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How Did Financial-Crisis-Based Criticisms of Market Efficiency Get It So Wrong?

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Abstract

In the aftermath of the financial crisis, market efficiency is being heavily criticized. However, the volatility-based criticisms rely on false grounds as efficiency and speculative bubbles are compatible. Indeed, the efficient market model is about rationality and information, not about stability. This model admits multiple solutions, as do most rational expectations models. One solution is the so-called fundamental one while the others are referred to as rational bubbles. Still, many practitioners, and even some financial academics, keep denying that speculative bubbles are compatible with efficient markets. This paper argues that not only would the recognition of efficient market multiplicity thwart irrationality-based theories, but it would also allow for further empirical developments taking full advantage of the power of diversity. The multiple price dynamics compatible with market efficiency represent a valuable asset largely underestimated by the profession.

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1. Introduction

Criticizing the Efficient Market Hypothesis (EMH) on the basis of highly volatile asset prices is conceptually wrong. More than twenty years after the economic literature has definitely proven the existence of multiple solutions to rational expectation models and the financial literature has logically followed with the rational bubble theory, most practitioners and numerous financial academics keep denying that speculative bubbles are compatible with efficient markets. EMH deserves more than ill-posed criticisms, it deserves extensive empirical testing based on the best available econometric methods.

The foundations of the Efficient Market Hypothesis (EMH) are regularly put into question with the main argument that EMH is much too restrictive to grasp the complexity of "real markets". Despite the remarkable case made by Fama (1998), in response to the anomaly-based charges made by the advocates of behavioural finance, EMH remains heavily challenged these days, especially in the aftermath of the deep confidence crisis toward the fundamental architecture of financial theory.

This paper argues that on top of being economically sound and abundantly documented by the facts (see Fama, 1965, 1970, and 1991; Jensen, 1978; Malkiel, 2003, and many others), EMH benefits from an asset that is mostly neglected, namely the wide spectrum of price dynamics offered by the multiplicity of solutions associated to rational expectations (RE) models. Surprisingly, proponents of the EMH tend to underestimate the power of this mathematical argument. The most prominent characteristic of multiplicity regarding financial valuation is the one that makes EMH compatible with the presence of speculative bubbles.

Despite their routine use in macroeconomic theory, multiple solutions models still suffer from a bad reputation[1] among economists who tend to view them as underspecified. In that line of thought, criteria meant to neutralize multiplicity in RE models flourish in the literature[2]. Our view goes in the opposite direction: there is nothing wrong with multiple solutions. Indeed, if a model-builder ends up finding multiple solutions, it simply means that the phenomenon at stake may take multiple shapes. Therefore, adding *ad hoc* restrictions with the sole aim to reduce the multiplicity damages the model as it makes the solution determination artificial. Moreover, econometrics offer estimation and testing techniques that reveal adequate to deal with the whole set of solutions to RE models.

The case for rational bubbles has been extensively made by LeRoy (2004) who observes that "*The identification of bubbles with irrationality is found not only in the financial media, but also in many professional discussions*" (p. 785) and further states that "*Bubbles can also be defined and analyzed in settings that do not involve irrationality*" (p. 785). He shows that despite the common belief that bubbles are irrational several rationally-based explanations are plausible. We here draw the logical consequence from this study, namely that rational bubbles need to be considered as compatible with the EMH as much as the no-bubble (or fundamental) solution.

[1] Multiple solutions are sometimes called « indeterminacies » (see, e.g., McCallum (2003)), which stresses the negative connotation associated to them.

[2] Those criteria refer to learning processes (Bray, 1982; Bray and Savin, 1986; Fourgeaud *et al.*, 1986), minimal variance (Taylor, 1977), expectational stability (Evans, 1985; Evans and Honkapohja, 2003), minimal state variables (McCallum, 1983 and 1999), among others (Onatski, 2006). While learning processes are arguably realistic regarding agents' behavior, purely statistical constraints look more like artificial restrictions introduced to get rid of the bothering multiplicity.

Although multiple solutions might seem disturbing for policy purposes and for market practitioners[3], they do actually represent an opportunity as they offer additional degrees of freedom for their empirical implementation. If a model built from all relevant assumptions ends up leaving the space for multiple solutions, then the data will "choose" its best candidate. Logically, additional criteria tend to deteriorate the quality of the adjustment without adding sensible economic content. Why would economists welcome *ad hoc* restrictions when it comes to RE solutions while at the same time they refuse limiting admissible parameters values without any good reason? Why should diversity be an asset for parameters but not for dynamic specifications? Arbitrary functions (martingales in the present situation) are not more offensive than arbitrary numbers. Moreover, macroeconometrics has now a long-standing tradition of estimating Euler equations under rational expectations without any prior restriction on the solutions set. Financial econometricians might well do the same.

Following this argument, we suggest reconsidering the very basics of market efficiency that is the simplest model for the price dynamics derived from the rational expectations hypothesis. This equation has multiple solutions: one is the so-called fundamental solution and all others are rational bubbles. In a rather masochistic way, the literature on market efficiency focuses on the fundamental solution, though rational bubbles are equally compatible with all the assumptions that make markets efficient. Indeed, since the Euler equation that constitutes the theoretical foundation for EMH admits an infinity of solutions, one should deal with all of them and explore the power of diversity. This can be seen as a consequence from Cochrane's statement that "(…) *for* testing*, it seems that everything volatility tests can do, Euler equations can do better*" (p.4 78).

[3] Actually, a single – but wrong – solution is much worse.

The rest of the paper is organized as follows. Section 2 presents the basic RE model derived from the EMH and its multiple solutions. The notion of fundamental solution is further examined in Section 3. Section 4 discusses the related econometric issues, namely RE model estimation and tests for bubbles. Section 5 concludes.

2. Multiple solutions under the EMH

According to the RE hypothesis introduced by Muth (1961), the economic agents form optimal predictions given their current information set. Optimality has to be understood in probabilistic terms: rational expectations minimize the quadratic forecast error given the available information[4]. More precisely, consider a future price p_{t+1} to be forecasted and I_t the set of all variables observable at time *t*. Then, the RE of p_{t+1} is its conditional expectation given the σ-algebra generated by all stochastic processes constituting the available information I_t:

$$E\left[p_{t+1} \left| I_t \right. \right] = \arg\min_{\hat{p}_{t+1} \in I_t} E\left(\hat{p}_{t+1} - p_{t+1} \right)^2 .$$

Notably, this definition is implicit. It provides no direct expression of the expectation in terms of the observable variables used in its formation. Actually, such an expression is even impossible to derive without reference to a dynamic generating process. In that sense, rational expectations are endogenous and need to be determined while solving the model at stake, not *before* as in the case of other expectation schemes (naïve expectations or adaptive expectations, for instance). Formally, a RE model is very different from standard dynamic models as its resolution requires dealing simultaneously with the explained variables and their expectations.

[4] For further theoretical formalism and computational issues associated to RE, see, e.g., Pesaran (1987) and Broze and Szafarz (1991).

In classical univariate linear stochastic recursive equations, one variable is explained by its past values and some exogenous factors. Given the adequate numbers of initial conditions, the dynamics of the process of interest is unique. On the opposite, in univariate linear RE models, one faces (at least) two endogenous variables in the same equation: the explained variable and its expectation. For this reason, when the expectation is not observable (which is usually the case), the model needs to be solved before any empirics may be considered. A large body of the econometric literature has been devoted to that theoretical problem.

Since the 1980ies, the economists have been equipped with the necessary tools to solve any univariate linear RE model[5] (Gouriéroux *et al.,* 1982; Broze *et al*., 1985). Under its modern form, the basic efficient market model has a rather simple structure[6], sometimes referred to as the Cagan model in the macro theory[7]. In this specification, the current price, p_t, of a given asset (say, a stock price) depends on its rational expectation, $E\left[p_{t+1}|I_t\right]$, and on an exogenous variable, x_t, representing the revenues associated to the detention of the asset (say, the present value of the expected next period dividend, $x_t = \frac{E\left[d_{t+1}|I_t\right]}{1+r}$) in the following way:

$$p_t = \frac{1}{1+r}E\left[p_{t+1}|I_t\right] + x_t \qquad (1)$$

The general solution of model (1) may be expressed either with an arbitrary martingale (Pesaran, 1981; Gouriéroux *et al.,* 1982), or with an initial condition and a martingale

[5] While Blanchard and Kahn (1980) provided the first hints on the solutions of multivariate linear models, the general solution was derived in the nineties (Broze *et al*., 1995; King and Watson, 1998). See Sims (2002) and Anderson (2008) on the computational issues related to solving those models.

[6] As we deal here with a simple RE model, we do not review the solution techniques for general models. Let us simply recall that the dimension of the multiplicity of solutions depends on the horizon of the predictions included in the specification (this horizon is equal to one in the efficient market model below).

[7] Cagan (1956) proposed a hyperinflation model which was later reformulated under the RE hypothesis. This model often serves as a benchmark in the RE literature (see, e.g. Taylor, 1991).

difference (Broze *et al.*, 1985). The martingale representation is well-adapted to represent bubbles (Flood and Garber, 1980; Blanchard, 1979; Blanchard and Watson, 1982) as it allows for isolating a particular solution, the so-called "fundamental solution", denoted p_t^F. Indeed, the general solution to eq. (1) writes:

$$p_t = p_t^F + (1+r)^t M_t \quad (2)$$

Where M_t is an arbitrary martingale[8] and:

$$p_t^F = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} E\left[x_{t+i} \middle| I_t\right] \quad (3)$$

Thus, the model admits the infinite set of solutions given by (2). As martingales are known to possess constant marginal mean and non-decreasing marginal variance, the second term in the right hand-side of (2) is stochastic and explosive. For that reason, it is defined as the « bubble component », b_t, of the price dynamics:

$$b_t = (1+r)^t M_t \quad (4)$$

A way to annihilate the bubble component in (2), or equivalently to impose that $M_t = 0$, is provided by the transversality condition:

$$\lim_{i\to\infty} \frac{1}{(1+r)^i} E\left[x_{t+i} \middle| I_t\right] = 0 \quad (5)$$

Indeed, under condition (5) the fundamental solution given by (3) is the unique solution to model (1). The transversality condition is necessary for finite horizon optimization problems without constraint on the final state (Michel, 1982 and 1990). However, as stock pricing is typically an infinite horizon problem[9], condition (4) is not a pre-requisite for solving the Euler

[8] A martingale is a stochastic process adapted to the information filtration, I_t, such that its rational expectations for any time in the future equates its current value : $\forall i \geq 0 : E\left[p_{t+i} \middle| I_t\right] = p_t$.

[9] The assertion is untrue for finite-horizon assets as bonds and derivatives. However, the pricing of those assets typically includes a final condition (value at maturity) which annihilates bubbles. Therefore, bubbles and multiple prices may only be present for no-maturity assets (see Roll, 2002).

equation. With the notable exception of Tirole[10] (1982), when (and if!) they justify the use of the transversality condition, authors tend to put forward stability considerations since eq. (5) imposes that the expected dividends grow less quickly than the riskless asset. Whether this condition is reasonable or not is a matter of appreciation, but it is by no means a mathematical request or a consequence of the EMH. Therefore, identifying the EMH with the fundamental solution, or equivalently imposing condition (4), remains arbitrary.

3. How fundamental is the fundamental solution?

An important drawback of limiting EMH to the fundamentals in (3) stems from the fact that fundamental solutions are model-specific. Indeed, as soon as the underlying equilibrium model becomes more complex than equation (1), defining the fundamental solution becomes trickier and no consensus has been reached yet on the criterion one should generalize condition (4). We interpret that indecisiveness as an additional piece of evidence on the interpretative confusion regarding the fundamental-versus-bubbles distinction.

Model (1) is intuitively appealing for stock valuation. It becomes more debatable for other markets, like exchange rates, real-estate, and commodities. Pricing in such markets leads to sophisticated models for which the very notion of fundamental price is far from obvious. For instance, regarding exchange rates, one might refer to financial no-arbitrage conditions like the covered interest rate parity, or alternatively turn to macroeconomic equilibrium conditions like the purchasing power parity. The chances are slim that those different approaches drive

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[10] Tirole (1982) shows that, even in an infinite-horizon framework, bubbles are impossible when the number of agents is finite. However, LeRoy (2003) argues that "*This assertion requires qualification, and the nature of the qualification depends on how infinite-date settings are modelled*" (p. 18)

the same fundamental exchange rate. Therefore, for a large class of financial assets, fundamental prices are model-specific.

Pushing the argument one step further, even for stocks, model (1) is not free from criticism. The interest rate *r* in (1) is supposed to be the (assumed constant) risk-adjusted rate, i.e., the agent's required rate of return for the stock at stake. Alternatively, *r* can be seen as the risk-free rate under a modified probability, interpreting then equation (1) as a one-step-forward no-arbitrage condition.[11] Under this framework, the forward-looking solution (3) may also be seen as the outcome of a no-arbitrage condition, but specified in an infinite time horizon. In other terms, the fundamental solution represents the fair stock price in a financial world where agents buy stocks with the sole perspective of keeping them forever. If enough traders contemplate transitory stock holding and/or speculate on uncertain future sales, then the no-arbitrage motivation for favoring solution (3) among all solutions to (1) disappears.

In consequence, the fundamental solution, if any, is model-specific, but the existence of multiple solutions is not. Any RE model involving future expectations exhibit multiple solutions.[12] Indeterminacy is the rule, not the exception. Even more, the recognition of multiple sensible price dynamics may be seen as a way to overcome the difficulty to isolate a fundamental solution. Indeed, if one admits that all solutions deserve consideration since they all fulfil the equilibrium condition derived from EMH, then there is no need to particularize one out of them as representing *the* fundamental price. Econometrics can do the job then.

[11] See, for instance, This Saint Jean (2008).
[12] With the exception of very special cases regarding the parameters configuration (See Broze *et al.*, 1995; King, and Watson, 1998)

4. The econometric side of the story

The estimation of RE models dates back to Sargent (1978) and has been extensively analysed by the econometric literature[13] as RE models exhibit a large spectrum of applications, mainly in macroeconomics. Those models have the well-known merit of overcoming the Lucas critique.[14]

In the financial econometric literature, tests have been proposed and implemented for the detection of speculative bubble dynamics (see, e.g., West, 1988). Starting with variance bound tests (Shiller, 1981), various methodologies have been assessed. However, as a matter of fact, the empirical results fail to converge. Typically, to any paper finding a bubble in a given price series, there is a counterpart finding no bubble (Gürkaynak, 2008).

Contradicting Shiller's (2003) argument against the EMH, theoretical evidence show that excess volatility is compatible with rational bubbles (Cochrane, 1991, Salge, 1997; Adam and Szafarz, 1993), that is with solutions to equation (1) or to any extension based on an equilibrium condition under RE. As the current debate focuses on issues such as irrationality and mispricing due to behavioral finance (see, e.g., Shleifer, 2000), recognizing that not all rejections of the no-bubble null hypothesis do validate irrational market explanations is a strong case in favour of market efficiency.

Theoretically, within the framework of model (1) the fundamental solution is well-defined. However, empirically, the problems associated to the identification of the fundamental

[13] Se, e.g., Taylor (1979), Lucas and Sargent (1981). Hansen and Singleton (1982), Broze and Szafarz (1991), and Tucci (2004).
[14] See Farmer (1991) for a discussion on Lucas critique and multiple solutions.

solution deeply hamper the econometric procedures as summarized by Flood and Hodrick (1990): "*It is our contention that no econometric test has yet demonstrated that bubbles are present in the data. In each case, misspecification of the model or alternative market fundamentals seems the likely explanation of the findings*" (p. 87). Indeed, as shown by equation (3), this solution is based on expected dividends that are unobservable and, therefore, assumptions are required for estimation and tests. Moreover, the basic model itself, because of its simplicity (linearity, constant interest rate, etc.), may suffer from misspecification. Those issues are likely to explain the poor consistency of the results found in the empirical literature on bubbles. They also raise serious doubts about their ability to ultimately reject or confirm the EMH. Of course, testing whether a given market is driven by the fundamentals may be of some interest *per se* and, in this respect, methodological improvements could reveal useful. However, this has little to do with the "EMH versus irrational markets" controversy.

If one concedes that the EMH may be summarized by a RE model like equation (1) - or a more sophisticated one of the same kind -, then one has also to admit that the EMH econometric validation relates RE estimation and test, irrespectively of the bubble issue. As any theoretical assumption, the EMH needs to be confronted to the data[15], but excess volatility and existence of bubbles are irrelevant for that matter. On the opposite, observable consequences drawn from equation (1) and its general solution (2) may reveal instrumental given the possibility to test for martingale characteristics.[16] Fortunately, the macroeconometric literature has produced a toolbox that financial empiricists may apply extensively, with the notable advantage of disposing from a data wealth that no macroeconomist could even dream of.

[15] See, e.g., Guesnerie (1993).

[16] This methodology appears as the new generation of the random-walk tests, as random walks are special cases of martingales.

5. Conclusion

In an efficient market, market prices may follow different paths. However, as shown by Cochrane (1991), variance bounds and stability are not part of the EMH and, therefore, one cannot reject market efficiency on the basis of excess volatility tests. Rational bubbles constitute possible outcomes of the efficient market dynamics. As stated by LeRoy (2003): « *Contrary to a widely-held opinion, asset price bubbles do not necessarily reflect a model specification that is exotic in any respect (unless assuming an infinite number of dates is considered exotic)* » (p. 2). Much more than bubbles is required to reject market efficiency and, to our knowledge, the empirical literature has not gone so far.

Going one step further, one can question the academic interest for bubbles. The rationality issue might indeed be more important to deal with, especially when it comes to the "*fair pricing*" interrogations raised by the media and the investors who are suffering from the recent financial crisis. Speculative bubbles provide an appealing imagery (that makes financial markets look like casinos), but they do not really meet those serious concerns as fairness is more adequately addressed through the notion of efficiency (and, more precisely, the level of efficiency of each market). Academics in finance who care for the already damaged reputation of their profession should pay more attention to efficiency than to bubbles.

Multiple solutions are sometimes regarded as a sign of under-specification. However, economists have no problem with models including parameters to be estimated and variables of which probability distributions are to be determined empirically. Actually, the extent of admissible indeterminacy in a theoretical model is not clear-cut. In other words, the set of characteristics to be left to econometric determination vary across models. In our opinion, for models built from EMH (as for most rational expectations models), an arbitrary martingale is part of this set.

Rational bubbles may look as price dynamics that lie far away from the expected-dividend-discount fundamental but conceptually there are not, because all share the common grounds of rational expectations pricing. Furthermore, as rational expectations are auto-referential (agents are assumed to know the model including the expectation formation), multiple solutions are a natural outcome. Starting from the Euler equation is in line with the econometricians' standard way of doing.

In summary, while some consider multiple price dynamics as a drawback, we argue that it represents a challenging opportunity for empirical work on the EMH. Whether an economic theory is to be judged on the degrees of freedom of its outcome is an interesting open epistemological question. But, when it comes to the efficient market paradigm there is no doubt that admitting rational bubbles within the family of admissible solutions offers a remarkable theoretically-based way to respond to the criticisms from the proponents of behavioral finance and other irrationality-based approaches who put forward the excess volatility observed on many stock exchanges, especially during financial crises. It is a pity that advocates of market efficiency do not take full advantage of such hardly refutable argument. As Cochrane (1991) puts it: « *I and others like me whose research is still devoted to extending rational economic models to account for anomalies may, in the end, be wrong, but at least we are not pig-headed in the face of clear contradictory evidence* » (pp. 482-3).

References

Adam, M.C. and A. Szafarz (1993), « Speculative bubbles and financial markets », *Oxford Economic Papers*, 44, 626-640.

Anderson, G.S. (2008), « Solving linear rational expectations models: A horse race », *Computational Economics*, 31, 2, 95-113.

Blanchard, O.J. and C.M. Kahn (1980), « The solution of linear difference models under rational expectations », *Econometrica*, 48, 1305-1311.

Blanchard, O.J. and M.W. Watson (1982), «Bubbles, rational expectations and financial markets », in P. Wachtel (Ed.) *Crises in the economics and financial system*, Lexington, MA: Lexington Books.

Blanchard, O.J. (1979), « Speculative bubbles, crashes and rational expectations », *Economics Letters*, 3, 387-389.

Bray, M.M. (1982), « Learning, estimation, and the stability of rational expectations », *Journal of Economic Theory*, 26, 2, 318-339.

Bray, M.M. and N. E. Savin (1986) « Rational expectations equilibria, learning, and model specification", *Econometrica*, 54, 5, 1129-1160.

Broze, L., Gouriéroux, C. and Szafarz, A. (1985), « Solutions of dynamic linear rational expectations models », *Econometric Theory*, 1, 341-368.

Broze, L., C. Gouriéroux, and A. Szafarz (1995), « Solutions of multivariate rational expectations models », *Econometric Theory*, 11, 2, 229-257.

Broze, L. and A. Szafarz (1984), « On linear models with rational expectations which admit a unique solution », *European Economic Review*, 24, 1, 103-111.

Broze, L. and A. Szafarz (1991), *The econometric analysis of non-uniqueness in rational expectations models*, North-Holland, Amsterdam.

Cagan, P. (1956), « The monetary dynamics of hyperinflation », in *Studies in the Quantity Theory of Money*, M. Friedman (Ed.), Chicago: Chicago University Press.

Cochrane, J.H. (1991), « Volatility tests and efficient markets », *Journal of Monetary Economics*, 27, 463-485.

Evans G.W. (1985) « Expectational stability and the multiple equilibria problem in linear rational expectations models », *Quarterly Journal of Economics*, 100, 4, 1217-1233.

Evans, G.W. and S. Honkapohja (2003), « Expectational stability of stationary sunspot equilibria in a forward-looking linear model », *Journal of Economic Dynamics and Control*, 28, 1,171-181.

Fama, E.F. (1965), « The behavior of stock market prices », *Journal of Business*, 38, 34-105.

Fama, E. (1970), «Efficient capital markets: A review of theory and empirical work», *Journal of Finance*, 25, 383-417.

Fama, E.F. (1991), « Efficient capital markets: II", *Journal of Finance*, 46, 1575-1617.

Fama, E.F. (1998), « Market efficiency, long-term returns, and behavioural finance », *Journal of Financial Economics*, 49, 283-306.

Farmer, R.E.A. (1991), « The Lucas Critique, Policy Invariance and Multiple Equilibria », *Review of Economic Studies*, 58, 2, 321-332.

Flood, R. P. and P. M. Garber (1980), « Market fundamentals versus price level bubbles: The first tests », *Journal of Political Economy*, 8, 745-770.

Flood, R. P. and R. J. Hodrick (1990), « On Testing for Speculative Bubbles », *Journal of Economic Perspectives*, 4, 2, 85-101.

Fourgeaud, C., Gourieroux, C. and Pradel, J. (1986), « Learning procedures and convergence to rationality », *Econometrica*, 54, 4, 845-868.

Gouriéroux, C., J.J. Laffont and A. Monfort (1982), « Rational expectations in linear models: Analysis of the solutions », *Econometrica,* 50, 2, 409-425.

Guesnerie, R. (1993), « Theoretical tests of the rational expectations hypothesis in economic dynamical models », *Journal of Economic Dynamics and Control*, 17, 5-6, 847-864.

Gürkaynak, R.S. (2008), « Econometric tests of asset price bubbles: Taking stock », *Journal of Economic Surveys*, 22, 1, 166-186.

Hansen, L.P. and K. J. Singleton (1982), « Generalized instrumental variables estimation of nonlinear rational expectations models », *Econometrica*, 50, 5, 1269-1286.

Jensen, M.C. (1978), « Some Anomalous Evidence Regarding Market Efficiency », *Journal of Financial Economics*, 6, 2/3, 95-101.

King, R.G and M.W. Watson (1998), « The solution of singular linear difference systems under rational expectations », *International Economic Review*, 39, 4,1015-1026.

LeRoy, S.F. (2003), « Rational Exuberance », working paper, University of California, Santa Barbara.

LeRoy, S.F. (2004), « Rational Exuberance », *Journal of Economic Literature*, 42, 3, 783-804.

Lucas R.E. and T.J. Sargent (Eds) (1981), *Rational Expectations and Econometric Practice,* Minneapolis: University of Minnesota Press.

Malkiel, B. (2003), « Passive investment strategies and efficient markets », *European Financial Management*, 9, 1, 1-10.

McCallum, B.T. (1983), « On non-uniqueness in rational expectations models: An attempt at perspective », *Journal of Monetary Economics*, 11, 139-168.

McCallum, B.T. (1999), « Role of the minimal state variable criterion in rational expectations models », *International Tax and Public Finance*, 6, 621–639.

McCallum, B.T. (2003), « Multiple-solution indeterminacies in monetary policy analysis », *Journal of Monetary Economics,* 50, 5, 1153-1175.

Michel, P. (1982), « On the transversality condition in infinite horizon optimal problems », *Econometrica*, 50, 4, 975-985.

Michel, P. (1990), « Some clarifications on the transversality condition », *Econometrica*, 58, 3, 705-723.

Muth, J.F. (1961), «Rational expectations and the theory of price movements», *Econometrica*, 29, 315-335.

Onatski, A. (2006), « Winding number criterion for existence and uniqueness of equilibrium in linear rational expectations models », *Journal of Economic Dynamics and Control,* 30, 2, 323-345.

Pesaran, M.H. (1981), « Identification of rational expectations models », *Journal of Econometrics*, 16, 3, 375-398.

Pesaran, M.H. (1987), *The limits to rational expectations*, Oxford: Basil Blackwell.

Roll, R. (2002), « Rational infinitely lived asset prices must be non-stationary », *Journal of Banking and Finance*, 26, 6, 1093-1097.

Salge, M. (1997), *Rational Bubbles*, Berlin: Springer-Verlag.

Sargent T.J. (1978), « Estimation of dynamic labor demand schedules under rational expectations », *Journal of Political Economy*, 86, 6, 1009-1044.

Shiller, R. (1981), « Do stock prices move too much to be justified by subsequent changes in dividends ? », *American Economic Review*, 71, 421-436.

Shiller R.J. (2003), « From efficient markets theory to behavioral finance », *Journal of Economic Perspectives*, 17, 1, 83-104.

Shleifer, A. (2000), *Inefficient markets: An introduction to behavioral finance*, Oxford: Oxford University Press.

Sims C.A. (2002), « Solving linear rational expectations models », *Computational Economics*, 20, 1-2, 1-20.

Taylor, J.B. (1977), « Conditions for unique solutions in stochastic macroeconomic models with Rational Expectations », *Econometrica*, 45, 6, 1377-1385.

Taylor, J.B. (1979), « Estimation and control of a macroeconomic model with rational expectations », *Econometrica*, 47, 5, 1267-1286.

Taylor, M.P (1991), « The hyperinflation model of money demand revisited », *Journal of Money, Credit & Banking*, 23, 3, 327-351.

This Saint Jean, I. (2008), « Valeur fondamentale et bulles rationelles », in C. Walter and E. Brian (Eds) *Critique de la valeur fondamentale,* Paris: Springer, 55-77.

Tirole, J. (1982), « On the possibility of speculation under rational expectations », *Econometrica*, 50, 5, 1163-1182.

Tucci, M.P. (2004), *The rational expectation hypothesis, time-varying parameters and adaptive control: A promising combination?*, Dordrecht: Springer.

West, K.D. (1988), « Bubbles, fads and stock price volatility tests: A partial evaluation », *Journal of Finance*, 43, 639-656.