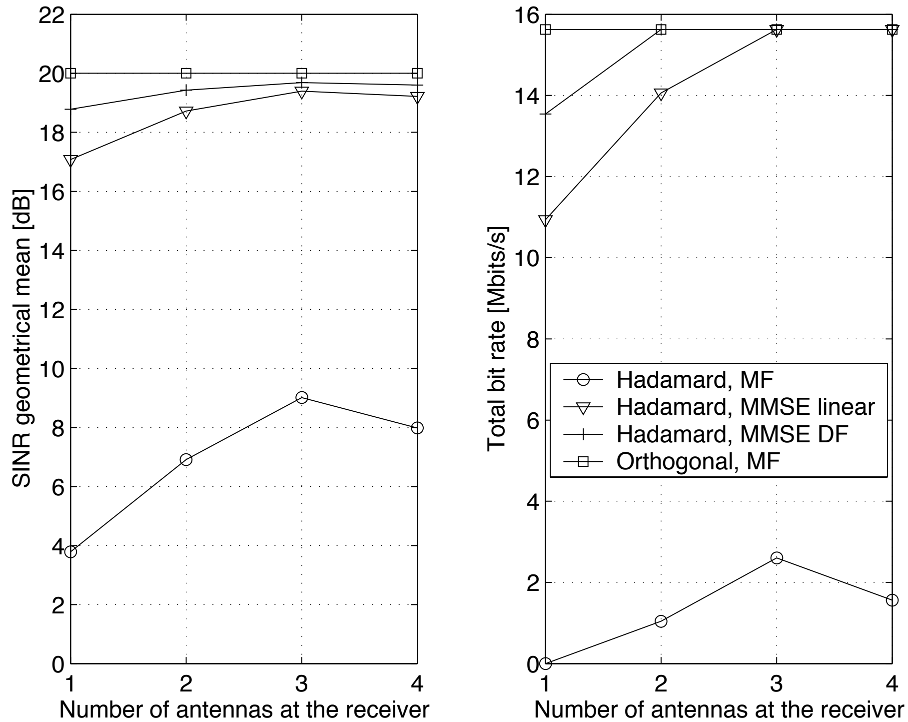

Figure 6: SINR geometrical mean and total bit rates as a function of the number of antennas at the receiver, $K = 4$, $N_c = 4$, $N = 5$, $N_d^e = 1$.

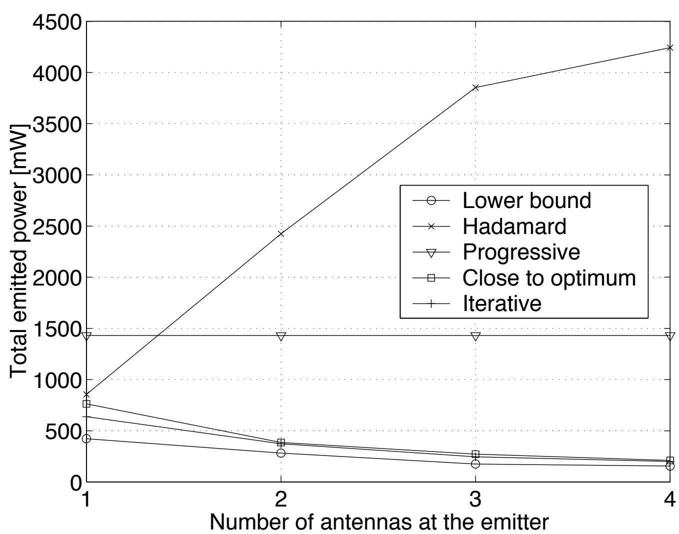

Figure 7: Total emitted power as a function of the number of antennas at the transmitters, $K = 4$, $N_c = 4$, $N = 5$, $N_d^r = 1$.

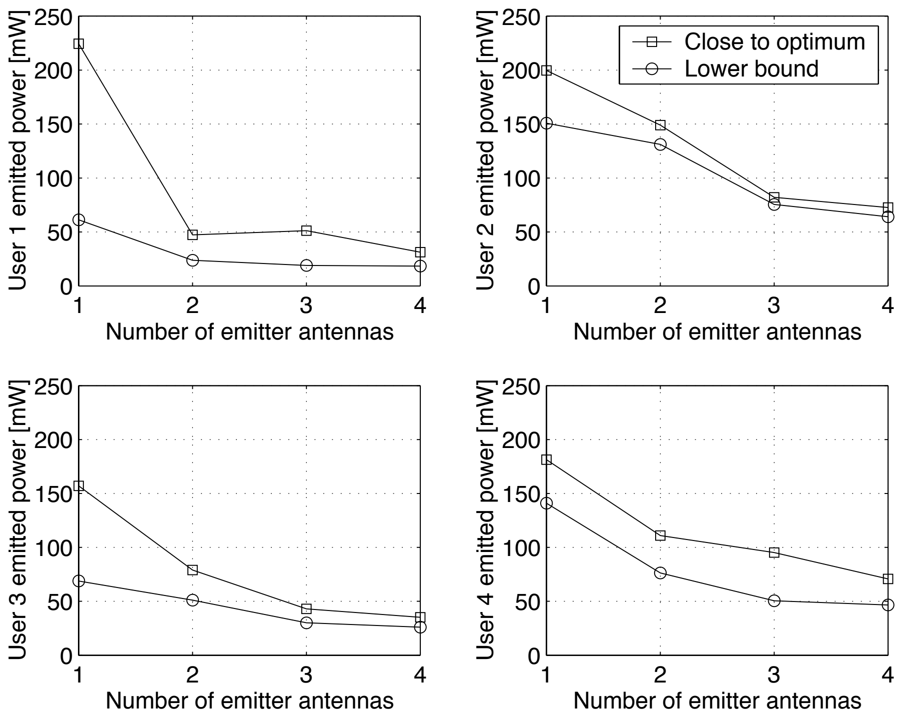

Figure 8: Emitted powers as a function of the number of antennas at the transmitters in case of the close to optimum solution, $K = 4$, $N_c = 4$, $N = 5$, $N_d^r = 1$.

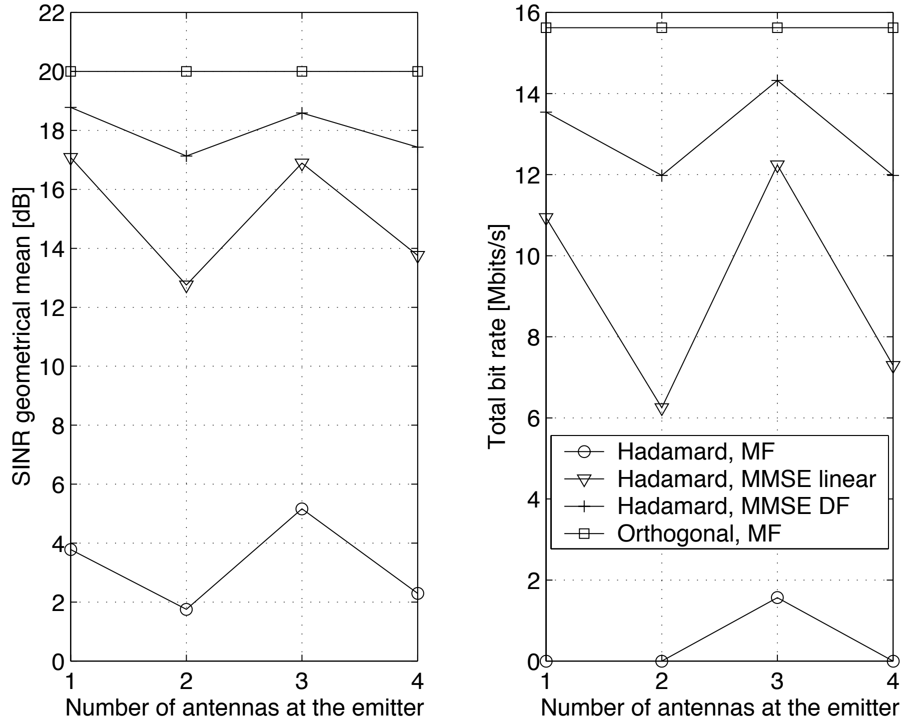

Figure 9: SINR geometrical mean and total bit rates as a function of the number of antennas at the transmitters, $K = 4$, $N_c = 4$, $N = 5$, $N_d^r = 1$.

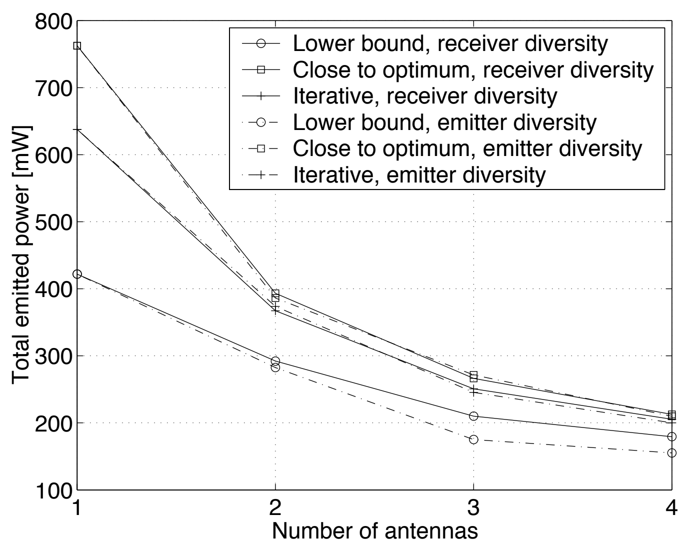

Figure 10: Total emitted power as a function of the number of antennas at the transmitters or at the receiver, $K = 4$, $N_c = 4$, $N = 5$.

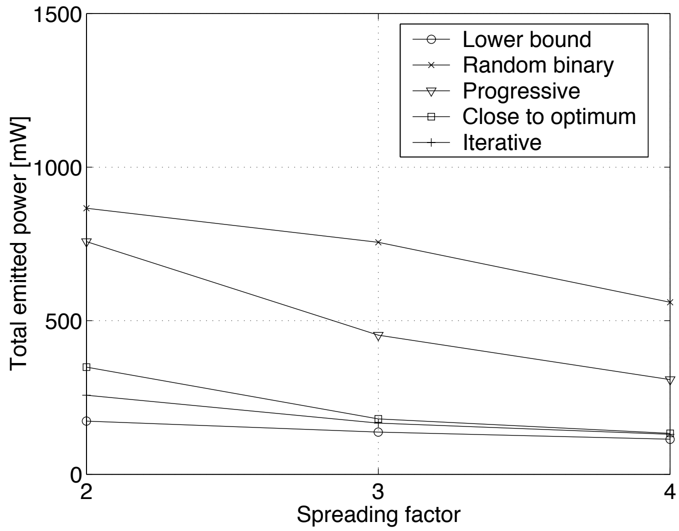

Figure 11: Total emitted power as a function of the spreading factor, $K = 4$, $N = 5$, $N_d^e = 2$, $N_d^r = 4$.

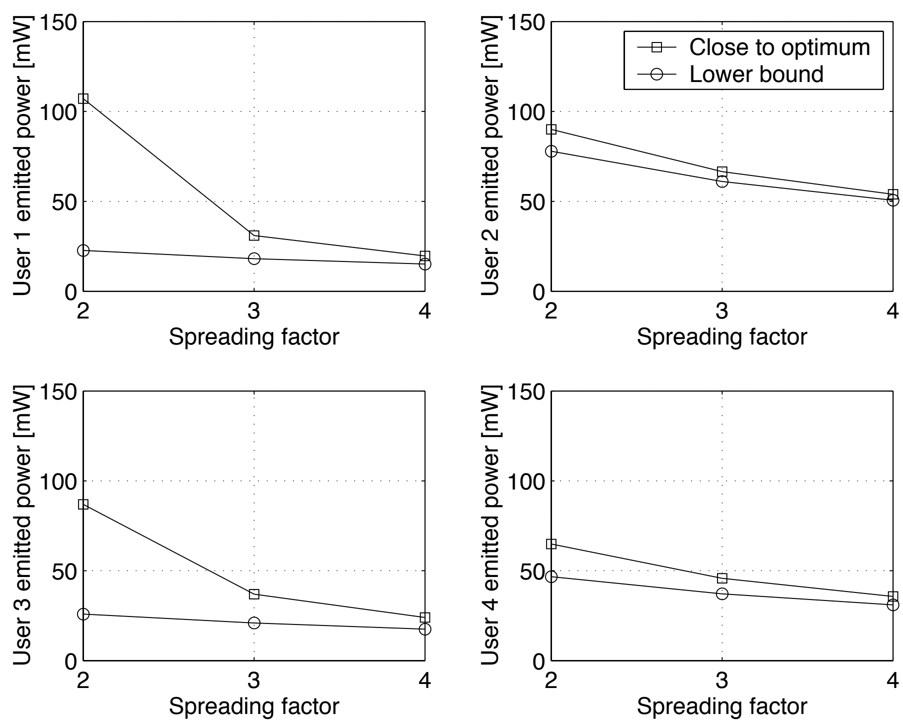

Figure 12: Emitted powers as a function of the spreading factor in case of the close to optimum solution, $K = 4$, $N = 5$, $N_d^e = 2$, $N_d^r = 4$.

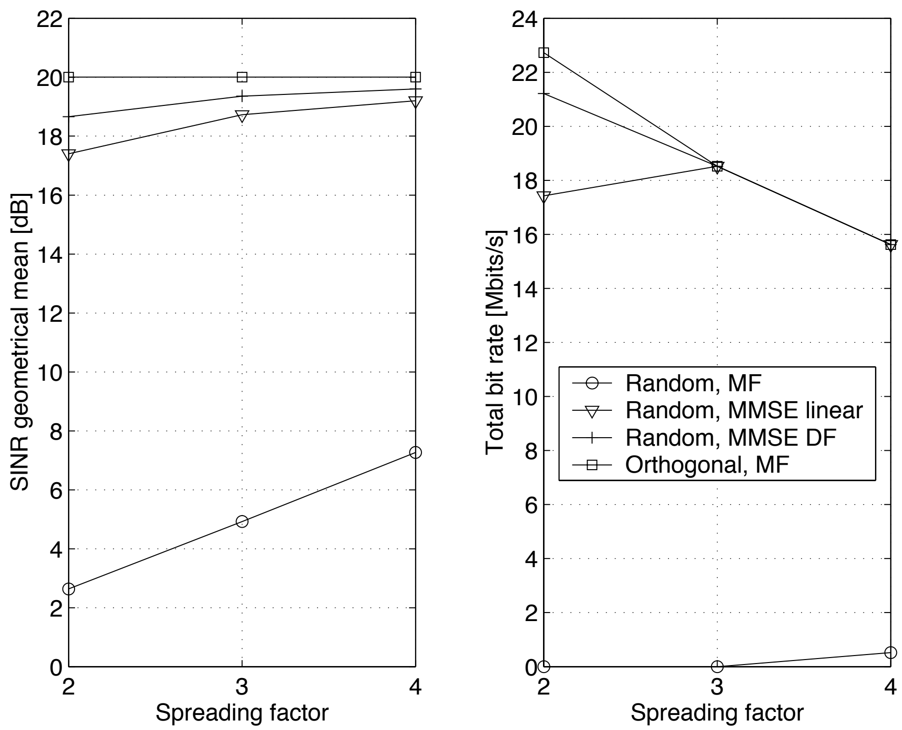

Figure 13: SINR geometrical mean and total bit rates as a function of the spreading factor, $K = 4$, $N = 5$, $N_d^e = 2$, $N_d^r = 4$.