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A general formula for the WACC

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Abstract

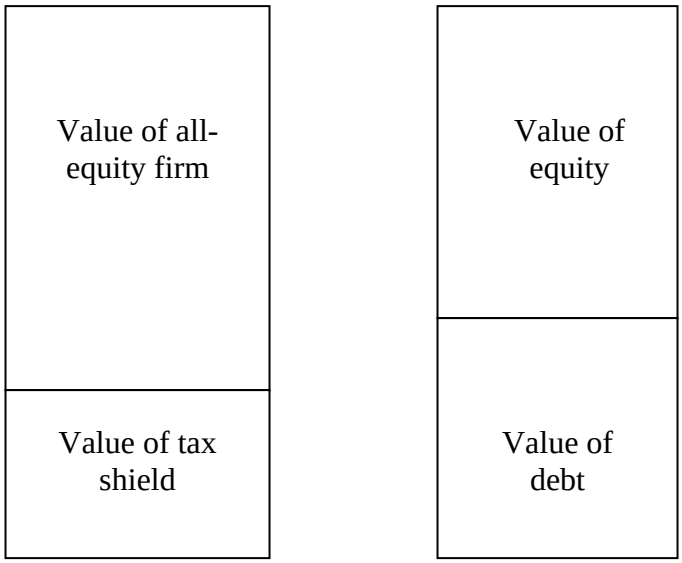
Recent controversies testify that the tax shield valuation remains a hot topic in the financial literature. Basically, two methods have been proposed to incorporate the tax benefit of debt in the present value computation: The adjusted present value (APV), and the classical weighted average cost of capital (WACC). This note clarifies the relationship between these two apparently different approaches by offering a general formula for the WACC. This formula encompasses earlier results obtained by Modigliani and Miller (1963) and Harris and Pringle (1985).

1. Introduction

Firms' interest expenses are tax-deductible. Therefore, debt increases the cash flows available to stockholders and bondholders by the amount of the tax reduction. A higher value for the firm follows. While this fact is straightforward to understand, the correct valuation of tax shields remains highly controversial (see, e.g. Goffin, 2004; Chrissos and Gillet, 2003; Farber et al., 2004). A recent paper by Fernandez (2004) has revived the debate which dates back from the sixties, when Modigliani and Miller (1963) have first pointed out the fiscal influence on the cost of capital.

This paper derives a general formula for the weighted average cost of capital (WACC). The argument starts from the market value balance sheet of a levered firm (Figure 1).

Figure 1: The market value balance sheet of a levered firm



Each side of the balance sheet provides a way to compute the firm's value. The left-hand box in Fig.1, i.e., the assets view of the balance sheet, leads to the separation of the indebted firm into two components, namely, its all-equity counterpart and the tax shield.

The right-hand box, i.e., the financing view of the balance sheet, expresses the value of the firm as the sum of the equity and debt present values.

This equivalence implies that the value, V, of the levered firm is given by:

$$V = VU + VTS = E + D\,, \tag{1}$$

where VU is the value of the equivalent all-equity firm (equal to the value of the unlevered free cash flows), VTS is the tax shield, E is the value of the equity, and D is the value of the debt.

From Eq. (1), by equating the expected returns resulting, one gets an expression the firm's return on equity which remains valid under any assumption regarding the tax shield and its discounting factor. The resulting return on equity may then be incorporated in the classical definition of the WACC (weighted average of the cost of the firm's assets and liabilities) leads to the general formula. Moreover, this paper shows that the general formula encompasses earlier results obtained by Modigliani and Miller (1963) and Harris and Pringle (1985).

The paper is organized as follows. Section 2 derives the return on equity from the balance sheet argument. In Section 3, the general formula for the WACC is obtained and illustrated by two special cases. Section 4 concludes.

2. The return on equity

The adjusted present value resulting from the asset view of the balance sheet (Fig. 1, left-hand side) uses different discounting factors for the unlevered cash flows and the tax shield. Thanks to Eq. (1), the value of the levered firm, V, is the value of the all-equity firm VU, plus the present value of the tax shield VTS:

$$V = VU + VTS \tag{2}$$

In Eq. (2), the two terms of the right-hand side are evaluated separately. The value of the all-equity firm VU is obtained by discounting the free cash flows at the opportunity cost of capital, denoted r_A. Tax savings discounted at the corresponding rate r_{TS} yield the present value of the tax shield. In practice, two polar cases may be considered depending on the risk associated to the firm's future debt level.

From Eq. (2), the value of the levered firm can be viewed as a portfolio composed of the unlevered firm and the tax shield. The expected return r_V, resp. the beta β_V, of a portfolio is the weighted average of the expected returns, resp. the betas, of its components, namely r_A, resp. β_A, for the unlevered firm, and r_{TS}, resp. β_{TS}, for the tax shield:

$$r_V = r_A \frac{VU}{V} + r_{TS} \frac{VTS}{V}, \tag{3}$$

and:

$$\beta_V = \beta_A \frac{VU}{V} + \beta_{TS} \frac{VTS}{V}. \tag{4}$$

From the investors' perspective, the value of the levered firm is the sum of the market value of equity, E, and the market value of debt, D:

$$V = E + D. \tag{5}$$

Starting from (5), the expected return is:

$$r_V = r_E \frac{E}{V} + r_D \frac{D}{V}, \tag{6}$$

where r_E, resp. r_D, is the expected return of the equity, resp. the debt. The betas are obtained accordingly by:

$$\beta_V = \beta_E \frac{E}{V} + \beta_D \frac{D}{V}. \tag{7}$$

As assets and liabilities are equal, we obtain from (5) and (6):

$$r_A \frac{VU}{V} + r_{TS} \frac{VTS}{V} = r_E \frac{E}{V} + r_D \frac{D}{V}, \tag{8}$$

and:

$$\beta_A \frac{VU}{V} + \beta_{TS} \frac{VTS}{V} = \beta_E \frac{E}{V} + \beta_D \frac{D}{V}. \tag{9}$$

Solving for r_E and β_E, the following general expressions are derived:

$$r_E = r_A + (r_A - r_D)\frac{D}{E} - (r_A - r_{TS})\frac{VTS}{E}, \tag{10}$$

and:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\frac{D}{E} - (\beta_A - \beta_{TS})\frac{VTS}{E}. \tag{11}$$

Eq. (10) and (11) provide a very general setting for computing the expected return and beta on equity. Indeed, as the discounting rate r_{TS} and the beta β_{TS} are not constrained, these two formulas remain valid whatever the specification of the tax shield. They are even applicable in the context of valuation through option models (Leland, 1994).

Two special cases deserve to be mentioned. First, if the tax shield has the same risk as the debt ($r_{TS} = r_D$ and $\beta_{TS} = \beta_D$), then (10) and (11) become:

$$r_E = r_A + (r_A - r_D)\frac{D - VTS}{E}, \tag{12}$$

and:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\frac{D - VTS}{E}. \tag{13}$$

Second, if the tax shield has the same risk level as the assets of the firm ($r_{TS} = r_A$ and

$\beta_{TS} = \beta_A$), then Eq. (10) and Eq. (11) simplify to:

$$r_E = r_A + (r_A - r_D)\frac{D}{E}, \tag{14}$$

and:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\frac{D}{E}. \tag{14}$$

The general expression of the return on equity provided by Eq. (10) will be used in the next section to formulate the WACC.

4. The WACC formula

The WACC is defined by the following standard formula:

$$WACC = r_E\left(1 - L\right) + r_D\left(1 - T_C\right)L, \tag{21}$$

with:

$$L = \frac{D}{V}.$$

From Eq. (13), the return on equity may be expressed as:

$$r_E = r_A + \left(r_A - r_D\right)\frac{D}{E} - \left(r_A - r_{TS}\right)\frac{VTS}{E}. \tag{22}$$

Substituting for r_E in Eq. (21) yields:

$$\boldsymbol{WACC = r_A\left(1 - \frac{VTS}{V}\right) - r_D T_C \frac{D}{V} + r_{TS}\frac{VTS}{V}}. \tag{23}$$

Eq. (23) provides a general formulation for the WACC whatever the hypotheses made on the risk structure of the future debt. It was made possible thanks to the previous balance sheet approach developed in Section 3. In this formula, the tax shield VTS and its discounting rate r_{TS} are left unconstrained. Therefore, Eq. (23) may not be subject to the Fernandez' (2002) criticism expressed in the paper entitled "The value of tax shield is NOT equal to the present value of tax shield". Indeed, in our setting, any tax shield value MUST BE the present value of tax shield, precisely because the discounting rate r_{TS} is the one to be applied to the tax shield.

The next section will show the Eq. (23) encompasses special cases examined by several authors.

5. Special cases

We will consider successively two rules regarding the debt future structure of the firm. The general WACC formula, Eq. (23), applies to each of them but the relationship between the WACC and the opportunity cost of capital r_A varies. Consequently, the expected return on equity also changes. The two chosen rules correspond to situations examined in the literature. Rule 1 refers to a constant debt level in the framework of constant perpetuities (Modigliani and Miller, 1963). Rule 2 assumes a constant proportion of debt (Harris and Pringle, 1985).

Rule 1 (Modigliani Miller): constant debt level

The hypotheses are twofold:

- The level of debt is constant
- The tax shield discounting rate is: $r_{TS} = r_D$.

Modigliani and Miller (1963) derive the following formulas for the WACC and the expected return on equity:

$$WACC = r_A(1 - T_C L), \tag{24}$$

where:

$$L = \frac{D}{V} = \frac{D}{D+E},$$

and:

$$r_E = r_A + (r_A - r_D)(1 - T_C)\frac{L}{1-L}. \tag{25}$$

Eq. (24) and Eq. (25) may be reinterpreted as special cases of our general specifications, namely Eq. (22) for the return on equity and Eq. (23) for the WACC. Indeed, under the two hypotheses,

$$VTS = \frac{T_C r_D D}{r_D} = T_C D. \tag{26}$$

Eq. (22) then becomes:

$$\begin{aligned} r_E &= r_A + \left(r_A - r_D\right)\frac{D}{E} - \left(r_A - r_D\right)\frac{T_C D}{E} = r_A + \left(r_A - r_D\right)\left(\frac{D}{E} - \frac{T_C D}{E}\right) \\ &= r_A + \left(r_A - r_D\right)\frac{D}{E}\left(1 - T_C\right), \end{aligned} \tag{27}$$

which is equivalent to Eq. (25) since:

$$\frac{L}{1-L} = \frac{\frac{D}{D+E}}{1 - \frac{D}{D+E}} = \frac{D}{E}. \tag{28}$$

Subsequently, Eq. (23) is transformed into:

$$WACC = r_A\left(1-\frac{T_C D}{V}\right) - r_D T_C \frac{D}{V} + r_D \frac{T_C D}{V} = r_A\left(1-T_C L\right) \tag{29}$$

Rule 2: constant debt ratio

This rule can be used for any set of free cash flows. Based on Miles and Ezzel (1980), Harris and Pringle's(1985) model supposes that all tax shield have the same risk as the firm's asset and should be discounted at the opportunity cost of capital.

Rule 2 (Harris and Pringle)

The hypothesis is:

- D is proportional to VU

As a consequence:

- $r_{TS} = r_A$

The results obtained by Harris and Pringle (1985) may be summarized in the following way for the WACC and the expected return on equity:

$$WACC = r_E(1-L) + r_D(1-T_C)L = r_A - r_D T_C L\,, \tag{30}$$

and:

$$r_E = r_A + (r_A - r_D)\frac{L}{1-L}\,. \tag{31}$$

The proof of these results from Eq. (22) and Eq. (23) goes as follows:

$$r_E = r_A + \left(r_A - r_D\right)\frac{D}{E} - \left(r_A - r_A\right)\frac{VTS}{E} = r_A + \left(r_A - r_D\right)\frac{D}{E}, \tag{32}$$

Then, thanks to Eq. (28), one has:

$$r_E = r_A + \left(r_A - r_D\right)\frac{L}{1-L}. \tag{33}$$

For the WACC, Eq. (23) yields:

$$\begin{aligned} WACC &= r_A\left(1 - \frac{VTS}{V}\right) - r_D T_C \frac{D}{V} + r_A \frac{VTS}{V} = r_A - r_D T_C \frac{D}{V} \\ &= r_A - r_D T_C L \end{aligned} \tag{34}$$

Conclusion

The WACC is a fundamental concept in corporate finance. Its basic definition, averaging the cost of capital coming from both the equity and the debt, looks simple. However, as a matter of fact, its practical implementation yields several questions, mostly linked to the distinction between book values and market values. This paper addresses more specifically the tax shield valuation and establishes a general formula which remains valid for any debt structure.

In this context, our contribution allows not only to confront the usual WACC computation to a more progressive and less synthetic one, but also to help firms to adapt the WACC approach to any chosen tax shield valuation model. In this sense, the WACC appears as a powerful and very adaptable concept. Nevertheless, several firms' peculiarities as intermediated credit, bankruptcy costs, international financial engineering projects, etc., are still easier to value separately rather than through a WACC. Therefore, the generalization of this paper to such complex situations remains a challenging issue.

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