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# Social responsibility and mean-variance portfolio selection

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JEL Classifications: G11, G14, G20

Keywords: Socially Responsible Investment, Portfolio Selection, Mean-variance Optimization, Linear Constraint, Socially Responsible Ratings.

CEB Working Paper N° 10/002
January 2010

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This version: January 2010.

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**Abstract**

In theory, investors choosing to invest only in socially responsible entities restrict their investment universe and should thus be penalized in a mean-variance framework. When computed, this penalty is usually viewed as valid for all socially responsible investors. This paper shows however that the additional cost for responsible investing depends essentially on the investors' risk aversion. Social ratings are introduced in mean-variance optimization through linear constraints to explore the implications of considering a social responsibility (SR) threshold in the traditional Markowitz (1952) portfolio selection setting. We consider optimal portfolios both with and without a risk-free asset. The SR-efficient frontier may take four different forms depending on the level of the SR threshold: a) identical to the non-SR frontier (i.e. no cost), b) only the left portion is penalized (i.e. a cost for high-risk-aversion investors only), c) only the right portion is penalized (i.e. a cost for low-risk aversion investors only) and d) the whole frontier is penalized (i.e. a positive cost for all the investors). By precisely delineating under which circumstances SRI is costly, those results help elucidate the apparent contradiction found in the literature about whether or not SRI harms diversification.

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**Acknowledgments**

The author thanks Marie Brière, Benjamin Lorent, Valérie Mignon, Kim Oosterlinck, Hugues Pirotte, Ombretta Signori, Ariane Szafarz and Kokou Topeglo for their helpful comments.

## 1) Introduction

In Markowitz's (1952) setting, portfolio selection is driven solely by financial parameters and the investor's risk aversion. This framework may however be viewed as too restrictive since, in the scope of Socially Responsible Investment (SRI)[1], investors also consider non-financial criteria. This paper explores the impact of such SRI concerns on mean-variance portfolio selection.

SRI has recently gained momentum. In 2007, its market share reached 11% of assets under management in the United States and 17.6% in Europe.[2] Moreover, by May 2009, 538 asset owners and investment managers, representing $18 trillion of assets under management, had signed the Principles for Responsible Investment (PRI)[3]. Within the SRI industry, initiatives are burgeoning and patterns are evolving rapidly.

In practice, SRI takes various forms. Negative screening consists in excluding assets on ethical grounds (often related to religious beliefs), while positive screening selects the best-SR rated assets (typically, by combining environmental, social, and governance ratings). Renneboog *et al.* (2008) describe "negative screening" as the first generation of SRI, and "positive screening" as the second generation. The third generation combines both screenings, while the fourth adds shareholder activism.

[1] SRI is defined by the European Sustainable Investment Forum (2008) as "a generic term covering ethical investments, responsible investments, sustainable investments, and any other investment process that combines investors' financial objectives with their concerns about environmental, social and governance (ESG) issues".
[2] More precisely, the Social Investment Forum (2007) assessed that 11% of the assets under management in the United States, that is $2.71 trillion, were invested in SRI, and according to the European Sustainable Investment Forum (2008), this share was 17.6% in Europe.
[3] The PRI is an investor initiative in partnership with the UNEP Finance Initiative and the UN Global Compact. The six principles for responsible investment advocate deep consideration for ESG criteria in the investment process (see PRI, 2009)

SRI financial performances are a fundamental issue. Does SRI perform as well as conventional investments? In other words, is doing “good” also doing well? A large body of empirical literature is devoted to the comparison between SR and non-SR funds. According to Renneboog *et al.* (2008), there is little evidence that the performances of SR funds differ significantly from their non-SR counterparts. Conversely, Geczy *et al.* (2006) find that restricting the investment universe to SRI funds can seriously harm diversification. Taken at face value, those statements seem hard to reconcile.

Within Markowitz’s (1952) mean-variance theoretical framework, negative screening implies that the SR efficient frontier and the capital market line will be dominated by their non-SR counterparts because asset exclusion restricts the investment universe. Farmen and Van Der Wijst (2005) notice that, in this case, risk aversion matters in the cost of investing responsibly. Positive screening corresponds to preferential investment in well-rated SRI assets without prior exclusion, with each investor being allowed to choose her own SR commitment (Landier and Nair, 2009). This translates into a trade-off between financial efficiency and portfolio ethicalness (Beal *et al.*, 2005). Likewise, Dorfleitner *et al.* (2009) propose a theory of mean-variance optimization including stochastic social returns within the investor’s utility function. However, to our knowledge, easily implementable mean-variance portfolio selection for second-generation SRI is still missing from the literature. Moreover, the impact of risk aversion on the cost of SRI has not been investigated so far. Our paper aims at filling those two gaps. By delineating the conditions under which SRI is costly, it will furthermore help elucidate the apparent contradiction found in the literature regarding SRI’s influence on diversification.

This paper measures the trade-off between financial efficiency and SRI in the traditional mean-variance optimization. We compare the optimal portfolios of an SR-insensitive investor and her SR-sensitive counterpart in order to assess the cost associated with SRI. Our contribution is twofold. First, we extend the Markowitz (1952) model[4] by imposing an SR threshold. This leads to four possible SR-efficient frontiers: a) the SR-frontier is the same as the non-SR frontier (i.e. no cost), b) only the left portion is penalized (i.e. a cost for high-risk-aversion investors only), c) only the right portion is penalized (i.e. a cost for low-risk aversion investors only), and d) the full frontier is penalized (i.e. a cost for all investors). Despite its crucial importance, practitioners tend to leave the investor's risk aversion out of the SRI story. Our paper on the other hand offers a fully operational mean-variance framework for SR portfolio management, a framework that can be used for all asset classes (stocks, bonds, commodities, mutual funds, etc.). It makes explicit the consequences of any given SR threshold on the determination of the optimal portfolio. To illustrate this, we complement our theoretical approach by an empirical application to emerging bond portfolios.

The rest of the paper is organized as follows. Section 2 proposes the theoretical framework for the SR mean-variance optimization in the presence of risky assets only. Section 3 adds a risk-free asset. Section 4 applies the SRI methodology to emerging sovereign bond portfolios. Section 5 concludes.

## 2) SRI portfolio selection (risky assets)

In this section, we explore the impact of considering responsible ratings in the mean-variance portfolio selection. To do so, we first assess the social responsibility of the optimal portfolios resulting from the traditional optimization of Markowitz (1952). Then we consider

[4] See Steinbach (2001) for a literature review on the extensions of the Markowitz (1952) model.

the case of an SR-sensitive investor who wants her portfolio to respect high SR standards, and we explore the consequences of such a constraint for optimal portfolios.

Consider a financial market composed of $n$ risky securities[5] $(i = 1, \dots, n)$. Let us denote by $\mu = [\mu_1, \quad \dots, \quad \mu_n]'$ the vector of expected returns and by $\Sigma = (\sigma_{ij})$ the $n \times n$ positive-definite covariance matrix of the returns. A portfolio $p$ is characterized by its composition, that is its associated vector $\omega_p = [\omega_{p1} \quad \omega_{p2} \quad \dots \quad \omega_{pn}]'$, where $\omega_{pi}$ is the weight of the $i^{th}$ asset in portfolio $p$, $\iota = [1 \quad \dots \quad 1]'$ and $\omega_p ' \iota = 1$.

In the traditional mean-variance portfolio selection (Markowitz, 1952), the investor maximizes her portfolio's expected return $\mu_p = \omega_p ' \mu$ for a given volatility or variance $\sigma_p^2 = \omega_p ' \Sigma \omega_p$, Let $\lambda > 0$ be the parameter accounting for the investor's level of risk aversion. The problem of the SR-insensitive investor is then written[6]:

**Problem 1**

$$\begin{aligned} &\max_{\{\omega\}} \quad \omega' \mu - \frac{\lambda}{2} \omega' \Sigma \omega \\ &subject \ \ to \quad \omega' \iota = 1 \end{aligned} \tag{1}$$

The solutions to Problem 1 form a hyperbola in the mean-standard deviation plane $(\mu_p, \sigma_p)$ and will be referred to here as the SR-insensitive efficient frontier.

Let us now add an SR rating independent from expected returns and volatilities. Typically, this is an extra-financial rating relating to environmental, social, or governance issues. It can

[5] Notations of Lo (2008) are used here.
[6] For sake of simplicity, short sales are allowed here.

also combine several ratings (Landier and Nair, 2009). Let $\phi_i$ be the SR rating associated with the $i$-th security and $\phi = [\phi_1 \quad \phi_2 \quad ... \quad \phi_n]'$. We assume that the rating is additive. Consequently, the rating $\phi_p$ of portfolio $p$ is given by:

$$\phi_p = \omega_p'\phi = \sum_{i=1}^{n} \omega_{pi}\phi_i \tag{2}$$

This linearity hypothesis (see Barracchini, 2007; Drut, 2009; Scholtens, 2009) is often used by practitioners to SR-rate financial indices[7]. The representation in eq. (2) holds for positive as well as negative screening[8].

Even when investors are SR-insensitive (thus facing problem 1), their optimal portfolios can be SR-rated. Proposition 1 expresses those ratings $\phi_p$ associated with SR-insensitive efficient portfolios.

**Proposition 1**

(i) Along the SR-insensitive frontier, the SR rating $\phi_p$ is a linear function of the expected return $\mu_p$:

$$\phi_p = \delta_0 + \delta_1 \ \mu_p \tag{3}$$

with $$\delta_0 = \frac{(\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\mu)(\mu'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2}$$

and $$\delta_1 = \frac{(\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2}.$$

(ii) If $\delta_1 > 0$, $\phi_p$ ranges from $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota}$ (for the minimum-variance portfolio) to $+\infty$ (when the expected return tends to the infinite).

[7] See for instance the Carbon Efficient Index of Standard & Poor's with the carbon footprint data from Trucost PLC.

[8] For negative screening, $\phi_i$ is binary and $\phi_p$ denotes the proportion of portfolio $p$ invested in the admissible assets.

(iii) If $\delta_1 < 0$, $\phi_p$ ranges from $\frac{\iota' \Sigma^{-1} \phi}{\iota' \Sigma^{-1} \iota}$ (for the minimum-variance portfolio) to $-\infty$ (when the expected return tends to the infinite).

**Proof:** see Appendix 1

Thus, Proposition 1 gives the SR rating $\phi_p$ of any portfolio lying on the SR-insensitive frontier. From the optimality conditions comes the fact that, along the efficient frontier, both the SR rating $\phi_p$ and the expected return $\mu_p$ are linear functions of the quantity $\frac{1}{\lambda}$, so it is straightforward that the SR rating $\phi_p$ can be written as a linear function of the expected return $\mu_p$, as in eq. (1). The direction of this link is determined by the sign of the parameter $\delta_1$. The parameter $\delta_1$ can take both signs because, for instance, the assets with the highest returns can be the best or the worst SR-rated. Furthermore, the sign of the parameter $\delta_1$ is crucial because it represents where the trade-off appears between risk aversion and SR rating. If $\delta_1 > 0$, resp. $\delta_1 < 0$, the riskier the optimal portfolio, the better, resp. the worse, its SR rating. In other words, if $\delta_1 > 0$, resp. $\delta_1 < 0$, the best SR-rated portfolios are at the top, resp. at the bottom, of the SR-insensitive frontier.

Consider now the case of an SR-sensitive investor. For instance, she requires a portfolio that is well-rated for environment. Henceforth, the SR rating $\phi_p$ is introduced in the mean-variance optimization by means of an additional linear constraint imposing a given threshold $\phi_0$ on $\phi_p$. For positive screening, the threshold value is left to the investor's discretion (Beal *et al.*, 2005, Landier and Nair, 2009). The SR-sensitive optimization is summarized by Problem 2.

**Problem 2**

$$\max_{\{\omega\}} \quad \omega'\mu - \frac{\lambda}{2}\omega'\Sigma\omega$$
$$subject\ to \quad \omega'\iota = 1 \tag{4}$$
$$\phi_p = \omega'\phi \geq \phi_0$$

We derive the analytical solutions to this problem by following Best and Grauer's (1990) methodology. Proposition 2 summarizes the results.

**Proposition 2**

The shape of the SR-sensitive efficient frontier depends on the sign of $\delta_1$ and on the threshold value $\phi_0$ in the following way:

| | $\delta_1 < 0$ | $\delta_1 > 0$ |
|---|---|---|
| $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$ | For $\lambda < \lambda_0$, the SR-sensitive frontier is another hyperbola lying below the SR-insensitive frontier.<br><br>For $\lambda > \lambda_0$, the SR-sensitive frontier is identical to the SR-insensitive frontier. | The SR-sensitive frontier is identical to the SR-insensitive frontier |
| $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$ | The SR frontier differs totally from the SR-insensitive frontier. | For $\lambda < \lambda_0$, the SR-sensitive frontier is identical to the SR-insensitive frontier.<br><br>For $\lambda > \lambda_0$, the SR-sensitive frontier is another hyperbola lying below the SR-insensitive frontier |

With $\lambda_0 = \dfrac{(\mu'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\mu)}{(\phi' - \phi_0\iota')\Sigma^{-1}\iota}$.

The associated expected return $E_0$ and the expected variance $V_0$ are:

$$E_0 = \frac{1}{\iota'\Sigma^{-1}\iota}(\mu'\Sigma^{-1}\iota + \frac{1}{\lambda_0}((\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\iota) - (\mu'\Sigma^{-1}\iota)^2)$$

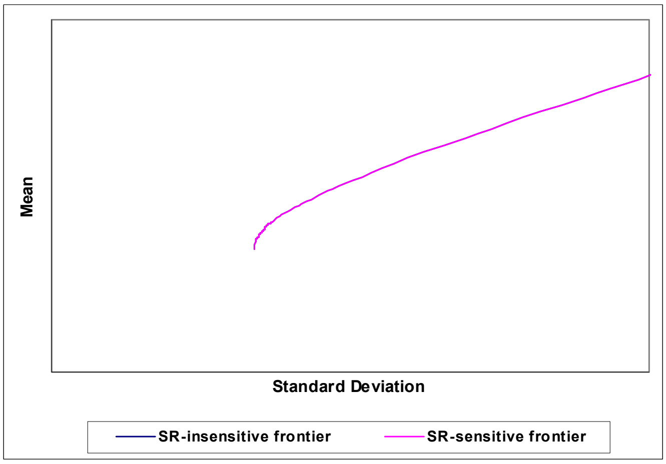
$$V_0 = \sigma_0^2 = \frac{1}{\iota'\Sigma^{-1}\iota}(1 + \frac{1}{\lambda_0^2}((\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\iota) - (\mu'\Sigma^{-1}\iota)^2)$$

**Proof:** see Appendix 2.

Proposition 2 makes explicit the situations in which there is an SRI cost. The impact of the constraint on the SR ratings depends on the parameter $\delta_1$ and on the strength of the constraint. As showed in Proposition 1, if $\delta_1 > 0$, resp. $\delta_1 < 0$, the best SR-rated portfolios are at the top, resp. at the bottom, of the SR-insensitive frontier: by consequence, the SR constraint impacts first the efficient frontier at the bottom, resp. at the top. In addition, the more the investor wants a well-rated portfolio, that is to say the higher the threshold $\phi_0$, the bigger the portion of the efficient frontier being displaced. In the case where the threshold $\phi_0$ is below the minimum rating of the SR-insensitive frontier, the efficient frontier is even not modified at all. We illustrate the four possible cases through Figures 1 to 4.

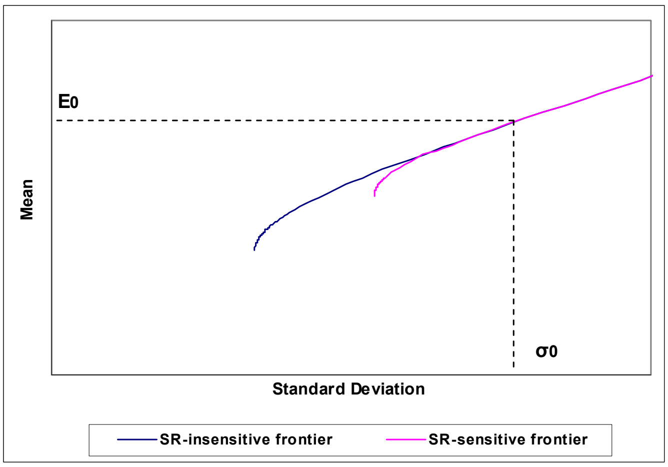
In the case where $\delta_1 > 0$ and $\frac{\iota'\sum^{-1}\phi}{\iota'\sum^{-1}\iota} > \phi_0$ (see Figure 1), the SR-sensitive and the SR-insensitive frontiers are the same and there is no SRI cost at all. This is the most favourable case.

**Figure 1** SR-sensitive frontier versus SR-insensitive frontier with $\delta_1 > 0$ and $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$



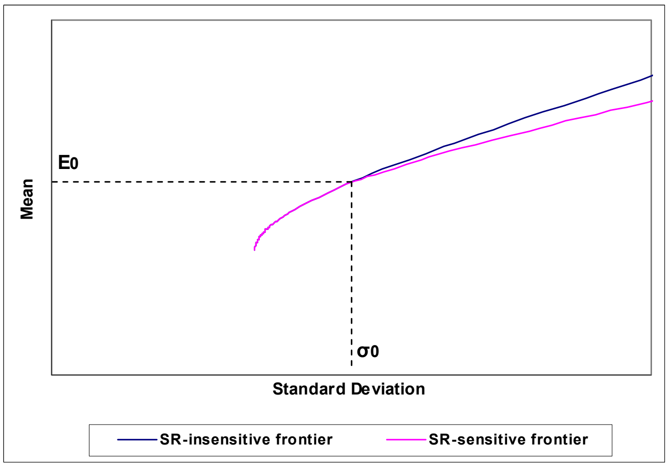

In the case where $\delta_1 > 0$ and $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$ (see Figure 2), the SR-sensitive and the SR-insensitive frontiers are the same above the corner portfolio defined by its expected return $E_0$ and its expected variance $V_0 = \sigma_0^2$. For portfolios with lower expected returns and variances, the SRI constraint induces less efficient portfolios. There is only an SRI cost for investors whose risk aversion parameter is above the threshold $\lambda_0$.

**Figure 2** SR-sensitive frontier versus SR-insensitive frontier with $\delta_1 > 0$ and $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$



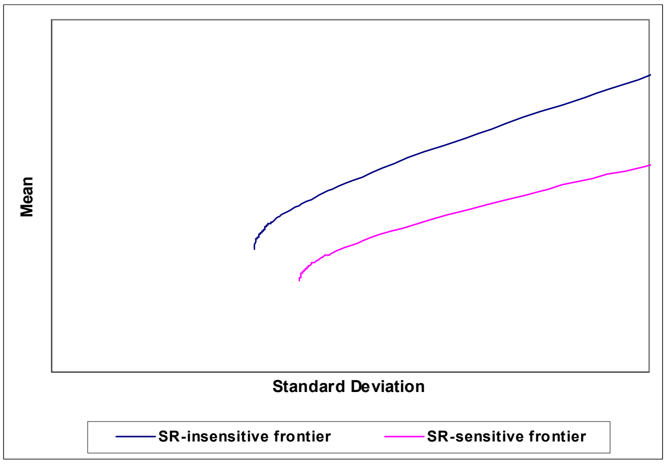
In the case where $\delta_1 < 0$ and $\dfrac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$ (see Figure 3), the SR-sensitive and the SR-insensitive frontiers are the same below the corner portfolio $(E_0, V_0)$. For portfolios with higher expected returns and variances, the SRI constraint induces less efficient portfolios. There is only an SRI cost for investors whose risk aversion parameter is below the threshold $\lambda_0$.

**Figure 3** SR-sensitive frontier versus SR-insensitive frontier with $\delta_1 < 0$ and $\dfrac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$




In the case where $\delta_1 < 0$ and $\dfrac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$ (see Figure 4), the SR-sensitive and the SR-insensitive frontiers are totally different. The SRI constraint induces less efficient portfolios for every investor. There is an SRI cost for everybody. This is the worst case.

**Figure 4** SR-sensitive frontier versus SR-insensitive frontier with $\delta_1 < 0$ and $\frac{\iota' \Sigma^{-1} \phi}{\iota' \Sigma^{-1} \iota} < \phi_0$




To sum up, while the investor's risk aversion is generally left out of the story in the SRI practice, we show in this Section that this parameter matters in the cost of responsible investing[9]. Indeed, we show that this SR cost depends on the link between SR ratings and financial returns and on the investor's risk aversion; four cases being possible: a) the SR-sensitive frontier is the same as the SR-insensitive frontier (i.e. no cost), b) only the left portion of the efficient frontier is penalized (i.e. a cost for high-risk-aversion investors only), c) only the right portion of the efficient frontier is penalized (i.e. a cost for low-risk aversion investors only), and d) the full frontier is penalized (i.e. a cost for all the investors).

## 3) Portfolio selection with a risk-free asset

In this section, we assume the existence of a risk-free asset and we explore, in this case, the impact of considering responsible ratings in the mean-variance portfolio selection. Indeed,

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[9] This section highlights the impact of a constraint on the portfolio rating in the mean-variance optimization. However, the cost of investing responsibly, if non-zero, may be non-significant. The significance of the mean-variance efficiency loss may be assessed using the test of Basak *et al.* (2002) or any spanning test (see de Roon and Nijman (2001) for a literature review).

the social responsibility of this risk-free asset should also be taken into account. So, we assess first the social responsibility of the optimal portfolios obtained by an SR-insensitive investor. And then we study whether an SR-sensitive investor is penalized by requiring portfolios with high SR standards.

Denote by $r$ the return of the risk-free asset and by $\omega_r$ the fraction of wealth invested in this risk-free asset. The standard mean-variance portfolio selection in the presence of a risk-free asset has been extensively studied by Lintner (1965) and Sharpe (1965). It corresponds to Problem 3.

**Problem 3**

The investor wants to solve the following program:

$$\begin{array}{ll} \max\limits_{\{\omega\}} & \omega'\mu + \omega_r r - \frac{\lambda}{2}\omega'\Sigma\omega \\ subject\ \ to & \omega'\iota + \omega_r = 1 \end{array} \tag{5}$$

In the mean-standard deviation plan, the set of optimal portfolios is referred as the well-known Capital Market Line (CML). As the investor does not consider responsible ratings in her optimization, we refer it here as “SR-insensitive capital market line”.

As in Section 2, we add responsible ratings to the story. Henceforth, we denote $\phi*$ as the responsible rating of the risk-free asset and the portfolio rating is defined as $\phi_p = \omega'\phi + \omega_r\phi*$. In the following, we seek to determine the portfolio ratings $\phi_p$ of the optimal portfolios on the “SR-insensitive capital market line”.

**Proposition 3**

(i) Along the SR-insensitive capital market line, the responsible rating $\phi_p$ is a linear function of the expected return $\mu_p$ :

$$\phi_p = \delta_0^* + \delta_1^* \ \mu_p \tag{6}$$

$$\text{with } \delta_0^* = \phi^* - r\,\frac{(\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota)}{(\mu - r\iota)' \Sigma^{-1} (\mu - r\iota)}$$

$$\text{and } \delta_1^* = \frac{(\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota)}{(\mu - r\iota)' \Sigma^{-1} (\mu - r\iota)}$$

(ii) If $\delta_1^* > 0$, $\phi_p$ ranges from $\phi^*$ (for the minimum-variance portfolio) to $+\infty$ (when the expected return tends to the infinite.)

(iii) If $\delta_1^* < 0$, $\phi_p$ ranges from $\phi^*$ (for the minimum-variance portfolio) to $-\infty$ (when the expected return tends to the infinite).

**Proof:** see Appendix 3

Proposition 3 attributes an SR rating of any portfolio of the SR-insensitive capital market line. From the optimality conditions comes the fact that, along the capital market line, both the SR rating $\phi_p$ and the expected return $\mu_p$ are linear functions of the quantity $\frac{1}{\lambda}$ It is therefore straightforward that the SR rating $\phi_p$ can be written as a linear function of the expected return $\mu_p$ as in eq. (6). It is striking that this relationship expressed by eq. (6) has the same form as eq. (3) in the case without a risk-free asset. Note that the portfolio of an infinitely risk averse investor would be fully invested in the risk-free asset and would have its SR rating $\phi^*$. Here, the direction of this link is determined by the sign of the parameter $\delta_1^*$. In the same way as in Section 2, the sign of the parameter $\delta_1^*$ is crucial because it represents where the trade-off appears between risk aversion and SR rating. If $\delta_1^* > 0$, resp. $\delta_1^* < 0$, the riskier the optimal portfolio, the better, resp. the worse, its SR rating. In other words, if $\delta_1^* > 0$, resp. $\delta_1^* < 0$, the best SR-rated portfolios are at the top, resp. at the bottom, of the SR-insensitive capital market line.

Similarly to Section 2, we now consider the case of SR investors wishing high SR standards and so, requiring the portfolio rating $\phi_p = \omega'\phi + \omega_r\phi^*$ to be above a threshold $\phi_0$. This corresponds to Problem 4.

**Problem 4**

The investor wants to solve the following program:

$$\begin{aligned} &\max_{\{\omega\}} \quad \omega'\mu + \omega_r r - \frac{\lambda}{2}\omega'\Sigma\omega \\ &subject\ to \quad \omega'\iota + \omega_r = 1 \\ &\qquad\qquad \phi_p = \omega'\phi + \omega_r\phi^* \geq \phi_0 \end{aligned} \tag{7}$$

In Problem 4, the constraints in the mean-variance optimization are also linear. Thus, we employ Best and Grauer’s (1990) methodology, as we did for Problem 2. Proposition 4 summarizes the results.

**Proposition 4**

The shape of the SR-sensitive capital market line depends on the sign of $\delta_1^*$ and on the threshold value $\phi_0$ in the following way:

| | $\delta_1^* < 0$ | $\delta_1^* > 0$ |
|---|---|---|
| $\phi^* > \phi_0$ | For $\lambda < \lambda_0^*$, the SR-sensitive capital market line is a hyperbola lying below the SR-insensitive capital market line.<br><br>For $\lambda > \lambda_0^*$, the SR-sensitive capital market line is identical to the SR-insensitive capital market line. | The SR-sensitive capital market line is the same as the SR-insensitive capital market line |
| $\phi^* < \phi_0$ | The SR-sensitive capital market line differs totally from the SR-insensitive capital market line and becomes a hyperbola. | For $\lambda < \lambda_0^*$, the SR-sensitive capital market line is identical to the SR-insensitive capital market line.<br><br>For $\lambda > \lambda_0^*$, the SR-sensitive capital market line is a hyperbola lying below the SR-insensitive capital market line. |

With $\lambda_0^* = \dfrac{(\mu - r\iota)'\Sigma^{-1}(\phi - \phi^* \iota)}{(\phi_0 - \phi^*)}$

The associated expected return $E_0^*$ and the expected variance $V_0^*$ are:

$$E_0^* = r + \frac{1}{\lambda_0^*}(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)$$

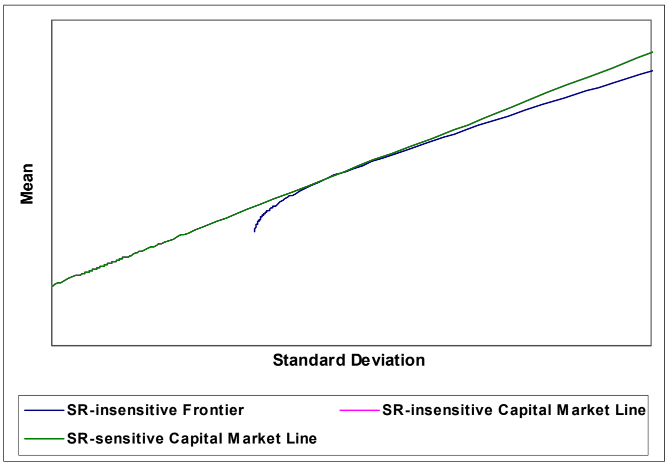
$$V_0^* = \sigma_0^{2*} = \frac{1}{\lambda_0^{*2}}(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)$$

**Proof:** see Appendix 4

Proposition 4 makes explicit the situations in which there is an SRI cost in the presence of a risk-free asset. As in Proposition 2, the impact of the constraint on the SR ratings depends on the parameter $\delta_1^*$ and on the strength of the constraint. As showed in Proposition 3, if $\delta_1^* > 0$, resp. $\delta_1^* < 0$, the best SR-rated portfolios are at the top, resp. at the bottom, of the SR-insensitive capital market line: by consequence, the SR constraint impacts first the capital market line at the bottom, resp. at the top. However, contrary to the case without a risk-free asset, the modified part of the capital market line has a different mathematical form: for this segment, the capital market line becomes a hyperbola in the mean-standard deviation plan. Figures 5 to 8 illustrate the four cases.

In the case where $\delta_1^* > 0$ and $\phi^* > \phi_0$ (see Figure 5), the SR-sensitive and the SR-insensitive capital market lines are the same and there is no SRI cost at all. This is the best case.

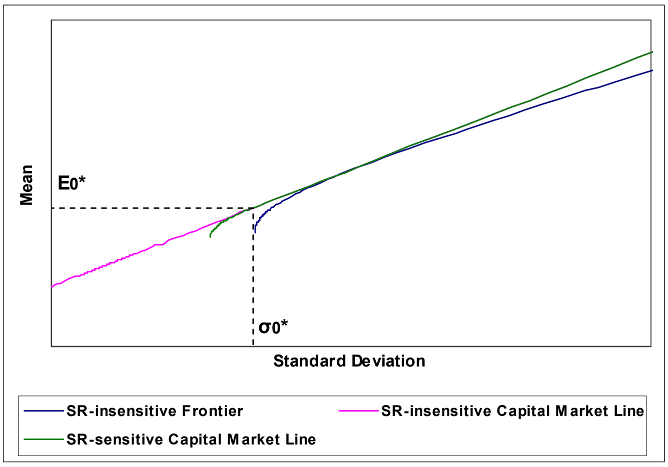
**Figure 5** SR-sensitive capital market line versus SR-insensitive capital market line with $\delta_1^* > 0$ and $\phi^* > \phi_0$




In the case where $\delta_1^* > 0$ and $\phi^* < \phi_0$ (see Figure 6), the SR-sensitive and the SR-insensitive capital market lines are the same for portfolios below the corner portfolio defined by its expected return $E_0^*$ and the expected variance $V_0^* = \sigma_0^{2*}$. Below this portfolio, the SR-sensitive capital market line becomes a hyperbola. There is an SRI cost only for investors with cold feet, that is to say with a risk aversion parameter above the threshold $\lambda_0^*$.

**Figure 6** SR-sensitive capital market line versus SR-insensitive capital market line with

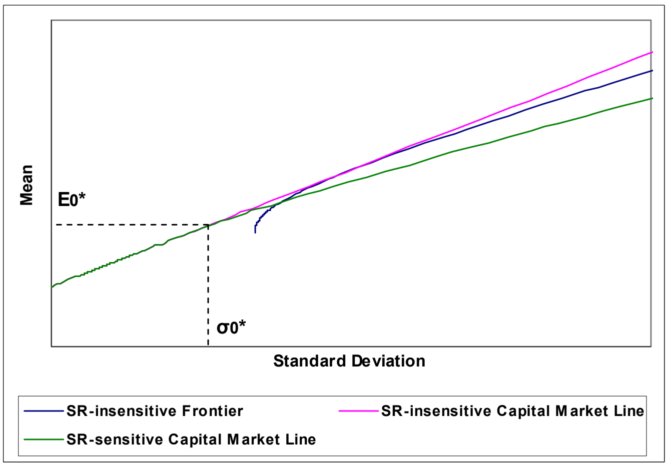
$$\delta_1^* > 0 \text{ and } \phi^* < \phi_0$$




In the case where $\delta_1^* < 0$ and $\phi^* > \phi_0$ (see Figure 7), the SR-sensitive and the SR-insensitive capital market lines are the same for portfolios above the corner portfolio $(E_0^*, V_0^*)$. Above this portfolio, the SR-sensitive capital market line becomes a hyperbola. There is an SRI cost only for investors with a risk aversion parameter below the threshold $\lambda_0^*$.

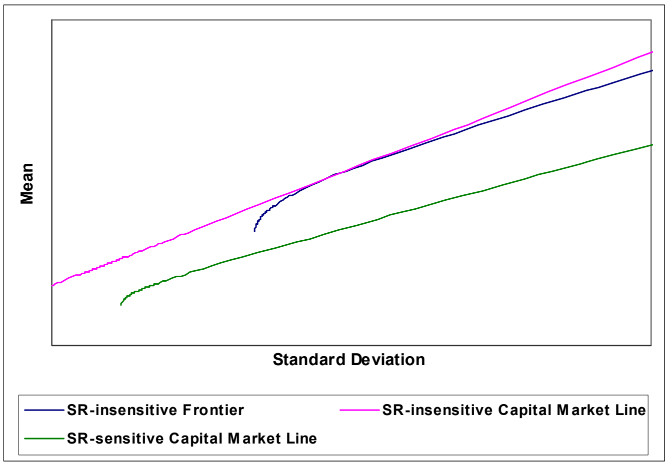
**Figure 7** SR-sensitive capital market line versus SR-insensitive capital market line with

$$\delta_1^* < 0 \text{ and } \phi^* > \phi_0$$



In the case where $\delta_1^* < 0$ and $\phi^* < \phi_0$ (see Figure 8), the SR-sensitive and the SR-insensitive capital market lines differ entirely. The SR-sensitive capital market line is no longer a line but a hyperbola. This is the most disadvantageous case: there is an SRI cost for all the investors.

**Figure 8** SR-sensitive capital market line versus SR-insensitive capital market line with $\delta_1^* < 0$ and $\phi^* < \phi_0$




In the mean-variance portfolio selection in the presence of a risk-free asset also, the investor's risk aversion matters in the SRI cost. In order to be ready-to-use for practitioners, we make explicit the four cases are possible: a) the SR-capital market line is the same as the SR-insensitive capital market line (i.e. no cost), b) only the left portion is penalized (i.e. a cost for high-risk-aversion investors only), c) only the right portion is penalized (i.e. a cost for low-risk aversion investors only), and d) the full capital market line is penalized (i.e. a cost for all the investors).

## 4) Application to an emerging bond portfolio

This section illustrates the results of Sections 2 and 3 by considering the case of a responsible US investor on the emerging bond market.

We consider the EMBI+ indices from JP Morgan as proxy for emerging bond returns. These indices track total returns for actively traded external debt instruments in emerging markets.[10] The indices are expressed in US dollars and taken at a monthly frequency from January 1994 to October 2009. They are extracted from Datastream. Descriptive statistics are available in Appendix 5.

In the same way as Scholtens (2009), we use the Environmental Performance Index (EPI) as responsible ratings. The EPI is provided jointly by the universities of Yale and Columbia in collaboration with the World Economic Forum and the Joint Research Centre of the European Commission. EPI focuses on two overarching environmental objectives: reducing environmental stress to human health and promoting ecosystem vitality and sound management of natural resources. These objectives are gauged using 25 performance indicators tracked in six well-established policy categories, which are then combined to create a final score. EPI scores attributed in 2008 are reported in Table 3.

[10] Argentina, Brazil, Bulgaria, Ecuador, Mexico, Panama, Peru, Philippines, Russia, Venezuela.

**Table 3** Environmental Performance Index 2008

| | |
|---|---|
| ARGENTINA | 81.78 |
| BRAZIL | 82.65 |
| BULGARIA | 78.47 |
| ECUADOR | 84.36 |
| MEXICO | 79.80 |
| PANAMA | 83.06 |
| PERU | 78.08 |
| PHILIPPINES | 77.94 |
| RUSSIA | 83.85 |
| VENEZUELA | 80.05 |
| Mean | 81.00 |
| Standard Deviation | 2.44 |
| UNITED STATES | 81.03 |

Sources: Universities of Yale and Columbia.

Here, the portfolio EPI is defined in the same way as in eq. (2). We start by estimating the portfolio EPI along the SR-insensitive frontier, which corresponds to estimating the relationship (3) of Proposition 1. We obtain the following estimates for the parameters $\delta_0$ and $\delta_1$:

$$\hat{\delta}_0 = 76.00 \qquad \hat{\delta}_1 = 0.30$$

As $\hat{\delta}_1 > 0$, the portfolio EPI increases with the expected return on the SR-insensitive efficient frontier: a 1%/year increase in expected returns corresponds to an increase of 0.30 in the EPI portfolio. The minimal EPI portfolio on the SR-insensitive frontier is obtained for the minimum-variance portfolio and is equal to $\frac{\iota'\hat{\Sigma}^{-1}\phi}{\iota'\hat{\Sigma}^{-1}\iota} = 78.26$.

As an illustration of Problem 2, we seek to determine the impact of SR attempts on the efficient frontier and we impose a set of constraints $\phi_p > \phi_0$ on the portfolio EPI. Figure 9 exhibits the SR-sensitive frontiers for several thresholds $\phi_0$. The corner portfolios for which there is a disconnect between the SR-sensitive and SR-insensitive frontiers are in Table 4.

**Figure 9** SR-sensitive frontiers versus SR-insensitive frontier for the EMBI+ indices, January 1994 to October 2009

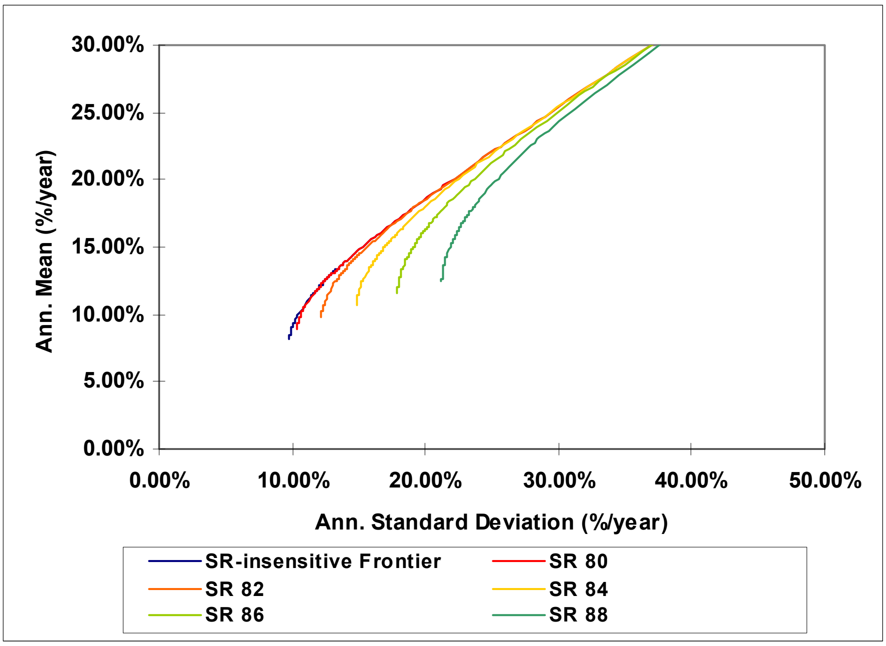



**Table 4** Corner portfolios for which the SR constraint is binding

| $\phi_0$ | $\lambda_0$ | Expected return $E_0$ (%/year) | Expected volatility $\sigma_0$ (%/year) |
|---|---|---|---|
| 82 | 3.14 | 20.13 | 22.22 |
| 84 | 2.05 | 26.84 | 32.17 |
| 86 | 1.52 | 33.55 | 42.48 |
| 88 | 1.21 | 40.26 | 52.93 |

As expected from Proposition 2, for $\phi_0 < \frac{\iota'\hat{\Sigma}^{-1}\phi}{\iota'\hat{\Sigma}^{-1}\iota} = 78.26$, the SR-sensitive frontier is the same as the SR-insensitive frontier. For $\phi_0 > \frac{\iota'\hat{\Sigma}^{-1}\phi}{\iota'\hat{\Sigma}^{-1}\iota} = 78.26$, the SR-sensitive frontier differs from the SR-insensitive frontier at the bottom and is the same at the top. For instance, with a threshold equal to 84 on the portfolio EPI, the SR-sensitive and the SR-insensitive frontiers are the same for expected returns above 26.84%/year and differ for expected returns below 26.84%/year. In the case of emerging bonds, improving the portfolio EPI costs more for investors with high risk aversion.

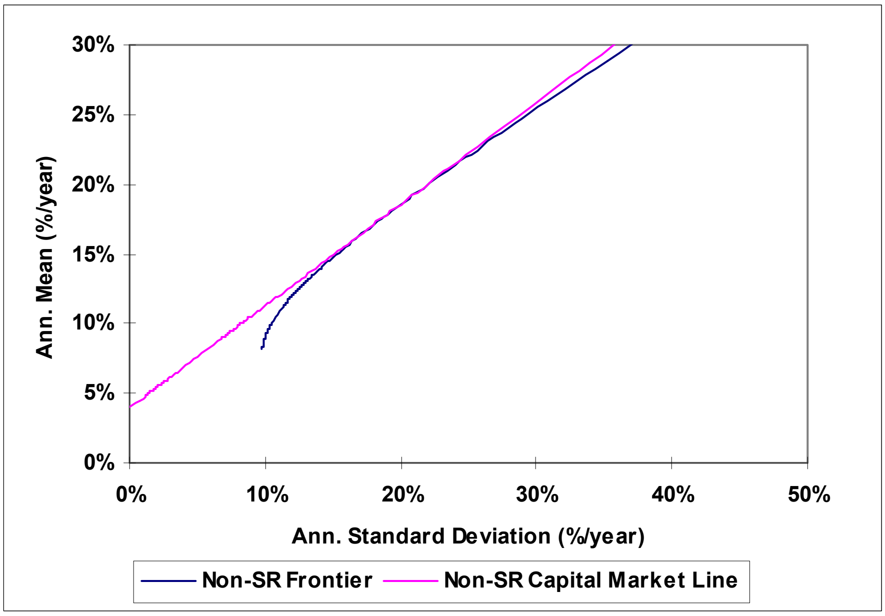
In order to illustrate Problems 3 and 4, we rely on the US 1-month interbank rate as a risk-free asset.[11] Its responsible rating corresponds to the EPI of the United States $\phi^* = 81.03$. Then, we estimate the parameters $\delta_0^*$ and $\delta_1^*$:

$$\hat{\delta}_0^* = 80.95 \qquad \hat{\delta}_1^* = 0.02$$

As $\hat{\delta}_1^* > 0$, the portfolio EPI increases with the expected return of the SR-insensitive capital market line. According to the estimations, for a 1%/year increase in expected returns, the EPI portfolio is 0.02 higher. The minimal EPI portfolio on the SR-insensitive frontier is obtained for the minimum-variance portfolio and is equal to $\phi^* = 81.03$. The SR-insensitive frontier is shown in Figure 10.

[11] This variable is extracted from Datastream.

**Figure10** SR-insensitive capital market line for the EMBI+ indices, January 1994 to October 2009




From now on, we consider SR investors wishing to adopt high environmental standards: thus, we impose a set of constraints $\phi_p > \phi_0$ in the same way as in Problem 4. Figure 11 exhibits the SR-sensitive capital market lines for several thresholds $\phi_0$, and Table 5 displays the corner portfolios.

**Figure 11** SR-sensitive capital market lines versus SR-insensitive capital market lines for the EMBI+ indices, January 1994 to October 2009

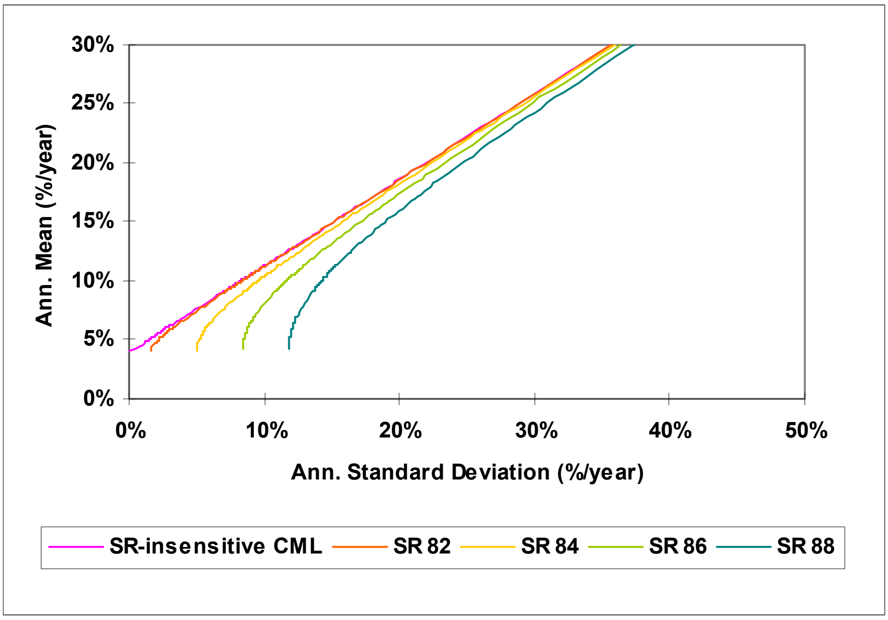


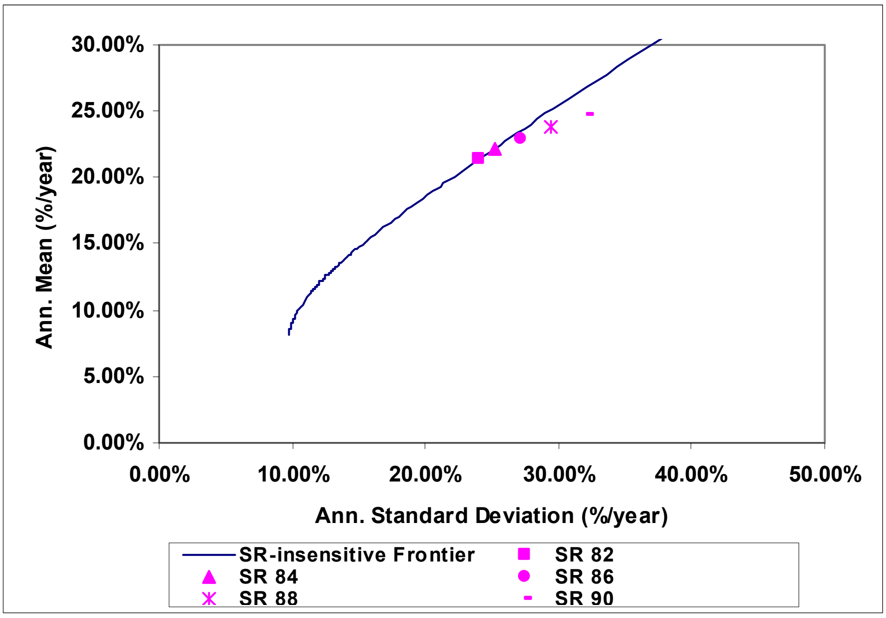

**Table 5** Corner portfolios for which the SR constraint is binding

| $\phi_0$ | $\lambda_0$ | Expected return $E_0$ (%/year) | Expected volatility $\sigma_0^*$ (%/year) |
|---|---|---|---|
| 82 | 1.16 | 50.01 | 63.06 |
| 84 | 0.38 | 144.97 | 193.08 |
| 86 | 0.23 | 239.93 | 323.11 |
| 88 | 0.16 | 334.89 | 453.13 |

As expected, for $\phi_0 < \phi^* = 81.03$, the SR-sensitive capital market line is the same as the SR-insensitive capital market line. For $\phi_0 > \phi^* = 81.03$, the SR-sensitive capital market line differs from the SR-insensitive one at the bottom and is the same at the top. Here also, the SRI cost appears for investors with high risk aversion. We notice that the corner portfolios have particularly high expected returns and volatilities (see Table 5): this can be explained by the particularly low sensitivity $\hat{\delta}_1^*$. For example, if we consider a threshold $\phi_0 = 82$, the corner portfolio has an expected return of 50.01%/year and an expected volatility of 63.06%/year, meaning that for expected returns below 50.01%/year, the SR-sensitive and SR-insensitive capital market lines are disconnected. However, we observe in Figure 11 that the SR-insensitive and SR-sensitive capital market lines are very close for expected returns slightly below 50.01%/year.

This numerical application highlights that the cost implied by high environmental requirements in an emerging bond portfolio differs according to the investor's risk aversion. In this particular case, it costs more to be green for investors with cold feet. Let us now focus on a typical investor. Sharpe (2007) suggests that the "representative investor" has a risk aversion parameter $\lambda = \frac{2}{0.7}$ in the traditional mean-variance optimization of eq. (1). We seek to determine the consequences of SR thresholds for this "representative investor" by computing the optimal portfolios for different thresholds on the portfolio EPI. Figure 12 displays these portfolios (the means and variances of the optimal portfolios are given in Appendix 6).

**Figure 12** Displaced optimal portfolios for the "representative investor"




For the "representative investor", the constraint on the portfolio EPI has no cost while the threshold is below 82.37, which is slightly above the average EPI rating of the sample's countries. When the threshold is above 82.37, an SRI cost appears and the optimal portfolio is no longer on the SR-insensitive frontier. This SRI cost rises with the strength of the constraint. In this case, the "representative investor" is directly concerned by the disconnect between SR-sensitive and SR-insensitive frontiers for reasonable SR thresholds.

## 5) Conclusion

The rapid growth of the SRI fund market have given birth to a burgeoning academic literature. Most academic studies show that there is little difference between financial returns of SRI funds and conventional funds. However, Geczy *et al.* (2006) highlight that limiting the investment universe to SRI funds can seriously harm diversification. To shed light on this debate, our paper aimed at modelling SRI in the traditional mean-variance portfolio selection framework (Markowitz, 1952).

In our study, SRI is introduced in the mean-variance optimization as a constraint on the average responsible rating of the underlying entities. Our results are detailed so that they are ready to use by practitioners. Indeed, we show that a threshold on the responsible rating may impact the efficient frontier in four different ways, depending on the link between the returns and the responsible ratings and on the strength of the constraint. The SR-sensitive efficient frontier can be: a) identical to the SR-insensitive efficient frontier (i.e. no cost at all), b) penalized at the bottom only (i.e. a cost for high risk-aversion investors only), c) penalized at the top only (i.e. a cost for low risk-aversion investors only), d) totally different from the SR-insensitive efficient frontier (i.e. a cost for every investor). In other words, if portfolio ratings increase (resp. decreases) with the expected return along the traditional efficient frontier, the SRI cost arises first at the bottom (resp. at the top) of the frontier. The results are the same in the presence of a risk-free asset. Our work highlights the fact that the investor's risk aversion clearly matters in the potential cost of investing responsibly, this cost being zero in some cases. We strongly believe that this finding could help portfolio managers of SRI funds.

As the calculations in our paper are very general, we believe it could find other applications in the asset management industry, notably for portfolio selection with asset liquidity constraints. However, one limitation of our study is that it assumes expected returns to be independent from responsible ratings: further research could focus on modelling the impact of an SRI constraint in the mean-variance optimization when expected returns and volatilities depend on responsible ratings.

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**Appendix 1**

Following Best and Grauer (1990), in the standard mean-variance case, the weights vector that solves the problem is:

$$\omega = \frac{\Sigma^{-1}\iota}{\iota'\Sigma^{-1}\iota} + \frac{1}{\lambda}\left[\Sigma^{-1}(\mu - \iota\frac{\iota'\Sigma^{-1}\mu}{\iota'\Sigma^{-1}\iota})\right]$$

The corresponding expected return $\mu_P$ stands as:

$$\mu_P = \mu'\omega = \frac{1}{\iota'\Sigma^{-1}\iota}\left[\iota'\Sigma^{-1}\mu + \frac{1}{\lambda}((\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)^2)\right]$$

And the expected variance $\sigma_P^2$ stands as:

$$\sigma_P^2 = \omega'\Sigma\omega = \frac{1}{\iota'\Sigma^{-1}\iota}\left[1 + \frac{1}{\lambda^2}((\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)^2)\right]$$

The corresponding responsible rating $\phi_P$ is also a linear function of $\frac{1}{\lambda}$:

$$\phi_p = \phi'\omega = \frac{1}{\iota'\Sigma^{-1}\iota}\left[\iota'\Sigma^{-1}\phi + \frac{1}{\lambda}((\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi)\right]$$

As the expected return is also a linear function of $\frac{1}{\lambda}$, it is possible to express the responsible rating as a linear function of the expected return:

$$\phi_p = \frac{(\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\mu)(\mu'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2} + \mu_p\frac{(\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2}$$

As a consequence, it possible to write $\phi_p = \delta_0 + \delta_1\,\mu_p$ with

$$\delta_0 = \frac{(\mu'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\mu)(\mu'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2}$$

$$\delta_1 = \frac{(\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\mu) - (\iota'\Sigma^{-1}\mu)^2}.$$

**Appendix 2**

The mean-variance optimization with the linear constraint on the portfolio rating has the same portfolio solutions as the standard mean-variance optimization, while the constraint is not binding. This is verified if the portfolio rating of the efficient portfolio from the traditional mean-variance optimization is above the threshold $\phi_0$. Two cases have to be distinguished: where $\delta_1 > 0$ and where $\delta_1 < 0$.

**Case $\delta_1 > 0$**

In this case, the portfolio rating increases linearly with the expected return in the traditional mean-variance optimization. The minimal portfolio rating is $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota}$. If $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$, the constraint on the portfolio rating is inactive and the solutions are the same as in the traditional mean-variance optimization. If $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$, the constraint on the portfolio rating is binding at the bottom of the efficient frontier and inactive at the top. Indeed, the constraint is binding for $\lambda > \lambda_0$ with:

$$\lambda_0 = \frac{(\mu'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\iota) - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\mu)}{(\phi' - \phi_0\iota')\Sigma^{-1}\iota}$$

In the mean-variance plan, this corner portfolio is the one for which the portfolio rating is $\phi_0$.

To compute the portion of the efficient frontier for which the linear constraint on the portfolio rating is binding, we apply the results of Best and Grauer (1990):

$$\mu_p = \gamma_0 + \frac{1}{\lambda}\gamma_1$$

$$\sigma_p^2 = \gamma_2 + \frac{1}{\lambda^2}\gamma_1$$

With

$$\gamma_0 = \frac{(\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\mu)(\phi'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\phi)^2 - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\phi)} + \phi_0 \frac{(\iota'\Sigma^{-1}\mu)(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\iota)(\mu'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\phi)^2 - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\phi)}$$

$$\gamma_1 = \mu'\Sigma^{-1}\mu + \frac{(\iota'\Sigma^{-1}\mu)^2(\phi'\Sigma^{-1}\phi) + (\mu'\Sigma^{-1}\phi)^2(\iota'\Sigma^{-1}\iota) - 2(\iota'\Sigma^{-1}\mu)(\mu'\Sigma^{-1}\phi)(\iota'\Sigma^{-1}\phi)}{(\iota'\Sigma^{-1}\phi)^2 - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\phi)}$$

$$\gamma_2 = \frac{-\phi'\Sigma^{-1}\phi + 2\phi_0(\iota'\Sigma^{-1}\phi) - (\iota'\Sigma^{-1}\iota)\phi_0^2}{(\iota'\Sigma^{-1}\phi)^2 - (\iota'\Sigma^{-1}\iota)(\phi'\Sigma^{-1}\phi)}$$

**Case** $\delta_1 < 0$

In this case, the portfolio rating decreases linearly with the expected return in the traditional mean-variance optimization. The maximal portfolio rating is $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota}$. If $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} < \phi_0$, the constraint on the portfolio rating is binding. If $\frac{\iota'\Sigma^{-1}\phi}{\iota'\Sigma^{-1}\iota} > \phi_0$, the constraint on the portfolio rating is binding at the top of the efficient frontier and inactive at the bottom. Indeed, the constraint is binding for $\lambda < \lambda_0$.

The equation of the portion of the efficient frontier for which the constraint is binding is the same as for the case $\delta_1 > 0$.

**Appendix 3**

Following Best and Grauer (1990), in the traditional mean-variance case, the portfolio solutions stand as:

$$\omega = \frac{1}{\lambda}\left[\Sigma^{-1}(\mu - r\iota)\right]$$

$$\omega_r = 1 - \frac{1}{\lambda}\left[\iota'\Sigma^{-1}(\mu - r\iota)\right]$$

In the mean-variance plan, these portfolios are:

$$\mu_p = \omega\mu + \omega_r r = r + \frac{1}{\lambda}\left[(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)\right]$$

$$\sigma_p^2 = \omega'\Sigma\omega = \frac{1}{\lambda^2}\left[(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)\right]$$

The responsible rating of the efficient portfolios is a linear function of $\frac{1}{\lambda}$:

$$\phi_p = \phi'\omega + \phi^*\omega_r = \phi^* + \frac{1}{\lambda}\left[(\phi - \phi^*\iota)'\Sigma^{-1}(\mu - r\iota)\right]$$

As the expected return $\mu_P$ is also a linear function of $\frac{1}{\lambda}$, it is possible to express the responsible rating $\phi_P$ of the efficient portfolios as a linear function of the expected return $\mu_P$:

$$\phi_p = \phi'\omega + \phi^*\omega_r = \phi^* + \frac{\mu_p - r}{(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)}\left[(\phi - \phi^*\iota)'\Sigma^{-1}(\mu - r\iota)\right]$$

As a consequence, it possible to write $\phi_p = \delta_0^* + \delta_1^* \mu_p$ with

$$\delta_0^* = \phi^* - r\frac{(\phi - \phi^*\iota)'\Sigma^{-1}(\mu - r\iota)}{(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)}$$

$$\delta_1^* = \frac{(\phi - \phi^*\iota)'\Sigma^{-1}(\mu - r\iota)}{(\mu - r\iota)'\Sigma^{-1}(\mu - r\iota)}$$

**Appendix 4**

The mean-variance optimization with the linear constraint on the portfolio rating has the same portfolio solutions as the standard mean-variance optimization while the constraint is not binding. This is verified if the portfolio rating of the efficient portfolio from the traditional mean-variance optimization is above the threshold $\phi_0$. Two cases have to be distinguished: where $\delta_1^* > 0$ and where $\delta_1^* < 0$.

**Case $\delta_1^* > 0$**

In this case, the portfolio rating increases linearly with the expected return in the traditional mean-variance optimization. The minimal portfolio rating is $\phi^*$. If $\phi^* > \phi_0$, the constraint on the portfolio rating is inactive and the solutions are the same as in the traditional mean-variance optimization: the Capital Market Line is not modified. If $\phi^* < \phi_0$, the constraint on the portfolio rating is binding at the bottom of the efficient frontier and inactive at the top.

Indeed, the constraint is binding for $\lambda > \lambda_0^*$ with:

$$\lambda_0^* = \frac{(\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota)}{\phi_0 - \phi^*}$$

The risk aversion parameter $\lambda_0^*$ corresponds to the portfolio with the expected return:

$$E_0^* = r + \frac{1}{\lambda_0^*} (\mu - r\iota)' \Sigma^{-1} (\mu - r\iota)$$

And the expected variance:

$$V_0^* = \frac{1}{\lambda_0^{*2}} (\mu - r\iota)' \Sigma^{-1} (\mu - r\iota)$$

To compute the portion of the efficient frontier for which the linear constraint on the portfolio rating is binding, we apply the results of Best and Grauer (1990):

$$\mu_p = r + \frac{(\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota)}{(\phi - \phi^* \iota)' \Sigma^{-1} (\phi - \phi^* \iota)} (\phi_0 - \phi^*) + \frac{1}{\lambda} ((\mu - r\iota)' \Sigma^{-1} (\mu - r\iota) - \frac{((\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota))^2}{(\phi - \phi^* \iota)' \Sigma^{-1} (\phi - \phi^* \iota)})$$

$$\sigma_p^2 = \frac{(\phi_0 - \phi^*)^2}{(\phi - \phi^* \iota)' \Sigma^{-1} (\phi - \phi^* \iota)} + \frac{1}{\lambda^2} ((\mu - r\iota)' \Sigma^{-1} (\mu - r\iota) - \frac{((\phi - \phi^* \iota)' \Sigma^{-1} (\mu - r\iota))^2}{(\phi - \phi^* \iota)' \Sigma^{-1} (\phi - \phi^* \iota)})$$

Indeed, when the constraint on the portfolio rating is binding, the Capital Market Line is no longer a linear function in the mean-standard deviation plan but a hyperbola.

**Case** $\delta_1^* < 0$

In this case, the portfolio rating $\phi_p$ decreases linearly with the expected return $\mu_p$ in the traditional mean-variance optimization. The maximum portfolio rating is $\phi^*$. If $\phi^* < \phi_0$, the constraint on the portfolio rating is always active: the entire Capital Market Line becomes a hyperbola in the mean-standard deviation plan. If $\phi^* > \phi_0$, the constraint on the portfolio rating is binding at the top of the Capital Market Line. The modification of the Capital Market Line occurs for the same risk aversion parameter $\lambda_0^*$ as for the case $\delta_1^* > 0$.

**Appendix 5** Descriptive statistics for the EMBI + indices in US dollars, January 1994 to October 2009

| | Ann. Mean | Ann. Std. Dev. | Sharpe Ratio | Skewness | Kurtosis | Maximum | Minimum |
|---|---|---|---|---|---|---|---|
| **ARGENTINA** | 4.94% | 28.84% | 0.03 | -1.07 | 9.37 | 33.80% | -43.90% |
| **BRAZIL** | 15.03% | 21.40% | 0.51 | -0.62 | 8.22 | 26.47% | -27.17% |
| **BULGARIA** | 15.10% | 20.26% | 0.55 | -0.98 | 13.96 | 25.77% | -36.38% |
| **ECUADOR** | 16.19% | 33.51% | 0.36 | -1.47 | 10.10 | 28.29% | -55.78% |
| **MEXICO** | 9.83% | 11.48% | 0.50 | -0.78 | 7.72 | 12.84% | -14.59% |
| **PANAMA** | 15.31% | 20.73% | 0.54 | 0.26 | 8.18 | 28.88% | -22.63% |
| **PERU** | 14.88% | 22.17% | 0.49 | -0.58 | 10.50 | 34.50% | -29.93% |
| **PHILIPPINES** | 7.82% | 11.91% | 0.32 | -2.27 | 13.41 | 7.75% | -20.44% |
| **RUSSIA** | 18.27% | 34.05% | 0.42 | -1.92 | 20.27 | 35.63% | -72.18% |
| **VENEZUELA** | 13.79% | 23.07% | 0.42 | -0.84 | 12.84 | 34.05% | -39.13% |

**Appendix 6** Optimal portfolios for a representative investor in the absence of a risk-free asset

| **Threshold $\phi_0$** | **Expected return (%/year)** | **Expected volatility (%/year)** |
|---|---|---|
| No constraint | 21.38 | 24.03 |
| 82 | 21.38 | 24.03 |
| 84 | 22.10 | 25.25 |
| 86 | 22.98 | 27.17 |
| 88 | 23.86 | 29.48 |