Novel Block Constructions using an Intrafix for CPM with Frequency Domain Equalization

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***Abstract*—To enable frequency domain equalization (FDE) for continuous phase modulation (CPM), both cyclicity of individual symbol blocks and phase continuity between different blocks have to be guaranteed. In this letter, we present new block constructions that use a subblock of data-dependent symbols, called *intrafix*, to satisfy both constraints for different CPM-FDE systems: using either a cyclic prefix or a training sequence (TS), both for precoded and nonprecoded CPM. The known symbols of a TS can be used to improve synchronization and channel estimation. Precoding can be applied to a certain class of CPM schemes to halve the bit error rate.**

***Index Terms*—Continuous phase modulation (CPM), frequency domain equalization (FDE), cyclic prefix, training sequence, precoding**

I. INTRODUCTION

Continuous phase modulation (CPM) has been proposed as an attractive modulation technique for 60 GHz communications in [1], [2] and [3] and in the IEEE 802.15.3c standardization committee [4]. However, demodulating CPM entails a high system complexity [5]. This complexity can be lowered by equalizing the channel in the frequency domain [6]. To enable frequency domain equalization (FDE) with only one complex multiplication per sample, cyclic symbol blocks have to be created. In linear single-carrier and orthogonal frequency division multiplexing (OFDM) systems, this can be achieved by simply inserting a cyclic prefix (CP) into each block of data.

However, it is known that just the insertion of a CP is not sufficient for CPM [7]. An extra subblock, below called *intrafix*, has to be inserted into each block to cope with the memory in the CPM waveform. Although the term was first coined in [8], the use of such an intrafix has been suggested by many authors. In [7], the use of two sequences of tail symbols within each block was proposed. In [9], it was suggested that only one such a sequence was sufficient. In [10], it was then shown that the modulation index of a CPM scheme imposes an additional constraint on the technique of [7]. In [11] finally, it was proven that the intrafix technique is applicable to any CPM scheme and it was shown how the intrafix has to be constructed.

In this letter, we complement [11] by presenting how the intrafix can be used in CPM-FDE systems using a training sequence (TS) rather than a CP. Using a TS of known symbols–sometimes also called unique word [12]–between blocks of data mitigates interblock interference (IBI) exactly like a CP does. However, an important advantage of a TS is that it can be used by the receiver to improve synchronization and channel estimation, whereas the CP is simply discarded [13]. Moreover, using a TS entails no equalization performance loss [13]. Finally, the net throughput for CP and TS systems is very similar, as illustrated below in this letter. For these reasons, the latest versions of the IEEE [4] and ECMA [14] standards for communications at 60 GHz indeed propose the use of a TS rather than a CP.

We also apply the intrafix theory to enable FDE of differentially precoded CPM systems. This precoding can be used with the specific, popular class of CPM schemes with modulation index $h = 1/2Q$, where Q is any integer [15]. It approximately halves the bit error rate (BER) compared to nonprecoded CPM without any extra overhead or complexity increase.

This letter is structured as follows. In Section II, we briefly review the interest of using a TS rather than a CP. We then introduce the CPM waveform in Section III and we explain in Section IV why an intrafix is needed to enable FDE for this CPM waveform, on top of the CP or TS. In Section V we subsequently show that two intrafixes are needed in each block to enable FDE of differentially precoded CPM systems. Finally, we present simulation results in Section VI and draw conclusions in Section VII.

II. CYCLIC PREFIX VERSUS TRAINING SEQUENCE

In this section, we explain the equivalence between using a CP and a TS. We also highlight the differences between the two systems. We conclude that a TS has important advantages over a CP.

We consider a block-based communication system as explained in [9]. Parameters with respect to the l^{th} block are denoted with a superscript (l). In the case of a CP, the last N_P symbols of each block of length N are copied in front of the block, creating blocks of length $N_T = N + N_P$, see Fig. 1 a). In the case of a TS, N_P known symbols are appended to each block of length $N - N_P$, creating blocks of length N, see Fig. 1 b). To avoid inter-block interference, the length N_P of the CP or TS is chosen such that $N_P > L_C$ where $L_C T$

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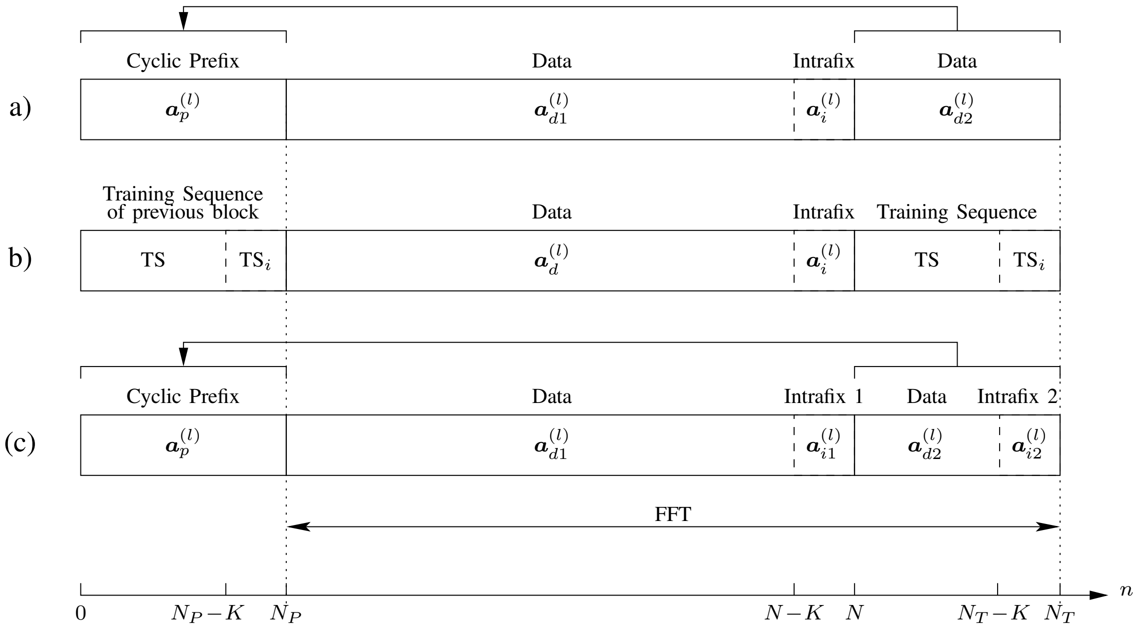

Fig. 1. Structure of an overall data block $\boldsymbol{a}^{(l)}$ in the case of (a) cyclic prefix, (b) training sequence and (c) cyclic prefix for precoded CPM with FDE.

is the length of the channel delay spread and T is the symbol duration.

To illustrate the similarity between the CP and TS approach, we have included the TS belonging to the previous block $\boldsymbol{a}^{(l-1)}$ in Fig. 1. This TS contains exactly the same symbols as the TS of block $\boldsymbol{a}^{(l)}$. However, we stress that blocks of $N_T = N + N_P$ symbols are transmitted in the case of a CP, of which N contain useful data. When using a TS, blocks of N symbols are transmitted, of which $N - N_P$ contain useful data. For both systems, a Fast Fourier Transform (FFT) of size N is calculated to enable FDE: when using a CP, it is simply discarded in the receiver. Therefore, the useful bit rate of a CP system R_{CP} relates to the useful bit rate of a TS system R_{TS} as:

$$\frac{R_{\text{CP}}}{R_{\text{TS}}} = \frac{N^2}{N^2 - N_P^2}. \tag{1}$$

For instance, if the CP or TS length is one quarter of the FFT length $N_P = N/4$, then the CP system has a 6.7% higher net throughput than the TS system.

However, as explained in [13], the known symbols of the TS can be used for channel estimation and synchronization. The symbols of the CP on the other hand, are not known at the receiver. Therefore, we have to include N_L known pilot carriers in the blocks when using a CP. The ratio between the useful bit rates then becomes:

$$\frac{R_{\text{CP}}}{R_{\text{TS}}} = \frac{N^2 - N_L N}{N^2 - N_P^2}. \tag{2}$$

Assuming a system with $N_L = 4$ pilot carriers, $N = 256$ and $N_P = 64$, the CP system only has a 5% higher net throughput than the TS system. If the channel dispersion length drops and the CP or TS size is reduced to $N_P = 16$, the TS system obtains a 1.2% higher net rate. We can conclude that the differences in net throughput between CP and TS systems are limited. However, the advantage of the known symbols in the TS is significant.

For CPM, an intrafix has to be inserted into each block on top of the CP or TS as we will see below. This intrafix of length K would then bring the ratio to

$$\frac{R_{\text{CP}}}{R_{\text{TS}}} = \frac{N^2 - N_L N - KN}{N^2 - N_P^2 - KN - N_P K}. \tag{3}$$

However, K is typically very small. For a binary CPM scheme with modulation index $h = 1/2$, the intrafix would contain only 1 symbol. Therefore, it typically changes the net rates by less than 1%.

In the next section, we will introduce CPM. We will point out that CPM waveforms contain memory, which has to be taken into account when inserting a CP or a TS into a block of sent symbols. Therefore, using a CP or TS is not as trivial for CPM as it is for common linear modulation techniques.

III. Continuous Phase Modulation

In this section, we briefly review the main characteristics of CPM. A transmitted CPM signal has the form:

$$s(t, \boldsymbol{a}) = \sqrt{\frac{2E_S}{T}} e^{j\,\phi(t,\boldsymbol{a})} \tag{4}$$

where $\boldsymbol{a}$ contains the sequence of M-ary amplitude shift keyed (ASK) symbols $a_n \in \{\pm 1, \pm 3, ..., \pm(M-1)\}$ [5]. Below, all vectors are written in boldface and we assume that the energy per symbol E_S is normalized to $E_S = 1$. The transmitted information is contained in the phase:

$$\phi(t, \boldsymbol{a}) = 2\pi h \sum_n a_n . \, q(t - nT) \tag{5}$$

where h is the *modulation index* and $q(t)$ is the *phase response*, related to the *frequency response* $f(t)$ by the relationship $q(t) = \int_{-\infty}^{t} f(\tau)\,\mathrm{d}\tau$. The pulse $f(t)$ is a smooth pulse shape over a finite time interval $0 \leq t \leq LT$ and zero outside, where L is an integer. The function $f(t)$ is normalized such that $\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t = \frac{1}{2}$.

The phase $\phi(t, \boldsymbol{a})$ during interval $nT \leq t \leq (n+1)T$ can then also be written as:

$$\phi(t, \boldsymbol{a}) = h\pi \sum_{i=0}^{n-L} a_i + 2\pi h \sum_{i=n-L+1}^{n} a_i \cdot q(t - iT). \tag{6}$$

In this expression, we distinguish two types of memory in the CPM signal: the *phase state*

$$\theta_n = h\pi \sum_{i=0}^{n-L} a_i \mod 2\pi, \tag{7}$$

and the *correlative state*

$$\boldsymbol{\sigma}_n = (a_{n-1}, a_{n-2}, \ldots, a_{n-L+1}). \tag{8}$$

Together, they form the *state* of the CPM signal $\boldsymbol{\chi}_n = (\theta_n, \boldsymbol{\sigma}_n)$ which captures all the memory. This memory has to be taken into account to enable FDE [9], as we will review in the next section.

IV. The Intrafix Enables CPM-FDE with a TS

For CPM waveforms, attaching a CP or a TS is not sufficient to enable FDE. To construct a data block which yields a cyclic CPM signal we also have to take the memory introduced by the CPM modulation into account. This memory is reflected by the state $\boldsymbol{\chi}_n^{(l)}$ of the modulator at symbol interval n in block l, as it was explained in Section III. From Fig. 1, it can be seen that the condition to get a cyclic CPM signal with period $N.T$ is

$$\boldsymbol{\chi}_{N_P}^{(l)} = \boldsymbol{\chi}_{N_T}^{(l)} \tag{9}$$

which is equivalent to:

$$\theta_{N_P}^{(l)} = \theta_{N_T}^{(l)} \tag{10}$$

and

$$\boldsymbol{\sigma}_{N_P}^{(l)} = \boldsymbol{\sigma}_{N_T}^{(l)}. \tag{11}$$

We therefore always have to satisfy both a phase state condition (10) and a correlative state condition (11), whether using a CP or a TS.

When using a CP, $\boldsymbol{a}_{d2}^{(l)} = \boldsymbol{a}_p^{(l)}$ by definition: the CP is merely a copy of the last symbols of the block. Therefore, (11) is automatically satisfied. When using a TS, the TS is obviously chosen the same for all blocks. Therefore, the correlative state condition (11) is also always satisfied in this case.

Satisfying the phase state condition (10) is not as trivial. At instant N_T, we have to bring the modulator back into the same phase state as where it was at instant N_P. Provided that N is chosen to be even–which is always the case to ensure an efficient implementation of the FFT–it was shown in [11] that this can be done by inserting an intrafix $\boldsymbol{a}_i^{(l)}$ of K data-dependent symbols into each block, where $K = \left\lceil \frac{p-1}{M-1} \right\rceil$. Here, $\lceil \cdot \rceil$ is the smallest integer greater than or equal to x and p is the denominator of the modulation index, written as $h = m/p$ where m and p have no common factors. Thanks to the equivalent model for a TS and a CP of Fig. 1, exactly the same technique can be applied for a TS: inserting $\boldsymbol{a}_i^{(l)}$ into $\boldsymbol{a}_d^{(l)}$ will guarantee that (10) is always satisfied.

It is possible to design the TS such that

$$\boldsymbol{\chi}_0^{(l)} = \boldsymbol{\chi}_{N_P}^{(l)} \tag{12}$$

by including an additional intrafix TS_i in the TS itself, as shown in Fig. 1 b). As the whole TS is composed of known symbols, this does not entail a loss of net data rate. Satisfying (12) is not necessary to guarantee cyclicity of the block. However, it has some other potential advantages. First, it renders the TS cyclic, yielding some attractive autocorrelation properties that could be exploited to improve synchronization [16]. Second, it ensures that

$$\theta_0^{(l)} = \theta_{N_P}^{(l)} = \theta_N^{(l)} = \theta_{N_T}^{(l)}. \tag{13}$$

This knowledge could be exploited in the Viterbi decoder that is present in any optimal CPM demodulator [17] to improve detection performance. Moreover, as we will see in the next section, it can be used to enable FDE of differentially precoded CPM systems.

V. Precoded CPM with FDE

In this section, we show how FDE can be applied to precoded CPM. Differential precoding can be used with the specific, popular class of CPM schemes with modulation index $h = 1/2Q$, where Q is any integer [15]. It approximately halves the BER compared to nonprecoded CPM without any extra overhead or complexity increase. However, it also imposes additional constraints on the block structure as we will see in this section. These constraints have to be taken into account to enable FDE.

We first introduce the CPM demodulator. Exploiting the Laurent decomposition [18], we can write (4) as a sum of $P = 2^{L-1}$ linearly modulated signals:

$$s(t) = \sum_{p=0}^{P-1} \sum_{n} b_{p,n}\, l_p(t - nT) \tag{14}$$

where the *pseudocoefficients* $b_{p,n}$ are given by

$$b_{p,n} = \exp\left[j\pi h \left(\sum_{i=0}^{n} a_i - \sum_{i=1}^{L-1} a_{n-i}\, \beta_{p,i} \right) \right] \tag{15}$$

with $\beta_{p,i}$ the i^{th} bit in the binary representation of p ($p = \sum_{i=1}^{L-1} 2^{i-1} \beta_{p,i}$). The Laurent pulses $l_p(t), p = 0, ..., P-1$ are real, with $l_p(t) = 0$ for $t < 0$ and $t > (L+1)T$. This linear representation of CPM (14) can be used to construct a CPM demodulator: it is known [17] that a Viterbi receiver can process the received CPM signal to generate a maximum likelihood estimate of the first pseudocoefficient:

$$\hat{b}_{0,n} = \exp\left(j\pi h \sum_{i=0}^{n} \hat{a}_i \right). \tag{16}$$

This vector of pseudocoefficients $\boldsymbol{b}_0 \doteq \left[\hat{b}_{0,0}, \hat{b}_{0,1}, \ldots, \hat{b}_{0,N_T} \right]$ is a sufficient statistic for $\boldsymbol{a}$: as stated in [15], $\boldsymbol{a}$ can be uniquely determined by $\boldsymbol{b}_0$. We can therefore extract the estimate of the input symbols $\hat{\boldsymbol{a}}$ from the estimate $\hat{\boldsymbol{b}}_0$ without losing any information. We will show in the next two sections how this can be done, both for nonprecoded and for precoded CPM.

If we do not apply any precoding, the detection goes as follows. Because $\left|\hat{b}_{0,n-1}\right| = 1$ it follows that $\hat{b}_{0,n-1}^{-1} = \hat{b}_{0,n-1}^{*}$ so that equation (16) can be written as

$$\exp\left(j\pi h\hat{a}_n\right) = \hat{b}_{0,n} \cdot \hat{b}_{0,n-1}^{*}. \tag{17}$$

This equation allows us to extract $\hat{\boldsymbol{a}}$ from $\hat{\boldsymbol{b}}_0$. However, we can see that one error in $\hat{\boldsymbol{b}}_0$ will on average cause two errors in $\hat{\boldsymbol{a}}$. We want to avoid this by applying a precoding.

In [15] it is shown that differential encoding can be applied to the class of binary CPM signals with $h = 1/(2Q)$ where Q is any integer, to annihilate the inherent differential decoding of CPM signals and thus improve error performance for coherent detection. In the precoder, the information bits $\boldsymbol{i}$, $i_n \in \{0, 1\}$ are first differentially encoded: $p_n = i_n \oplus i_{n-1}$ where $\oplus$ represents the modulo 2 addition. As we will explain below, we assume that

$$\hat{i}_{-1} = 0. \tag{18}$$

Because $\hat{i}_i \oplus \hat{i}_i = 0$, (16) can then be written as

$$\hat{b}_{0,n} = \exp\left(-j\frac{\pi}{2Q}n\right) \cdot \exp\left(j\frac{\pi}{Q}\hat{i}_n\right). \tag{19}$$

Contrary to (17), one error in $\hat{\boldsymbol{b}}_0$ will now on average cause only one error in $\hat{\boldsymbol{i}}$. This is what we wanted to obtain by precoding the information bits.

We assume that we want to combine precoding with FDE using a CP. However, the intrafix technique can also be applied to enable precoded CPM with FDE using a TS. From Fig. 1(c), it can be seen that the condition to get a cyclic CPM signal with period NT after discarding the CP is (9), also in this case where we add precoding. Indeed, this constraint is imposed to enable FDE and is the same for both nonprecoded and precoded CPM.

However, an extra constraint is imposed by the precoding. At the beginning of each block (i.e. at $n = 0$), we have to initialize the encoder by setting $i_{-1} = 0$, such that assumption (18) is valid. Nevertheless, in the receiver the CP is discarded. Therefore, we do not start decoding at $n = 0$ but at $n = N_P$. To enable correct decoding from this instant onwards, we have to bring the transmitter into the same state at $n = N_P$ as where it was at $n = 0$. We therefore also have to satisfy

$$\boldsymbol{\chi}_0^{(l)} = \boldsymbol{\chi}_{N_P}^{(l)}, \tag{20}$$

or, because $\boldsymbol{a}_p^{(l)} = \left[\boldsymbol{a}_{d2}^{(l)}; \boldsymbol{a}_{i2}^{(l)}\right]$ by definition,

$$\boldsymbol{\chi}_N^{(l)} = \boldsymbol{\chi}_{N_T}^{(l)}. \tag{21}$$

We can satisfy both (9) and (21) easily by inserting two intrafixes into each block as shown in Fig. 1(c). We first insert the second intrafix $\boldsymbol{a}_{i2}^{(l)}$ to satisfy (21). This intrafix enables correct decoding of the precoded CPM. As it is explained in [11], this intrafix is therefore calculated such that

$$h\pi\left(\sum_{n=0}^{N_P-K-1} a_{d2,n}^{(l)} + \sum_{n=0}^{K-1} a_{i2,n}^{(l)}\right) = 0 \mod 2\pi. \tag{22}$$

Next, the first intrafix $\boldsymbol{a}_{i1}^{(l)}$ is inserted to satisfy (9). From [11], we can derive that it has to be calculated such that:

$$h\pi\left(\sum_{n=0}^{N-N_P-K-1} a_{d1,n}^{(l)} + \sum_{n=0}^{K-1} a_{i1,n}^{(l)}\right) = 0 \mod 2\pi. \tag{23}$$

This intrafix guarantees cyclicity of the overall block, thus enabling FDE of the precoded CPM waveform.

VI. Simulation Results

Bit error rate (BER) simulations have been performed with both a CP and a TS using the CPM-FDE receiver described in [8]. The minimum mean square error (MMSE) channel equalizer was chosen. The binary 3-RC CPM scheme ($f(t) = (1 - \cos\frac{2\pi t}{LT})/2LT$ with $L = 3$) was simulated. Modulation indices $h = 0.5$ and $h = 0.25$ were chosen. As CPM-FDE has recently been proposed for high-rate communications at 60 GHz [3], we have performed simulations in a 60 GHz multipath environment, as described in [8]. A huge bandwidth is available at 60 GHz, so the bit rate is $R_b = 1$ Gbit/s. A blocksize $N = 256$ and CP or TS length $N_P = 64$ were simulated. The receiver lowpass (LP) filter is modeled as a raised cosine filter with roll-off factor $R = 0.5$.

Fig. 2 shows the BER performance in the 60 GHz environment, both with a CP (square markers) and a TS (circle markers). Comparing the TS and CP curves, it can be seen that the CP performance is slightly better. This is because in the case of a CP, $\frac{N}{N+N_P}$ of the total available energy is available for useful bits, whereas in the case of a TS this is only $\frac{N-N_P}{N}$. For $N = 256$ and $N_P = 64$, this yields 80% for the CP, and only 75% for the TS. This “loss” explains the small gap between the CP and TS curves. For two reasons we can conclude that both the CP and the TS constructions satisfy all requirements, both for the $h = 0.5$ (dashed lines) and the $h = 0.25$ (solid lines) cases. First, the CP or TS is sufficiently long. Otherwise interblock interference (IBI) would not be mitigated, causing BER flooring. Second, the intrafix is correctly calculated in both cases, yielding a cyclic CPM signal. This can again be seen from the fact that no BER flooring occurs, so that we can conclude that the FD equalizer operates correctly.

Also shown are the BER of the precoded $h = 0.25$ and $h = 0.5$ CPM schemes in the 60 GHz environment (diamond markers). As can be seen, precoding approximately reduces the BER by half. We can therefore conclude that the precoding and the FDE were combined correctly.

Finally, to corroborate the use of a TS, the BER performance in a nondispersive environment of the block synchronization algorithm presented in [13], applied to CPM, is shown. The synchronization with the TS was done using 10 training sequences. Synchronization using a CP can only use 2 sequences because the CP content changes with the block's

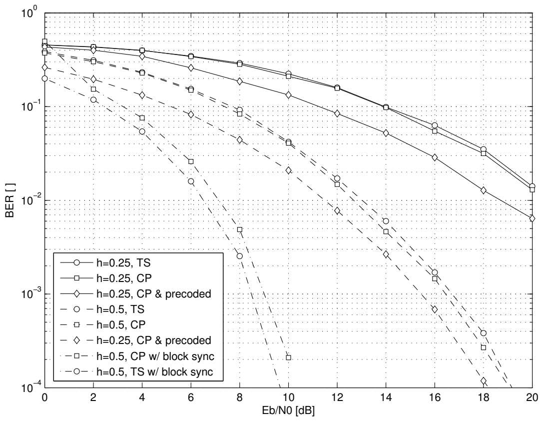

Fig. 2. BER performance of the $3RC$, $h = 0.5$ (dashed lines) and $h = 0.25$ (solid lines) schemes in a 60 GHz multipath environment with a CP (square markers), a TS (circle markers) and with precoding and a CP (diamond markers). The dash-dotted lines represent the performance with block synchronization in a nondispersive environment using a CP and a TS.

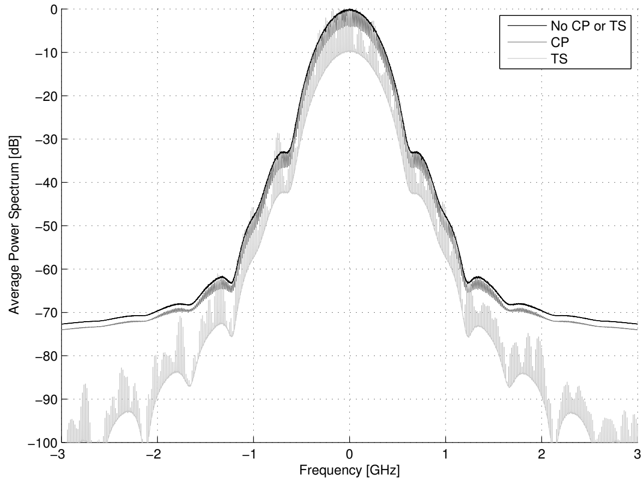

Fig. 3. PSDs of the $h = 0.5$ scheme for three cases: without a CP or TS, with a CP and with a TS. A CP introduces a ripple of about 4 dB, a TS up to 10 dB.

data. Therefore, the TS-based synchronization outperforms the CP-based synchronization, especially at low SNR. A further discussion of the error performance of this synchronization algorithm can be found in [19].

Fig. 3 shows the power spectral density (PSD) of the $h = 0.5$ scheme for three cases: without a CP or TS, with a CP and with a TS. A system with $N = 64$ and $N_P = 16$ was chosen to keep the computation time acceptable and to compare the results to [20]. The FFT size was set to 8192 to obtain a high resolution and a Hamming window was applied before taking the FFT to avoid introducing false frequencies. The highest value of the PSDs was normalized to 0 dB to emulate a situation where the PSDs have to fit below a spectral mask imposed by a regulator. Comparable to what is observed in [20] for OFDM, the CP introduces a strong ripple of about 4 dB into the PSD. Therefore, the transmit power has to be reduced to fit below the spectral mask. The TS introduces an even bigger ripple of about 10 dB. On the other hand, using a TS lowers the sidelobes compared to the systems without a CP or TS and with a CP. Ripple minimization would therefore be an interesting design criterion for constructing the TS.

VII. Conclusion

To enable low-complexity frequency domain equalization for CPM, cyclic blocks have to be created. At the same time, phase continuity between different blocks has to be guaranteed to retain the favorable properties of the CPM waveform. We have shown in this letter that these two requirements can be satisfied at the same time by inserting an intrafix into each block in addition to either a cyclic prefix or a training sequence. To apply this technique to precoded CPM, a second intrafix has to be inserted into the cyclic prefix or training sequence, enabling correct decoding at the receiver. To validate our new algorithms, we have presented simulation results in a 60 GHz enviroment.

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