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Keywords: Ownership, Corporate Governance, Control, Banzhalf Index.

CEB Working Paper N° 06/023
2006

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Abstract

Today, the shareholding structure of companies is sometimes so complex that it can be difficult to find back their actual owners and controllers. In particular, in continental Europe and in Asia, control tunnelling appears frequently through pyramidal structure. After describing the ownership structure through a graph association, this paper analyses the voting game at stake in the race for control. It compares existing methods and algorithms to identify the owners and controllers of a firm in a pyramidal structure without cross-ownership. As a real life example, the case of the Belgian retail company, Colruyt, is used to apply these different methods and compare their results. Furthermore, it shows how the ownership structure allows to the Colruyt family to keep the control of Colruyt even if there are discordances inside the family.

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1. Introduction

The control of a company is reached through voting rights in the General Assembly. As shown in Ehrhardt and Nowak (2003), the economic literature has well documented the fact that control over a firm often provides private benefits (see for example Chung and Kim, 1999; Chueng, 2005). However, by ignoring the indirect ownership, the direct voting right picture does not always identify the dominant shareholders who control the firm[1] (see Brioschi et al., 1989 ; Bedchuk et al., 2000 ; Chapelle, 2001 ; Thesmar, 2001 ; Faccio and Lang, 2002). In other words, indirect ownership coming from pyramidal structures may imply that the dominant shareholders do not have the direct control. Moreover, the cross-ownership increases the difficulty to evaluate the integrated ownership and control of a company.

While it is evident that the control is achieved with more than 50% of direct voting rights, it may be much more difficult to establish or determine control issues in the case of indirect voting rights. Furthermore, in empirical literature, control thresholds are often taken below 50%, depending on the country and the type of company. According to La Porta et al. (1999), it seems that the level of 20% is sufficient in most cases to control a firm. The example of Colruyt in section 4 of this paper will show how this threshold can influence our perception of control.

To analyse the links between different companies through a shareholding structure, this essay will rely on graph theory. To this end, the network of companies will be represented by a graph in which every firm and/or shareholder corresponds to a vertex. In the graph of direct shareholdings, the oriented arc between two vertexes stands for the share participation of one company in another. A value between 0 and 1 is attributed to every arc of the graph in order to represent the fraction of shares owned directly by one vertex of the arc in the other vertex of the arc. From the latter, different graphs of the same shareholding structure can be created. For instance, the graph of integrated ownership may be defined in a similar way. The computation of integrated ownership from direct ownership is well described in literature (see Ellerman, 1991; Flath, 1991; Chapelle and Szafarz, 2005). Graphs giving control of firms on other firms will be constructed further on. In order to derive these graphs from the direct shareholding structure, the graph theory will be used.

The problem of governance implies different firms and people. They all have different goals and they interact together to increase their own utility. The theory that models this situation is the game theory (see Mann and Shapley, 1960; Shubik, 1982; Gambarelli and

[1] We do not consider here shares with multiple voting rights.

Owen, 1994; Crama et al., 2004). This theory studies voting games, where each player votes to influence the final decision. Some players have more influence or chance to modify the final decision (see Crama et al., 2003). In this theory, the Banzhaf index measures in a binary voting game the player's probability to have a decisive vote when every other player has one chance on two to vote "yes" or "no".

In a shareholding structure, the voting rules are complex and the number of voices of a shareholder is not proportional to the number of shares he has directly and indirectly in the company he wants to govern. Indeed, if an ultimate shareholder owns indirectly some shares of a firm but does not control the intermediary firms, these shares will not provide him any voting rights and control over the final firm.

The first section of this article gives the definition corresponding to the problem of governance in terms of graph theory and game theory. The second part of the paper describes the existing models that provide a measure of control of an ultimate shareholder in a company. This section also compares their respective methodologies and interpretations. Section 3 applies the methodologies to a real case study: Colruyt. The last part draws conclusions.

2. Definitions

This section will give some definitions to model the problem of control in a share holding structure. There are three sets of definitions. The first set allows to describe any shareholding structure with the graph theory. The second set models voting games using the game theory. The last set is relative to notions of control and power of control.

A shareholding structure consists in a network of firms linked together through shares participations. The graph of direct ownership represents this network.

D1: The *graph of direct shareholding* is the graph $G=(V,S,A)$, where V is the vertex set representing the different firms of the structure, $S \subseteq V*V$ is the arc set, and $A=(a_{ij})$ is a real matrix with entries positive and smaller than or equal to 1. a_{ij} represents for each arc $(i,j)\in S$ the stock fraction of firm j owned by firm i and 0 if $(i,j)\notin S$.

D2: In graph theory, if $(i,j)\in S$, i is a *predecessor* of j and j is a *successor* of i (i is a *direct owner* or *direct shareholder* of j). If j has no predecessor, then j is a *source*.

D3: The *matrix of direct ownership* $A=(a_{ij})$ has been defined above. Baldone *et al.* (1996) define *integrated ownership* as the sum of all participations (direct and indirect). If y_{ij} represents the integrated ownership of i in j, the matrix Y is computed as:

$$Y=\sum_{i=1}^{\infty} A^i=(I-A)^{-1} A$$

D4: The matrix Y cleaned from double counting gives us the *matrix of integrated ownership* (Chapelle and Szafarz (2005)) $Z=(z_{ij})$ and Z is computed as:

$$Z=\left(diag(I-\tilde{A})\right) Y \qquad (1)$$

where $\tilde{A}=(a_j)$ and $a_j=\sum_{i=1}^{n} a_{ij}$.

In a shareholding structure, the phenomenon of *cross-ownership* appears when a firm directly or indirectly possesses its own shares. In graph theory, cross-ownership corresponds to a *cycle*.

D5: A *cycle (or a loop)* in graph $G=(V,S,A)$ is a sequence $\{(i_1,i_2),(i_2,i_3),(i_3,i_4),\ldots,(i_{k-1},i_k)\}\subset S$ such that $i_1=i_k$. This means that firm i_1 holds through other firm's shares of itself.

D6: A structure without cross-ownership is called a *pyramidal structure*.

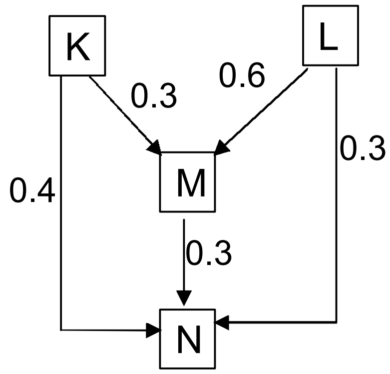
D7: In most cases, since not all shareholders are known, it appears that : $\sum_{i=1}^{n} a_{ij}<1$. The fraction $1-\sum_{i=1}^{n} a_{ij}$ of firm j is called its float. It represents the part of shares owned by unknown or small shareholders.

The following example illustrates the definitions from the first set and shows also the difference between the notions of direct ownership and integrated ownership.

Example 1:

The example 1, illustrated in Fig.1, contains 4 firms K, L, M and N. The two firms K and L are the sources of the graph. K has 30% of M and 40% of N since L has 60% of M and 30% of N. M owns 30% of N and is possessed at 30% by K, at 60% by L and at 10% by unknown share holders. Those 10% is the float. Fig. 1 gives the graph of the shareholding structure and the correspondent matrix.

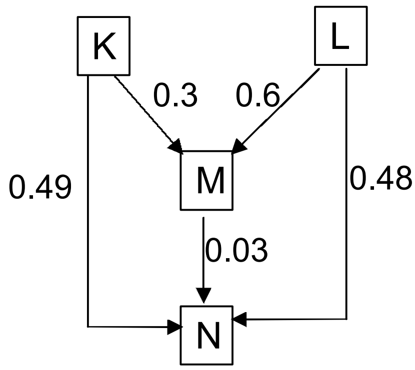
Figure 1 : Graph and matrix of direct ownership structure in example 1



$$A = \begin{pmatrix} 0 & 0 & 0.3 & 0.4 \\ 0 & 0 & 0.6 & 0.3 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

From equation (1), the matrix of integrated ownership can be computed to obtain the graph of Fig 2. In this graph, the coefficient 0.03 that comes from M in N corresponds to the fraction of N's shares owned indirectly by the unknown shareholders of M.

Figure 2 : Graph and matrix of integreted ownership structure in example 1



$$Z = \begin{pmatrix} 0 & 0 & 0.3 & 0.49 \\ 0 & 0 & 0.6 & 0.48 \\ 0 & 0 & 0 & 0.03 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The notion of control is linked to voting power. For instance, a given shareholder may be deterred from any control because s/he faces either a majority shareholder or a hostile coalition. In order to formalize such events, game theory offers adequate tools. Therefore, the second set of definitions introduces now notions of voting game.

Consider a given firm. A binary voting procedure ("1" or "0") is drawn from its shareholders' structure in the following way. The winning issue is "1" if the majority of direct votes[2] are "1." Now, let us suppose that the same voting scheme is applied for all firms of the structure at stake. As the vote of any firm depends upon its own shareholders, indirect owners

[2] votes coming from direct shareholders (under the one share-one vote principle)

influence the votes and thus control firms that may lie far away down the pyramid. This is precisely what happens when considering control links in a pyramid.

In this approach, every shareholder is considered as a player in a voting game (see e.g. Bilbao (2000), Felsenthal and Machover (1998), Owen (1995), Shubik (1982)). The following definitions from game theory will be needed:

D8: A *simple voting game* with n players is a game where each player votes 0 or 1 and the result is 0 if the majority is 0 and 1 if the majority is 1.

D9: The *weighted majority game* is a voting game where each player may have a different weight w_i.

The weighted majority game can be modelled with a Boolean function $f:\{0,1\}^n \rightarrow \{0,1\}$ such that for each $X = (x_1, x_2, x_3, \ldots, x_n) \in \{0,1\}^n$:

$$f(X) = \begin{cases} 1 & if \sum_{i=1}^{n} w_i x_i > 0.5 \\ 0 & if \sum_{i=1}^{n} w_i x_i < 0.5 \end{cases}$$

D10: The *direct game* g_j associated with firm j, is the weighted majority game where the players are the predecessors of j, and the weights are a_{ij}. The *indirect game* v_j associated with the firm j (not a source of the graph), is defined as:

$$v_j(X) = g_j(v_{i_1}(X), v_{i_2}(X), \ldots, v_{i_k}(X)),$$

where $X = (x_1, x_2, \ldots, x_n) \in \{0,1\}^n$, $\{i_1, i_2, \ldots, i_k\}$ is the set of predecessors of j, and if j is a source, $v_j(X) = x_j$.

The definition D10 introduces a voting game through a shareholding structure. In it, every firm is considered as a player, but some players do not control their own vote because $v_j(X) = x_j$.

The game theory also defines coalitions as sets of players voting in the same way.

D11: A coalition is a set of players $H \subset V$ and $H \neq \phi$ with a vector X_H such that $x_i = 1$ if $i \in H$ and 0 if $i \notin H$ and $v_i(X_H) \neq 1$. H is a *winning coalition* on j if $v_j(X_H) = 1$.

Now that the voting game is well defined, the next definitions introduce notions of 'control' and 'measure of control types.' The effective control defined below comes from the matrix model and is based on the notion of majority. The latter definition of the Banzhaf index comes from the game theory, and use the notion of decisive votes.

D12: As defined by Chapelle and Szafarz (2006), the *effective control* is obtained by applying a *majoritization rule* to the matrix of direct voting rights (supposed to be the matrix A here). The matrix of effective control $C = \left(c_{ij}\right)$ is thus given by:

$$c_{ij} = \begin{cases} 1 & \text{if } a_{ij} > 0.5 \\ 0 & \text{if } \exists k \neq i : a_{kj} > 0.5 \\ a_{ij} & \text{otherwise} \end{cases}$$

From the effective control, it is easy to compute the integrated control in a similar way than the integrated ownership. This definition of control corresponds to a full control ($c_{ij} = 1$ means that the decision of *j* is determined by the vote of *i*). In example 1, company L reaches the full control of N, through the control of M. However, in many real cases, no one has full control over a firm. The *Banzhaf power index* is a measure of partial control (capacity of a firm to influence the decisions of another firm).

D13: The *Banzhaf power index* (Banzhaf, 1965) of a player is equal to the probability that his vote is decisive under the following assumption: for each other players, the probability of voting "1" is ½ and of voting "0" is ½. The vote *i* is *decisive* for a winning coalition H if $H \backslash \{i\}$ is not a winning coalition. Some properties of this index are describe by Dubey et al. (1979).

3. Models and Algorithms

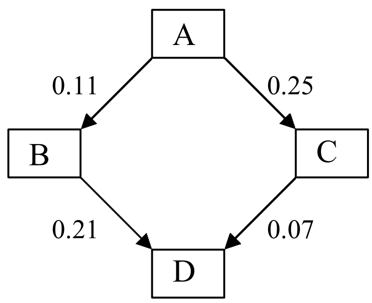
This section aims to develop models and algorithm to measure the control of players in firms. The first model is the weakest link method which provides a kind of measure of control. The second model based on matrix consolidation gives the effective controller of a firm. The last model computes the Banzhaf power index of players in an indirect voting game.

The Weakest link method:

The weakest link model was initially proposed by Claesens *et al.* (2000) to analyse complex pyramidal shareholding structures in Asia. This model defines the weakest link through a chain of voting rights from a parent shareholder to a final firm as the smallest shareholding (i.e. if A has 25% of firm B and B has 7% of firm C then, the weakest link is 7%). Then, the control power is given by the sum of the weakest links of all the chains going from a parent shareholder to the final firm.

On the example shown in fig. 3, the weakest link in the chain A → B → D is 11% and in the chain A → C → D is 7%. The firm A controls 18% of D.

Figure 3 : **Weakest link method**



This model can be applied on example 1 Fig.1. There are two chains to go from K to N. The first chain goes directly from K to N with a link of 40%. The second chain goes from K to M to N with as weakest link 30%. So K has control of 70% (40% + 30%) of N. In a similar way L get control of 60% (30% + 30%) of N.

The matrix consolidation method :

The matrix consolidation method uses an iterative algorithm to establish the graph of integrated control. The model assumes that the control is reached as soon as a firm has more voting right than a given threshold T_C.

The first step provides the matrix *B of direct control.*

$$b_{ij} = \begin{cases} 1 & \text{if } i \text{ control directly } j \text{ or } a_{ij} > T_C \\ 0 & \text{if } \exists k \neq i : k \text{ control directly } j \\ a_{ij} & \text{otherwise} \end{cases}$$

The second step build from B, a new matrix $C^{(1)}$ defined as:

$$c_{ij}^{(1)} = \begin{cases} 1 & \text{if } \exists \text{ a chain of 1 in } B \text{ from } i \text{ to } j \\ 0 & \text{if } \exists k \neq i : \exists \text{ a chain of 1 in } B \text{ from } k \text{ to } j \\ b_{ij} & \text{otherwise} \end{cases}$$

Now, the iteration builds from the matrix $C^{(h)}$ a new matrix $D^{(h)}$, and from this last the matrix $C^{(h+1)}$ can be constructed in the following way.

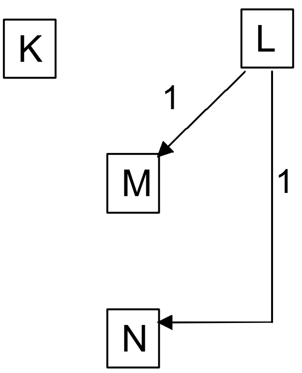
$$d_{ij}^{(h)} = \begin{cases} 1 & \text{if } c_{ij}^{(h)} + \sum\limits_{l: c_{il}^{(h)}=1} c_{lj}^{(h)} \geq T_C \\ 0 & \text{if } \exists k \neq j : \text{if } c_{ik}^{(h)} + \sum\limits_{l: c_{il}^{(h)}=1} c_{lk}^{(h)} \geq T_C \\ c_{ij}^{(h)} & \text{otherwise} \end{cases}$$

and

$$c_{ij}^{(h+1)} = \begin{cases} 1 & \text{if } \exists \text{ a chain of 1 in } D^{(h)} \text{ from } i \text{ to } j \\ 0 & \text{if } \exists k \neq i : \exists \text{ a chain of 1 in } D^{(h)} \text{ from } k \text{ to } j \\ b_{ij} & \text{otherwise} \end{cases}$$

This model applied on the example 1 with a control threshold of 50% gives as result that L control M and reach then the control of N. This result is shown in fig. 4.

Figure 4 : Graph and matrix of control structure in example 1



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In this case, the result corresponds to the effective control because the decision of N corresponds to the will of L.

The voting game approach:

This model computes the Banzhaf power index of ultimate shareholder into a firm with a Monte-Carlo algorithm (originally suggested by Mann and Shapley (1960)). In this model, the ultimate shareholders are the players.

Let N be the set of players considered here as the source of the Graph G, and t a target in the graph. The model aims to compute for a player $j \in N$ the Banzhaf power index $Z_G(j,t)$ corresponding to the probability that the vote of j is decisive in t. The value of this index is given by:

$$Z_G(j,t) = \frac{1}{2^{n-1}} \left(\sum_{X \in \{0,1\}^n : x_j = 1} v_t(X) - \sum_{X \in \{0,1\}^n : x_j = 0} v_t(X) \right) \tag{2}$$

The practical problem to compute this value is that it exist 2^n vectors X, and for each vector we have to evaluate $v_t(X)$. If n is small we can compute directly this index, but if n is big, we use a Monte Carlo simulation by taking a sample set S of vectors uniformly over $\{0,1\}^n$. The index is approximated by:

$$Z_G(j,t) \approx \frac{1}{2^{|S|}} \left(\sum_{X \in S: x_j = 1} v_t(X) - \sum_{X \in S: x_j = 0} v_t(X) \right) \tag{3}$$

where $|S|$ is the cardinal of S.

In a pyramidal structure, $v_t(X)$ is quite easy to compute. It only requires an assumption about the float.

In this model, we consider vectors X containing all the firms in the graph ($X \in \{0,1\}^{|V|}$). For the Monte Carlo simulation, we take uniformly a sample S of such vector. For each vector $X^0 \in S$, we compute $v_t(X)$ with an iterative algorithm:

$$x_j^{n+1} = \begin{cases} g_j(X^n) & \text{if } j \in V \setminus N \\ x_j^n & \text{if } j \in N \end{cases}$$

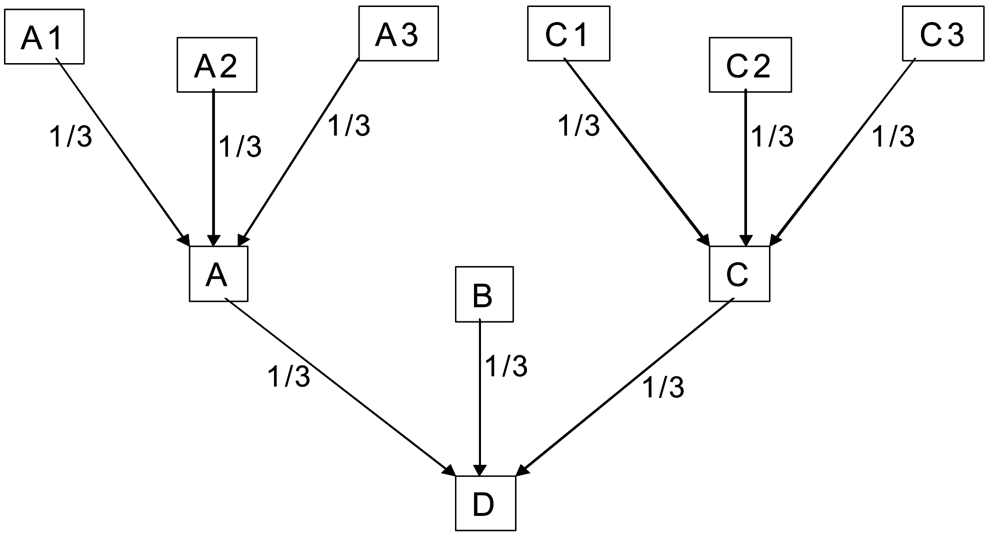
This means that at the round $n+1$, the vote of j is defined by the majority of votes of the predecessors of j at the round n. When $v_t(X)$ is computed for each $X^0 \in S$, we can easily evaluate $Z_G(j,t)$, with formula (3).

The application of this method on example 1, shows that only B has full control on D with a Banzhaf index of 1. Thus, in order to emphasize the interest of this approach, a more complex example will be needed:

Example 2:

Figure 5 includes 10 vertex among which 7 are sources of the graph (A1, A2, A3, C1, C2, C3 and B). The participations of A, B and C in D is 1/3 for each, and A1, A2 and A3 (respectively C1, C2 and C3) has a share participation of 1/3 in a (respectively C).

Figure 5 : Graph of direct ownership structure of example 2



Each of the three firms A, B and C has a Banzhaf index in d of ½. In the same way, A1, A2 and A3 has a Banzhaf index in A of 0.5 as well C1, C2 and C3 in C. This means that A1 has one chance in two to be decisive in the vote of A, which has itself one chance in two to be decisive in the vote of D. So, a1 has one chance in four to be decisive in D. It is the same for A2, A3, C1, C2 and C3.

Now, if company A3 and C1 are merging, then the Banzhaf power index in D of this new entity is the sum of the index of A3 and C1, 0.5 (=0.25+0.25). But what is interesting to observe, is that the Banzhaf power index of B decrease from 0.5 to 0.375. This is due to the fact that A and C has a greater chance to vote in the same way in the general assembly of D, and decrease so the power of B.

Comparison of the methods:

The weakest link applied on example 1, give to K a control of 70% in N and to L a control of 60% in N. The first objection to this model is that those percentages have no much signification. Indeed, the sum of the control of K and L in N is 130%. The second objection, is that this model seems to announce that K has more control in N than L. But in reality, by controlling M, L has the full control of N. However, this model is simple to implement and give in most cases a gross idea of who has the control.

The matrix consolidation method has the advantage to be relatively simple and works in all cases with a pyramidal structure without cross-ownership. This method Furthermore, with a control threshold of 50%, it will establish full control with all firms that have full control. Besides, the algorithm will only give those firms. There are two limitations to this model: first, it uses a threshold that is not always defined. Second, if there is no full control of one “player”, this model does not measure the control power of a “player.”

The voting game approach is probably the most complete and complex model. It allows to compute the Banzhaf index. This index has the advantage to give a measure of the voting power of a player especially if there is no full control of one of them. Furthermore, this index relies on the whole shareholding structure. As shown on example 2, a small change in the graph can modify the influence of all the players. However, we can see two main disadvantages to this approach.

The first one is the complexity of the algorithm. Indeed, the exact algorithm to compute the Banzhaf power index has a complexity of 2^n, and the computation time become huge if the number of companies increase. To overcome this inconvenient, a Monte-Carlo algorithm can be use to approach the value of this index.

The second one is the dependence of this index to unknown data. There are two kind of unknown data. The first kind is the links that exist between different players in the graph. For example, if an unknown shareholder owned two firms that are sources in the graph, those two firms will act in the same way and this will influence the Banzhaf index of the different players. Thus this model makes the assumption that all players are fully independent. The second kind of unknown data is the float. Because the float acts are unknown, assumptions about the votes of the float must be drawn. There are various ways of modelling the float. A first option would be to consider the float as a unique player or a coalition. This means that the whole float votes in the same way. A second possibility may be to imagine that one half the float votes "1" and the other half votes "0." This is equivalent to assuming that the float does not take part in the voting process. This float can then be modelled in two ways: on the one hand, as two players, each with half of the floats' votes, one player voting "1" and the second player voting "0;" on the other hand, the participation of the float can be redistributed to the other shareholders. A third way would be to consider the float as the sum of small players without or with small coalitions. The fraction of float voting "1" will be a random variable taking values between 0 and 1.[3]

The easiest model to implement is the weakest link model. But this model does not give an exact idea of who are the controllers. The second model of Chapelle and Szafarz is easy to implement. Furthermore, with threshold of 50%, it gives the real governor of a firm whatever the other players and the float do. But if there is no governor with a threshold of 50%, we can decrease the threshold. In this case, we are not certain to find a controller. But if we find one, he will not be an "absolute" controller, because a coalition of all the other players will be a winning coalition. Moreover, in this case we have no idea of the influence the other players. The most difficult method to implement is the method of Crama et al.. This method gives the Banzhaf power index which represents a measure of the control power. The comparison of the three models has been done on a real case: Colruyt.

The three models work well in pyramidal structure. However, in a structure with cross-ownership it is much more complicated. Only the voting game approach can be adapted to those structures. We have to make assumptions on companies' vote in a cycle. The problem with the method of Crama and al. (2004) is that the iterative computation of $v_t(X)$ does not always converge if we have cycle in the graph. Indeed, if we consider for example two firms "1" and "2" that owns mutually 100% of each other. If we suppose initially $X^0 = (1,0)$ (the vote of "1" is 1 and the vote of "2" is 0 at the round 0), we get successively $X^1 = (0,1)$, $X^2 = (1,0)$,….

[3] For Instance, if the float consists of n people each having a chance p of voting "1", then $x_{float} = \frac{B(n,p)}{n}$.

4. The case Colruyt:

This section goes over a real case: Colruyt. We will apply on this case the Algorithm of Chapelle and Szafarz to find the controller of Colruyt. We will vary the control threshold to show how this threshold can influence our perception of control. We will then compute the Banzhaf index of each ultimate owner and compare this result to the control determined by the algorithm of Chapelle and Szafarz.

In 1925, the baker Franz Colruyt opened his wholesale trade. In 1950 he founded the food wholesaler company S.A. Ets Franz Colruyt. In 1958, Jo Colruyt and his brothers (the children of Franz Colruyt) took the management of the firm and in 1964, they opened the first supermarkets. The name Colruyt was given to the supermarkets in 1976, one year before the IPO of the company. In 1994, Jef Colruyt, the son of Jo Colruyt, took the succession.

Today, "Establishments Franz Colruyt N.V. is a Belgian holding company whose subsidiaries are engaged in the operation of a retail network of 170 stores. The Company's subsidiaries include Bio-Planet, a bio-supermarket operator offering organic food and ecological non-food products; Okay, an operator of shops combining the service of a grocer with the assortment of a discount shop; Ripotot, an operator of shops in France, and Colruyt Export, a supplier of goods by container to wholesalers and supermarkets worldwide." [4]

Colruyt has several other subsidiaries like Dreamworld, Infoco, Dolmen Computer Applications etc… With a capitalisation of 4 billions euro, a turn over of 4.4 billions euro and net income after tax of 218 millions euro, Colruyt is a member of the Bel-20.

The main shareholders of the group are members of the Colruyt family. They own directly 6.23%. They have indirectly 22.74% through "Halse Investering Maatschappij" and 15.28% through "Distributie Investering Maatschappij". Four owners of "Halse Investering Maatschappij" were found in the available data: "Distributie Investering Maatschappij", Farik, Anima and Herbeco. These four companies represent about 40% of the share of the company. Farik, Anima and Herbeco have about 40% of "Distributie Investering Maatschappij" together. Farik, the smallest of the three companies is a portfolio management firm, Anima is a financial firm and Herbeco is an architect office. The owner and administrators of those three firms are all Colruyt members' family (See appendix A). Each one is owned by a different branch of the family. The other shareholders of "Halse Investering Maatschappij" and "Distributie Investering Maatschappij" are cousins of the family (they are not represented in the available data).

[4] http://today.reuters.com/stocks/Overview.aspx?symbol=COLRt.BR&chart=5.

The other big shareholder of Colruyt is the group Sofina. The group Sofina is a Belgian financial holding engaged in distribution and consumer product, telecommunication and energy production. The group owns indirectly through his filial Rebelco 5.65% of Colruyt.

The personnel of Colruyt has 3.11% and the group Colruyt has directly and indirectly a total of 2.63% of its own shares. [5]

Figure 6: Colruyt group : graph of direct ownership

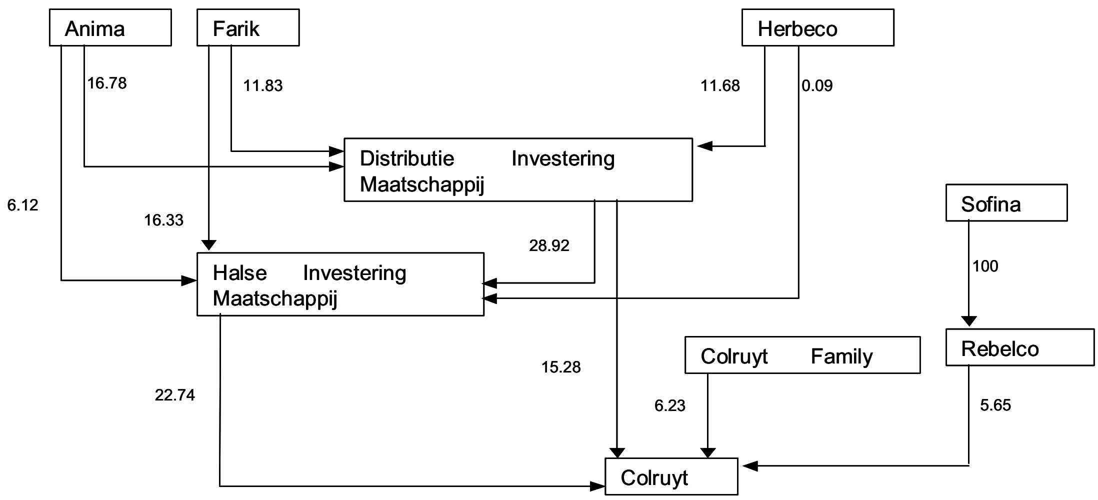


Figure 6 shows a simplified direct shareholding structure of the group corresponding to matrix $A = (a_{ij})$. The graph takes into account only the participation exceeding 5%.

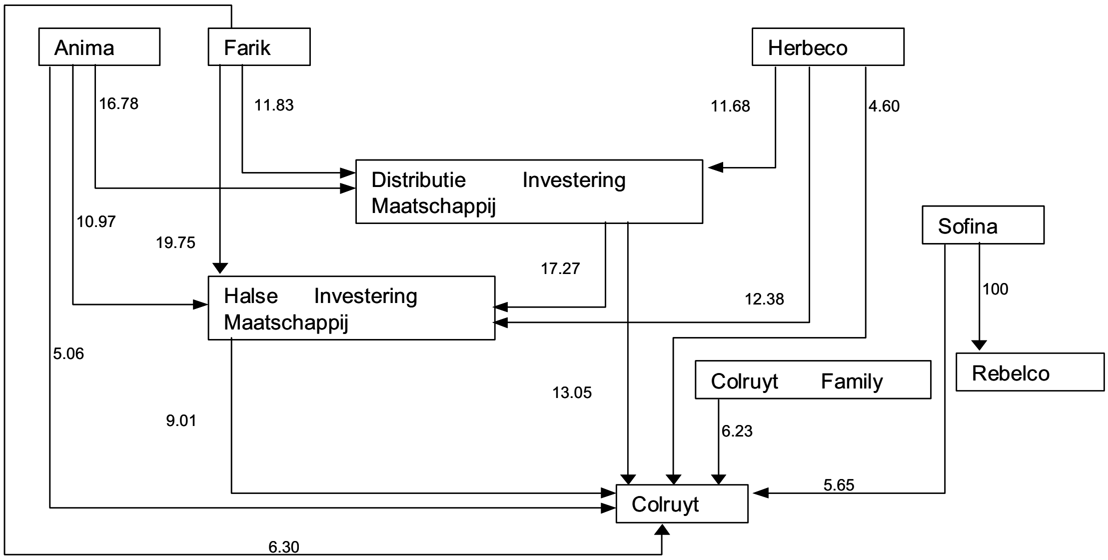
By computing the integrated ownership with (1) we get the graph in figure 7. The interpretation of this graph is not obvious. Indeed, the 9.01% that "Halse Investering Maatschappij" has in Colruyt for example, represent the fraction of the shares owned by "Halse Investering Maatschappij" that is not owned by shareholders of this firm in the graph.

All this previous analyse concerns the ownership. We observe that the five ultimate shareholders (Anima, Farik, Herbeco, Colruyt Family and Sofina) have similar number of shares (around 5%). Among those five shareholders, Farik has slightly more shares than the other shareholders. But is it the same situation about the control? Do all the five shareholders

[5] The data come from the program Amadeus developed at "Bureau Van Dijk" and have been updated in January 2005.

have the same influence in the AG? Does Farik have a slightly higher influence than the others? To answer those questions, we must analyse the control of Colruyt.

Figure 7 : Colruyt group : graph of integrated ownership



To measure the control of the firm, we will apply the different methods. The weakest link model is drawn from the graph of direct ownership. To measure the control percentage of Anima in Colruyt, we have to take into account all the ways going from Anima to Colruyt. We have three ways. The first way goes from Anima to “Halse Investering Maatschappij” to Colruyt and has as weakest link a value of 6.12%. The second way goes from Anima to “Distributie Investering Maatschappij” to “Halse Investering Maatschappij” to Colruyt and has as weakest link 16.78%. The last way goes from Anima to “Distributie Investering Maatschappij” to Colruyt and has as weakest link 15.28%. So, following this method the percentage of control of Anima in Colruyt is 38.18%. The weakest link model will so gives as control percentage of Farik in Colruyt 39.99% (11.83+11.83+16.33), for Herbeco 23.45% (0.09+11.68+11.68) and for Sofina 5.65%.

This result shows that Anima and Farik has similar control’s percentage and Herbeco has a lower control percentage. This is due to the fact that Herbeco has nearly no direct influence in “Halse Investering Maatschappij”.

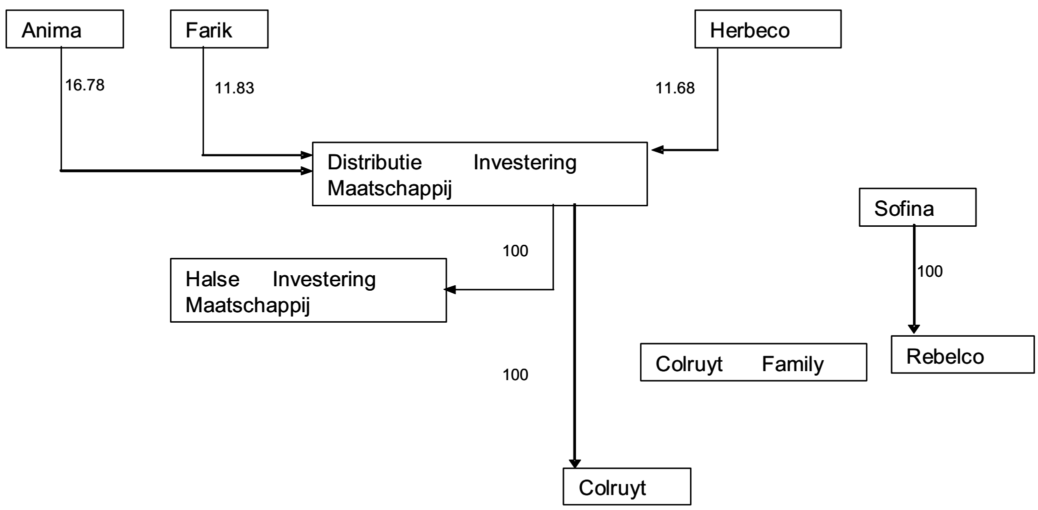
The matrix consolidation method, developed by Chapelle and Szafarz (2006), consists to consider that full control is reached as soon as a firm has more than a threshold (*TC*) of shares. The case Colruyt will be used to show how this threshold can influence the perception of control. The threshold will vary from 16.33% to 50%. Below 16.33% the method will not work because two firms have more than 16.33% in “Halse Investering Maatschappij”.

To analyse the governance of Colruyt, we must make assumptions about the float. Indeed in the data we have an important float. This model does not take the float into account but in the last model, for computing the Banzhaf index, we will suppose that the float does not take part to the vote.

For $28.92\% < TC \leq 50\%$, only Sofina has the control on Rebelco and has so 5.65% of control in Colruyt.

For $16.78\% < TC \leq 28.92\%$, the "Distributie Investering Maatschappij" takes control over "Halse Investering Maatschappij", and reaches so the majority of voting rights over Colruyt. This case is represented in figure 8. In term of control, we observe that "Distributie Investering maatschappij" has a very strong influence over Colruyt. This means the control of Colruyt pass through this firm.

Figure 8 : Colruyt group : control for 16.78%<TC<28.92%



For $16.33 < TC \leq 16.78\%$, Anima takes the control of "Distributie Investering Maatschappij" and reaches so control over "Halse Investering Maatschappij" and Colruyt as shown in figure 9. This result is not realistic. Indeed, Farik and Herbeco form a winning coalition if they both agree on the decision to take.

The analysis of those results shows in the first case that a high threshold is not restrictive enough to give an idea of who controls the company. In the second case, it becomes easy to see that control can be reach through the "Distributie Investering Maatschappij". But in this situation, because a lot of shareholders are unknown and the threshold is smaller than 50%. Another company or a coalition could reach control of "Halse Investering Maatschappij" with about 40% of voting right. In the last case which is not restrictive enough, it seems that Anima has full control over the three firms, "Distributie Investering Maatschappij", "Halse Investering Maatschappij" and Colruyt. In fact it is not the case because if Farik and Herbeco agree on the decision, they both have the majority of votes and Anima has no control over Colruyt. Actually, if two of the three companies (Anima, Farik and Herbeco) agree on the decision to take, this decision will be taken.

Figure 9 : Colruyt group : control for 16.33% <TC<28.78%

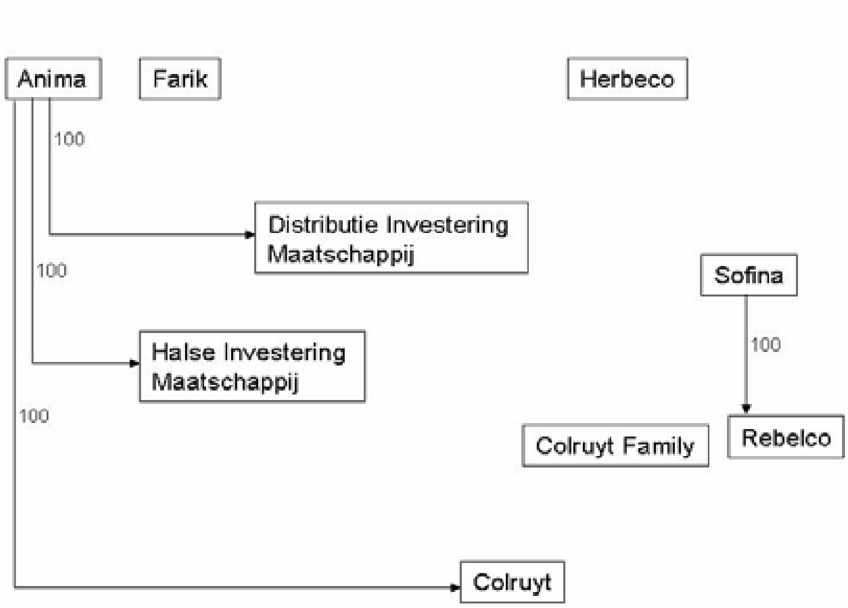
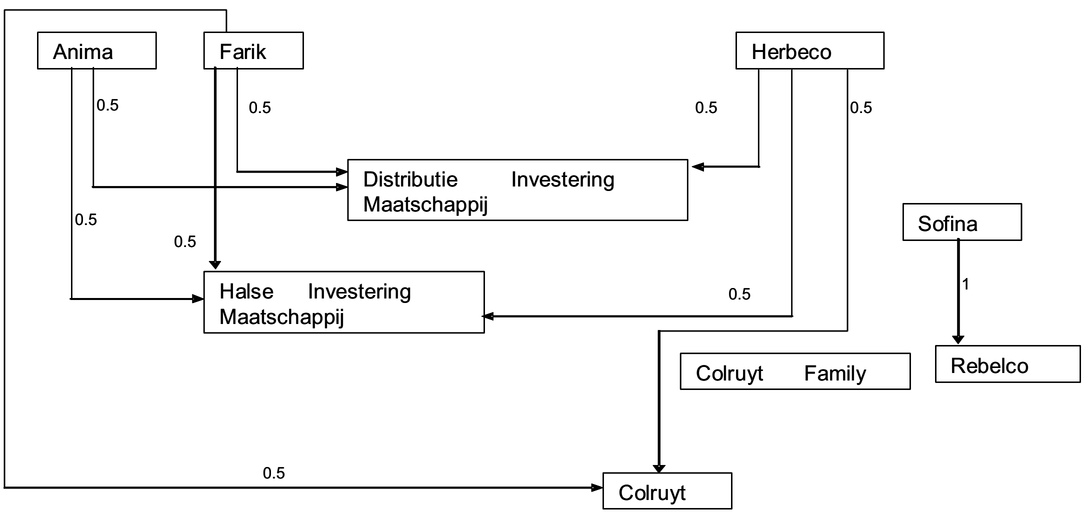


Figure 10 : Colruyt group : the Banzhaf power index



This result is also observed with the Banzhaf power index. For the computation of this index, the assumption that the float does not take part to the voting game has been made, and

the player considered here are: Anima, Farik, Herbeco, Colruyt Family and Sofina. The result is similar to the result obtained in the second case of previous model. The three first firms have each a Banzhaf index of 0.5, and the other players 0. This means that each of the three firms has one chance on two that his vote is a decisive one.

The administrators and shareholders of the three controlling firms are all different members of the Colruyt Family. Some of them are also administrator of the firms "Distributie Investering Maatschappij", "Halse Investering Maatschappij" and Colruyt.

The validity of the analysis depends heavily on unknown data. Indeed, if a member of the Colruyt family has 30% of "Distributie Investering Maatschappij", his vote has a great value and the analysis above becomes obsolete. Furthermore, if one person has the control of two of the three controller firms, the third firm has no control power on Colruyt. The control of one person on two of the three controlling firms can also be partial, if for example this person has 1/3 of the voting rights in each of the two firms. In this last case because the two firms have a great probability to vote in the same way, the power of the third controlling firm is decreased.

Another observation that can be made is that because of the ownership structure, the Colruyt family will act as a unique player. Indeed, they get the control of Coltuyt through one firm: "Distributie Investering Maatschappij". Even if there are disagreements between members of the family, the decision of Colruyt will come from negotiations between the members of the family and the other players has no control power. Practically, the Colruyt family holds a meeting to discuss the investment and strategy of the group.

We have first compute on the case Colruyt the integrated ownership from the direct ownership and observe that 5 ultimate owners have similar numbers of shares in Colruyt. We have then analysed the control of the Colruyt in three different ways. The weakest link approach gives a gross idea of the influence of the different shareholders. The matrix consolidation method has shown that there is no one who has full control of Colruyt with more than 50% of voting rights. But with a control threshold below 28.92%, the governing firm in this model becomes "Distributie Investering Maatschappij". And the control of this firm is shared between Farik, Anima and Herbeco. This corresponds to the conclusion of the previous section: we announce that with a low control threshold, we could find a controller that has not the full control because the other firms form a winning coalition. The last method computes the Banzhaf power index and gives the same result: a coalition between two of the three last firms is a winning coalition.

5. Conclusions:

This paper describes and compares different methods to determine the actual control of a firm. The methods are then applied to a real case: the Belgian retail company, Colruyt.

The first part of this paper has formalized the problem along two different lines. First, the graph theory allows defining an ownership structure in mathematical terms. Second, the voting game has been described with the game theory. To measure the power of a player in a voting game we compute the Banzhaf power index.

The second part of the article describes three existing methods to evaluate the "control power of a firm". The first method is the "weakest link method". This method gives only a rough idea of the control power of a firm. The second method has been developed by Chapelle and Szafarz (2006). This method develops an algorithm to establish the controller of a firm by considering full control is reached as soon as a firm has more than a given threshold of voting rights. When a firm gets control of another firm, it gets his voting rights over its subsidiary. The last method of Crama et al. uses a Monte-Carlo algorithm working on a structure with cross-ownership to compute the Banzhaf power index.

The third part uses first the weakest link method the example of Colruyt. It can be observe that even if theoretically this method may give wrong result, on this practical case, this model allow to identify the different actor having an influence and to give a kind of measure of this influence (without hypothesis on the float). The algorithm of the matrix consolidation method was applied on the example with different thresholds. On this case the best result was given with an intermediary threshold. A too high threshold does not allow to determine who the controllers are. The model with a too low threshold define a controlling firm that has a lot of influence on Colruyt, but who is not the only firm to have an influence. An average threshold determine a firm that have the greatest influence in the general assembly, but do not allow to determine the ultimate shareholder in the graph that have the strongest influence on Colruyt. This result was close of the result given by the Banzhaf index. The Banzhaf power index defines three firms in the graph that are controlling firms, all with the same influence.

This paper has shown on an example how the control threshold can influence the perception of controlling firm and compare this result to the measure of Banzhaf index. The first limitation of the models developed is the difficulties to model the control through the cross-ownership. The second limitation is the bias of the results due to the unknown float. Indeed, in our example, the large float does not allow us to affirm with certitude that only the three firms (Farik, Anima and Herbeco) have a control over Colruyt.

One way to improve the existing models is to evaluate the bias created by the float. A second improvement to the existing models concerns the evaluation of control through structures with cross-ownership.

Acknowledgments:

The Author is grateful to Ariane Chapelle and Ariane Szafarz for their useful advices, to Caroline Dewagter and Jean-Philippe Devos for their help in the redaction of this paper. Special thanks also to the "Bureau Van Dijk" for giving access to Osiris, Amadeus and the ownership data.

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Appendix A:

ETABLISSEMENTS FR. COLRUYT - ETABLISSEMENTEN FR. COLRUYT

BOARD MEMBERS & OFFICERS

Name	Function
FRANCISCUS (FRANS) JR COLRUYT	Administrator
FARIK	Administrator
HERBECO	Administrator
JOZEF MARIA DAMIAAN COLRUYT	Administrator
ANIMA	Administrator
FRANCOIS GILLET	Administrator

FARIK

BOARD MEMBERS & OFFICERS

Name	Function
FRANCISCUS (FRANS) JR COLRUYT	Chairman/Managing Director
KAROLIEN COLRUYT	Administrator
LUCIA COLRUYT	Administrator

ANIMA

BOARD MEMBERS & OFFICERS

Name	Function
JEF COLRUYT	Administrator
MARIAN COLRUYT	Administrator
WIM COLRUYT	Administrator
MARGARETHA DE MAN	Administrator

HALSE INVESTERINGSMAATSCHAPPIJ

BOARD MEMBERS & OFFICERS

Name	Function
FRANCISCUS (FRANS) JR COLRUYT	Administrator
JOSEPH COLRUYT	Administrator
PIET ANDRE MARIA COLRUYT	Administrator
AGNES COLRUYT	Administrator
CECILE COLRUYT	Administrator
ELISABETH COLRUYT	Administrator
HENRY COLRUYT	Administrator
JULES COLRUYT	Administrator
MARIE-JEANNE COLRUYT	Administrator
THERESA COLRUYT	Administrator

DISTRIBUTIE INVESTERINGSMAATSCHAPPIJ

BOARD MEMBERS & OFFICERS

Name	Function
FRANCISCUS (FRANS) JR COLRUYT	Administrator
JOZEF MARIA DAMIAAN COLRUYT	Administrator
PIET ANDRE MARIA COLRUYT	Administrator
WIM COLRUYT	Administrator