

### **Large Bayesian VARs**

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## Large Bayesian VARs<sup>1</sup>

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### Abstract

This paper shows that Vector Autoregression with Bayesian shrinkage is an appropriate tool for large dynamic models. We build on the results by De Mol, Giannone, and Reichlin (2008) and show that, when the degree of shrinkage is set in relation to the cross-sectional dimension, the forecasting performance of small monetary VARs can be improved by adding additional macroeconomic variables and sectoral information. In addition, we show that large VARs with shrinkage produce credible impulse responses and are suitable for structural analysis.

JEL Classification: C11, C13, C33, C53

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### Non-technical summary

Vector Autoregressions (VAR) are standard tools in macroeconomics and are widely used for structural analysis and forecasting. In contrast to e.g. structural models they do not impose restrictions on the parameters and hence provide a very general representation allowing to capture complex data relationships. On the other hand, this high level of generality implies a large number of parameters even for systems of moderate size. This entails a risk of over-parametrization since, with the typical sample size available for macroeconomic applications, the number of unrestricted parameters that can reliably be estimated is rather limited. Consequently, VAR applications are usually based only on a small number of variables.

The size of the VARs typically used in empirical applications ranges from three to about ten variables and this potentially creates an omitted variable bias with adverse consequences both for structural analysis and for forecasting, see e.g. Christiano, Eichenbaum, and Evans (1999) or Giannone and Reichlin (2006). For example, Christiano, Eichenbaum, and Evans (1999) point out that the positive reaction of prices in response to a monetary tightening, the so called price puzzle, is an artefact resulting from the omission of forward looking variables, like the commodity price index. In addition, the size limitation is problematic for applications which require the study of a larger set of variables than the key macroeconomic indicators, such as disaggregate information or international data. In the VAR literature, a popular solution to analyse relatively large data sets is to define a core set of indicators and to add one variable, or group of variables, at a time (the so called marginal approach), see e.g. Christiano, Eichenbaum, and Evans (1996) or Kim (2001). With this approach, however, comparison of impulse responses across models is problematic.

In this paper we show that by applying Bayesian shrinkage, we are able to handle large unrestricted VARs and that therefore VAR framework can be applied to empirical problems that require the analysis of more than a handful of time series. For example, we can analyse VARs containing the wish list of any macroeconomist (see e.g. Uhlig, 2004) but it is also possible to extend the information set further and include the disaggregated, sectorial and geographical indicators. Consequently, Bayesian VAR is a valid alternative to factor models or panel VARs for the analysis of large dynamic systems.

We use priors as proposed by Doan, Litterman, and Sims (1984) and Litterman (1986a) and we show that, contrary to common wisdom, further restrictions are unnecessary and that shrinkage is indeed sufficient to deal with large models provided that the

tightness of the priors is increased as we add more variables. Our study is empirical, but builds on the asymptotic analysis in De Mol, Giannone, and Reichlin (2008) which analyses the properties of Bayesian regression as the dimension of the cross-section and the sample size go to infinity. That paper shows that, when data are characterized by strong collinearity, which is typically the case for macroeconomic time series, by setting the degree of shrinkage in relation to the model size, it is indeed possible to control for over-fitting while preserving the relevant sample information. The intuition of this result is that, if all data carry similar information (near collinearity), the relevant signal can be extracted from a large data set despite the stronger shrinkage required to filter out the unsystematic component. In this paper we go beyond simple regression and study the VAR case.

We evaluate forecasting accuracy and perform a structural exercise on the effect of a monetary policy shock for systems of different sizes: a small VAR on employment, inflation and interest rate, a VAR with the seven variables considered by Christiano, Eichenbaum, and Evans (1999), a twenty variables VAR extending the system of Christiano, Eichenbaum, and Evans (1999) by key macro indicators, such as labor market variables, the exchange rate or stock prices and finally a VAR with hundred and thirty one variables, containing, beside macroeconomic information, also sectoral data, several financial variables and conjunctural information. These are the variables used by Stock and Watson (2005a) for forecasting based on principal components, but contrary to the factor literature, we model variables in levels to retain information in the trends. We also compare the results of Bayesian VARs with those from Factor Augmented VAR (FAVAR) of Bernanke, Boivin, and Eliasz (2005).

We find that the largest specification outperforms the small models in forecast accuracy and produces credible impulse responses, but that this performance is already obtained with the medium size system containing twenty key macroeconomic indicators.

### 1 Introduction

Vector Autoregressions (VAR) are standard tools in macroeconomics and are widely used for structural analysis and forecasting. In contrast to e.g. structural models they do not impose restrictions on the parameters and hence provide a very general representation allowing to capture complex data relationships. On the other hand, this high level of generality implies a large number of parameters even for systems of moderate size. This entails a risk of over-parametrization since, with the typical sample size available for macroeconomic applications, the number of unrestricted parameters that can reliably be estimated is rather limited. Consequently, VAR applications are usually based only on a small number of variables.

The size of the VARs typically used in empirical applications ranges from three to about ten variables and this potentially creates an omitted variable bias with adverse consequences both for structural analysis and for forecasting, see e.g. Christiano, Eichenbaum, and Evans (1999) or Giannone and Reichlin (2006). For example, Christiano, Eichenbaum, and Evans (1999) point out that the positive reaction of prices in response to a monetary tightening, the so called price puzzle, is an artefact resulting from the omission of forward looking variables, like the commodity price index. In addition, the size limitation is problematic for applications which require the study of a larger set of variables than the key macroeconomic indicators, such as disaggregate information or international data. In the VAR literature, a popular solution to analyse relatively large data sets is to define a core set of indicators and to add one variable, or group of variables, at a time (the so called marginal approach), see e.g. Christiano, Eichenbaum, and Evans (1996) or Kim (2001). With this approach, however, comparison of impulse responses across models is problematic.

To circumvent these problems, recent literature has proposed ways to impose restrictions on the covariance structure so as to limit the number of parameters to estimate. For example, factor models for large cross-sections introduced by Forni, Hallin, Lippi, and Reichlin (2000) and Stock and Watson (2002b) rely on the assumption that the bulk of dynamic interrelations within a large data set can be explained by few common factors. Those models have been successfully applied both in the context of forecasting (Bernanke and Boivin, 2003; Boivin and Ng, 2005; D'Agostino and Giannone, 2006; Forni, Hallin, Lippi, and Reichlin, 2005, 2003; Giannone, Reichlin, and Sala, 2004; Marcellino, Stock, and Watson, 2003; Stock and Watson, 2002a,b) and structural analysis (Stock and Watson, 2005b; Forni, Giannone, Lippi, and Reichlin, 2008; Bernanke, Boivin, and Eliasz,

2005; Giannone, Reichlin, and Sala, 2004). For datasets with a panel structure an alternative approach has been to impose exclusion, exogeneity or homogeneity restrictions as as in e.g. Global VARs (cf. di Mauro, Smith, Dees, and Pesaran, 2007) and panel VARs (cf. Canova and Ciccarelli, 2004).

In this paper we show that by applying Bayesian shrinkage, we are able to handle large unrestricted VARs and that therefore VAR framework can be applied to empirical problems that require the analysis of more than a handful of time series. For example, we can analyse VARs containing the wish list of any macroeconomist (see e.g. Uhlig, 2004) but it is also possible to extend the information set further and include the disaggregated, sectorial and geographical indicators. Consequently, Bayesian VAR is a valid alternative to factor models or panel VARs for the analysis of large dynamic systems.

We use priors as proposed by Doan, Litterman, and Sims (1984) and Litterman (1986a). Litterman (1986a) found that applying Bayesian shrinkage in the VAR containing as few as six variables can lead to better forecast performance. This suggests that overparametrization can be an issue already for systems of fairly modest size and that shrinkage is a potential solution to this problem. However, although Bayesian VARs with Litterman's priors are a standard tool in applied macro (Leeper, Sims, and Zha, 1996; Sims and Zha, 1998; Robertson and Tallman, 1999), the imposition of priors has not been considered sufficient to deal with larger models. For example, the marginal approach we described above has been typically used in conjunction with Bayesian shrinkage, see e.g. Mackowiak (2006, 2007). Litterman himself, when constructing a forty variable model for policy analysis, imposed (exact) exclusion and exogeneity restrictions in addition to shrinkage, allowing roughly ten variables per equation (see Litterman, 1986b).

Our paper shows that these restrictions are unnecessary and that shrinkage is indeed sufficient to deal with large models provided that, contrary to the common practise, we increase the tightness of the priors as we add more variables. Our study is empirical, but builds on the asymptotic analysis in De Mol, Giannone, and Reichlin (2008) which analyses the properties of Bayesian regression as the dimension of the cross-section and the sample size go to infinity. That paper shows that, when data are characterized by strong collinearity, which is typically the case for macroeconomic time series, by setting the degree of shrinkage in relation to the model size, it is indeed possible to control for over-fitting while preserving the relevant sample information. The intuition of this result is that, if all data carry similar information (near collinearity), the relevant signal can be extracted from a large data set despite the stronger shrinkage required to filter out the unsystematic component. In this paper we go beyond simple regression and study

the VAR case.

We evaluate forecasting accuracy and perform a structural exercise on the effect of a monetary policy shock for systems of different sizes: a small VAR on employment, inflation and interest rate, a VAR with the seven variables considered by Christiano, Eichenbaum, and Evans (1999), a twenty variables VAR extending the system of Christiano, Eichenbaum, and Evans (1999) by key macro indicators, such as labor market variables, the exchange rate or stock prices and finally a VAR with hundred and thirty one variables, containing, beside macroeconomic information, also sectoral data, several financial variables and conjunctural information. These are the variables used by Stock and Watson (2005a) for forecasting based on principal components, but contrary to the factor literature, we model variables in levels to retain information in the trends. We also compare the results of Bayesian VARs with those from Factor Augmented VAR (FAVAR) of Bernanke, Boivin, and Eliasz (2005).

We find that the largest specification outperforms the small models in forecast accuracy and produces credible impulse responses, but that this performance is already obtained with the medium size system containing the twenty key macroeconomic indicators. This suggests that for the purpose of forecasting and structural analysis it is not necessary to go beyond the model containing only the aggregated variables. On the other hand, this also shows that the Bayesian VAR is an appropriate tool for forecasting and structural analysis when it is desirable to condition on a large information set.

Given the progress in computing power (see Hamilton, 2006, for a discussion), estimation does not present any numerical problems. More subtly, shrinkage acts as a regularization solution of the problem of inverting an otherwise unstable large covariance matrix (approximately two thousand times two thousands for the largest model of our empirical application).

The paper is organized as follows. In Section 2 we describe the priors for the baseline Bayesian VAR model and the data. In Section 3 we perform the forecast evaluation for all the specifications and in Section 4 the structural analysis on the effect of the monetary policy shocks. Section 5 concludes. The Appendix provides some more details on the data set and the specifications and results for a number of alternative specifications to verify the robustness of our findings.

### 2 Setting the priors for the VAR

Let  $Y_t = (y_{1,t} y_{2,t} \dots y_{n,t})'$  be a potentially large vector of random variables. We consider the following VAR(p) model:

$$Y_t = c + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + u_t,$$
(1)

where  $u_t$  is an *n*-dimensional Gaussian white noise with covariance matrix  $\mathbb{E}u_tu_t' = \Psi$ ,  $c = (c_1, ..., c_n)'$  is an *n*-dimensional vector of constants and  $A_1, ..., A_p$  are  $n \times n$  autoregressive matrices.

We estimate the model using the Bayesian VAR (BVAR) approach which helps to overcome the curse of dimensionality via the imposition of prior beliefs on the parameters. In setting the prior distributions, we follow standard practice and use the procedure developed in Litterman (1986a) with modifications proposed by Kadiyala and Karlsson (1997) and Sims and Zha (1998).

Litterman (1986a) suggests using a prior often referred to as the Minnesota prior. The basic principle behind it is that all the equations are "centered" around the random walk with drift, i.e. the prior mean can be associated with the following representation for  $Y_t$ :

$$Y_t = c + Y_{t-1} + u_t$$
.

This amounts to shrinking the diagonal elements of  $A_1$  toward one and the remaining coefficients in  $A_1, ..., A_p$  toward zero. In addition, the prior specification incorporates the belief that the more recent lags should provide more reliable information than the more distant ones and that own lags should explain more of the variation of a given variable than the lags of other variables in the equation.

These prior beliefs are imposed by setting the following moments for the prior distribution of the coefficients:

$$\mathbb{E}[(A_k)_{ij}] = \begin{cases} \delta_i, & j = i, k = 1 \\ 0, & \text{otherwise} \end{cases}, \quad \mathbb{V}[(A_k)_{ij}] = \begin{cases} \frac{\lambda^2}{k^2}, & j = i \\ \vartheta \frac{\lambda^2}{k^2} \frac{\sigma_i^2}{\sigma_j^2}, & \text{otherwise} \end{cases}.$$
 (2)

The coefficients  $A_1, ..., A_p$  are assumed to be a priori independent and normally distributed. As for the covariance matrix of the residuals, it is assumed to be diagonal, fixed and known:  $\Psi = \Sigma$  where  $\Sigma = \text{diag}(\sigma_1^2, ..., \sigma_n^2)$ . Finally, the prior on the intercept is diffuse.

Originally, Litterman sets  $\delta_i = 1$  for all i, reflecting the belief that all the variables are characterized by high persistence. However, this prior is not appropriate for variables believed to be characterized by substantial mean reversion. For those we impose the prior belief of white noise by setting  $\delta_i = 0$ .

The hyperparameter  $\lambda$  controls the overall tightness of the prior distribution around the random walk or white noise and governs the relative importance of the prior beliefs with respect to the information contained in the data. For  $\lambda=0$  the posterior equals the prior and the data do not influence the estimates. If  $\lambda=\infty$ , on the other hand, posterior expectations coincide with the Ordinary Least Squares (OLS) estimates. We argue that the overall tightness governed by  $\lambda$  should be chosen in relation to the size of the system. As the number of variables increases the parameters should be shrunk more in order to avoid overfitting. This point has been shown formally by De Mol, Giannone, and Reichlin (2008).

The factor  $1/k^2$  is the rate at which prior variance decreases with increasing lag length and  $\sigma_i^2/\sigma_j^2$  accounts for the different scale and variability of the data. The coefficient  $\vartheta \in (0,1)$  governs the extent to which the lags of other variables are "less important" than the own lags.

In the context of the structural analysis we need to take into account possible correlation among the residual of different variables. Consequently, Litterman's assumption of fixed and diagonal covariance matrix is somewhat problematic. To overcome this problem we follow Kadiyala and Karlsson (1997) and Robertson and Tallman (1999) and impose a Normal inverted Wishart prior which retains the principles of the Minnesota prior. This is possible under the condition that  $\vartheta = 1$ , which will be assumed in what follows. Let us write the VAR in (1) as a system of multivariate regressions (see e.g. Kadiyala and Karlsson, 1997):

$$Y_{T \times n} = X_{T \times k} B_{k \times n} + U_{T \times n}, \tag{3}$$

where  $Y = (Y_1, ..., Y_T)'$ ,  $X = (X_1, ..., X_T)'$  with  $X_t = (Y'_{t-1}, ..., Y'_{t-p}, 1)'$ ,  $U = (u_1, ..., u_T)'$ , and  $B = (A_1, ..., A_p, c)'$  is the  $k \times n$  matrix containing all coefficients and k = np + 1. The Normal inverted Wishart prior has the form:

$$vec(B)|\Psi \sim N(vec(B_0), \Psi \otimes \Omega_0)$$
 and  $\Psi \sim iW(S_0, \alpha_0)$ , (4)

where the prior parameters  $B_0$ ,  $\Omega_0$ ,  $S_0$  and  $\alpha_0$  are chosen so that prior expectations and variances of B coincide with those implied by equation (2) and the expectation of  $\Psi$  is equal to the fixed residual covariance matrix  $\Sigma$  of the Minnesota prior, for details see

Kadiyala and Karlsson (1997).

We implement the prior (4) by adding dummy observations. It can be shown that adding  $T_d$  dummy observations  $Y_d$  and  $X_d$  to the system (3) is equivalent to imposing the Normal inverted Wishart prior with  $B_0 = (X'_d X_d)^{-1} X'_d Y_d$ ,  $\Omega_0 = (X'_d X_d)^{-1}$ ,  $S_0 = (Y_d - X_d B_0)'(Y_d - X_d B_0)$  and  $\alpha_0 = T_d - k$ . In order to match the Minnesota moments, we add the following dummy observations:

$$Y_{d} = \begin{pmatrix} \operatorname{diag}(\delta_{1}\sigma_{1}, \dots, \delta_{n}\sigma_{n})/\lambda \\ 0_{n(p-1)\times n} \\ \dots \\ \operatorname{diag}(\sigma_{1}, \dots, \sigma_{n}) \\ \dots \\ 0_{1\times n} \end{pmatrix} \qquad X_{d} = \begin{pmatrix} J_{p} \otimes \operatorname{diag}(\sigma_{1}, \dots, \sigma_{n})/\lambda & 0_{np\times 1} \\ \dots \\ 0_{n\times np} & 0_{n\times 1} \\ \dots \\ 0_{1\times np} & \epsilon \end{pmatrix}$$
(5)

where  $J_p = \text{diag}(1, 2, ..., p)$ . Roughly speaking, the first block of dummies imposes prior beliefs on the autoregressive coefficients, the second block implements the prior for the covariance matrix and the third block reflects the uninformative prior for the intercept ( $\epsilon$  is a very small number). Although the parameters should in principle be set using only prior knowledge we follow common practice (see e.g. Litterman, 1986a; Sims and Zha, 1998) and set the scale parameters  $\sigma_i^2$  equal the variance of a residual from a univariate autoregressive model of order p for the variables  $y_{it}$ .

Consider now the regression model (3) augmented with the dummies in (5):

$$Y_* = X_* B_{t \times n} + U_*,$$

$$T_* \times n \to T_* \times n$$
(6)

where  $T_* = T + T_d$ ,  $Y_* = (Y', Y'_d)'$ ,  $X_* = (X', X'_d)$  and  $U_* = (U', U'_d)'$ . To insure the existence of the prior expectation of  $\Psi$  it is necessary to add an improper prior  $\Psi \sim |\Psi|^{-(n+3)/2}$ . In that case the posterior has the form:

$$vec(B)|\Psi, Y \sim N\left(vec(\tilde{B}), \Psi \otimes (X'_*X_*)^{-1}\right)$$
 and  $\Psi|Y \sim iW\left(\tilde{\Sigma}, T_d + 2 + T - k\right), (7)$ 

with  $\tilde{B} = (X'_*X_*)^{-1}X'_*Y_*$  and  $\tilde{\Sigma} = (Y_* - X_*\tilde{B})'(Y_* - X_*\tilde{B})$ . Note that the posterior expectation of the coefficients coincides with the OLS estimates of the regression of  $Y_*$  on  $X_*$ . It can be easily checked that it also coincides with the posterior mean for the Minnesota setup in (2). From the computational point of view, estimation is feasible since it only requires the inversion of a square matrix of dimension k = np + 1. For the large data set of hundred and thirty variables and thirteen lags k is smaller than 2000. Adding dummy observations works as a regularisation solution to the matrix inversion

problem.

The dummy observation implementation will prove useful for imposing additional beliefs. We will exploit this feature in Section 3.3.

### **2.1** Data

We use the data set of Stock and Watson (2005a). This data set contains 131 monthly macro indicators covering broad range of categories including, inter alia, income, industrial production, capacity, employment and unemployment, consumer prices, producer prices, wages, housing starts, inventories and orders, stock prices, interest rates for different maturities, exchange rates, money aggregates. The time span is from January 1959 through December 2003. We apply logarithms to most of the series with the exception of those already expressed in rates. For non-stationary variables, considered in first differences by Stock and Watson (2005a), we use the random walk prior, that is we set  $\delta_i = 1$ . For stationary variables, we use the white noise prior, that is  $\delta_i = 0$ . The description of the data set, including the information on the transformations and the specification of  $\delta_i$  for each series, is provided in the Appendix A.

We analyse VARs of different sizes. We first look at the forecast performance. Then we identify the monetary policy shock and study impulse response functions as well as variance decompositions. The variables of special interest include a measure of real economic activity, a measure of prices and a monetary instrument. As in Christiano, Eichenbaum, and Evans (1999), we use employment as an indicator of real economic activity measured by the number of employees on non-farm payrolls (EMPL). The level of prices is measured by the consumer price index (CPI) and the monetary instrument is the Federal Funds Rate (FFR).

We consider the following VAR specifications:

- SMALL. This is a small monetary VAR including the three key variables;
- CEE. This is the monetary model of Christiano, Eichenbaum, and Evans (1999). In addition to the key variables in SMALL, this model includes the index of sensitive material prices (COMM PR) and monetary aggregates: non-borrowed reserves (NBORR RES), total reserves (TOT RES) and M2 money stock (M2);
- MEDIUM. This VAR extends the CEE model by the following variables: Personal Income (INCOME), Real Consumption (CONSUM), Industrial Production (IP),

Capacity Utilization (CAP UTIL), Unemployment Rate (UNEMPL), Housing Starts (HOUS START), Producer Price Index (PPI), Personal Consumption Expenditures Price Deflator (PCE DEFL), Average Hourly Earnings (HOUR EARN), M1 Monetary Stock (M1), Standard and Poor's Stock Price Index (S&P); Yields on 10 year U.S. Treasury Bond (TB YIELD) and effective exchange rate (EXR). The system contains in total 20 variables.

• *LARGE*. This specification includes all the 131 macroeconomic indicators of Stock and Watson's dataset.

It is important to stress that since we compare models of different size, we need to have a strategy for how to choose the shrinkage hyperparameter as models become larger. As the dimension increases, we want to shrink more, as suggested by the analysis in De Mol, Giannone, and Reichlin (2008) in order to control for over-fitting. A simple solution is to set the tightness of the prior so that all models have the same in-sample fit as the smallest VAR estimated by OLS. By ensuring that the in-sample fit is constant, i.e. independent of the model size, we can meaningfully compare results across models.

### 3 Forecast evaluation

In this section we compare empirically forecasts resulting from different VAR specifications.

We compute point forecasts using the posterior mean of the parameters. We write  $\hat{A}_{j}^{(\lambda,m)}$ , j=1,...,p and  $\hat{c}^{(\lambda,m)}$  for the posterior mean of the autoregressive coefficients and the constant term of a given model (m) obtained by setting the overall tightness equal to  $\lambda$ . The point estimates of the h-steps ahead forecasts are denoted by  $Y_{t+h|t}^{(\lambda,m)} = \left(y_{1,t+h|t}^{(\lambda,m)},...,y_{n,t+h|t}^{(\lambda,m)}\right)'$ , where n is the number of variables included in model m. The point estimate of the one-step-ahead forecast is computed as  $\hat{Y}_{t+1|t}^{(\lambda,m)} = \hat{c}^{(\lambda,m)} + \hat{A}_{1}^{(\lambda,m)}Y_{t} + ... + \hat{A}_{p}^{(\lambda,m)}Y_{t-p+1}$ . Forecasts h-steps ahead are computed recursively.

In the case of the benchmark model the prior restriction is imposed exactly, that is  $\lambda = 0$ . Corresponding forecasts are denoted by  $Y_{t+h|t}^{(0)}$  and are the same for all the specifications. Hence we drop the superscript m.

To simulate real-time forecasting we conduct an out-of-sample experiment. Let us denote by H the longest forecast horizon to be evaluated, and by  $T_0$  and  $T_1$  the beginning and the end of the evaluation sample, respectively. For a given forecast horizon h, in each

period  $T = T_0 + H - h, ..., T_1 - h$ , we compute h-step-ahead forecasts,  $Y_{T+h|T}^{(\lambda,m)}$ , using only the information up to time T.

Out-of-sample forecast accuracy is measured in terms of Mean Squared Forecast Error (MSFE):

$$MSFE_{i,h}^{(\lambda,m)} = \frac{1}{T_1 - T_0 - H + 1} \sum_{T=T_0 + H - h}^{T_1 - h} \left( y_{i,T+h|T}^{(\lambda,m)} - y_{i,T+h} \right)^2.$$

We report results for MSFE relative to the benchmark, that is

$$RMSFE_{i,h}^{(\lambda,m)} = \frac{MSFE_{i,h}^{(\lambda,m)}}{MSFE_{i,h}^{(0)}}.$$

Notice that a number smaller than one implies that the VAR model with overall tightness  $\lambda$  performs better than the naive prior model.

We evaluate the forecast performance of the VARs for the three key series included in all VAR specifications (Employment, CPI and the Federal Funds Rate) over the period going from  $T_0 = Jan70$  until  $T_1 = Dec03$  and for forecast horizons up to one year (H = 12). The order of the VAR is set to p = 13 and parameters are estimated using for each T the observations from the most recent 10 years (rolling scheme).

The overall tightness is set to yield a desired average fit for the three variables of interest in the pre-evaluation period going from Jan60 (t=1) until Dec69 ( $t=T_0-1$ ) and then kept fixed for the entire evaluation period. In other words for a desired Fit,  $\lambda$  is chosen as

$$\lambda_m(Fit) = \arg\min_{\lambda} \left| Fit - \frac{1}{3} \sum_{i \in \mathcal{I}} \frac{\mathrm{msfe}_i^{(\lambda, m)}}{\mathrm{msfe}_i^{(0)}} \right|,$$

where  $\mathcal{I} = \{EMPL, CPI, FFR\}$  and  $\text{msfe}_i^{(\lambda,m)}$  is an in-sample one-step-ahead mean squared forecast error evaluated using the training sample  $t = 1, ..., T_0 - 1^2$ . More precisely:

$$\operatorname{msfe}_{i}^{(\lambda,m)} = \frac{1}{T_0 - p - 1} \sum_{t=n}^{T_0 - 2} (y_{i,t+1|t}^{(\lambda,m)} - y_{i,t+1})^2,$$

<sup>&</sup>lt;sup>1</sup>Using all the available observations up to time T (recursive scheme) does not change the qualitative results. Qualitative results remain the same also if we set p = 6.

<sup>&</sup>lt;sup>2</sup>To obtain the desired magnitude of fit the search is performed over a grid for  $\lambda$ . Division by  $\mathrm{msfe}_i^{(0)}$  accounts for differences in scale between the series.

where the parameters are computed using the same sample  $t = 1, ..., T_0 - 1$ .

In the main text we report the results where the desired fit coincides with the one obtained by OLS estimation on the small model with p = 13, that is for

$$Fit = \frac{1}{3} \sum_{i \in \mathcal{I}} \frac{\text{msfe}_i^{(\lambda, m)}}{\text{msfe}_i^{(0)}} \bigg|_{\lambda = \infty, m = \text{SMALL}}.$$

In Tables B.1 and B.2 in the Appendix we present the results for a range of in-sample fits and show that they are qualitatively the same provided that the fit is not below 50%.

Table 1 presents the relative MSFE for forecast horizons h = 1, 3, 6 and 12. The specifications are listed in order of increasing size and the last row indicates the value of the shrinkage hyperparameter  $\lambda$ . This has been set so as to maintain the in-sample fit fixed, which requires the degree of shrinkage,  $1/\lambda$ , to be larger the larger is the size of the model.

Table 1: Relative MSFE, BVAR

SMALL CEE MEDIU

		SMALL	CEE	MEDIUM	LARGE
	EMPL	1.14	0.67	0.54	0.46
h=1	CPI	0.89	0.52	0.50	0.50
	FFR	1.86	0.89	0.78	0.75
	EMPL	0.95	0.65	0.51	0.38
h=3	CPI	0.66	0.41	0.41	0.40
	FFR	1.77	1.07	0.95	0.94
	EMPL	1.11	0.78	0.66	0.50
h=6	CPI	0.64	0.41	0.40	0.40
	FFR	2.08	1.30	1.30	1.29
	EMPL	1.02	1.21	0.86	0.78
h=12	CPI	0.83	0.57	0.47	0.44
	FFR	2.59	1.71	1.48	1.93
	λ	$\infty$	0.262	0.108	0.035

Notes: Table reports MSFE relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h and different models. SMALL, CEE, MEDIUM and LARGE refer to the VARs with 3, 7, 20 and 131 variables, respectively.  $\lambda$  is the shrinkage hyperparameter and is set so that the average in-sample fit for the three variable of interest is the same as in the SMALL model estimated by OLS. The evaluation period is 1971-2003.

Three main results emerge from the Table. First, adding information helps to improve

the forecast for all variables included in the table and across all horizons. However, and this is a second important result, good performance is already obtained with the medium size model containing twenty variables. This suggests that for macroeconomic forecasting, there is no need to use many sectoral and conjunctural information, beyond the twenty important macroeconomic variables since results do not improve significantly although they do not get worse<sup>3</sup>. Third, the forecast of the federal funds rate does not improve over the simple random walk model beyond the first quarter. We will see later that by adding additional priors on the sum of the coefficients these results, and in particular those for the federal funds rate, can be substantially improved.

### 3.1 Parsimony by lags selection

In VAR analysis there are alternative procedures to obtain parsimony. One alternative method to the BVAR approach is to implement information criteria for lag selection and then estimate the model by OLS. In what follows we will compare results obtained using these criteria to those obtained from the BVARs.

Table 2 presents the results for SMALL and CEE. We report results for p=13 lags and for the number of lags p selected by the BIC criterion. For comparison, we also recall from Table 1 the results for the Bayesian estimation of the model of the same size. We do not report estimates for p=13 and BIC selection for the large model since for that size the estimation by OLS and p=13 is unfeasible. However, we recall in the last column the results for the large model estimated by Bayesian approach.

These results show that for the model SMALL, BIC selection results in the best forecast accuracy. For the larger *CEE* model, the classical VAR with lags selected by BIC and the BVAR perform similarly. Both specifications are, however, outperformed by the large Bayesian VAR.

### 3.2 The Bayesian VAR and the Factor Augmented VAR (FAVAR)

Factor models have been shown to be successful at forecasting macroeconomic variables with a large number of predictors. It is therefore natural to compare forecasting results based on the Bayesian VAR with those produced by factor models where factors are

<sup>&</sup>lt;sup>3</sup>However, due to their timeliness, conjunctural information, may be important for improving early estimates of variables in the current quarter as argued by Giannone, Reichlin, and Small (2008). This is an issue which we do not explore here.

Table 2: Relative MSFE, OLS and BVAR

			SMALL			CEE		LARGE
		p = 13	p = BIC	BVAR	p = 13	p = BIC	BVAR	BVAR
h=1	EMPL	1.14	0.73	1.14	7.56	0.76	0.67	0.46
	CPI	0.89	0.55	0.89	5.61	0.55	0.52	0.50
	FFR	1.86	0.99	1.86	6.39	1.21	0.89	0.75
h=3	EMPL	0.95	0.76	0.95	5.11	0.75	0.65	0.38
	CPI	0.66	0.49	0.66	4.52	0.45	0.41	0.40
	FFR	1.77	1.29	1.77	6.92	1.27	1.07	0.94
h=6	EML	1.11	0.90	1.11	7.79	0.78	0.78	0.50
	CPI	0.64	0.51	0.64	4.80	0.44	0.41	0.40
	FFR	2.08	1.51	2.08	15.9	1.48	1.30	1.29
h=12	EMPL	1.02	1.15	1.02	22.3	0.82	1.21	0.78
	CPI	0.83	0.56	0.83	21.0	0.53	0.57	0.44
	FFR	2.59	1.59	2.59	47.1	1.62	1.71	1.93

Notes: Table reports MSFE relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h and different models. SMALL, CEE refer to the VARs with 3 and 7 variables, respectively. Those systems are estimated by OLS with number of lags fixed to 13 or chosen by the BIC. The evaluation period is 1971-2003. For comparison the results of Bayesian estimation of the two models and of the large model are also provided.

estimated by principal components.

A comparison of forecasts based, alternatively, on Bayesian regression and principal components regression has recently been performed by De Mol, Giannone, and Reichlin (2008) and Giacomini and White (2006). In those exercises, variables are transformed to stationarity as is standard practice in the principal components literature. Moreover, the Bayesian regression is estimated as a single equation.

Here we want to perform an exercise in which factor models are compared with the standard VAR specification in the macroeconomic literature where variables are treated in levels and the model is estimated as a system rather than as a set of single equations. Therefore, for comparison with the VAR, rather than considering principal components regression, we will use a small VAR (with variables in levels) augmented by principal components extracted from the panel (in differences). This is the FAVAR method advocated by Bernanke, Boivin, and Eliasz (2005) and discussed by Stock and Watson (2005b).

More precisely, principal components are extracted from the large panel of 131 variables. Variables are first made stationary by taking first differences wherever we have

imposed a random walk prior  $\delta_i = 1$ . Then, as principal components are not scale invariant, variables are standardised and the factors are computed on standardised variables, recursively at each point T in the evaluation sample.

We consider specifications with one and three factors and look at different lag selection for the VAR. We set p=13, as in Bernanke, Boivin, and Eliasz (2005) and we also consider the p selected by the BIC criterion. Moreover, we consider Bayesian estimation of the FAVAR (BFAVAR), taking p=13 and choosing the shrinkage hyperparameter  $\lambda$  that results in the same in-sample fit as in the exercise summarized in Table 1.

Results are reported in Table 3 (the last column recalls results from the large Bayesian VAR for comparison).

		FA	VAR 1 fact	or	FA	VAR 3 fact	ors	LARGE
		p = 13	p = BIC	BVAR	p = 13	p = BIC	BVAR	BVAR
h=1	EMPL	1.36	0.54	0.70	3.02	0.52	0.65	0.46
	CPI	1.10	0.57	0.65	2.39	0.52	0.58	0.50
	FFR	1.86	0.98	0.89	2.40	0.97	0.85	0.75
h=3	EMPL	1.13	0.55	0.68	2.11	0.50	0.61	0.38
	CPI	0.80	0.49	0.55	1.44	0.44	0.49	0.40
	FFR	1.62	1.12	1.03	3.08	1.16	0.99	0.94
h=6	EMPL	1.33	0.73	0.87	2.52	0.63	0.77	0.50
	CPI	0.74	0.52	0.55	1.18	0.46	0.50	0.40
	FFR	2.07	1.31	1.40	3.28	1.45	1.27	1.29
h=12	EMPL	1.15	0.98	0.92	3.16	0.84	0.83	0.78
	CPI	0.95	0.58	0.70	1.98	0.54	0.64	0.44
	FFR	2.69	1.43	1.93	7.09	1.46	1.69	1.93

Table 3: Relative MSFE, FAVAR

Notes: Table reports MSFE for the FAVAR model relative to that from the benchmark model (random walk with drift) for employment (EMPL), CPI and federal funds rate (FFR) for different forecast horizons h. FAVAR includes 1 or 3 factors and the three variables of interest. The system is estimated by OLS with number of lags fixed to 13 or chosen by the BIC and by applying Bayesian shrinkage. The evaluation period is 1971-2003. For comparison the results from large Bayesian VAR are also provided.

The Table shows that the FAVAR is in general outperformed by the BVAR of large size and that therefore Bayesian VAR is a valid alternative to factor based forecasts, at least to those based on the FAVAR method.<sup>4</sup> We should also note that BIC lag selection generates the best results for the FAVAR while the original specification of Bernanke, Boivin, and Eliasz (2005) with p = 13 performs very poorly due to its lack of parsimony.

<sup>&</sup>lt;sup>4</sup>De Mol, Giannone, and Reichlin (2008) show that for regressions based on stationary variables, principal components and Bayesian approach lead to comparable results in terms of forecast accuracy.

### 3.3 Prior on the sum of coefficients

The literature has suggested that improvement in forecasting performance can be obtained by imposing additional priors that constrain the sum of coefficients (see e.g. Sims, 1992; Sims and Zha, 1998; Robertson and Tallman, 1999). This is the same as imposing "inexact differencing" and it is a simple modification of the Minnesota prior involving linear combinations of the VAR coefficients, cf. Doan, Litterman, and Sims (1984).

Let us rewrite the VAR of equation (1) in its error correction form:

$$\Delta Y_t = c - (I_n - A_1 - \dots - A_p)Y_{t-1} + B_1 \Delta Y_{t-1} + \dots + B_{p-1} \Delta Y_{t-p+1} + u_t.$$
 (8)

A VAR in first differences implies the restriction  $(I_n - A_1 - \cdots - A_p) = 0$ . We follow Doan, Litterman, and Sims (1984) and set a prior that shrinks  $\Pi = (I_n - A_1 - \cdots - A_p)$  to zero. This can be understood as "inexact differencing" and in the literature it is usually implemented by adding the following dummy observations (cf. Section 2):

$$Y_d = \operatorname{diag}(\delta_1 \mu_1, \dots, \delta_n \mu_n) / \tau \qquad X_d = ((1 \ 2 \ \dots \ p) \otimes \operatorname{diag}(\delta_1 \mu_1, \dots, \delta_n \mu_n) / \tau \quad 0_{n \times 1}) . (9)$$

The hyperparameter  $\tau$  controls for the degree of shrinkage: as  $\tau$  goes to zero we approach the case of exact differences and, as  $\tau$  goes to  $\infty$ , we approach the case of no shrinkage. The parameter  $\mu_i$  aims at capturing the average level of variable  $y_{it}$ . Although the parameters should in principle be set using only prior knowledge, we follow common practice<sup>5</sup> and set the parameter equal to the sample average of  $y_{it}$ . Our approach is to set a loose prior with  $\tau = 10\lambda$ . The overall shrinkage  $\lambda$  is again selected so as to match the fit of the small specification estimated by OLS.

Table 4 reports results from the forecast evaluation of the specification with the sum of coefficient prior. They show that, qualitatively, results do not change for the smaller models, but improve significantly for the *MEDIUM* and *LARGE* specifications. In particular, the poor results for the federal funds rate discussed in Table 1 are now improved. Both the *MEDIUM* and *LARGE* models outperform the random walk forecasts at all the horizons considered. Overall, the sum of coefficient prior improves forecast accuracy, confirming the findings of Robertson and Tallman (1999).

<sup>&</sup>lt;sup>5</sup>See for example Sims and Zha (1998).

Table 4: Relative MSFE, BVAR with the prior on the sum of coefficients

		SMALL	CEE	MEDIUM	LARGE
	EMPL	1.14	0.68	0.53	0.44
h=1	CPI	0.89	0.57	0.49	0.49
	FFR	1.86	0.97	0.75	0.74
	EMPL	0.95	0.60	0.49	0.36
h=3	CPI	0.66	0.44	0.39	0.37
	FFR	1.77	1.28	0.85	0.82
	EMPL	1.11	0.65	0.58	0.44
h=6	CPI	0.64	0.45	0.37	0.36
	FFR	2.08	1.40	0.96	0.92
	EMPL	1.02	0.65	0.60	0.50
h=12	CPI	0.83	0.55	0.43	0.40
	FFR	2.59	1.61	0.93	0.92

Notes for Table 1 apply. The difference is that the prior on the sum of coefficients has been added. The tightness of this prior is controlled by the hyperparameter  $\tau = 10\lambda$ , where  $\lambda$  controls the overall tightness.

# 4 Structural analysis: impulse response functions and variance decomposition

We now turn to the structural analysis and estimate, on the basis of BVARs of different size, the impulse responses of different variables to a monetary policy shock.

To this purpose, we identify the money shock by using a recursive identification scheme adapted to a large number of variables. We follow Bernanke, Boivin, and Eliasz (2005), Christiano, Eichenbaum, and Evans (1999) and Stock and Watson (2005b) and divide the variables in the panel into two categories: slow- and fast-moving. Roughly speaking the former group contains real variables and prices while the latter consists of financial variables (the precise classification is given in the Appendix A). The identifying assumption is that slow-moving variables do not respond contemporaneously to a monetary policy shock and that the information set of the monetary authority contains only past values of the fast-moving variables.

The monetary policy shock is identified as follows. We order the variables as  $Y_t = (X_t, r_t, Z_t)'$ , where  $X_t$  contains the  $n_1$  slowly moving variables,  $r_t$  is the monetary policy instrument and  $Z_t$  contains the  $n_2$  fast moving variables and we assume that the monetary policy shock is orthogonal to all other shocks driving the economy. Let  $B = CD^{1/2}$  be the  $n \times n$  lower diagonal Cholesky matrix of the covariance of the residuals of the

reduced form VAR, that is  $CDC' = \mathbb{E}[u_t u_t'] = \Psi$  and  $D = \text{diag}(\Psi)$ .

Let  $e_t$  be the following linear transformation of the VAR residuals:  $e_t = (e_{1t}, ..., e_{nt})' = C^{-1}u_t$ . The monetary policy shock is the row of  $e_t$  corresponding to the position of  $r_t$ , that is  $e_{n_1+1,t}$ .

The Structural VAR can hence be written as

$$A_0Y_t = \nu + A_1Y_{t-1} + \dots + A_pY_{t-p} + e_t, \quad e_t \sim WN(0, D),$$

where 
$$\nu = C^{-1}c$$
,  $\mathcal{A}_0 = C^{-1}$  and  $\mathcal{A}_j = C^{-1}A_j$ ,  $j = 1, ..., p$ .

Our experiment consists in increasing contemporaneously the federal funds rate by one hundred basis points.

Since we have just identification, the impulse response functions are easily computed following Canova (1991) and Gordon and Leeper (1994) by generating draws from the posterior of  $(A_1, ..., A_p, \Psi)$ . For each draw  $\Psi$  we compute B and C and we can then calculate  $A_j, j = 0, ..., p$ .

We report the results for the same overall shrinkage as given in Table 4. Estimation is based on the sample 1961-2002. The number of lags remains 13. Results are reported for the specification including sum of coefficients priors since it is the one providing the best forecast accuracy and also because, for the *LARGE* model, without sum of coefficients prior, the posterior coverage intervals of the impulse response functions become very wide for horizons beyond two years, eventually becoming explosive (cf. the Appendix, Figure C.1). For the other specifications, the additional prior does not change the results.

Figure 1 displays the impulse response functions for the four models under consideration and for the three key variables. The shaded regions indicate the posterior coverage intervals corresponding to 90 and 68 percent confidence levels. Table 5 reports the percentage share of the monetary policy shock in the forecast error variance for chosen forecast horizons.

Results show that, as we add information, impulse response functions slightly change in shape which suggests that conditioning on realistic informational assumptions is important for structural analysis as well as for forecasting. In particular, it is confirmed that adding variables helps in resolving the price puzzle (on this point see also Bernanke and Boivin, 2003; Christiano, Eichenbaum, and Evans, 1999). Moreover, for larger models the effect of monetary policy on employment becomes less persistent, reaching a peak at about one year horizon. For the large model, the non-systematic component

of monetary policy becomes very small, confirming results in Giannone, Reichlin, and Sala (2004) obtained on the basis of a factor model. It is also important to stress that impulse responses maintain the expected sign for all specifications.

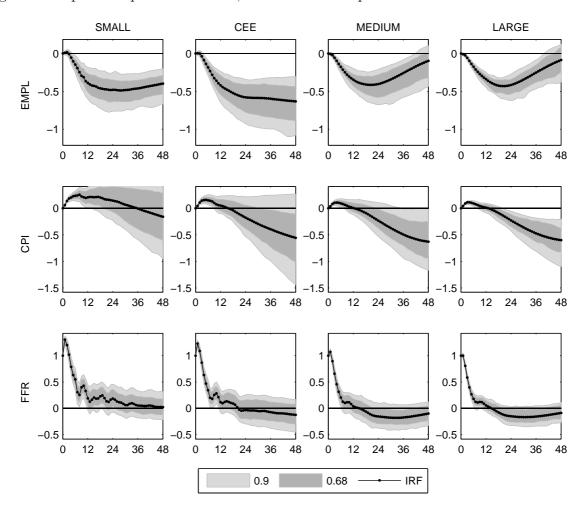


Figure 1: Impulse response functions, BVAR with the prior on the sum of coefficients

Notes: Figure presents the impulse response functions to a monetary policy shock and the corresponding posterior coverage intervals at 0.68 and 0.9 level for employment (EMPL), CPI and federal funds rate (FFR). SMALL, CEE, MEDIUM and LARGE refer to the VARs with 3, 7, 20 and 131 variables, respectively. The prior on the sum of coefficients has been added with the hyperparameter  $\tau = 10\lambda$ .

The same features can be seen from the variance decomposition, reported in Table 5. As the size of the model increases, the size of the monetary shock decreases. This is not surprising, given the fact that the forecast accuracy improves with size, but it highlights an important point. If realistic informational assumptions are not taken into consideration, we may mix structural shocks with miss-specification errors. Clearly, the assessment of the importance of the systematic component of monetary policy depends

Table 5: Variance decomposition, BVAR with the prior on the sum of coefficients

	Hor	SMALL	CEE	MEDIUM	LARGE
EMPL	1	0	0	0	0
	3	0	0	0	0
	6	1	1	2	2
	12	5	7	7	5
	24	12	14	13	8
	36	18	19	14	7
	48	23	23	12	6
CPI	1	0	0	0	0
	3	3	2	1	2
	6	7	5	3	3
	12	6	3	1	1
	24	2	1	1	1
	36	1	2	3	2
	48	1	3	5	3
FFR	1	99	97	93	51
	3	90	84	71	33
	6	74	66	49	21
	12	46	39	30	14
	24	26	21	18	9
	36	21	17	16	7
	48	18	15	16	7

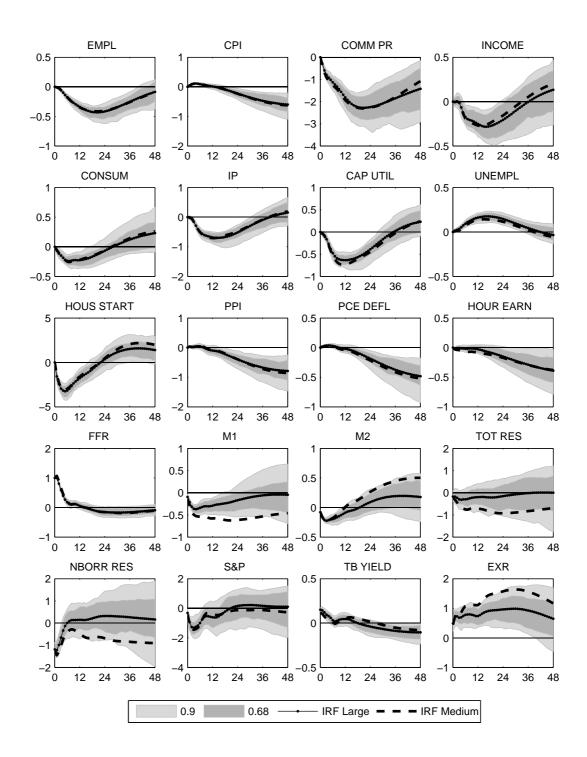
Notes: Table reports the percentage share of the monetary policy shock in the forecast error variance for chosen forecast horizons for employment (EMPL), CPI and federal funds rate (FFR). SMALL, CEE, MEDIUM and LARGE refer to the VARs with 3, 7, 20 and 131 variables, respectively. The prior on the sum of coefficients has been added with the hyperparameter  $\tau=10\lambda$ . The models have been estimated in the sample 1961-2002.

on the conditioning information set used by the econometrician and this may differ from that which is relevant for policy decisions. Once the realistic feature of large information is taken into account by the econometrician, the estimate of the size of the non-systematic component decreases.

Let us now comment on the impulse response functions of the monetary policy shock on all the twenty variables considered in the *MEDIUM* model. Impulse responses and variance decomposition for all the variables and models are reported in the Appendix, Tables C.1-C.4.

Figure 2 reports the impulses for both the MEDIUM and LARGE model as well as the posterior coverage intervals produced by the LARGE model.

Figure 2: Impulse response functions for model MEDIUM and LARGE, BVAR with the prior on the sum of coefficients



Notes: Figure presents the impulse response functions to a monetary policy shock from MEDIUM and LARGE specifications for all the variables included in MEDIUM. The posterior coverage intervals at 0.68 and 0.9 level correspond to the LARGE specification. The prior on the sum of coefficients has been added with the hyperparameter  $\tau=10\lambda$ .

Let us first remark that the impulse responses are very similar for the two specifications and in most cases those produced by the MEDIUM model are within the coverage intervals of the LARGE model. This reinforces our conjecture that a VAR with 20 variables is sufficient to capture the relevant shocks and the extra information is redundant.

Responses have the expected sign. First of all, a monetary contraction has a negative effect on real economic activity. Beside employment, consumption, industrial production and capacity utilization respond negatively for two years and beyond. By contrast, the effect on all nominal variables is negative. Since the model contains more than the standard nominal and real variables, we can also study the effect of monetary shocks on housing starts, stock prices and exchange rate. The impact on housing starts is very large and negative and it lasts about one year. The effect on stock prices is significantly negative for about one year. Lastly, the exchange rate appreciation is persistent in both nominal and real terms as found in Eichenbaum and Evans (1995).

### 5 Summary

This paper assesses the performance of Bayesian VAR for monetary models of different size. We consider standard specifications in the literature with three and seven macroe-conomic variables and also study a VARs with twenty and a hundred and thirty variables. The latter considers sectoral and conjunctural information in addition to macroeconomic information. We examine both forecasting accuracy and structural analysis of the effect of a monetary policy shock.

The setting of the prior follows standard recommendations in the Bayesian literature except for the fact that the overall tightness hyperparameter is set in relation to the model size. As the model becomes larger, we increase the overall shrinkage so as to maintain the same in-sample fit across models and guarantee a meaningful model comparison.

Overall, results show that a standard Bayesian VAR model is an appropriate tool for large panels of data. Not only a Bayesian VAR estimated over one hundred variables is feasible, but it produces better forecasting results than the typical seven variables VAR considered in the literature. The structural analysis on the effect of the monetary shock shows that a VAR based on twenty variables produces results that remain robust when the model is enlarged further.

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# Description of the data set

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Log RW prior	×××××× ××××××××××××× ×××××××××××××××××
MEDIUM	××××××××××××××××××××××××××××××××××××××
CEE	×××
SMALL	××
Slow/Fast	x x x x x x x x x x x x x x x x x x x
Series	BEMPLOYERS ON NORMARIA PARFOLLS: TOTAL PHYNEE  PROPERTY ALL PROBACHES THANSE READ STREAMS TO THE PHYNEE  PROPERTY ALL PRODUCTION (N.C.) AND PRICES (1998—100) [EG-40]  PRESONAL INCOME IESS THANSE READ PRICES (1998—100) [EG-40]  PREAL CONSIGNATION (AC)  THAN ALL OMNINATION (AC)  THAN ALL WORKERS OF PRODUCTION NOST WAS A OVER (%A)  THEAL CONSIGNATION (AC)  THAN ALL WORKERS OF PRODUCTION NOST WAS A OVER (%A)  PREDADUCE B PRICE TO THE CALL WORKERS OF PRIV NOWFAM PAYPOLLS - GOODS PRODUCING PRESONAL INCOME (AR. BILL CHAIN 2006) \$  PRODUCE B PRICE TO THE CALL WORKERS OF PRIV NOWFAM PAYPOLLS - GOODS PRODUCING PRESONAL INCOME (AR. BILL CHAIN 2006) \$  PRODUCE B PRICE TO THE CALL WORKERS OF PRIV NOWFAM PAYPOLLS - GOODS PRODUCING PRESONAL INCOME (AR. BILL CHAIN 2006) \$  PRODUCTION NUES. THE PRODUCTION NUES. THE PRODUCTS AND THE PRODUCTS OF THE PRODUCT OF THE PRO
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HOUSES AUTHORIZED BY BUILD. PERMITS:WEST(THOUS.U.,S.A.) PURCHASING MANAGERS, INDEX (SA) NAPM NEW ORDERS INDEX (PERCENT) NAPM VENDOR DELLVERIES INDEX (PERCENT)	NAPM INVENTORIES INDEX (PERCENT)  MFRS' NEW ORDERS, CONSUMER GOODS AND MATERIALS (BIL. CHAIN 1982 \$)  MFRS' NEW ORDERS, DURABLE GOODS INDUSTRIES (BIL. CHAIN 2000 \$)  MFRS' NEW ORDERS, DURABLE GOODS INDUSTRIES (BIL. CHAIN 1982 \$)  MFRS' NEW ORDERS, DURABLE GOODS INDUS. (BIL. CHAIN 1982 \$)  MANUFACTURING AND TRADE INVENTORIES (BIL. CHAIN 2000 \$)  RATIO, MFG. AND TRADE INVENTORIES (BIL. CHAIN 2000 \$)  PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (1982=100,SA)  PRODUCER PRICE INDEX:INTERMED MAT:SUPPLIES & COMPONENTS(1982=100,SA)	PRODUCER PRICE INDEX:CRUDE MATERIALS (1982=100,SA)  NAPM COMMODITY PRICES INDEX (PERCENT)  CPI-U: APPAREL & UPKEEP (1982-84=100,SA)  CPI-U: TRANSPORTATION (1982-84=100,SA)  CPI-U: MEDICAL CARE (1982-84=100,SA)  CPI-U: COMMODITIES (1982-84=100,SA)	CPL-U: DVRABLES (1982-84=100,SA)  CPL-U: SERVICES (1982-84=100,SA)  CPL-U: ALL ITEMS LESS FOOD (1982-84=100,SA)  CPL-U: ALL ITEMS LESS SHEUTER (1982-84=100,SA)  CPL-U: ALL ITEMS LESS MIDICAL CARE (1982-84=100,SA)  CPL-U: ALL ITEMS LESS MIDICAL CARE (1982-84=100,SA)  POEL INPL PR DEFL:PCE: DVRABLES (1987=100)	PCE, IMPL PR DEFL:PCE; NONDURABLES (1996=100) PCE, IMPL PR DEFL:PCE; SERVICES (1987=100) AVG HRLY EARNINGS OF PROD OR NONSUP WORKERS ON PRIV NONFARM PAYROLLS - CONSTRUCTION AVG HRLY EARNINGS OF PROD OR NONSUP WORKERS ON PRIV NONFARM PAYROLLS - MANUFACTURING U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)		MONEY STOCK MAZMIL O'NTE RESERVES EUROS (JPREB) DMRFS&SAKSM TIME DEP(BIL \$,SA) DEPOSITORY INST RESERVES:TOTAL, ADJ FOR RESERVE REQ CHGS(MIL, S,SA) DEPOSITORY INST RESERVES:AONBORROWED, ADJ, RES REQ CHGS(MIL, S,SA) MONEY STOCK, MI(CURR, TRAV CKS,DEM DEP(OTHER CK'ABLE DEP)(BILL,S,SA) S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1914-43=10) INTEREST RATE: U.S.TREASURY CONST MATURITIES,1A*R.(% PER ANN,NA) MONEY STOCK: MA(MAZH-LG TIME BEP, TERM RP'S&INST ONLY MAMFS)(BIL,S,SA) MONEY STOCK: MA(MAZH-LG TIME BEP, TERM RP'S&INST ONLY MAMFS)(BIL,S,SA) MONEY STOCK: MA(MAZH-LG TIME BEP, TERM RP'S&INST ONLY MAMFS)(BIL,S,SA) MONEY STOCK: MA(MAZH-LG TIME BEP, TERM RP'S&INST ONLY MAMFS)(BIL,S,SAA) MONEY STOCK: MA(MAZH-LG TIME BEP, TERM RP'S&INST ONLY MAMFS)(BIL,S,SAA) MONEY STOCK: MA(MAZH-LG TIME BERNER REQUIREMENT CHANGE (GIJ) MONEY STOCK: MA(MAZH-LG) MONEY STOCK: D'NTSTANDING IN OBSO DOLLARS (BCI) WYLLY RP LG COM'L BANKE;NET CREDIT OF PRESENVE (BL) CONNERRICAL & INDUSTRIAL LOANS OUSTINGEN (GIJ9) MATIO, CONSUMER (CREDIT OUTSTANDING IN OUTSTAND
PMI PMI PMNO PMDEL	PMNV A0M008 A0M007 A1M092 A1M092 A0M070 PWFCSA PWMSA	PWCMSA PMCP PU83 PU84 PU85	PUCD PUS PUXF PUXHS PUXM GMDCD	GMDCN GMDCS CES277 CES278 HHSNTN	FYFF	FM2 FMRA FMRABA FMRNBA FM1 FYGT10 EXRUS FM3 FM3 FM3 FM3 FM2 FCLNQ FCNG FCNG FYGM6 FYGT1 FYGT1 FYGM3 FY

# B Relative MSFE for different in-sample fits

Table B.1: Relative MSFE, BVAR

		Fit	SMALL	CEE	MEDIUM	LARGE
	EMPL	0.25	1.02	0.96	0.98	0.99
		0.5	0.58	0.59	0.53	0.48
		0.75	1.14	0.81	0.58	0.46
h=1	CPI	0.25	0.73	0.73	0.74	0.74
		0.5	0.54	0.48	0.50	0.50
		0.75	0.89	0.63	0.51	0.51
	FFR	0.25	1.03	1.02	1.02	1.02
		0.5	0.92	0.90	0.87	0.90
		0.75	1.86	1.04	0.80	0.73
	EMPL	0.25	1.06	0.97	1.01	1.02
		0.5	0.55	0.55	0.48	0.43
		0.75	0.95	0.81	0.56	0.38
h=3	CPI	0.25	0.65	0.65	0.67	0.67
		0.5	0.47	0.35	0.38	0.39
		0.75	0.66	0.52	0.44	0.42
	FFR	0.25	1.09	1.07	1.07	1.07
		0.5	1.10	0.99	1.02	1.10
		0.75	1.77	1.39	0.97	0.92
	EMPL	0.25	1.12	1.02	1.06	1.06
		0.5	0.64	0.62	0.56	0.51
		0.75	1.11	1.01	0.72	0.50
h=6	CPI	0.25	0.62	0.63	0.65	0.64
		0.5	0.49	0.33	0.34	0.36
		0.75	0.64	0.53	0.42	0.43
	FFR	0.25	1.18	1.15	1.16	1.16
		0.5	1.28	1.08	1.28	1.42
		0.75	2.08	1.91	1.38	1.26
	EMPL	0.25	1.28	1.13	1.19	1.19
		0.5	0.70	0.73	0.69	0.69
		0.75	1.02	1.45	0.94	0.82
h=12	CPI	0.25	0.64	0.65	0.66	0.66
		0.5	0.57	0.40	0.38	0.38
		0.75	0.83	0.72	0.51	0.49
	FFR	0.25	1.35	1.29	1.30	1.30
		0.5	1.46	1.24	1.50	1.84
		0.75	2.59	2.64	1.59	1.93
		0.25	0.0032	0.0021	0.0012	0.0005
	$\lambda$	0.5	0.2209	0.0626	0.0326	0.0124
		0.75	$\infty$	0.5990	0.1772	0.0538

Notes for Table 1 apply. The difference is that the shrinkage hyperparameter  $\lambda$  was set so that the in-sample fit for the three variables of interest equals 0.25, 0.5 or 0.75.

Table B.2: Relative MSFE, BVAR with the prior on the sum of coefficients

		Fit	SMALL	CEE	MEDIUM	LARGE
	EMPL	0.25	1.00	1.01	1.01	1.01
		0.5	0.57	0.53	0.50	0.46
		0.75	1.14	0.9	0.59	0.45
h=1	CPI	0.25	0.96	0.96	0.96	0.96
		0.5	0.53	0.51	0.49	0.50
		0.75	0.89	0.7	0.52	0.50
	FFR	0.25	1.00	1.00	1.00	1.00
		0.5	0.91	0.87	0.80	0.83
		0.75	1.86	1.19	0.77	0.73
	EMPL	0.25	0.99	1.00	1.00	1.00
		0.5	0.50	0.43	0.42	0.38
		0.75	0.95	0.83	0.55	0.37
h=3	CPI	0.25	0.94	0.95	0.95	0.95
		0.5	0.44	0.41	0.38	0.40
		0.75	0.66	0.55	0.42	0.38
	FFR	0.25	1.01	1.01	1.01	1.01
		0.5	1.14	0.96	0.86	0.90
		0.75	1.77	1.79	0.89	0.82
	EMPL	0.25	0.98	0.99	0.99	0.99
		0.5	0.52	0.47	0.48	0.42
		0.75	1.11	0.90	0.67	0.45
h=6	CPI	0.25	0.92	0.93	0.93	0.93
		0.5	0.46	0.40	0.36	0.38
		0.75	0.64	0.57	0.39	0.36
	FFR	0.25	1.02	1.02	1.02	1.02
		0.5	1.23	0.92	0.89	0.95
		0.75	2.08	2.32	1.09	0.94
	EMPL	0.25	0.97	0.98	0.98	0.98
		0.5	0.53	0.51	0.52	0.47
		0.75	1.02	0.82	0.68	0.53
h=12	CPI	0.25	0.90	0.91	0.91	0.91
		0.5	0.56	0.45	0.40	0.43
		0.75	0.83	0.74	0.47	0.41
	FFR	0.25	1.01	1.01	1.01	1.01
		0.5	1.32	0.86	0.78	0.82
		0.75	2.59	2.49	1.11	0.99

Notes for Tables 1 and 4 apply. The difference is that the shrinkage hyperparameter  $\lambda$  was set so that the in-sample fit for the three variables of interest equals 0.25, 0.5 or 0.75.

# C Impulse response functions and variance decomposition

SMALL CEE MEDIUM LARGE 0.5 0.5 0.5 0.5 0 -0.5 -0.5 -0.5 -0.5 12 36 48 48 0.5 0.5 0.5 0.5 0 CPI -0.5-0.5 -0.5 -0.5 0 48 0 12 24 36 48 0 12 36 48 0 0.5 0.5 0.5 0.5 FFR 0 -0.5 -0.5 -0.5 -0.5 0 36 48 0 12 24 36 0 36 48 0 24 0.9 0.68

Figure C.1: Impulse response functions, BVAR

Notes for Figure 1 apply. The difference is that the prior on the sum of coefficients has not been imposed.

Table C.1: IRF and variance decomposition for model SMALL, BVAR with the prior on the sum of coefficients

			Imp	pulse Re	esponse		Variance Decomposition							
Lag/Hor	0	3	6	12	24	36	48	1	3	6	12	24	36	48
CES002	0	-0.01	-0.15	-0.39	-0.48	-0.46	-0.40	0	0	1	5	12	18	23
PUNEW	0	0.18	0.24	0.20	0.14	-0.01	-0.16	0	3	7	6	2	1	1
FYFF	1	1.02	0.55	0.19	0.18	0.05	0.02	99	90	74	46	26	21	18

Notes: Table reports the values of impulse responses and the percentage share of the monetary policy shock in the forecast error variance for chosen forecast horizons for for all the variables in the model (the explanations for the mnemonics are given in the Appendix A). The prior on the sum of coefficients has been added with the hyperparameter  $\tau = 10\lambda$ . The model has been estimated in the sample 1961-2002.

Table C.2: IRF and variance decomposition for model CEE, BVAR with the prior on the sum of coefficients  ${\cal C}$ 

			Impu	lse Res		Variance Decomposition								
Lag/Hor	0	3	6	12	24	36	48	1	3	6	12	24	36	48
CES002	0	-0.04	-0.18	-0.42	-0.58	-0.60	-0.63	0	0	1	7	14	19	23
PUNEW	0	0.14	0.15	0.05	-0.18	-0.39	-0.56	0	2	5	3	1	2	3
PSM99Q	0	-0.95	-1.36	-2.00	-2.17	-1.63	-1.36	0	2	4	6	10	12	12
FYFF	1	0.87	0.34	0.12	-0.03	-0.08	-0.13	97	84	66	39	21	17	15
FM2	-0.09	-0.41	-0.42	-0.26	-0.10	-0.11	-0.17	4	13	15	11	4	2	1
FMRRA	0.03	-0.46	-0.77	-0.54	-0.08	0.14	0.27	0	0	2	2	1	1	1
FMRNBA	-1.07	-1.21	-0.54	-0.32	0.21	0.16	0.20	3	4	3	2	1	1	1

Notes for Table C.1 apply.

Table C.3: IRF and variance decomposition for model MEDIUM, BVAR with the prior on the sum of coefficients

			Impu	lse Res	ponse				Vari	ance	Deco	mpos	ition	
Lag/Hor	0	3	6	12	24	36	48	1	3	6	12	24	36	48
CES002	0	-0.06	-0.18	-0.35	-0.40	-0.26	-0.10	0	0	2	7	13	14	12
PUNEW	0	0.10	0.09	0.01	-0.23	-0.49	-0.63	0	1	3	1	1	3	5
PSM99Q	0	-0.92	-1.25	-1.94	-2.27	-1.79	-1.08	0	2	4	7	12	13	12
A0M051	0	-0.01	-0.19	-0.27	-0.16	0.05	0.20	0	0	0	3	3	2	1
A0M224_R	0	-0.14	-0.25	-0.22	-0.06	0.14	0.26	0	1	3	6	4	2	3
IPS10	0	-0.15	-0.51	-0.71	-0.48	-0.08	0.19	0	0	2	8	10	7	6
A0M082	0	-0.18	-0.53	-0.74	-0.45	-0.02	0.24	0	0	2	11	16	11	9
LHUR	0	0.02	0.07	0.14	0.11	0.02	-0.06	0	0	1	5	11	9	7
HSFR	0	-2.86	-2.99	-1.61	0.47	2.05	1.98	0	$^2$	6	8	5	6	8
PWFSA	0	0.02	0.07	-0.09	-0.39	-0.72	-0.86	0	0	0	0	1	3	5
GMDC	0	0.02	0.02	-0.04	-0.22	-0.41	-0.52	0	0	0	0	1	3	5
CES275	0	-0.04	-0.07	-0.08	-0.17	-0.29	-0.38	0	0	1	1	1	$^2$	3
FYFF	1	0.66	0.20	0.04	-0.16	-0.18	-0.10	93	71	49	30	18	16	16
FM2	-0.07	-0.23	-0.18	0.01	0.27	0.45	0.50	2	6	5	3	2	3	4
FMRRA	-0.20	-0.60	-0.75	-0.72	-0.92	-0.83	-0.70	0	1	2	3	4	4	4
FMRNBA	-1.26	-1.02	-0.41	-0.48	-0.69	-0.84	-0.90	3	4	3	$^2$	$^2$	$^2$	2
FM1	-0.13	-0.50	-0.54	-0.58	-0.61	-0.53	-0.46	2	8	11	10	9	8	7
FSPCOM	-0.41	-1.45	-1.11	-0.38	-0.12	-0.12	-0.28	0	2	$^2$	1	1	1	0
FYGT10	0.12	0.05	0.01	0.05	0.02	-0.05	-0.08	4	2	$^2$	1	1	1	1
EXRUS	0.45	0.79	1.08	1.11	1.57	1.57	1.17	2	4	5	8	10	12	12

Notes for Table C.1 apply.

Table C.4: IRF and variance decomposition for model LARGE, BVAR with the prior on the sum of coefficients  $\,$ 

Lag/Hor				ılse Resi		9.2	40	-				mposi		
ana	0	3	6	12	24	36	48	1	3	6	12	24	36	48
CES002	0	-0.06	-0.18	-0.34	-0.42	-0.25	-0.09	0	0	2	5	8	7	6
PUNEW	0	0.11	0.09	0.01	-0.23	-0.47	-0.60	0	2	3	1	1	2	3
PSM99Q	0	-0.80	-1.13	-1.81	-2.25	-1.85	-1.42	0	2	4	6	8	8	7
A0M051	0	-0.01	-0.17	-0.25	-0.21	0.00	0.13	0	0	0	1	1	1	1
$A0M224_{-}R$	0	-0.15	-0.24	-0.23	-0.07	0.12	0.23	0	0	2	3	2	2	2
IPS10	0	-0.16	-0.49	-0.68	-0.52	-0.09	0.16	0	0	2	5	5	4	4
A0M082	0	0.03	0.09	0.16	0.15	0.04	-0.03	0	0	1	4	6	6	5
LHUR	0	-0.16	-0.48	-0.64	-0.39	0.02	0.23	0	0	1	5	5	4	3
HSFR	0	-2.95	-3.17	-1.77	0.36	1.52	1.39	0	1	4	5	4	3	4
PWFSA	0	0.03	0.04	-0.10	-0.40	-0.67	-0.79	0	0	0	0	1	2	3
GMDC	0	0.04	0.03	-0.02	-0.19	-0.37	-0.48	0	0	0	0	0	2	3
CES275	0	-0.01	-0.02	-0.03	-0.15	-0.29	-0.39	0	0	0	0	0	1	2
A0M052	0	-0.02	-0.14	-0.18	-0.15	0.00	0.10	0	0	0	1	1	1	1
A0M057	0	-0.25	-0.53	-0.60	-0.43	-0.03	0.22	0	0	2	4	5	4	3
A0M059	0	-0.17	-0.35	-0.26	-0.01	0.31	0.47	0	0	0	1	1	1	1
IPS11	o o	-0.15	-0.42	-0.60	-0.46	-0.06	0.18	0	0	1	4	4	3	3
IPS299	0	-0.12	-0.38	-0.57	-0.44	-0.07	0.17	0	0	1	2	3	3	2
IPS12	0	-0.12	-0.31	-0.33	-0.03	0.27	0.36	0	0	0	1	1	1	1
IPS13	0	-0.11	-0.31	-0.33	-0.03	0.66	0.30 $0.87$	0	0	0	1	1	1	1
IPS18	0	-0.25 -0.05	-0.09	-0.12	-0.03	0.00	0.37 $0.15$	0	0	0	0	0	0	0
IPS25	0				-0.01 -1.31	-0.74	-0.25		0	0	$\frac{0}{2}$	4	4	
	ı	-0.14	-0.59	-1.14				0		-				3
IPS32	0	-0.16	-0.57	-0.77	-0.57	-0.12	0.15	0	0	1	3	3	3	2
IPS34	0	-0.23	-0.88	-1.26	-0.92	-0.16	0.31	0	0	1	3	4	3	2
IPS38	0	-0.28	-0.62	-0.77	-0.45	-0.14	-0.03	0	0	1	4	5	4	3
IPS43	0	-0.19	-0.58	-0.80	-0.57	-0.07	0.22	0	0	1	4	5	4	3
IPS307	0	-0.36	-0.16	-0.37	-0.37	-0.27	-0.22	0	0	0	0	0	0	0
IPS306	0	0.15	-0.17	0.02	0.10	0.22	0.28	0	0	0	0	0	0	0
PMP	0	-1.17	-1.32	-0.51	0.46	0.53	0.32	0	1	4	5	4	4	4
LHEL	0	-0.99	-1.63	-2.19	-1.84	-0.47	0.52	0	1	3	5	6	6	5
LHELX	0	-0.02	-0.03	-0.05	-0.04	-0.01	0.01	0	1	2	5	6	5	5
LHEM	0	-0.04	-0.12	-0.23	-0.25	-0.14	-0.04	0	0	1	3	6	5	5
LHNAG	0	-0.04	-0.12	-0.24	-0.26	-0.14	-0.03	0	0	0	3	5	4	4
LHU680	0	0.01	-0.01	0.17	0.35	0.25	0.07	0	0	0	0	4	5	4
LHU5	0	0.63	1.14	1.19	0.50	-0.18	-0.54	0	0	0	1	1	1	1
LHU14	0	0.74	1.91	2.62	1.77	0.14	-0.84	0	0	0	2	3	2	2
LHU15	0	0.55	1.54	4.55	5.19	2.10	-0.68	0	0	0	2	5	5	4
LHU26	0	0.45	1.74	4.12	3.48	0.64	-1.32	0	0	0	1	2	2	2
LHU27	ő	0.76	1.37	5.10	7.06	3.69	0.08	0	0	0	1	4	4	3
A0M005	0	1.89	3.09	2.88	1.07	-0.37	-0.82	0	1	2	4	4	3	3
CES003	0	-0.10	-0.31	-0.59	-0.67	-0.37	-0.02	0	0	1	4	7	7	6
	0		0.11					0	0	0	0	0	0	
CES006	I	0.40		0.37	-0.36	-0.69	-0.82							1
CES011	0	-0.23	-0.44	-0.75	-0.87	-0.44	-0.02	0	0	1	3	5	4	3
CES015	0	-0.09	-0.31	-0.61	-0.65	-0.35	-0.12	0	0	1	4	7	7	6
CES017	0	-0.12	-0.40	-0.79	-0.88	-0.48	-0.15	0	0	1	4	8	8	7
CES033	0	-0.05	-0.15	-0.32	-0.29	-0.14	-0.06	0	0	0	1	2	2	1
CES046	0	-0.04	-0.09	-0.18	-0.25	-0.18	-0.07	0	0	1	3	4	4	3
CES048	0	-0.03	-0.11	-0.24	-0.31	-0.19	-0.04	0	0	0	2	4	4	3
CES049	0	-0.04	-0.10	-0.25	-0.37	-0.30	-0.16	0	0	1	2	5	6	5
CES053	0	-0.03	-0.11	-0.22	-0.23	-0.08	0.06	0	0	0	2	3	2	2
CES088	0	-0.02	-0.06	-0.13	-0.23	-0.19	-0.09	0	0	0	1	2	2	2
CES140	0	-0.04	-0.07	-0.09	-0.15	-0.13	-0.04	0	0	0	0	1	1	1
A0M048	0	-0.09	-0.24	-0.37	-0.40	-0.22	-0.06	0	0	1	3	4	4	5
CES151	0	0.00	-0.05	-0.04	-0.01	0.02	0.03	0	0	0	1	2	2	2
CES155	0	-0.02	-0.05	-0.05	-0.02	0.02	0.03	0	0	1	2	2	2	2
AoM001	0	-0.01	-0.06	-0.06	-0.01	0.03	0.04	0	0	0	2	2	2	5
PMEMP	0	-0.62	-1.15	-0.80	0.11	0.42	0.37	0	0	2	5	$\overline{4}$	$\overline{4}$	5
HSNE	ő	-2.14	-2.84	-1.55	0.33	1.73	1.97	0	0	1	2	1	1	2
HSMW	0	-3.79	-4.65	-2.14	-0.25	1.08	1.28	0	1	2	4	4	3	4
HSSOU	0	-3.19 -2.73	-4.03 -2.81	-2.14 -1.47	0.65	1.54	1.17	0	1	2	3	2	2	3
	l .	-2.75 -2.75								1	3 2	$\frac{2}{2}$	$\frac{2}{2}$	2
HSWST	0		-2.81	-2.22	0.15	1.63	1.57	0	0					
HSBR	0	-3.21	-3.13	-1.98	0.36	1.56	1.42	0	2	4	5	4	3	4
HSBNE	0	-2.49	-2.91	-1.93	0.34	1.88	2.10	0	1	2	3	2	2	3
HSBMW	0	-4.36	-4.77	-2.54	-0.34	1.12	1.41	0	2	4	5	5	4	4
HSBSOU	0	-2.92	-2.95	-1.72	0.55	1.50	1.15	0	1	2	3	2	2	2
HSBWST	0	-2.80	-2.42	-2.26	0.19	1.60	1.50	0	0	1	1	1	1	2

PMI	0	-0.98	-1.19	-0.66	0.27	0.52	0.37	0	1	3	5	4	4	4
PMNO	0	-1.43	-1.35	-0.37	0.52	0.57	0.32	0	1	3	4	3	4	4
PMDEL	0	-0.63	-1.13	-1.16	-0.11	0.66	0.59	0	0	0	1	2	2	2
PMNV	0							0	0	0	1	2	2	$\frac{2}{2}$
		-0.37	-0.54	-0.86	-0.04	0.38	0.35							
A0M008	0	-0.52	-0.91	-0.90	-0.36	0.26	0.50	0	0	1	3	3	2	2
A0M007	0	-0.86	-1.38	-1.58	-1.05	-0.16	0.28	0	0	1	3	4	3	2
A0M027	0	-1.15	-1.99	-2.71	-2.56	-1.59	-0.87	0	0	0	1	2	2	2
A1M092	0	-0.30	-0.54	-1.00	-1.54	-1.14	-0.44	0	0	1	2	4	4	4
A0M070	0	0.06	0.03	-0.23	-0.40	-0.25	-0.03	0	0	0	1	3	3	2
A0M077	0	0.00	0.01	0.01	0.00	0.00	0.00	0	0	1	3	2	1	1
l .	1													
PWFCSA	0	0.01	0.03	-0.16	-0.49	-0.77	-0.87	0	0	0	0	1	2	3
PWIMSA	0	0.00	-0.08	-0.29	-0.66	-0.93	-1.00	0	0	0	0	1	3	3
PWCMSA	0	0.03	-0.29	-0.80	-1.16	-1.17	-1.13	0	0	0	0	1	1	1
PMCP	0	-1.13	-1.84	-1.70	-0.70	-0.13	0.26	0	0	1	4	4	4	4
PU83	0	0.03	-0.01	-0.06	-0.22	-0.37	-0.48	0	0	0	0	1	2	3
PU84	0	0.08	0.05	-0.07	-0.26	-0.56	-0.77	0	0	0	0	0	1	2
PU85	0	0.05	0.07	0.11	0.04	-0.14	-0.31	0	0	0	1	0	0	1
	1													
PUC	0	0.06	0.03	-0.07	-0.33	-0.57	-0.67	0	0	0	0	1	2	3
PUCD	0	0.03	-0.02	0.04	-0.07	-0.25	-0.38	0	0	0	0	0	0	1
PUS	0	0.17	0.17	0.12	-0.09	-0.35	-0.52	0	1	3	2	1	1	2
PUXF	0	0.11	0.09	0.03	-0.18	-0.44	-0.61	0	1	1	1	0	1	3
PUXHS	0	0.07	0.06	-0.01	-0.22	-0.43	-0.56	0	0	0	0	0	2	3
PUXM	0	0.12	0.10	0.01	-0.24	-0.49	-0.61	0	1	1	1	1	2	3
	1													
GMDCD	0	0.01	0.02	0.03	-0.08	-0.27	-0.40	0	0	0	0	0	0	1
GMDCN	0	0.06	0.03	-0.09	-0.38	-0.62	-0.71	0	0	0	0	1	3	4
GMDCS	0	0.02	0.03	0.02	-0.07	-0.21	-0.34	0	0	0	0	0	0	1
CES277	0	0.02	0.07	0.09	-0.05	-0.24	-0.38	0	0	0	0	0	0	1
CES278	0	0.00	-0.04	-0.06	-0.16	-0.27	-0.37	0	0	0	0	0	1	1
HHSNTN	0	-0.94	-0.89	-0.37	0.87	1.08	0.76	0	0	1	1	1	1	2
	1							1						- 1
FYFF		0.58	0.18	0.05	-0.15	-0.16	-0.09	51	33	21	14	9	7	7
FM2	-0.08	-0.22	-0.20	-0.09	0.09	0.19	0.18	1	3	3	2	1	1	1
FMRRA	-0.17	-0.30	-0.30	-0.20	-0.15	-0.02	0.00	0	0	0	0	0	0	0
FMRNBA	-1.19	-0.65	0.01	0.15	0.32	0.26	0.16	1	1	1	1	0	0	0
FM1	-0.09	-0.36	-0.34	-0.29	-0.17	-0.06	-0.05	0	2	3	2	1	1	1
FSPCOM	-0.31	-1.31	-0.99	-0.61	0.16	0.17	0.11	0	1	1	0	0	0	0
FYGT10	0.15	0.09	0.03	0.05	-0.04	-0.09	-0.10	1	1	1	0	0	0	1
	1							1						- 1
EXRUS	0.49	0.68	0.78	0.74	0.96	0.93	0.65	1	2	2	3	3	4	3
FM3	-0.05	-0.15	-0.15	-0.10	-0.03	0.07	0.12	0	1	1	1	0	0	0
FM2DQ	-0.07	-0.24	-0.21	-0.05	0.29	0.58	0.67	1	2	2	1	1	2	3
FMFBA	-0.03	-0.15	-0.19	-0.17	-0.15	-0.15	-0.15	0	0	1	1	1	1	1
FCLNQ	0.38	0.27	0.13	-0.11	-0.80	-0.78	-0.36	1	1	1	0	0	1	1
FCLBMC	8.25	-2.08	-2.32	-2.27	-1.08	0.11	0.93	0	0	0	0	0	0	0
-	1										-			
CCINRV	0.02	-0.08	-0.19	-0.40	-0.53	-0.16	0.32	0	0	0	1	3	2	1
A0M095	0.00	-0.01	-0.01	-0.04	-0.05	0.02	0.09	0	0	0	0	1	1	1
FSPIN	-0.20	-1.30	-1.00	-0.56	0.24	0.20	0.14	0	0	1	0	0	0	0
FSDXP	0.01	0.07	0.05	0.03	-0.02	-0.02	-0.01	0	1	1	1	1	1	1
FSPXE	-0.57	-1.28	-0.75	0.58	2.46	1.88	0.96	0	0	0	0	1	1	1
CP90	0.80	0.47	0.11	0.04	-0.16	-0.18	-0.11	22	18	13	9	7	6	6
FYGM3	0.59	0.31	0.04	-0.02	-0.16	-0.17	-0.12	12	11	8	6	4	4	4
FYGM6	1	0.31 $0.28$		0.02		-0.17 -0.15		1	8					- 1
	0.53		0.04		-0.14		-0.10	10		6	5	4	4	4
FYGT1	0.50	0.26	0.04	0.01	-0.13	-0.15	-0.11	6	5	4	3	2	3	3
FYGT5	0.24	0.14	0.05	0.05	-0.05	-0.10	-0.10	2	1	1	1	1	1	1
FYAAAC	0.10	0.08	0.04	0.05	-0.03	-0.09	-0.12	1	1	1	0	0	1	1
FYBAAC	0.11	0.11	0.08	0.09	-0.01	-0.10	-0.14	1	1	1	1	1	1	1
SCP90	-0.20	-0.07	-0.03	0.02	0.02	0.01	0.01	3	4	5	4	4	4	4
sFYGM3	-0.20	-0.17	-0.03	0.02	0.02	0.01	0.01	6	10	9	8	7	7	7
	1													
sFYGM6	-0.43	-0.21	-0.05	0.02	0.07	0.05	0.02	7	11	10	9	8	8	8
sFYGT1	-0.45	-0.22	-0.05	0.03	0.08	0.05	0.02	4	7	7	6	6	6	6
sFYGT5	-0.73	-0.34	-0.04	0.07	0.16	0.10	0.02	8	11	10	8	7	8	7
sFYGT10	-0.82	-0.38	-0.05	0.08	0.18	0.12	0.02	9	12	11	8	8	8	7
sFYAAAC	-0.89	-0.40	-0.04	0.09	0.19	0.12	0.02	13	15	13	10	9	9	8
sFYBAAC	-0.89	-0.40	-0.04	0.03 $0.12$	0.13 $0.21$	0.12	-0.01	11	14	11	8	8	8	7
								1						
EXRSW	0.82	1.16	1.05	0.81	1.21	1.08	0.60	1	1	1	1	1	2	2
EXRJAN	0.60	0.73	1.13	1.12	1.05	0.69	0.28	0	1	1	1	1	1	1
EXRUK	-0.68	-1.15	-1.22	-1.15	-1.77	-2.08	-1.74	1	1	2	2	3	4	4
EXRCAN	0.11	0.22	0.05	0.08	0.06	0.26	0.42	0	0	0	0	0	0	1

Notes for Table C.1 apply.