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The Too Discrete Charm of Mathematics

As a participant to this ICM, you are of course already convinced of the following obvious facts: mathematics is a living science, it is useful, it is fun, and it leads to a variety of careers. You are also aware presumably that these facts are mostly unknown to the public, whose only experience of mathematics is from school – for better or worse.

Even fellow scientists of other fields and policy makers sometimes see mathematics as a collection of tools, ready to be taken "off the shelf" and used in science without further development.

This general misconception about mathematics itself and its interactions with so many fields of activity is obviously harmful to its future development.

Luckily, over the last twenty years, various mathematicians have started to take seriously the task of popularising the interest and usefulness of mathematical research and this trend is accelerating.

A number of successful plays, films or novels with a mathematical background have recently appeared, and we should both encourage and use this wave of interest in our field and profession.

This question will be discussed at the present ICM during a panel discussion on the theme „Should mathematicians care about communicating to broad audiences?" organised by the European Mathematical Society. The present text merely describes my own enjoyable efforts in that direction.

Mathematics and theatre

In August 2001, I attended a performance of the play "Proof" by David Auburn in New York. It was a huge success, earning the author the Pulitzer Prize and running for more than three years on Broadway.

In June 2002, the theatre company "Le Rideau de Bruxelles" announced that it would be translating and staging the play in French in September. On an impulse, I wrote to the directors of the theatre (whom I didn't know at all) offering assistance

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in case they had any mathematical question about the contents of the play.

I don't know if I really expected an answer, but as it happened their enthusiastic reply led me to an intriguing four-year involvement with the theatre.

The play "Proof" is centred on the world of research mathematicians, and one of the characters is loosely inspired by the fate of John Nash, who suffered a serious mental breakdown at the age of thirty, after producing major mathematical contributions. The play is about the proof of a major theorem that this character may or may not have obtained during remissions of his illness, and I find it remarkable that the play could interest, even thrill, both critics and public, when the idea of a proof is so far from their usual interests.

The reference to the character's illness had led the theatre people in Brussels to think that a mathematician necessarily skirted the thin line at the limit of madness.

And at the first meeting I had with the director of the play and the actors, they explained candidly that having read the play they wanted "to see and possibly to touch a mathematician in the flesh". After four hours of intense discussion, I like to believe they concluded that a mathematician could be pretty normal after all.

But their questions – coming from outside the mathematical world – can give us some idea of what we should explain when attempting to popularise mathematics. Firstly, the play being about a long proof of a theorem, they wanted to understand what a proof is, how it looks like, how it differs from a very long numerical computation – and this is not an easy question.

Secondly, mathematicians traditionally present their results or their field in final form. Thus we start with definitions, to proceed with statements then proofs.

But this hides the most appealing part of our work, namely the non-deductive and non-formally logical search for the solution to a problem by various attempts from different angles, by intuition more than deduction. Stressing the role of intuition and the search for elegance and beauty in mathematics struck a chord with the actors and director, and showed them how comparable mathematical and artistic creation could be.

Thirdly, since they were about to take on the roles of mathematicians, they wanted to understand what makes us tick and were genuinely interested in our lives, problems and motivations.

Quite strangely, it was only after many encounters that I understood that they knew there were university professors in mathematics in Brussels, but did not realise that this play about "mathematicians" would be about these people. The two images they had simply did not coincide.

Since then, I always mention to prospective or new students that all the teachers they will meet in their mathematical studies are also researchers, and that, in mathematics, elegance and intuition are as important as deduction.

But the theatre story didn't stop there. It was quickly agreed that the director and I would make a joint presentation before one performance, from the points of view of mathematics and the theatre. We planned for one presentation, but ended up giving more than ten, allowing me to talk about the existence of mathematical research, its enjoyment and possible careers to around 1500 high school students.

In 2003, I read a review of another play with mathematical contents, " Partition" by Ira Hauptman, played at that time only in Berkeley. I suggested to my new friends at the Rideau de Bruxelles that they might also produce it, and they did.

In the author's words, it is a fantasy based on the meeting of G.H. Hardy and S. Ramanujan in Cambridge. It has a solid basis in historical fact, based on Hardy's writings, and a fantasy part with the presence on stage of Fermat and Ramanujan's familiar goddess, Namagiri.

The translation and production of the play in Brussels was an irresistible call to immerse myself in the writings of Hardy, the notebooks of Ramanujan and the Riemann hypothesis, as an amateur discovering new vistas.

Entering a large bookshop in London, I was very pleasantly surprised to find on the main shelf of the science department three different books, each fully devoted to explaining the Riemann hypothesis to a general audience, and some more including it as a chapter.

The play was as successful as "Proof" in Brussels and again I made a series of presentations with Jules-Henri Marchant, director of the theatre and of the production of "Partition".

Hardy's "A Mathematician's Apology" contains sufficiently many strong opinions (some still valid, some less so) to start interesting debates. On some days, to illustrate or contradict his belief that the mathematics of prime numbers is beautiful and immortal, whereas literature is not, Jules-Henri Marchant would end up reciting and analysing Rimbaud's poetry, to which I would recite and analyse Euclid's proof that the number of primes is infinite, or a formula from Ramanujan's notebook. Hopefully, the beauty of each was equally perceived.

These presentations again reached 1500 high school students (and the EU Commissioner for research Philippe Busquin) and I also found myself talking about theatre and (mostly) mathematics on radio and television.

In 2005, „Proof" was made into a film by John Madden, with Gwyneth Paltrow, Jake Gyllenhaal and Anthony Hopkins in the leading roles. The film presumably reached a larger audience than the play, but in terms of creating an activity aimed at presenting mathematics, a theatre performance offers much better opportunities.

Mathematics and research

What is mathematical research and why is it worth pursuing? If we want to convince both young people about to choose their field of study and decision makers in charge of research funds, we must explain this, and to explain it I believe we must show many examples.

Luckily, examples abound which are relatively simple to explain, and more and more can be picked up from books and collections of papers published today.

I always prefer to present subjects that I must first learn myself: then I don't assume anything is easy or known, since I didn't know about it before, and I can share my enthusiasm for my recent discovery. Well, here are some examples.

The play "Partition" got me studying the basis of the Riemann hypothesis, and I found it quite easy to explain elements of prime numbers at different levels.

Bernoulli's formula:

$$\frac{\pi^2}{6} = \frac{1}{(1-\frac{1}{2^2})\cdot(1-\frac{1}{3^2})\cdot(1-\frac{1}{5^2})\cdot(1-\frac{1}{7^2})\cdot(1-\frac{1}{11^2})\ \ldots}$$

is a nice starting point, showing that the length of a circle is somehow encoded in the sequence of all prime numbers, which is used in the infinite product of the right hand side. To me, this already shows beauty and magic in mathematics, with this relation between a priori totally distinct classical notions of geometry and arithmetic.

The Riemann zeta function is obtained from the right hand side by replacing the squares by powers by a real or complex number (with real value greater than one), then by analytic continuation. Thus the Riemann hypothesis can be loosely explained, together with some idea of its relation with prime numbers. And it has the spectacular value of being part both of Hilbert's list of 23 problems in 1900 and of the Clay Foundation's list of the seven Millennium problems a century later.

For those concerned with the usefulness of mathematics, prime numbers are a striking example to give: seen by Hardy as beautiful because unconnected to any useful reality, prime numbers are in fact used regularly by most members of any audience you could address. Indeed, the RSA cryptography of bankcards and electronic commerce is based on the impossibility of factoring a product of two very large prime numbers in a reasonable time.

Other examples would include:

- differential equations, leading both through numerical analysis to the amazing precision of the control of satellites (able to collect pieces of a comet and bring them back), and theoretical speculation, like the fact admitted today that if the moon were not present, the axis of the earth would be unstable-thus making the presence of life, in particular our own, much less likely
- partial differential equations, and for instance the 1822 Navier-Stokes equations of fluid mechanics, which led both to one of the Clay Millennium problems and to the modelling of airbags at the Institute of Applied Mathematics in Kaiserslautern. This modelling itself leads to new questions in mathematics, since the boundary of the domain studied (the airbag) deforms very quickly, requiring more refined mesh free techniques of numerical analysis
- Fourier series, evolving into wavelets, leading to amazingly efficient image treatment, used by some in medical imaging and by all on their computer screens or electronic cameras
- the future construction of ITER, a prototype of a toroidal fusion reactor, that will take the best

The play „Partition"

part of twenty years, involves both modern mathematical modelling of the behaviour of plasma and the basic fact that it must be a torus (and not for instance a sphere) by an old topological result showing that the sphere does not carry any non-zero continuous tangent vector field.

These stories can be adapted to different styles and levels of audience, and aim to illustrate at the same time the beauty of mathematics, its permanence as an endeavour of humanity, its pervasiveness in most fields of activity, the way it progresses both by its internal logic and by dynamic interactions with problems in other fields.

Mathematics and funding

At the founding meeting of the European Mathematical Society in 1990, the Council created a committee dealing with relations with the European Commission. It seemed convenient to have a member of that committee living in Brussels, and being the only person in the room with that qualification I soon became increasingly involved both with the EMS and the EC.

The interaction between the mathematical community and the European Commission has been a slow process, which often brought useful results when they were least expected. It must be stressed that Philippe Busquin, the former Commissioner for Research, made a tremendous job of promoting scientific research and making it a strong component of Europe's political agenda. His successor, Janez Potocnik, shares the same positive vision.

However, a number of scientific programmes do not yet integrate the potentialities of mathematics, or recognise the need to create new mathematics to be applied later.

The twentieth century witnessed major mathematical progress: old problems have been solved thanks to the strong input of new ideas, new methods, new fields. The development of computing capacities allows problems to be tackled that previously remained intractable.

Mathematics is now a subject developing both by internal logic and by interaction with a growing number of domains in science, industry and services. To name the most obvious, mathematics has a very dynamic symbiosis with physics, engineering, biology (genomics), finance, computer science, telecommunications, logistics …

This again is not sufficiently perceived by society, policy makers, and even fellow scientists. Mathematics still appears in some programmes as a subset of physical and engineering sciences, ignoring the general picture.

It was often said that mathematicians need only paper and pen to make their discoveries (forgetting even the indispensable waste paper basket). But this assumed that there *were* mathematicians, with access to comprehensive collections of books and journals.

Today, nothing can be taken for granted, and we have to explain and convince others that covering the needs of the development of mathematics is good value for society. And these needs are quite easy to list: positions at all levels of career, flexibility to organise workshops and intensive courses of all sorts, full access to the literature (impossible on paper, but made possible by electronic storage and communication), computing power where it is relevant, easy international mobility and contacts, research programmes aimed at the core of mathematics and at interdisciplinary interactions, education of future generations of teachers and researchers.

Mathematics and careers

Young people ready to embark on a four or five-year course of study are rightfully focussed on their career perspectives. This is probably where the popular reputation of mathematics in Belgium (and presumably in several other countries as well) is at its lowest. In between a research career in astrophysics or molecular biology, and a financially rewarding one in engineering or business, mathematics seems simply invisible.

The general belief seems to be that a mathematics degree leads one to teach mathematics in a secondary school, and this does not currently appeal to many students

An effective measure we took at the Université Libre de Bruxelles (first in 1989 and a second time in 2001) was to contact as many of our graduates as we could, over the last ten promotions, asking them what job they were currently holding.

The first result of the survey was that almost all graduates had quite quickly found a job connected with mathematics, and only a small and fast-decreasing percentage were secondary school teachers (37 % in 1989, 15 % in 2001). This is of course a major problem for the school system, but for graduates it means they find more appealing jobs.

The second is that over 40 % of the graduates work in private companies, and near Brussels this mostly means banks, insurance companies and the pharmaceutical industry.

Finally, a surprisingly large percentage (35 % in the 2001 study) found support for an initial research career, starting with a Ph.D.

Publication of these results, together with some description of the appeal of mathematics, in a major daily newspaper was each time followed by an increase in recruitment (which doesn't look like a mere coincidence).

These studies were carried out on a rather small number of students, and I believe that similar studies made in other cities would yield somewhat different results depending on the local job market. Indeed, although mobility is much talked about, most students will first look for a job in their own region and a precise and verifiable survey proved itself an effective argument for the choice of a course of study. In most cities, the main trends would presumably be the same.

Actually, the greatest difficulty in such an information campaign is to convince a journalist that it is worth writing such an article. Sometimes it can be achieved by personal contact, or by sending emails and accepting many won't be answered, or riding a wave like the theatre productions mentioned above. In fact these led to three articles and a one-hour radio programme purely on the meaning of mathematical research. Generally speaking, publishers of newspapers want to publish news, so we should be ready to react promptly when news connected with mathematics gives us an opportunity.

To conclude

For the future development of mathematics, we need to make it more visible at all levels, as a fast moving science, well integrated with developments in many fields of activities, driven both by its internal logic and by the potentiality of applications in an ever greater variety of areas. Examples abound and can be presented on many occasions. An endless task? An exciting task!