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Implementing a Structural Valuation Model of Swap Credit-Sensitive Rates

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Implementing a Structural Valuation Model of Swap Credit-Sensitive Rates*

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Abstract

Currency and interest rate swaps are subject to a complex, two-sided default risk. Several theoretical papers have addressed the problem of pricing swap credit risk. I propose a complete implementation procedure of the structural line of research in theoretical credit risk analysis in order to attempt to evaluate an OTC contract such as the swap contract. It is shown how structural models can enable us to extract the whole credit risk information from scarce data if of good quality, which leads to the problem of mixing accounting and available financial data from traded prices. Ther analytical results are therefore benchmarked against actual transaction data. Although the results are not very satisfactory for swap pricing, the procedure provides interesting insights in some parameter estimations linked to the credit worthiness of the firm that show to be consistent indicators, useful for credit risk management purposes.

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1 Introduction

Swaps, the most widely used off-balance sheet derivative products–and most widely used derivative products as a whole–, are subject to credit risk as they are not traded on exchanges[1]. Their credit risk is complex to analyze as both counterparties are subject to non-null default probabilities. It is becoming crucial to price this credit risk for at least two reasons. First, markets become more complete and lower credit ratings access the swap markets (through the recent Credit Risk Swaps for example)[2]. Second, as their risk management systems get more sophisticated as far as market risk is concerned, financial institutions become more aware of the weakness of their credit risk exposure calculation and of the need to value this exposure, rather than ration it through credit lines.

Many theoretical developments have appeared in the last few years that allow for more precise pricing of credit risk on fixed-income instruments (with possible applications to swaps). There are two major lines of research in this field. One follows the basic principle of Merton [1974] and refine it by adding stochastic interest rates and more complex bankruptcy rules (see notably Shimko and alii [1993] and Longstaff and Schwartz [1994 and 1995]). This line of research follows traditional financial economics and contingent claim analysis. We thus call it the "structural" approach to credit risk pricing. It has the advantage of being fully analytical. Another line of research has evolved somewhat differently and takes the default process as an exogenous variable (see Jarrow and Turnbull [1995], Lando [1994] and Duffie and Huang [1996]). This second line of research is also based on arbitrage free arguments. It thus differs from past actuarial results that prevailed for a long time in banks' research departments (see Duffee [1995 a and b]). It differs from the first line of research (our so-called "structural" line of research) notably because the event of default is given exogenously to the model rather than being deducted from the process of the firm's value (or a related process). Because of this difference, the "structural" approach is conceptually more satisfying. The question then arises of whether it can be directly used in practice or whether it should remained confined to theoretical analysis of the credit risk problem. See Cossin [1997] for a survey of the credit risk literature.

The applicability of the first strand of research, the so-called "structural" approach, is tested here. The second strand of research is clearly –and necessarily– more

[1] There were $17,712 billion of swap or swap related products at the end of 1995 versus $ 9,185 billion of exchange traded derivatives (futures and options) (BIS report August 1996).

[2] Total outstanding credit risk derivatives contracts (mostly swaps) were valued at around $40 billion for the beginning of 1996 (see Charles Smithson, Credit Derivatives (II), Risk, June 1996).

applicable but has the disadvantage of being fitted to past data rather than to analytical solutions of process evolution. Applying one of these models (the more natural choice being Duffie and Huang [1996]) would certainly result in successfully fitting our data. Unfortunately, the predictive power of well fitted models is well-known to be rather low[3]. Moreover, they do not provide any recipe for the estimation of their parameters, while structural models provide a financial underlying framework that governs the use that can be made of them. I defend the view that a full analytical model, in the line of Merton [1974] and Black and Cox [1976], although being less good at fitting the data, would rely on a better and more complete rationale and would thus be a better tool for calculating future credit risk spreads. From the earlier study (see Cossin and Pirotte [1998]), the choice of the Longstaff and Schwartz [1995] model was made again, focusing more on the most efficient way to estimate the parameters. Indeed, some of them are already a result independently of the swap pricing objective. Although this line of research deserves special attention, the specific chosen model has its limitations, stressed underneath. It is has been considered since 1995, as the best choice among structural models in the current literature, but it is today criticised on its mathematical derivation. Because of this recent evidence, the results computed through the Longstaff and Schwartz [1995] model should be taken with care and have been still computed in the background as a reference. For the sake of rigor, we will present swap rates computed in the hypothesis of a null volatility of the interest rates, what is acceptable here in Switzerland since this volatility is very low (around 2%. The tests here below will provide the numerical proof.). The results tend to minimize, in our context, the bias that the Longstaff and Schwartz model could introduce in the swap pricing.

This research is thus constrained to be a joint test of the line of research described and of this specific model. The analysis will bring forward some difficulties that financial engineering is currently overcoming (notably the dual credit risk problem) and some deeper problems that arise when trying to fit real data with an advanced theoretical model. It is definitely a test of the practicability and rationality of the implementation allowed by a structural model rather than the sole precision in pricing that motivates the present study. In particular, the structural methodology will produce interesting results for continuous credit risk monitoring.

A medium size bank (MSB) provided us with a sample of interest rate and currency swaps actual transaction prices with some confidential counterparty information, a unique database currently in academic research as far as we know. It thus

[3] And more recently, different studies have attested the limited predictive quality of historical series of defaults and rating changes (see KMV's evidence in Crouhy and Mark [1998] for example).

allows for a more precise treatment of credit risk than done before, even though our sample remains small. In the case of this particular paper it allows to test both the ease of implementation and the practical validity of a model of credit risk pricing.

The paper proceeds by presenting briefly the analytical framework chosen, then by describing the data and the implementation method used, in particular for the estimation of the underlying parameters of such a structural model, before presenting the results of the estimation of the parameters and the final pricing with its comparison to the transaction prices. Conclusions follow.

2 Description of the Analytical Valuation Framework

In this paper, the final objective is to compute swap credit spreads analytically after having identified and estimated the inputs necessary to their calculation, in order to compare these spreads to our transaction data. In the present section, we describe the Longstaff and Schwartz [1995] model that will be used for this final pricing, stressing its basic assumptions and shortcomings. It is also shown then in which context we are still able to use the model given a major criticism made on its derivation, for the sake of rigor.

The Longstaff and Schwartz [1995] published paper treats risky debt valuation in a classical manner (i.e., in the spirit of Merton [1974] and Black and Cox [1976] but with stochastic interest rates) while the 1994 working paper included a section on swap valuation using the same framework. Their model provides closed-form expressions for the fixed and the floating sides of the swap contract (with both sides presenting some credit risk)[4]. Although the model has some shortcomings (for example: simplistic debt structure of the firm, no true two-sided default, the usual Vasicek framework limitations, etc.), it is one of the most advanced models currently in the classical line of credit risk analysis. It views credit risk as equivalent to a short position on a put option just as in the original Merton [1974] paper, but allowing default to happen prior to the maturity of the contract. The model relies on the definition of two basic processes for r, the short-term riskless interest rate which dynamics are assumed to be those of a Vasicek [1977] model, and for V, the total value of the assets of the firm which is assumed to follow a geometric Brownian motion:

$$dV = \mu V\,dt + \sigma_v V\,dZ_1 \tag{1}$$

[4]The double sided credit risk is not introduced perfectly though as the stopping time (end of contract linked to one party's default) of one counterparty does not depend on the stopping time of the other counterparty (see Duffie and Huang [1996]).

$$dr = (\xi - \beta r)\,dt + \eta\,dZ_2 \tag{2}$$

where μ, σ, ξ, β and η are constants and Z_1 and Z_2 are standard Wiener processes with instantaneous correlation ρdt.

The authors also make two major assumptions. First the value of the firm is independent from its capital structure. Second, while in most papers using option pricing frameworks bankruptcy time is defined as the moment when V, the value of the firm, reaches the face value of the debt, here a threshold value K is defined similarly to Black and Cox [1976]. Default occurs when V reaches K which gives more flexibility to define the time of financial distress in the calculation. X, the ratio of V over K, is a sufficient statistic for the riskiness of the firm. The authors then show that the value of a risky fixed discount bond (and so the value of the discount factors for the fixed payments of the swap) can be expressed as:

$$P(X, r, T) = D(r, T) - \omega\,D(r, T)\,Q(X, r, T), \tag{3}$$

where $D(r, T)$ is the value of the riskless discount bond given by Vasicek [1977] and ω is the loss value if default does occur. ω is related to the priority rules under distress. $Q(X, r, T)$ is an expression that can be thought of as the probability of default under the risk-neutral measure. For a riskless bond, ω and Q are equal to zero and in this case $P(X, r, T) = D(r, T)$. Moreover it is assumed that financial distress triggers the default of all of the firm's debt. $P(X, r, T)$ is decreasing in ω, r and T and increasing in X.

For the floating-rate payments of the floating side of the swap, the authors show that:

$$F(X, r, \tau, T) = P(X, r, T)\,R(r, \tau, T) + \omega\,D(r, T)\,G(X, r, \tau, T) \tag{4}$$

where τ is the time when the floating-rate is determined ($\tau \leqslant T$). If default occurs prior to T, the payoff on the claim is $(1 - \omega)r$ and that is what is reflected in the equation through the expressions $R(r, \tau, T)$ and $\omega\,D(r, T)\,G(X, r, \tau, T)$. (See Longstaff and Schwartz [1995] for the full expressions of Q, R and G).

But, the derivations of the cumulated probabilities $Q(X, r, T)$ and $G(X, r, \tau, T)$ seem to pose a mathematical problem. In a risk-neutral world, the drift μ of equation 1 becomes r_t, and r_t is precisely assumed to be stochastic here. Therefore, the expectation of the first passage time of V_t under K is influenced by the behavior of r_t not only through the correlation coefficient between the two processes (dr_t and dV_t/V_t), but also because of the drift of the risk-neutral expression of V_t. There is no mean to recover a one-dimensional problem. The expected time of default cannot therefore be considered as a point on a line but on a surface of possible passage-times.

Furthermore, there is already a missing element in their Vasicek's bond price but with an insignificant impact[5].

One way to encompass this problem of bi-dimensionality is to realize that, as it will be empirically obtained below, the volatility of interest rates is very low here in Switzerland (around 2%). Therefore, assuming a null volatility ($\eta = 0$) is not a very restrictive assumption and it allows us to still use the Longstaff and Schwartz model consistently. Indeed, the results show that a low volatility of interest rates has merely no effect on the pricing results compared to the null case.

Despite these considerations, Longstaff and Schwartz [1995] remains an interesting model not only for the integration of stochastic interest rates but also because of the possibility of early default, i.e. before T.

Assuming that a counterparty always makes its contractual payments if solvent (in particular, payment by a counterparty is not contingent on whether the other counterparty is solvent), the value of the fixed-rate payments is given by:

$$k \sum_{i=1}^{N} P(X, r, T_i) \tag{5}$$

and the value of the floating-rate payments is given by:

$$\sum_{i=1}^{N} F(X, r, T_i - \tau, T_i) \tag{6}$$

The fixed-rate coupon k which equates the two expressions is the swap rate. In other words, the rate k for which the value of the fixed rate payments equals the value of the floating rate payments is what is called the swap rate (we assume here a par swap).

Such a model, although it presents many advances over past results, still presents some limitations. Some researchers reproach the use of a Vasicek interest rate model as being too simple. Financial engineering should allow professionals to introduce more complex models rather straightforwardly. Nonetheless, some tests in the line of Chan, Karolyi, Longstaff and Sanders [1992] based on Swiss interest rates data (which we use later) tend to confirm the validity in practice of the Vasicek set-up (see Bruand [1998a]). Nonetheless, as such, the paper does not deal very well with the netting of flows in swap contracts. Because defaults have been so rare in swaps, it is difficult to say whether the measure given by the model is too conservative or

[5] The transcription of the Vasicek price of a credit riskfree zero-coupon bond, in the paper of Longstaff and Schwartz [1995], is not exact. More precisely, a term $(\exp(-\beta T) - 1)^2$ is replaced by $(\exp(-2\beta T) - 1)$ which is inaccurate. The difference can be shown however to be very small in the overall calculation (around 1 or 2 basis points).

whether settlements in court may actually reach these values. But the true weakness of the Longstaff and Schwartz paper probably lies in its treatment of the dual credit risk of swaps. Indeed, although double sided credit risk is integrated in the paper, default from one side does not trigger default on the other side. In this sense, the model is not a true dual credit risk paper such as Duffie and Huang [1996] and Hübner [1997] are in the other line of research.

Once these weaknesses have been acknowledged, we have to recognize the fact that there is no better alternative model in the classical line of research, and that some of the problems (notably default triggers and default values) are endemic to this classical line of research. As explained before though, this may be the price to pay to have a good underlying theory. One should remember in this regard that the more recent models and empirical studies still find comparative statics very much in line with those of the old Merton [1974] paper, which has proved that having the right, simple model at the start may still be acceptable beyond refinements and engineering of the real details.

The goal is thus now to use the kind of framework proposed by Longstaff and Schwartz [1995], with and without the additional assumption of a deterministic process for the interest rate, to avoid the mathematical criticism that is being made, while providing an implementation methodology that combines both the technologies of the modelling of the term structure of interest rates and that of the structural modelling of credit risk.

3 Data Description

We obtained confidential swap transaction data from a medium size bank which is state-guaranteed. We argue that, given this state-guaranty, the rating of this counterparty is indeed higher than AAA, but not yet perfectly riskless however. The confidential information we had available for each transaction was the type of transaction (bid or offer), the fixed rate (which corresponds to the swap transaction rate) with the initial LIBOR rate for IRS, the rating of the client (or the type of client when no rating is available), the amount traded, the begin date and the maturity date of the contract. The transactions considered did not involve collaterals (which became current practice only later). The interest rate swaps section of our data set includes transactions initiated between March 7, 1990 and December 15, 1994, with maturities of 3 up to 10 years, with an average of 8 years and for an average amount of close to $50 millions. All data considered in this study are Swiss Francs data. (See Cossin and Pirotte [1997] for a full description of the data base as well as a discussion of the problems linked to marking to market and credit lines and why they do not affect our data here). The implementation of the framework presented above is being made on a subsample of 23 swap transactions for which we have the counterparties' identity and where no collateralization occurred. As described in Cossin and Pirotte [1997], the sample, although unique in the academic world by 1997, has the double disadvantage of being small and of being of rather high credit risk quality. It includes 3 counterparties rated Aaa, Aa1 and Aa2 respectively. Nonetheless, it provides a unique basis for the study of swap pricing on actual transaction data (as it is extremely rare to obtain confidential data from swap market participants).

Some of the pertinent results to this study found in Cossin and Pirotte [1997] are that there is credit risk pricing in these data, that the spread between Aaa's and Aa's is of 1.16 bp and the spread between Aaa's and a group of counterparties rated Aa2 or less is of 3.8 bp with a high significance. This in itself justifies trying to obtain credit risk spread measures from a theoretical model. Another interesting result obtained (but one contradictory to the theory) is that, as far as interest swaps are concerned at least, terms (e.g., the notional amount of the contract or the maturity of the contract) do not affect the spreads to the interbank market (see Cossin and Pirotte [1997] for an empirical investigation on the key determinants of swap spreads).

Financial and accounting data necessary for the analysis were obtained from DataStream:

- Libor, Libid, Eurocurrency, IRS quoted rates,
- Swiss government bond data,
- market capitalizations and a set of balance sheet' values allowing the computation of diverse debt levels for the different counterparties: the total value of assets, the outstanding debt amount (long term), the total value of creditors (short-term and long-term) and the total value of debtors (short-term and long-term).

The quality of the IRS data was checked against quotes provided for part of the period considered by a major swap broker (Gottex). No significant difference or clear outliers where found. Sun, Sundaresan and Wang [1993] compared Data Stream and DRI Libor and Libid data and concluded that the two sources provided rates that are not "economically different from each other". Market capitalizations and some other available values from the balance sheets were checked against the Swiss stock guide edited by UBS, and the overall coherency of the accounting values was verified in every case.

4 The Valuation Methodology

The final objective is to compute a theoretical price from the model described above. But, some inputs must be previously estimated and the procedure, which consists in four parts, is provided below. It will be noticed that some of these inputs are important results per se and that the swap pricing is finally just another application among others that could rely on them.

First, the three parameters of Vasicek's model have to be estimated: ξ , β and η. This was achieved through a GMM procedure (Generalized Method of Moments) following the study made in Chan, Karolyi, Longstaff and Sanders[1992]. A set of four moments was computed for the estimation of the three parameters; this "over-estimation" theoretically leads to stronger estimates[6].

There is, however, a precision to make. In Vasicek's price of a credit riskfree bond, which is given by

$$D(r,T) = \exp(A(T) - B(t)r), \tag{7}$$

where

$$A(T) = \left(\frac{\eta^2}{2\beta^2} - \frac{\alpha}{\beta}\right) T + \left(\frac{\eta^2}{\beta^3} - \frac{\alpha}{\beta^2}\right) \exp\left((-\beta T) - 1\right) - \left(\frac{\eta^2}{4\beta^3}\right) \left(1 - \exp\left(-\beta T\right)\right)^2, \tag{8}$$

and

$$B(T) = \frac{1 - \exp\left(-\beta T\right)}{\beta}, \tag{9}$$

Longstaff and Schwartz [1995] consider α as the resultant of the sum to ξ, the drift of the interest rate process, of a total market premia of interest rate risk, say λ^*[7]. Vasicek assumes this premia to be constant over time and many authors consider it equal to zero following the hypothesis of local expectations. A typical calibrating alternative is to fit α on the market price of Swiss government bonds of the same maturity as the swap which is being priced[8].

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[6] The method proposed by Chan, Karolyi, Longstaff and Sanders[1992] allows to unrestrict the power of r such that processes with a volatility parameter dependent on the level of interest rates, are also included. In our case, since we focus on an Ornstein-Uhlenbeck process, this fourth parameter was set to zero.

[7] The market premia is represented by λ^* to differentiate it from λ, the market premia per unit of volatility, i.e. the increase in expected instantaneous rate of return on a bond per additional unit of risk following Vasicek's development.

[8] A market premium could be calculated following Campbell [1986] and Engle, Lilien and Robbins [1987] but no significant "calibration improvement" should be obtained as we fit α to a government bond of the same maturity.

However, Vasicek [1977] shows that, in theory, the unit market risk premia must be independent of the maturity of the bond to be priced to rule out arbitrage opportunities. Nevertheless, his derivation

Second, the structural riskiness of the firm is mainly related to the "distance-from-default", represented here by the ratio $X = V/Fe^{-rT}$, F being a threshold value (such that the firm defaults if V ends below F at maturity) equal to a considered level of the face value of the debt of the firm, and to the volatility of the assets returns, σ_v. Also, we do not have any particular indication of the estimated maturity of the debt burden of the firm, which is essential for the use of Merton's call to value the equity of the firm as a call on the assets. For the estimation of these values, a three-step iterative procedure has been implemented:

1. Using the swap maturity, the volatility of the return on the traded shares and the book value of assets as starting guesses for T^* (the estimated overall maturity of the debt burden), σ_v and V, the market value of the assets of the firm is estimated such that the market value of the firm's equity on that day fits the value of a call option on the value of the assets of the firm[9]:

$$V^* \text{such that } E = f\left(V^*, F^i, r(T^*), \sigma_E, T^*\right), \tag{10}$$

where V^* stands for the binding market value of the assets of the firm, F^i is the face value of the debt level i taken into consideration, and $r(T^*)$ is the spot interest rate for the maturity T^*, calculated through Vasicek's model coherently to the parameters estimated earlier[10].
Five different threshold values F^i $(i = 1, ..5)$ are computed using five different estimations of the face value of the debt burden, for the sake of completeness. The main objective is to understand the impact that could have the threshold on the pricing or on the riskiness indicators. Would the choice be crucial, it will be then interesting to understand the financial motivations to prefer some threshold. Geske and Delianedis [1998] test two different methodologies to see the impact of the shorter reimbursement periods of the short-term debt while,

of the bond price is made on the hypothesis of a constant interest rate risk premia through time. This is a reduction of the interest rate market as shown by Bruand [1998b]. Our procedure is thus mainly justified by a calibration argument on real bond market prices.

Another proposition, following the discussion with Hélyette Geman, member of the jury of the present thesis, is simply to use the properties of the Vasicek model. In short, in the standard notation, the process of dr is written as

$$dr_t = a(b - r_t)dt + \eta dZ_t.$$

And, b, the long run mean to which r_t is assumed to revert, might not be necessarily calibrated with the other parameters through a GMM procedure but using standard regression techniques and the fact that, from Vasicek's model itself, $R(\infty) = b + \frac{\lambda\eta}{a} - \frac{\eta^2}{2a^2}$, where λ is the market premia per unit of risk. Thanks to this last equation, we do not need to estimate λ directly but through the estimation of $R(\infty)$.

[9] All derivations are based on the risk-neutral probability measure and so are the results obtained.

[10] * indicates an iterated value for the variable to which it is associated.

here, we test different static starting barriers under the same methodology. Given the scarcity of accounting data of good quality, we decided to define different measures taking into account the fact that the counterparties are financial institutions and that the relative duration of their assets and liabilities will play a crucial role for their potentiality of default. Five thresholds were defined:

Threshold 1: Half the long-term debt plus the remaining value of the foreign capital, i.e. the short term debt. This comes directly from KMV'methodology[11]. As reported in Crouhy and Mark [1998], KMV has observed from a sample of several hundred companies that default usually happens when the assets level is somewhere between the value of total liabilities and the value of short-term debt.

Threshold 2: The debt-equity ratio calculated stricto-sensu by Datastream applied to the accounting value of assets.

Since we unfortunately rely on accounting data with some degree of imprecision, taking this ratio computed from market data sources can be a good alternative.

Threshold 3: The value of the long-term debt plus the amount of fixed term deposits.

Since banks could typically face a problem of duration mismatching in their balance sheet for their activity of lending and borrowing, our idea is to add a shorter or a longer current asset category to the structural debt level of the firm and to oppose these results to those using KMV's evidenced methodology. This threshold is also provided for the completeness of the study and the comparison with threshold 4.

Threshold 4: The value of the long-term debt plus the amount of non-fixed term deposits.

Comparing thresholds 3 and 4 will lead us to the answer to the question: will we attain different credit values if we take more volatile deposited funds? Of course, depending on the proportion of these two categories into the liability side of the firm, we can have quite different values or not. For counterparties with ratings Aaa and Aa1, the fixed term-deposits are

[11] KMV Corporation is a company founded by Stephen Kealhofer, John McQuown and Oldrich Vasicek that has developped a framework for credit risk pricing and monitoring based on Merton's model. Two main items of their approach is the inference of the volatility of assets and the value of the firm form stock market data and the mapping of their "structural" distances-to-default (DD) to the actual probabilities of default for a given time horizon. Indeed, DDs allow them to recategorize firms into classes and compare this classification with that of Moody's or Standard & Poors.

much more numerous than the non-fixed ones. For these counterparties, threshold 3 is 3 times higher than threshold 4. For the state-guaranteed (MSB) and for the Aa2-rated counterparties, both thresholds are almost equivalent. Therefore comparing the effects of these two thresholds can be interesting. In effect, given these differences, we could "à priori" think that well-rated counterparties try to (and can) avoid non-fixed term deposits while this is not the case for the Aa2 one. And for the state-guaranteed counterparty, this risk can be just part of its social policy since it is a state-owned bank with the corresponding endorsement.

Threshold 5: The value of the assets minus the amount of short term debtors. Since we are dealing with financial institutions, the credit risk arising because of their debt burden can be thought of as being related to their investment strategy and to the risk of their own debtors, mainly the medium and long-term ones[12]. Instead of looking at short-term debtors, we could think that a situation such like a geographical krash or recession, for example, will typically correspond to a structural problem of bad debt accounts that could weaken its long-term viability.

To be consistent with our earlier definition of the event of default, we must make a precision on the difference between the stock-based insolvency (when the value of the assets falls below the value of the liabilities as defined earlier), and the flow-based insolvency or liquidity-driven default (when the firm cannot honor its payments). Liquidity-driven default reflects the incapability of the firm to finance its due payments. And this incapability will ultimately reflect its incapability to raise new funds. Therefore, in a frictionless world, as mentioned in Saá-Requejo and Santa-Clara [1997], liquidity-driven default can only happen due to stock-based insolvency. But in presence of market frictions, both thresholds may correspond to different levels. I do not think that threshold 5 is contradictory with the previous definition of X; it is simply a refinement based on the liquidity of the assets' side of the balance sheet.

It has to be noticed that we will obtain significant differences whenever the starting threshold levels really differ from each other, since the iterative proce-

[12] In the same domain, an exaustive study that compares the standard Merton approach to a refined view of the capital structure of the firm in terms of maturities and two different thresholds (Merton's total debt or taking into account short-term and long-term debt maturity specificities) is provided by Delianedis and Geske [1998]. Their application is with respect to the rating transition matrices and expectations of defaults.

dure will certainly produce very similar values for starting values too close to each other.

2. The estimation of the volatility of the assets value can be inferred by applying Itô's lemma to Merton's equity pricing and reverting it as a function for σ_v^* (as shown in Ronn and Verma [1986]):

$$\sigma_v^* = \frac{\sigma_e\, E}{V^*\, N(x)}\,,\; x = \frac{\ln\left(X^*\right) + \sigma_v^{*2}\, T^*/2}{\sigma_v^* \sqrt{T^*}}, \tag{11}$$

   where $X^* = V^*/F^i e^{-rT}$ for every F^i defined above ($i = 1, ..5$) and x is the "distance-from-default" in units of volatility.

3. The initial guess for T is then modified on the basis of the expected time (τ_b) for V to reach a new threshold level defined by the strict amount of long-term debt of the firm (B) lower than any of the F^i thresholds used in equations 10 and 11, which are based on Merton's potential default at maturity. To maintain an overall consistency when using two different perspectives of default (equations 10 and 12) to obtain two sets of estimates that are jointly used in the same equations, $\{V^*, \sigma_v^*\}$ and T^*, the same threshold should not be used in the definition of the early default[13]. The first-passage time density when the barrier is approached from above by V can be expressed as[14]

$$q(\tau_b) = \frac{-ln\left(B/V^*\right)}{\sigma_v^* \sqrt{\tau_b^3}} n\left(h_2(\tau_b)\right), \tag{12}$$

   where $n(\cdot)$ is the standard normal probability density function and

$$h_2(\tau_b) = \frac{ln\left(\frac{B}{V^* e^{-r(T^*)T^*}}\right)}{\sigma_v^* \sqrt{T^*}} - \frac{1}{2}\sigma_v^* \sqrt{T^*}. \tag{13}$$

   The expected time to default on the long-term debt is therefore given by the following integral:

$$T^* = I\!\!E_0\left[\tau_b\right] = \int_{t_0}^{\infty} \tau_b\, q(\tau_b)\, d\tau_b, \tag{14}$$

   choosing a sufficiently high value for the upper bound of τ_b.

This iterative procedure is performed until convergence of the σ_v^* value.

[13] Indeed, we can think that it is given a little more flexibility to shareholders during the life of the contract than at the "structural term-horizon" of the firm.

[14] A derivation of this expression can be found in Rich [1994], Karatzas and Schreve [1994] and in appendix 10.3 of Pirotte [1999].

The remaining parameter to be estimated is ρ, the correlation between the dynamics of dr and dV/V which is at the center of the main contribution of Longstaff and Schwartz [1995][15]. In our case, since the equity value is by construction the "market informative" source that leads to the final determination of V and σ_v, it has been straightforwardly decided to estimate this statistic directly from Euro-currency rates and stocks returns time series[16].

Once the earlier estimations obtained, fixed and floating discounts are calculated and allow us to derive the theoretical swap transaction rates.

The "production-line" of this methodology shows the difficulty of using this type of models given that it requires a mixture of trading data which is easily available, with accounting numbers that are not or at least which informational quality is not proved. The results are presented in the next section along with their analysis.

[15] Here, the relationship between dr and dV/V is explicitely and exogenously defined through a constant correlation parameter. Today, other studies try to model endogenously this correlation as stemming from an equilibrium model, for example.

[16] The iterative procedure used here computes $r(T)$ from Vasicek's model without any relationship with V, which implies that we indirectly assume that the information or the variance shocks flow from r to V.

Trade n.	Date	Rating of the Counterparty	Maturity	Traded rate	Quoted rate
1	03.07.92	Bank Aaa	10	7.51	7.52
2	01.09.92	Bank Aaa	9	7.14	7.19
3	02.09.92	Bank Aaa	9	7.15	7.21
4	02.09.92	Bank Aaa	10	7.13	7.21
5	03.09.92	Bank Aaa	10	7.14	7.27
6	08.12.93	Bank Aaa	10	4.27	4.31
7	07.03.94	Bank Aaa	9	4.73	4.64
8	25.04.94	Bank Aaa	10	5.30	5.36
9	29.04.94	Bank Aaa	10	5.25	5.28
10	14.06.94	Bank Aaa	10	5.63	5.90
11	02.11.94	Bank Aaa	4	5.50	5.59
12	15.12.94	Bank Aaa	10	5.86	5.80
13	02.12.93	Bank Aa1	10	4.32	4.33
14	03.12.93	Bank Aa1	10	4.30	4.33
15	04.03.94	Bank Aa1	5	4.66	4.50
16	30.06.94	Bank Aa1	10	5.68	5.84
17	14.09.94	Bank Aa1	10	5.90	5.96
18	20.09.94	Bank Aa1	8	5.70	5.93
19	20.10.94	Bank AA1	6	5.72	5.77
20	24.12.92	Bank Aa2	5	5.72	5.77
21	07.03.94	Bank Aa2	9	4.73	4.64
22	28.10.94	Bank Aa2	7	5.79	5.82
23	28.10.94	Bank Aa2	10	5.92	5.94

Table 1: Sample of IRS transactions available for the analysis. Rates are in percent. The traded rates are the rates that MBS agreed with its counterparties while the quoted rates are market quotes prevailing at that time and extracted from Datastream. In all reported swaps, MSB was receiving the fixed rate.

5 Results and Analysis

Table 1 provides the description of the sample of swap transactions to be priced, along with the transacted price and the price quoted standardly on the marketplace. Thus it requires to estimate 23 sets of parameters for the interest rate and the assets value processes at the dates of contract origination. This table will also be helpful in comparing the theoretical prices to real market conditions at those dates. All swaps are swaps where MSB receives the fixed leg from its counterparties.

GMM was performed on 625 weekly values of 1-month Euro-currency interest rates prevailing before the swap transaction dates. The purpose of choosing such a long historical time series is to obtain consistent values for the parameters governing

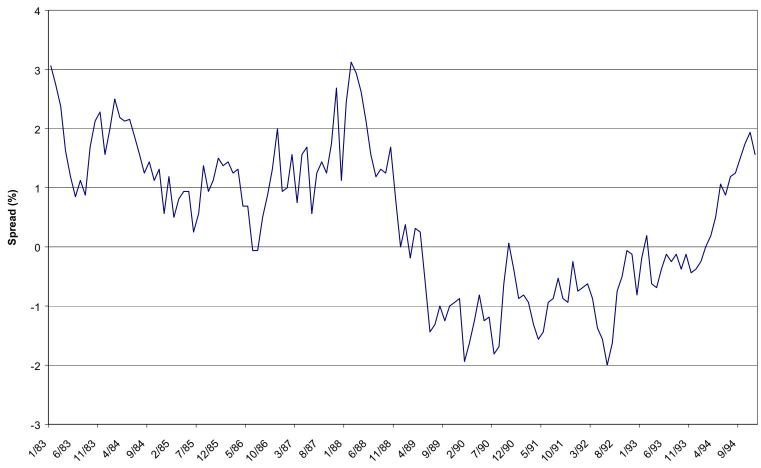

Figure 1: This figure provides the evolution of the term spread (long-term minus short term rates) in Euro-currency rates from January 1983 to December 1994.

the mean reversion property of the Ornstein-Uhlenbeck process, α and β. Figure 1 visually confirms the necessity of covering long time periods of historical data in order to reliably estimate the long-term target of the process.

Results are shown in table 2 and table 3, in columns 3 to 5. All these estimates were annualized. Some results are valid for different swap trades since the dates of contract origination are either the same (but contracts of different maturities) or are close to each other so that the same weekly data will apply to them. It can be seen that while β and η values can be argued to be significant at the 10% confidence level, it is clear that ξ values do not appear to be significant in 75% of the cases. So far, we have made the hypothesis of a null market risk premia, implying that $\alpha = \xi$. A way to circumvent these problems is simply to fit the α parameter such that the theoretical Vasicek price fits the market price on the contracting date of a Swiss government bond maturing in the same year[17] and considered to be totally riskless.

The last column of tables 2 and 3 shows the fitted values obtained for α with this calibration.

Results are coherent with what the theory predicts about them in the sense that β will have a negative impact on the basis of equation 2 (changes in r_t are related to differences with its long-run mean), which is necessary for the Vasicek's process to be mean-reverting[18]. β can be quite high in absolute value implying a high reversion

[17] See footnote 8 for a detailed explanation.

[18] The Ornstein-Uhlenbeck process defined in equation 2 can be compared to another formulation which is more intuitive:

$$dr = a\,(b - r)\;dt + \eta\,dZ_2.$$

Date	1 month Euro-rate (%)	ξ	β	η	Swap Maturity	α
03.07.92	9.1875	0.0571	0.8798	0.0330	10	0.0606
		(2.28)	(2.04)	(9.13)		
01.09.92	7.6875	0.0587	0.9300	0.0329	9	0.0631
		(2.37)	(2.20)	(8.99)		
02.09.92	7.6875				9	0.0630
02.09.92	7.6875				10	0.0616
03.09.92	7.6875				10	0.0615
08.12.93	4.3125	0.0447	0.9914	0.0266	10	0.0439
		(2.03)	(2.85)	(8.85)		
07.03.94	4.3125	0.0216	0.5452	0.0244	9	0.0245
		(1.04)	(1.71)	(8.01)		
25.04.94	4.0625	0.0235	0.5497	0.0243	10	0.0258
		(1.14)	(1.73)	(7.89)		
29.04.94	3.9375	0.0245	0.5633	0.0242	10	0.0282
		(1.21)	(1.80)	(7.90)		
14.06.94	3.9375	0.0204	0.5018	0.0239	10	0.0282
		(1.03)	(1.63)	(7.75)		
02.11.94	3.6875	0.0244	0.5393	0.0235	4	0.0345
		(1.28)	(1.79)	(7.56)		
15.12.94	3.9375	0.0247	0.5375	0.0234	10	0.0340
		(1.32)	(1.82)	(7.48)		

Table 2: Estimation of the parameters of the term structure of interest rates for the first 12 swap transaction dates of our sample. The three first parameters result from the GMM estimation performed on series of 625 historical 1-month Euro-currency rates that were prevailing just before the contracting date. The fourth parameter results from the calibration to a Swiss government bond of the same maturity as that of the swap considered. T-stats are shown in parentheses.

Date	1-month Euro rate (%)	ξ	β	η	Swap Maturity	α
02.12.93	4.6250	0.0444	0.9762	0.0266	10	0.0459
		(2.01)	(2.82)	(8.83)		
03.12.93	4.6250	0.0444	0.9762	0.0266	10	0.0426
		(2.01)	(2.82)	(8.83)		
04.03.94	4.3125	0.0216	0.5452	0.0244	5	0.0216
		(1.04)	(1.71)	(8.01)		
30.06.94	4.1250	0.0223	0.5208	0.0239	10	0.0308
		(1.13)	(1.70)	(7.70)		
14.09.94	3.8750	0.0220	0.5101	0.0236	10	0.0327
		(1.15)	(1.70)	(7.58)		
20.09.94	4.0000	0.0238	0.5317	0.0236	8	0.0334
		(1.25)	(1.77)	(7.56)		
20.10.94	3.7500	0.0259	0.5595	0.0236	6	0.0360
		(1.36)	(1.86)	(7.56)		
24.12.92	6.3125	0.0563	0.9671	0.0319	5	0.0612
		(2.33)	(2.42)	(9.99)		
07.03.94	4.3125	0.0216	0.5452	0.0244	9	0.0245
		(1.04)	(1.71)	(8.01)		
28.10.94	3.6875	0.0254	0.5532	0.0235	7	0.0363
		(1.33)	(1.84)	(7.56)		
28.10.94	3.6875				10	0.0338

Table 3: Estimation of the parameters of the term structure of interest rates for the last 11 swap transaction dates of our sample. The three first parameters result from the GMM estimation performed on series of 625 historical 1-month Euro-currency rates that were prevailing just before the contracting date. The fourth parameter results from the calibration to a Swiss government bond of the same maturity as that of the swap considered. T-stats are shown in parentheses.

	X values (for the different threshold levels)				
Counterparties	Threshold 1	Threshold 2	Threshold 3	Threshold 4	Threshold 5
State-guaranteed	1.117-1.218	1.089-1.191	1.120-1.221	1.115-1.215	1.138-1.235
Aaa	1.346-2.779	1.307-2.722	1.255-2.427	1.569-3.518	1.536-2.777
Aa1	1.311-1.349	1.230-1.272	1.314-1.381	1.671-1.844	1.287-1.317
Aa2	1.126-1.766	1.102-1.733	1.300-1.774	1.254-1.758	1.226-1.748

Table 4: X ranges for the different counterparties and threshold levels accross the contracting dates. State-Guaranteed means the counterparty with whom all others have traded. Again, we argue that this guaranty makes this counterparty to have an implicit rating higher than AAA but some residual risk still remains however.

	σ_v values in % (for the different threshold levels)				
Counterparties	Threshold 1	Threshold 2	Threshold 3	Threshold 4	Threshold 5
State-guaranteed	0.77-2.52	0.59-2.25	0.79-2.55	0.75-2.50	0.87-2.66
Aaa	4.82-22.77	4.49-22.68	4.01-22.22	6.40-23.70	6.18-22.45
Aa1	7.90-11.38	7.33-11.21	7.92-11.45	10.25-12.71	7.70-11.30
Aa2	2.43-14.00	2.13-14.61	6.92-13.86	7.04-14.14	6.87-14.32

Table 5: σ_v ranges for the different counterparties and threshold levels accross the contracting dates. The volatility estimates are annual values.

effect which is coherent with the inversion of the term structure evidenced in figure 1.

Tables 4 and 5 provide the results for the second part of our implementation dedicated to the estimation of the "riskiness indicators" (X) and the volatility of the returns on the assets value of the firm, σ_v. The range of estimates of the variable that links both processes for dr and dV/V, namely the correlation parameter ρ, is also shown aside for each counterparty in table 6.

Counterparties	ρ ranges
State guaranteed	-0.1647 to 0.0076
Aaa	-0.1891 to 0.0089
Aa1	-0.1824 to -0.0443
Aa2	-0.1880 to -0.0317

Table 6: ρ ranges on the different transaction dates for the different counterparties.

It appears clearly that the correlation between interest rates and the compounded returns on V is negative, which is in line with the relationship between bond values and the level of interest rates. This is particularly important for the current counterparties which are financial institutions and thus very-sensitive to interest-rate spreads.

b is the long-term value to which the short-term process converges and a is the speed of reversion. A strict comparison can show that $\alpha = ab$ and that $\beta = b$.

The values of X together with those of σ_v obtained are quite compatible with credit ratings while giving interesting insights. While the state-guaranteed counterparty has the lowest X, it also has from far the lowest σ_v[19]. This is consistent with a firm that is able to take on more lending and borrowing because of the backyard guaranty of the state and its particular social mission but is also more stable and conservative in its investment strategy. While the volatility of asset returns remains quite stable across the sample for the Aa1 counterparty, it can merely double for the Aa2 firm or be five times bigger at some dates for the Aaa firm. But this also coincides then with a wide "distance-from-default" and coherently a longer expected maturity of the liabilities of the firm[20]. This is consistent with the fact that more aggressive firms should hold more financing flexibility while targeting a higher growth potential. Alternative combinations of X's and σ_v's can lead to the same overall "riskiness". Remembering that the estimated σ_v was based on an iteration on σ_E, it means that more volatile stocks do not necessarily lead to a "higher credit risk" conclusion. The only way to be able to compare these sets of values cross-sectionally would be to compute the "distance-from-default" (DD) in volatility units, as shown in equation 11 with x. Table 7 provides such measures. It can be deduced that, as evidenced by KMV, there seems to be a strong overlapping of the values across ratings, so that it is more the modal value that will define the belonging of a counterparty to a given rating class. Second, as with any test of a model, there is no way to know without any other numbers, that DD values are more accurate than the ratings or viceversa. It seems that the coherence is not really respected between classes Aa1 and Aa2 unless for the fourth threshold' case. At least, the state-guaranteed MSB seems really to have a superior creditworthiness on the sole basis of the "distance-from-default".

Tables 8, 9, 10, 11 and 12 present the results for the computation of the swap rates with the Longstaff and Schwartz [1995] model but assuming that $\eta = 0$ what we argue is realistic since the GMM estimations produced numbers between 2 and 3.5% (see Tables 2 and 3). Moreover, the swap rates produced with the Longstaff and Schwartz methodology without making this restriction produced exactly the same results. Hence, neither $\eta = 0$ is a restrictive assumption, nor can the Longstaff and Schwartz model be really misleading (given its criticized derivation) in such a context. Thus, the following comments will be applicable to the two alternatives.

[19] As it is precised in previous footnotes, while we assume that the state-guaranty of this counterparty gives to it an implicit rating that is higher than AAA, we cannot ascertain that it is perfectly riskless however, since it is also financed by private capital.

[20] Not shown here but available on demand. It has also to be noticed that there are more values for the Aaa counterparty than for the Aa1 and Aa2 ones, in the computed ranges, because we have focused on the dates of the data points available to us. A more general test on a wider and equivalent range of days for every counterparty would be certainly useful.

Counterparties	"Distances-from-default" (for the different threshold levels) Threshold 1	Threshold 2	Threshold 3	Threshold 4	Threshold 5
State-guaranteed	1.97 to 5.21	1.61 to 4.48	2.05 to 5.16	1.92 to 5.00	2.35 to 5.36
	2.98	2.51	2.97	2.83	3.81
Aaa	0.78 to 1.77	0.76 to 1.68	0.65 to 1.56	1.00 to 2.18	0.71 to 2.13
	1.36	1.31	1.19	1.69	1.51
Aa1	0.68 to 0.94	0.52 to 0.73	0.75 to 0.97	1.82 to 2.18	0.61 to 0.92
	0.83	0.65	0.89	2.04	0.77
Aa2	0.69 to 2.05	0.49 to 1.52	0.75 to 1.13	0.64 to 1.75	0.58 to 1.26
	1.12	0.87	0.94	0.99	0.84

Table 7: Ranges and means of "distances-from-default" for the different counterparties and threshold levels accross the contracting dates. Means are provided below the range but are only indicative since they are computed on a specific realized sample with varying size for each counterparty.

Unfortunately, although the pricing seems to be quite close to traded rates for some of the trades, swap pricing following this methodology and the pricing equations presented above do not clearly give reliable values (see tables 8, 9, 10, 11 and 12). The swap rates obtained by the model are in most cases lower than traded and quoted prices (14 to 17 cases out of 23), thereby meaning that MSB has received fixed rates that were more favorable than the theoretical rates. Looking at the credit spreads, it becomes obvious that this difference is mainly due to a difference at the riskfree level already. But, what is a riskfree swap? If we look at the DD values above, we can argue that if our MSB state-guaranteed firm should be rated well above the Aaa counterparty, then MSB could represent a really riskfree firm. So that would be the price of a MSB vs. a MSB-like swap. Knowing that MSB is receiving the fixed leg, the traded price has to be higher to MSB since it is a swap against a less well rated counterparty. That is why the credit spread is positive but even with this spread, the risky swap price is still lower than the traded price. Then, the conclusion can be twofold: either Longstaff and Schwartz ' model produces too low credit risk premias or/and the inaccuracy of the one-factor model of the term structure of interest rates produces almost systematically values that are lower than they should be. There still remains a third alternative: MSB has obtained a really favorable rate. This seems doubtful since the swap OTC market is very liquid and the counterparties are very active and large in the world marketplace. On the other hand, the concentration of the market on big "players" and some remaining market inefficiencies in the pricing of risky swaps, could explain some systematic bias.

Credit spreads are in fact lower than 2 bps for the majority of the trades in the sample. Curiously, some credit spreads jump to 20 or more than 30 bps and are not therefore very satisfactory since we have a highly rated sample.

Some credit spreads however are relatively high. But if we look already at the difference between the traded rates and the quotes, we find out that some differences can rise to 27 bps. Then it becomes difficult to identify the difference as a theoretical gap or a market gap.

Comparing the values under different thresholds does not change the concluding remarks, although the pricing with the fourth threshold seems to produce a higher number of negative differences with respect to traded prices. It is important for precise pricing purposes since the inclusion of less short-term debt, for example, into the threshold value retained assumes that we would be looking at a further forward probability of default[21]. It depends then on the term-horizon of the investor who is willing to invest in this debt or enter into the swap agreement.

[21] A well applied discussion of this issue is done by Delianedis and Geske [1998].

	Rating of the Counterparty	Swap riskfree rate	Swap risky rate	Swap Credit Spread	Difference with traded price	Difference with quoted price
1	Bank Aaa	7.4223	7.4223	0.0002	-8.7700	-9.7700
2	Bank Aaa	7.0757	7.0757	0.0007	-6.4285	-11.7619
3	Bank Aaa	7.0645	7.0645	0.0007	-8.0532	-14.8865
4	Bank Aaa	6.9141	6.9142	0.0010	-21.5847	-29.5847
5	Bank Aaa	6.9054	6.9054	0.0010	-23.4637	-36.4637
6	Bank Aaa	4.4487	4.4495	0.0768	17.9511	13.9511
7	Bank Aaa	4.4360	4.4367	0.0727	-29.3319	-19.9985
8	Bank Aaa	4.5335	4.5380	0.4501	-76.1992	-82.1992
9	Bank Aaa	4.7707	4.7742	0.3465	-47.5799	-50.5799
10	Bank Aaa	5.2067	5.2142	0.7460	-41.5832	-68.5832
11	Bank Aaa	5.1364	5.1943	5.7926	-30.5684	-39.5684
12	Bank Aaa	5.7939	6.1670	37.3076	30.7022	36.7022
13	Bank Aa1	4.7391	4.7653	2.6210	44.5280	43.5280
14	Bank Aa1	4.4391	4.4685	2.9346	16.8471	13.8471
15	Bank Aa1	4.0845	4.0984	1.3961	-56.1557	-40.1557
16	Bank Aa1	5.4990	5.7151	21.6096	3.5131	-12.4869
17	Bank Aa1	5.7977	5.9228	12.5056	2.2760	-3.7240
18	Bank Aa1	5.6636	5.7580	9.4322	5.7954	-16.8713
19	Bank Aa1	5.5357	5.5601	2.4390	-15.7419	-20.9919
20	Bank Aa2	6.4159	6.4159	0.0001	69.8367	64.5867
21	Bank Aa2	4.4360	4.4490	1.3006	-28.1039	-18.7706
22	Bank Aa2	5.7056	5.7900	8.4485	0.0040	-2.9960
23	Bank Aa2	5.5677	5.6893	12.1653	-23.0674	-25.0674

Table 8: Pricing results with the first threshold. Results for the theoretical price following the parameter estimation and the application of the closed-form pricing equation of Longstaff and Schwartz [1995] to fixed and floating debt. The swap credit spread is simply the difference (in basis points) between the riskfree theoretical price and the risky one. The two last columns provide the difference (in basis points) of the computed prices with the market references, i.e. the traded price and the standard quote available that day.

	Rating of the Counterparty	Swap riskfree rate	Swap risky rate	Swap Credit Spread	Difference with traded price	Difference with quoted price
1	Bank Aaa	7.4223	7.4223	0.0002	-8.7701	-9.7701
2	Bank Aaa	7.0757	7.0757	0.0007	-6.4285	-11.7619
3	Bank Aaa	7.0645	7.0645	0.0007	-8.0532	-14.8865
4	Bank Aaa	6.9141	6.9142	0.0009	-21.5847	-29.5847
5	Bank Aaa	6.9054	6.9054	0.0010	-23.4638	-36.4638
6	Bank Aaa	4.4487	4.4497	0.1002	17.9745	13.9745
7	Bank Aaa	4.4360	4.4367	0.0782	-29.3263	-19.9930
8	Bank Aaa	4.5335	4.5384	0.4919	-76.1573	-82.1573
9	Bank Aaa	4.7707	4.7745	0.3802	-47.5462	-50.5462
10	Bank Aaa	5.2067	5.2149	0.8180	-41.5112	-68.5112
11	Bank Aaa	5.1364	5.2013	6.4944	-29.8667	-38.8667
12	Bank Aaa	5.7939	6.1938	39.9825	33.3770	39.3770
13	Bank AA1	4.7391	4.7882	4.9092	46.8162	45.8162
14	Bank AA1	4.4391	4.4932	5.4096	19.3220	16.3220
15	Bank AA1	4.0845	4.1161	3.1643	-54.3875	-38.3875
16	Bank AA1	5.4990	5.9097	41.0668	22.9703	6.9703
17	Bank AA1	5.7977	6.0345	23.6788	13.4492	7.4492
18	Bank AA1	5.6636	5.8502	18.6567	15.0198	-7.6468
19	Bank AA1	5.5357	5.5864	5.0665	-13.1144	-18.3644
20	Bank Aa2	6.4159	6.4159	0.0001	69.8367	64.5867
21	Bank Aa2	4.4360	4.4654	2.9454	-26.4591	-17.1257
22	Bank Aa2	5.7056	5.9259	22.0366	13.5922	10.5922
23	Bank Aa2	5.5677	5.8672	29.9554	-5.2773	-7.2773

Table 9: Pricing results with the second threshold. Results for the theoretical price following the parameter estimation and the application of the closed-form pricing equation of Longstaff and Schwartz [1995] to fixed and floating debt. The swap credit spread is simply the difference (in basis points) between the riskfree theoretical price and the risky one. The two last columns provide the difference (in basis points) of the computed prices with the market references, i.e. the traded price and the standard quote available that day.

	Rating of the Counterparty	Swap riskfree rate	Swap risky rate	Swap Credit Spread	Difference with traded price	Difference with quoted price
1	Bank Aaa	7.4223	7.4223	0.0002	-8.7701	-9.7701
2	Bank Aaa	7.0757	7.0757	0.0007	-6.4285	-11.7619
3	Bank Aaa	7.0645	7.0645	0.0007	-8.0532	-14.8865
4	Bank Aaa	6.9141	6.9142	0.0009	-21.5848	-29.5848
5	Bank Aaa	6.9054	6.9054	0.0010	-23.4638	-36.4638
6	Bank Aaa	4.4487	4.4502	0.1481	18.0224	14.0224
7	Bank Aaa	4.4360	4.4377	0.1733	-29.2312	-19.8979
8	Bank Aaa	4.5335	4.5423	0.8786	-75.7706	-81.7706
9	Bank Aaa	4.7707	4.7775	0.6754	-47.2510	-50.2510
10	Bank Aaa	5.2067	5.2208	1.4110	-40.9182	-67.9182
11	Bank Aaa	5.1364	5.2605	12.4066	-23.9544	-32.9544
12	Bank Aaa	5.7939	6.3934	59.9465	53.3410	59.3410
13	Bank Aa1	4.7391	4.7648	2.5754	44.4824	43.4824
14	Bank Aa1	4.4391	4.4680	2.8849	16.7974	13.7974
15	Bank Aa1	4.0845	4.0947	1.0192	-56.5325	-40.5325
16	Bank Aa1	5.4990	5.6683	16.9306	-1.1658	-17.1658
17	Bank Aa1	5.7977	5.8957	9.7981	-0.4315	-6.4315
18	Bank Aa1	5.6636	5.7364	7.2736	3.6367	-19.0300
19	Bank Aa1	5.5357	5.5542	1.8486	-16.3323	-21.5823
20	Bank Aa2	6.4159				
21	Bank Aa2	4.4360	4.4467	1.0710	-28.3335	-19.0002
22	Bank Aa2	5.7056	5.7723	6.6779	-1.7666	-4.7666
23	Bank Aa2	5.5677	5.6657	9.8031	-25.4296	-27.4296

Table 10: Pricing results with the third threshold. Results for the theoretical price following the parameter estimation and the application of the closed-form pricing equation of Longstaff and Schwartz [1995] to fixed and floating debt. The swap credit spread is simply the difference (in basis points) between the riskfree theoretical price and the risky one. The two last columns provide the difference (in basis points) of the computed prices with the market references, i.e. the traded price and the standard quote available that day. No sufficient data on face values was available for bank Aa2 in 91-92 (transaction n. 20).

	Rating of the Counterparty	Swap riskfree rate	Swap risky rate	Swap Credit Spread	Difference with traded price	Difference with quoted price
1	Bank Aaa	7.4223	7.4223	0.0003	-8.7700	-9.7700
2	Bank Aaa	7.0757	7.0757	0.0008	-6.4284	-11.7618
3	Bank Aaa	7.0645	7.0645	0.0008	-8.0531	-14.8864
4	Bank Aaa	6.9141	6.9142	0.0012	-21.5845	-29.5845
5	Bank Aaa	6.9054	6.9054	0.0012	-23.4635	-36.4635
6	Bank Aaa	4.4487	4.4490	0.0226	17.8969	13.8969
7	Bank Aaa	4.4360	4.4361	0.0119	-29.3926	-20.0593
8	Bank Aaa	4.5335	4.5349	0.1415	-76.5078	-82.5078
9	Bank Aaa	4.7707	4.7718	0.1098	-47.8166	-50.8166
10	Bank Aaa	5.2067	5.2093	0.2552	-42.0741	-69.0741
11	Bank Aaa	5.1364	5.1525	1.6085	-34.7526	-43.7526
12	Bank Aaa	5.7939	5.9726	17.8621	11.2566	17.2566
13	Bank AA1	4.7391	4.7462	0.7164	42.6234	41.6234
14	Bank AA1	4.4391	4.4473	0.8188	14.7313	11.7313
15	Bank AA1	4.0845	4.0853	0.0829	-57.4689	-41.4689
16	Bank AA1	5.4990	5.5258	2.6757	-15.4207	-31.4207
17	Bank AA1	5.7977	5.8139	1.6202	-8.6094	-14.6094
18	Bank AA1	5.6636	5.6745	1.0881	-2.5487	-25.2154
19	Bank AA1	5.5357	5.5384	0.2668	-17.9141	-23.1641
20	Bank Aa2	6.4159				
21	Bank Aa2	4.4360	4.4514	1.5488	-27.8558	-18.5224
22	Bank Aa2	5.7056	5.8097	10.4102	1.9657	-1.0343
23	Bank Aa2	5.5677	5.7153	14.7599	-20.4728	-22.4728

Table 11: Pricing results with the fourth threshold. Results for the theoretical price following the parameter estimation and the application of the closed-form pricing equation of Longstaff and Schwartz [1995] to fixed and floating debt. The swap credit spread is simply the difference (in basis points) between the riskfree theoretical price and the risky one. The two last columns provide the difference (in basis points) of the computed prices with the market references, i.e. the traded price and the standard quote available that day. No sufficient data on face values was available for bank Aa2 in 91-92 (transaction n. 20).

	Rating of the Counterparty	Swap riskfree rate	Swap risky rate	Swap Credit Spread	Difference with traded price	Difference with quoted price
1	Bank Aaa	7.4223	7.4223	0.0003	-8.7700	-9.7700
2	Bank Aaa	7.0757	7.0757	0.0008	-6.4284	-11.7618
3	Bank Aaa	7.0645	7.0645	0.0008	-8.0531	-14.8864
4	Bank Aaa	6.9141	6.9142	0.0012	-21.5845	-29.5845
5	Bank Aaa	6.9054	6.9054	0.0012	-23.4635	-36.4635
6	Bank Aaa	4.4487	4.4490	0.0266	17.9009	13.9009
7	Bank Aaa	4.4360	4.4372	0.1247	-29.2798	-19.9465
8	Bank Aaa	4.5335	4.5402	0.6686	-75.9807	-81.9807
9	Bank Aaa	4.7707	4.7759	0.5132	-47.4133	-50.4133
10	Bank Aaa	5.2067	5.2175	1.0828	-41.2464	-68.2464
11	Bank Aaa	5.1364	5.2267	9.0282	-27.3329	-36.3329
12	Bank Aaa	5.7939	6.2839	48.9932	42.3878	48.3878
13	Bank AA1	4.7391	4.7668	2.7765	44.6835	43.6835
14	Bank AA1	4.4391	4.4702	3.1040	17.0164	14.0164
15	Bank AA1	4.0845	4.1044	1.9899	-55.5619	-39.5619
16	Bank AA1	5.4990	5.7839	28.4839	10.3874	-5.6126
17	Bank AA1	5.7977	5.9624	16.4728	6.2431	0.2431
18	Bank AA1	5.6636	5.7902	12.6561	9.0193	-13.6474
19	Bank AA1	5.5357	5.5691	3.3410	-14.8399	-20.0899
20	Bank Aa2	6.4159				
21	Bank Aa2	4.4360	4.4559	1.9922	-27.4123	-18.0790
22	Bank Aa2	5.7056	5.8453	13.9712	5.5267	2.5267
23	Bank Aa2	5.5677	5.7520	18.4329	-16.7998	-18.7998

Table 12: Pricing results with the fifth threshold. Results for the theoretical price following the parameter estimation and the application of the closed-form pricing equation of Longstaff and Schwartz [1995] to fixed and floating debt. The swap credit spread is simply the difference (in basis points) between the riskfree theoretical price and the risky one. The two last columns provide the difference (in basis points) of the computed prices with the market references, i.e. the traded price and the standard quote available that day. No sufficient data on face values was available for bank Aa2 in 91-92 (transaction n. 20).

6 Conclusion

The present analysis has attempted to implement a theoretical model of credit risk pricing to evaluate its use on transaction data. The credit risk pricing model studied here is somewhat cumbersome to implement mainly because of the problem of informative quality of the data when putting together market traded data and accounting numbers.

Although it is difficult to accept the model for pure swap pricing purposes when looking at the theoretical prices obtained (which tends to put the model rather than the market pricing into question), it provides, along with the methodology inherent to structural modelling, useful theoretical insights for credit monitoring and management. Further investigation on larger samples of data would be necessary to give definite answers about the validity of the model and its main weaknesses though.

Points of main concern in the implementation have been the threshold level and the associated maturity of liabilities that constitute it. It is not clear that a given threshold level will have the same significance whatever is the activity and the industrial sector of the firm. The rest of the riskiness behavior of the firm is assumed to be entirely explained by the volatility of the assets of the firm. Depending on whether we assume the volatility of equity returns to be related only to the systematic risk of the stock market or also to other economic variables in the sense of the APT, the volatility can be assumed to contain more or less information about the sensitivity of firm' returns to its financial and economic environment.

Furthermore, the model of Longstaff and Schwartz prices fixed and floating debt while its swap extension views swaps simply as back-to-back loans. Therefore we miss some co-behavior of the swap legs when sensitivity to credit worthiness is taken into account. But, as an advance in the integration of market and credit risks into a structural model of debt pricing, this model is still very appealing. A positive conclusion is that the problem for which a reproach if being made to Longstaff and Schwartz with respect to their derivation looses all its importance when using the model in low-volatile interest rate environments. In such places, the real pricing problem is definitely the capacity to put in place a model of two-sided default risk rather than to be more accurate in the pricing impact of the stochastic interest rates.

In the case of swap pricing, it would be interesting to obtain comparative research in alternative theoretical models, notably true two-sided default models. Hopefully, more advanced models of swap credit risk pricing will arise to overcome the difficulties of the model tested here, without giving up on the endogeneity of the bankruptcy (or downgrading) process. The now frequent use of collaterals (as well as marking to market, credit lines, etc...) make the difficulties in building up pertinent models all the more important. It may thus become even more difficult to evaluate who is right, of those who think the models are not good enough yet to value the real credit spreads and those who think, like Hull and White [1992], that "players are not receiving adequate compensation for the credit risk they are bearing." It can be mentioned then that simulation is still a viable technique that would enable the pricer to conserve the structural underlying framework while characterizing the exact payoff' profile of the investment.

Finally, the scarcity of good quality transaction data on swaps has been a large impediment to analytical advances or confirmations in the field. At a time when markets on credit risk products start to develop, better knowledge and more consensus in credit risk pricing would certainly prove valuable to professionals. Hopefully, swap market operators will trust academics with large sample of confidential transaction data and give them a chance to advance both empirical and theoretical research in the field, at the advantage of both sides.

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