Rudolf Wolf to Alfred Wolfer: The Transfer of the Reference Observer in the International Sunspot Number Series (1876–1893)

Shreya Bhattacharya (shreya.bhattacharya@oma.be)
Royal Observatory of Belgium
Laure Lefevre
Royal Observatory of Belgium
Theodosios Chatzistergos
Max-Planck-Institut für Sonnensystemforschung
Hisashi Hayakawa
Nagoya University
Maarten Jansen
Université Libre de Bruxelles

Research Article

Keywords: Sun, sunspots, sunspot number

Posted Date: October 10th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3407219/v1

Additional Declarations: No competing interests reported.

Rudolf Wolf to Alfred Wolfer: The Transfer of the Reference Observer in the International Sunspot Number Series (1876–1893)

S. Bhattacharya [1,2] · **L. Lefèvre** [1] · **T. Chatzistergos** [3] · **H. Hayakawa** [4,5,6,7] · **M. Jansen** [2]

Abstract
In 1876 Alfred Wolfer started observing the Sun and recording properties of sunspots alongside Rudolf Wolf. Their observations became the basis for the construction of the Sunspot Number series. After Wolf's death in 1893, Wolfer became the primary observer for the Sunspot Number series. Even though the observations of Wolf and Wolfer had an overlap of almost 17 years (1876–1893), this shift of primary observer

✉ S. Bhattacharya
shreya.bhattcharya@oma.be

L. Lefèvre
laure.lefevre@oma.be

T. Chatzistergos
chatzistergos@mps.mpg.de

H. Hayakawa
hisashi@nagoya-u.jp

M. Jansen
maarten.jansen@ulb.ac.be

1 World Data Center SILSO, Royal Observatory of Belgium, 3 Avenue Circulaire, 1180, Bruxelles, Belgium

2 Departments of Mathematics and Computer Science, Université Libre de Bruxelles (ULB), Boulevard du Triomphe Campus Plaine, CP213, Bruxelles, Belgium

3 Max-Planck-Institut für Sonnensystemforschung, Justus-von-Liebig-weg 3, 37077 Göttingen, Germany

4 Institute for Space-Earth Environmental Research, Nagoya University, Nagoya, 4648601, Japan

5 Institute for Advanced Research, Nagoya University, Nagoya, 4648601, Japan

6 Space Physics and Operations Division, RAL Space, Science and Technology Facilities Council, Rutherford Appleton Laboratory, Harwell Oxford, Didcot, Oxfordshire, OX11 0QX, UK

7 Nishina Centre, Riken, Wako, 3510198, Japan

from Wolf to Wolfer seems to have led to inconsistencies in the Sunspot Number series primarily due to inhomogeneities in Wolf's observations. To address this issue we digitise *Mittheilungen* (Wolf's Journals) and analyze their tabulated datasets. These journals include the raw sunspot data from various observers that Zürich Observatory used to compile the Sunspot Number (SNV1). These datasets have been used as source data for the construction of the first version of the Sunspot Number (SNV1) series, but they were not digitally accessible for a recalibration of the sunspot number series until recently. We have also acquired external datasets from recent archival investigations for contemporaneous sunspot observations. In this study, we use the Mittheilungen dataset to produce a new recalibration of the sunspot number series covering 1816–1944, using three major observers (Schwabe, Weber, and Wolfer) as backbones. The availability of the raw data allows us to identify issues in the determination of the scaling factors or k-factors, between the records of different observers, but also the use of modern techniques for cross-calibrations. Our reconstruction for the years 1816–1944 is done with a novel method inspired by Chatzistergos et al. (2017, A&A, 602, A69) allowing to eliminate inconsistencies that resulted from the application of erroneous k-factors.

Keywords: Sun, sunspots, sunspot number

1. Introduction

Sunspot counting has been one of the longest ongoing scientific experiments in human history and forms one of the longest direct measurements of solar activity (Owens, 2013) with almost 413 years of data (Vaquero and Vázquez, 2009; Arlt and Vaquero, 2020a). Long term datasets of solar activity are extremely important to improve our understanding of the solar forcing on the terrestrial climate (Gray et al., 2010; Solanki, Krivova, and Haigh, 2013; IPCC, 2021; Chatzistergos, Krivova, and Yeo, 2023). It is essential to access an extensive collection of stable sunspot observations and sophisticated methodologies to stitch them together in a single series for reconstructions of long-term solar activity (Clette et al., 2023).

The first version of the sunspot number series (SNV1), also known as the "Zürich series" or the "Wolf Number" was constructed in the mid-19th century by Rudolf Wolf. He compiled sunspot observations made by various astronomers and created a continuous record of Sunspot Number (SN) going back to the early 18th century (Friedli, 2016; Clette et al., 2023; Clette et al., 2021). Wolf standardized the observations by scaling a combination of the number of spots and groups of an observer to a primary observer, using so-called k-factors or k-coefficients. What we call "Wolf number" (W) are a combination of the number of groups and spots observed during any particular day, according to the definition of Wolf (Wolf, 1848) following the relation $W = 10g + f$, where g is the number of sunspot groups and f the number of individual spots (Izenman, 1985). The Zürich sunspot number series was widely adopted in the 19th century and has been used to date as a standard measure of solar activity.

Around 1876, Alfred Wolfer became Wolf's assistant and thus had an influence on the International Sunspot Number (SN) (Svalgaard, 2010). Wolfer was an amateur astronomer turned professional in Switzerland, made over 40,000 sunspot observations over a period of 50 years, and contributed to our understanding of the solar activity on the decadal to centennial timescales. Additionally, he was a member of the Swiss Astronomical Society and the Royal Astronomical Society. After Wolf's death in 1893, Wolfer became a primary observer thereafter in the construction of the SNV1 series. In 1894, Wolfer calculated his scaling factor, $k = 0.6$, to uniformly calibrate his counts to Wolf's observations (Svalgaard, 2010; Friedli, 2020). Despite Wolf and Wolfer's parallel observations for 17 years (1876–1893), the validity of the 0.6 scaling factor is in question, mainly because Wolf changed his telescopes and SN calculation methods several times and suffered from degraded eyesight towards the end of his life (Friedli, 2016; Usoskin, Kovaltsov, and Chatzistergos, 2016; Friedli, 2020). Several studies have already demonstrated a nonlinear relationship between Wolf and Wolfer's counts (Friedli, 2020; Usoskin, Kovaltsov, and Chatzistergos, 2016). These studies have also shown that it ends up with a significant overestimation/underestimation of the series to stitch their datasets together with a linear k-factor. A thorough re-analysis of the raw data is needed to identify and account for potential inconsistencies and discrepancies in the series.

For this aim, over the past decade the Solar-Physics community developped extensive collaborations to improve the reliability and homogeneity of the sunspot data and their products (Clette et al., 2016). This involves sunspot workshops (https://ssnworkshop.fandom.com/wiki/Home), engaging in the Solar Physics topical issue on SN re-calibration (Friedli, 2016), and a working group in the International Space Science Institute (ISSI) on recalibration of the SN (https://www.issibern.ch/teams/sunspotnoser/) (Clette et al., 2023). There have also been a lot of archival investigations (Hayakawa et al., 2019, 2020; Carrasco et al., 2020; Hayakawa et al., 2021b; Arlt et al., 2013; Carrasco et al., 2021a; Hayakawa et al., 2021a) and state-of-the-art calibration techniques (Usoskin et al., 2016a; Svalgaard and Schatten, 2016; Chatzistergos et al., 2017; Usoskin, Kovaltsov, and Kiviaho, 2021; Willamo, Usoskin, and Kovaltsov, 2017; Cliver and Ling, 2016). Data collection and their stability assessment is an important part for the re-calibration of the sunspot series. The WDC-SILSO team has been working actively on the stability assessment of observations (Mathieu et al., 2019) through the VAL-U-SUN project (http://www.sidc.be/valusun/: 2017–2021), and is now working on the data collection through the FARSUN project (https://www.sidc.be/FARSuN: 2023–2026) which are both Belgian BRAIN projects (https://www.belspo.be/belspo/brain-be/index_fr.stm).

Up until recently the raw data for individual sunspot numbers were digitally sparse for the period before 1980, i.e. when the SILSO database starts. The absence of digital accessibility to such raw data significantly restricted the ability to undertake adjustments and reanalysis of the sunspot number series and thus – particularly – the correction of SN had been challenging owing the scarcity of digital raw data, with only the composite series having undergone such changes. This restriction does not apply to data collected after 1980 that was already in digital format. SN Version 2 (SNV2), which was published in 2015 (Clette and Lefèvre, 2016) as an important step towards the ongoing sunspot number re-calibration (Clette et al., 2023), only concentrated on corrections on the SN series rather than reconstructions. At that time, SNV2 was rescaled to Alfred Wolfer by omitting the 0.6 factor that had always been used after Wolf.

In parallel to the reconstruction efforts for the SN, multiple efforts were made to homogenize the Group Number Series (GN), which includes daily counts of sunspot groups. The first GN series was produced by Hoyt and Schatten (1998a,b). Their series covers the period from 1610 to 1998 and widely used as a standard GN reconstructions until 2015. Since then, a number of alternative GN series have been presented (Svalgaard and Schatten, 2016; Usoskin et al., 2016a; Chatzistergos et al., 2017; Usoskin, Kovaltsov, and Kiviaho, 2021; Willamo, Usoskin, and Kovaltsov, 2017; Cliver and Ling, 2016).

The first ever attempt to eliminate the k-factor was done by Usoskin et al. (2016b), who produced a GN reconstruction based on the monthly active day fraction of the observers. Chatzistergos et al. (2017) succeeded in developing a modern non-parametric calibration method (inspired by Usoskin et al., 2016a) to calibrate the records of observers to those of primary observers. Notably, the method used by Chatzistergos et al. (2017) has been applied to the reconstruction of Group Sunspot Numbers (GN) and has not been used in the reconstruction of Sunspot Numbers (SN) yet. The current study is the first to attempt to use this methodology to reconstruct SN in this way. This is a key step to evaluate the potential of using this cross-calibration approach to broaden the scope of the SN reconstruction.

This is made possible by the recent encoding efforts for digital-format datasets of the *"Mittheilungen der Naturforschenden Gesellschaft in Berne"* and *"Mitteilungen der Edgenossischen Sternwarte Zurich"*. These digital datasets contain a wealth of information and have been digitized at the Royal Observatory of Belgium (https://www.astro.oma.be/en/), particularly the WDC-SILSO, between 2017 and 2019 (Clette et al., 2021). Apart from that, a wealth of data has been located in the Zurich archive covering 1610–1980, which have not been recovered in a digital tabulated format yet (Clette et al., 2021). These journals include sunspot records as far back as Harriot in 1610 (Wolf, 1861). The journal was maintained from 1848 (Wolf, 1848) until his death in 1893 (Wolf and Wolfer, 1894). In addition to the digitisation of the *Mittheilungen*, recent studies have partly revised and added individual sunspot and group observations on the basis of archival investigations from original materials (Carrasco et al., 2020; Hayakawa et al., 2020; Carrasco et al., 2022, 2021a; Carrasco, 2021; Hayakawa et al., 2021a).

Hence, this study aims to eliminate the potential discrepancy around the Wolf-Wolfer scale transition based on a non-linear approach for the cross-calibration of the SN series (Chatzistergos et al., 2017). We intend to produce the SN Series from 1816 to 1944 (SNV2.3) using all the exploitable raw data that are available to date in digital format such as Arlt et al. (2013); Carrasco (2021); Hayakawa et al. (2020). We present our primary datasets in Section 2. Stability checks of the primary observers are presented in Section 3. In Section 4 we present the SN Version 2.2 which we constructed mainly for comparison purposes as a continuation of the series by Bhattacharya et al. (2023) using k-factors for the period 1849–1944. Our main reconstruction is called SNV2.3, and is in fact, the first part of what will be SNV3, using the non-parametric approach by Chatzistergos et al. (2017): it is presented in Section 6 along with comparisons to previous versions of SN. Finally we summarise our results and draw our conclusions in Section 7.

2. Data Sources

In this section we present all the datasets that we use throughout this study.

2.1. The *Mittheilungen*

Professor Rudolf Wolf had collected databases of sunspot counts from multiple observers in the *Mittheilungen*. This dataset remains our primary source of sunspot data. In 1843, Wolf established a journal called "Mittheilungen der Naturforschenden Gesellschaft in Berne," which later became "Mitteilungen der Edgenossischen Sternwarte Zurich" when he relocated to Zurich in 1849. Every year, Professor Wolf published his collections in the latter publication, including sunspot observations dating as far back as Harriot's records in 1610 (Wolf, 1861). The journal was active from 1848 (Wolf, 1848) until Professor Wolf's death in 1893 (Wolf and Wolfer, 1894). In addition to his own records and observations, he collected sunspot observations from his fellow European scientists and auxiliary observers, and compiled them in these journals (Bhattacharya et al., 2023). Throughout this article, these journals will consistently be referred to as "the Mittheilungen". Note that, Wolf's successors kept recording the data in the *Mittheilungen* till 1979.

Rudolf Wolf also made observations of his own between 1848 and 1893, but he started reporting his data in the *Mittheilungen* only in 1849. We refer to the data from Wolf's standard telescope as "Wolf's data" henceforth to distinguish them with his data performed with his portable telescope (see Section 3.2.3). We also refer to the Wolfer's data in the *Mittheilungen* as "Wolfer's data" in the rest of the text. In 1919, Wolfer stopped publishing data from observers other than Zürich observers in *Mittheilungen*, while Waldmeier stopped publication altogether and started to preserve the original data only in the archives from 1945 onward (Clette et al., 2021). Therefore, what we have encoded in the *Mittheilungen* stops in 1945, rather than in 1979.

2.2. The Source Books

Along with the *Mittheilungen*, other records written by Wolf were found in loose, unbound form in 2015 at the ETH Archives in Zürich (Bhattacharya et al., 2023, 2021; Friedli, 2016; Clette et al., 2021). Before his discoveries were printed in the *Mittheilungen*, Wolf first documented them in these source tables (hereafter the Source Books). Contrary to the *Mittheilungen*, the Source Books offer a lot of supplementary material. Wolf kept thorough records of information about the observers used on particular days, their related k-factors, among other annotations (Wolf, 1878b; catalogue entry Hs368:46) that were not included in the *Mittheilungen*. The Source Books from 1849 to 1877 that Friedli (2016) digitalized are now available on the Wolf Society website (http://www.wolfinstitute.ch/home.html). These digital records make it possible to pinpoint the times that Wolf sought out specific observers to fill in his observational gaps. The source books include also data for earlier times, whereas those before 1849 have not been encoded yet unfortunately. Since Wolfer started to be part of the SN series in 1876, only 2 years of data are available in the Source Books for Wolfer - 16 observations in 1876 with a k-factor of 0.64 and 27 observations in 1877 with a factor of 0.86. The time span of each observer is shown in Figure 1. Note that for each observer only those days are available when their observations were used to fill gaps in Wolf's data. Furthermore, the source books revealed that Wolf interpolated the existing records to fill in gaps for some days for which he could not find any record from other observers (Friedli, 2016). These interpolated values are indicated in Figure 1 without a name for the observer (https://www.wolfinstitute.ch/rawdata.html).

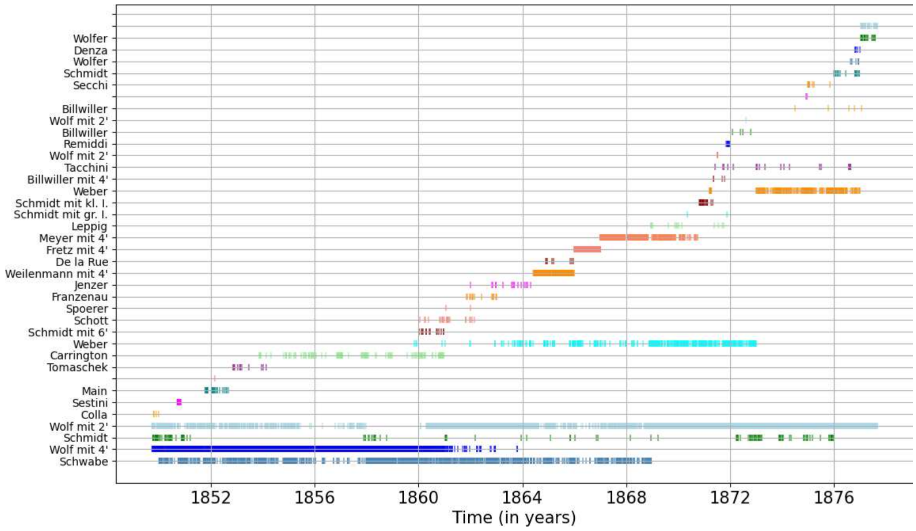

Figure 1. The time span of each of the identified observer in the Source Books. The rows without an observer's name correspond to extrapolated values.

2.3. Recounts of Other Observers

As a part of the re-calibration efforts put together by the global Solar Physics community, we also have at our disposal the recounted datasets of various observers from different epochs. These datasets are the result of the recounts done by various researchers from the original drawings of the observers. We provide a list of the datasets that were recovered between 1816 and 1944. The columns in Table 1 give the name of the observer, their period of observation, whether spot counts are available, whether group counts are available, any remarks related to the dataset, whether it is included in this study, the corresponding backbone if included, and the reference, respectively.

Table 1. List of observers with reanalyzed data available for the years 1816–1944

Observer	Period	Spots	Groups	Remarks	Included	Backbone	Reference
Karl von Lindener	1800–1827	Yes	Yes		Yes	Tevel	Hayakawa et al. (2023a)
Stephan Prantner	1804–1844	Yes	Yes		Yes	Tevel	Hayakawa et al. (2021c)
Thaddäus Derfflinger	1804–1824	Yes	Yes		Yes	Tevel	Hayakawa et al. (2020)
Cornelis Tevel	1816–1836	Yes	Yes	Better points than Mittheilungen	Yes	Backbone	Carrasco (2021)
Samuel Heinrich Schwabe	1826–1868	Yes	Yes	Consistent data points	Yes	Backbone	Arlt et al. (2013)
Antonio Colla	1830–1843	Yes	Yes	25 data points, not enough data points	No		Carrasco et al. (2020b)
Toubei Kunitomo	1835–1836	Yes	Yes		Yes	Schwabe	Fujiyama et al. (2019)
William Cranch Bond	1847–1849	No	Yes	Cannot be used	No		Carrasco et al. (2020a)
Angelo Secchi	1853–1878	No	Yes	Spot counts are not included	No		Ermolli et al. (2023)
Richard Carrington	1853–1861	Yes	Yes	Continuous data	Yes	Schwabe	Bhattacharya et al. (2021)
Gustav Spoerer	1861–1894	Yes	Yes	0 sunspots days are missing	Yes	Weber	Diercke, Arlt, and Denker (2015)
Ventosa (Madrid Observatory Period 1)	1868–1896	Yes	Yes		Yes	Wolfer	Aparicio et al. (2014)
D. E. Hadden	1891–1931	Yes	Yes	Not continuous data but can be included	Yes	Wolfer	Carrasco et al. (2013)
Aguilar (Madrid Observatory Period 2)	1906–1923	No	Yes	Only Monthly group numbers are available	No		Aparicio et al. (2014)
Kodaikanal Observatory	1906–2023	Yes	Yes		Yes	Wolfer	Sivaraman, Gupta, and Howard (1993)[1]
Valencia Observatory	1920–1928	Yes	Yes	Continuous data	Yes	Wolfer	Carrasco et al. (2014)
Stonyhurst College Observatory	1921–1935	No	Yes	Spot counts are not included	No		Carrasco et al. (2021b)
Katsue Misawa	1921–1934	Yes	Yes	Continuous data	Yes	Wolfer	Hayakawa et al. (2023b)

Note that in this study, we adopt a naming convention for these datasets. They are identified by the name of the observer, followed by the first author of the corresponding published study. For instance, Stephan Prantner's data is denoted as Prantner (Hayakawa).

It is worth mentioning that even though datasets from Stephan Prantner (Hayakawa et al., 2021c), Thaddäus Derfflinger (Hayakawa et al., 2020), and Karl von

Lindener extend back to 1804 and 1800 respectively, we have intentionally limited our reconstruction efforts in this study to the year 1816. This decision aligns with the Tevel backbone (refer to Section 4), as data coverage before 1816 is notably deficient (see Figure 8).

3. Stability of long running observers

In this section we are discussing the stability of some key long running observers. That is important to understand the issues with the Wolf-Wolfer transition, but also to ascertain the stability of the observers that we want to use as backbones in our new reconstruction (see Sect. 4). However, we do not study the stability of Schwabe's records since this was already presented by Bhattacharya et al. (2023).

3.1. Stability of Wolfer's observations

We check the stability of Wolfer using simple ratio between his records and those from other observers (see Figure 2). There are 103 observers overlapping with Wolfer listed in the *Mittheilungen* for the time period 1876–1928. However, we chose only the 9 observers who have more than 20 years of overlapping observations with Wolfer.

It is evident from Figure 2 that Wolfer's data do not show any significant trend when compared with other long term observers. Note that, we analysed the ratios with the Mann-Kendall test to detect and quantify the trends on the ratios (Kendall, 1962). We find in general no trend, or minutely decreasing/increasing trends, for the ratios hence, it is safe to conclude that Wolfer can be treated as a stable observer during any future reconstruction.

3.2. Stability of Wolf's Observations

Rudolf Wolf observed sunspots between 1848 and 1893 but started reporting his data from 1849 onward. Over the years, he used several telescopes of which we were able to find clues in the *Mittheilungen* (Friedli, 2016; Bhattacharya et al., 2021, 2023). Among these telescopes, he used observations from his standard refractor with an aperture of 37 Parisian lines, a focus length of 48 Parisian inches, and eyepieces for 64- and 144-fold magnification (Wolf, 1848; Friedli, 2016), as the reference of the International Sunspot Series and calibrated all other observers to this telescope's observations.
In the course of his observations Wolf also changed his telescopes twice as mentioned above (Friedli, 2016, 2020), once in 1861 and in 1890. Therefore, we first discuss the stability of Wolf's counts compared to Wolfer and Bilwiller over their entire period, and then we discuss the stability of Wolf's counts separately for those made with his standard and portable telescopes.

3.2.1. Wolfer's k-factors

From 1877 to 1888 Wolf used to calculate Wolfer's k-factor twice a year, while for the years 1889–1893 Wolfer's k-factor was calculated every quarter of the year, that is

Table 2. List of the k-factors (and number of days in parentheses) tabulated in the *Mittheilungen* for Wolfer and Billwiller for 1876–1893.

Date	Semester (No.of Obs)	Wolf (No.of Obs)	Billwiller (No.of Obs)	Wolfer (No.of Obs)
1876-01-01		1.5(261)	0.74(20)	0.64(20)
1877-01-01	sem 1 (N_obs)	1.5(259)	1.01(32)	0.86(133)
1877-07-01	sem 2 (N_obs)		0.84(41)	0.82 (110)
1878-01-01	sem 1 (N_obs)	1.5 (282)	0.89 (31)	0.67 (131)
1878-07-01	sem 2 (all obs used) (N_obs)		1.05 (67)	0.78 (286)
1879-01-01	sem 1 (N_obs)	1.5 (280)	1.73 (34)	0.65 (119)
1879-07-01	sem 2 (all obs used) (N_obs)		0.77 (45)	0.69 (119)
1880-01-01	sem 1 (N_obs)	1.5 (270)		0.74 (109)
1880-07-01	sem 2 (all obs used) (N_obs)			0.76 (109)
1881-01-01	sem 1 (N_obs)	1.5 (280)		0.67 (132)
1881-07-01	sem 2 (all obs used) (N_obs)			0.69 (111)
1882-01-01	sem 1 (N_obs)	1.5 (278)		0.62 (136)
1882-07-01	sem 2 (all obs used) (N_obs)			0.67 (104)
1883-01-01	sem 1 (N_obs)	1.5 (290)		0.64 (126)
1883-07-01	sem 2 (all obs used) (N_obs)			0.54 (120)
1884-01-01	sem 1 (N_obs)	1.5 (290)		0.54 (123)
1884-07-01	sem 2 (all obs used) (N_obs)			0.52 (115)
1885-01-01	sem 1 (N_obs)	1.5 (284)		0.54 (126)
1885-07-01	sem 2 (all obs used) (N_obs)			0.56 (117)
1886-01-01	sem 1 (N_obs)	1.5 (295)		0.58 (131)
1886-07-01	sem 2 (all obs used) (N_obs)			0.54 (128)
1887-01-01	sem 1 (N_obs)	1.5 (299)		0.51 (138)
1887-07-01	sem 2 (all obs used) (N_obs)			0.51 (105)
1888-01-01	sem 1 (N_obs)	1.5 (285)		0.49 (112)
1888-07-01	sem 2 (all obs used) (N_obs)			0.43 (113)
1889-01-01	quarter 1 (N_obs)	1.5 (286)		0.64 (110)
1889-04-01	quarter 2 (N_obs)			0.72 (127)
1889-07-01	quarter 3 (N_obs)			0.61 (132)
1889-10-01	quarter 4 (N_obs)			0.52 (94)
1890-01-01	quarter 1 (N_obs)	1.5 (288)		0.66 (94)
1890-04-01	quarter 2 (N_obs)			0.48 (136)
1890-07-01	quarter 3 (N_obs)			0.44 (147)
1890-10-01	quarter 4 (N_obs)			0.48 (111)
1891-01-01	quarter 1 (N_obs)	1.5 (310)		0.54 (114)
1891-04-01	quarter 2 (N_obs)			0.52 (143)
1891-07-01	quarter 3 (N_obs)			0.53 (147)
1891-10-01	quarter 4 (N_obs)			0.58 (138)
1892-01-01	quarter 1 (N_obs)	1.5 (278)		0.64 (117)
1892-04-01	quarter 2 (N_obs)			0.62 (130)
1892-07-01	quarter 3 (N_obs)			0.6 (126)
1892-10-01	quarter 4 (N_obs)			0.64 (98)
1893-01-01	quarter 1 (N_obs)	1.5 (258)		0.63 (94)
1893-04-01	quarter 2 (N_obs)			0.52 (126)
1893-07-01	quarter 3 (N_obs)			0.51 (139)
1893-10-01	quarter 4 (N_obs)			0.54 (85)

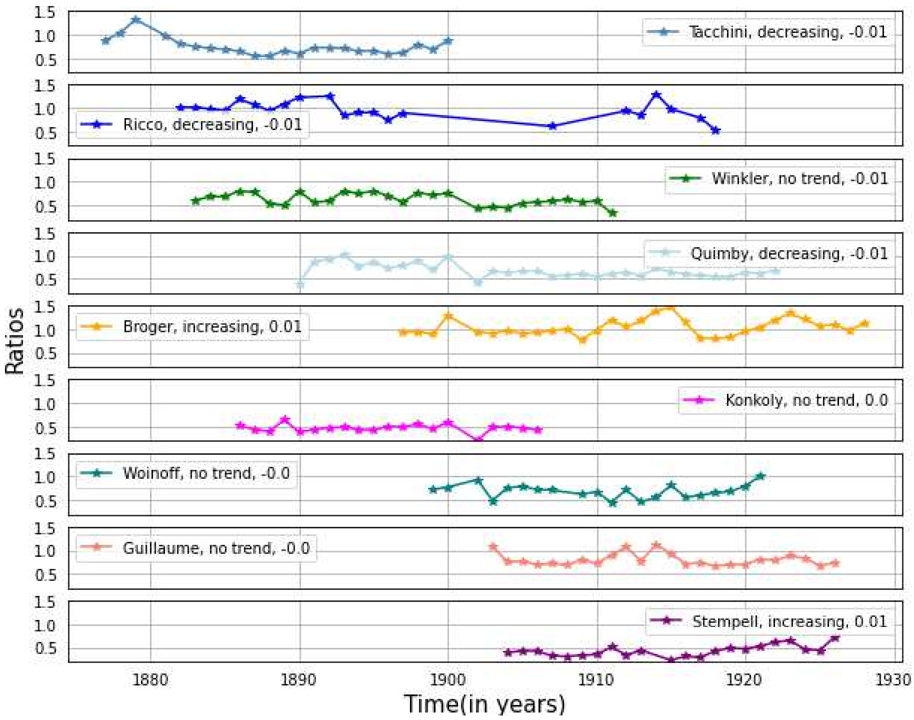

Figure 2. Annual ratios of Sunspot Number (SN) counts by Wolfer to those from contemporaneous observers as listed in each panel. The observers are chosen to have an overlap of at least 20 years with Wolfer. The x and y-axes represent time period and ratios, respectively. The trend along with the slope (given in yr^{-1}) indicated in each panel is determined by the Mann-Kendall test.

four k-factors in a year (Wolf, 1890; Friedli, 2020).

An evolution of Wolfer's and Billwiller's k-factors can be seen in Figure 3. In Figure 3 the panels show the tabulated k-factors of Wolfer and Billwiller, indicated in the legend, in the *Mittheilungen* as calculated by Wolf as well as the yearly k-factors calculated by us from the raw data available in the *Mittheilungen*. It is evident from Figure 3 that the quality of Wolf's observations were declining till 1888 and were not stable until 1890 when he changed to a telescope with higher magnification (Friedli, 2020). As a result, Wolfer's k-factors show a steady decrease between roughly 1878 and 1888. Considering that we established in the previous subsection that Wolfer's counts appear stable, it is possible that Wolf realized that his counts were degrading around 1877 and thus, he combined all three observers (Wolf + Billwiller + Wolfer) to serve as the reference series. Note that the scale of recalculated k-factors are different from the one tabulated in the*Mittheilungen*. This discrepancy is partly due to the fact that we recalculated the k-factors using raw data, whereas Wolf most likely scaled his portable telescope's data to his standard telescope using a 1.5 scaling factor and then calculated the k-factors for Billwiller and Wolfer. Another contributing factor to the differences is the variable sampling for computing the k-factors by Wolf compared to our calculation on annual basis.

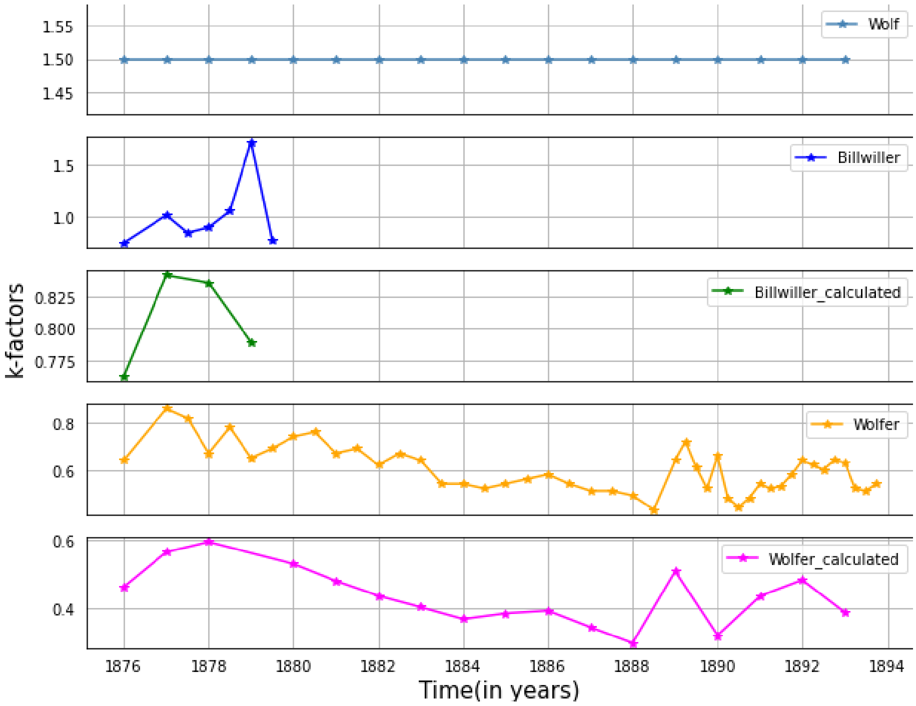

Figure 3. k-factors with respect to Wolf's records. The upper panel shows the k-factors for Wolf, followed by those for Billwiller as tabulated in the *Mittheilungen*, for Billwiller as calculated by us from the raw sunspot data found in the *Mittheilungen*, Wolfer as tabulated in the *Mittheilungen*, and Wolfer as calculated by us from the raw sunspot data. Note that our calculation of k-factors is done on annual basis, while those tabulated in the *Mittheilungen* are for irregular intervals. The x-axis gives the time and y-axis gives the k-factors. The tabulated k-factors in the *Mittheilungen* can be seen in Table 2.

Table 2 presents a comprehensive list of the k-factors recorded in the *Mittheilungen* for Wolf, Wolfer, and Billwiller during the period from 1876 to 1893. The table provides details of the date, semester or quarter of k-factor calculation, and the corresponding number of observations used for each observer by Wolf to calculate the k-factors. However, we note that this is the case only for the observations of Wolfer and Billwiller. The number of observations listed for Wolf for each year is the total number of observations Wolf performed over that year, while the k-factor he used for his counts was determined with merely the data over the years from 1859 to 1861 (Friedli, 2016, 2020).

3.2.2. Wolf's Standard Telescope (1849–1864)

Wolf used his Standard telescope (Wolf-SM, henceforth) from 1849 till 1864. Note that Wolf did not distinguish observers for the period of 1849–1859 in the *Mittheilungen* (Bhattacharya et al., 2021, 2023), hence there were no ways to know which observer was used on which day. However, this issue was overcome with the digitisation of his Source Books (see Section 2.2). Nonetheless, the Source Books identify the observers only on days when their observations were used to fill gaps. Therefore, we find no daily overlap with any of the observers either in the *Mittheilungen* or in the Source Books with that of Wolf's Standard Telescope.

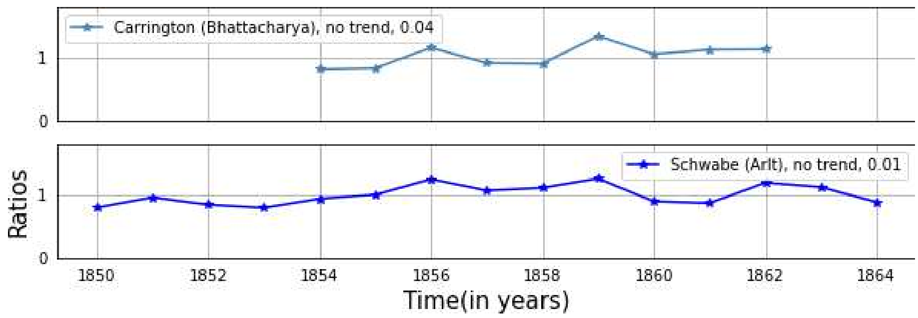

Figure 4. The panels show the yearly ratios of the Sunspot Numbers (SN) of Carrington (Bhattacharya) and Schwabe (Arlt) to those by Wolf. The observers' names are given as labels in each panel. The x and y-axes represent the time period and ratio, respectively. The trend along with the slope (given in yr^{-1}) determined by the Mann-Kendall test are also indicated in each panel.

We use Carrington (Bhattacharya) and Schwabe (Arlt) data (see Section 2.3) as they overlap with Wolf. Thus, we have a daily comparison with Wolf's data. We followed the same procedure as described in Section 3.1, for plotting the simple ratios and as can be seen from Figure 4, data from Wolf's standard telescope does not show any significant trends. Hence we can consider the dataset stable.

3.2.3. Wolf's Portable Telescopes (1864–1893)

Wolf used his portable telescope (40/700 mm Parisian refractor) from 1861 to 1889. He computed it's k-factor to be 1.5 to his standard telescope, but the computation was based on a rather limited number of comparisons during the years from 1859 to 1861 (Friedli, 2016, 2020). Later, he changed his telescope in 1890 to a higher magnification (42/800 mm Fraunhofer refractor), possibly to compensate for his degrading eyesight, but he kept unchanged his k-factor (1.5) to bring his observations to his previous scale. This can be one of the possible reasons why the relationship between Wolfer's and Wolf's counts has been reported to be non-linear (Friedli, 2020; Usoskin, Kovaltsov, and Chatzistergos, 2016).

Another noteworthy trend we find in Wolf's (Portable Telescope) data is Wolf's knack to record an even number of spots. Figure 5 portrays data from Wolf's portable telescope from the Source Books over 1849–1859 and from the *Mittheilungen* over 1859–1893. We show the early data from the Source books because the data in the *Mittheilungen* do not differentiate observers over 1849–1859. For reasons still unknown at this point, Wolf recorded less and less odd sunspot counts with complete omission in the year 1893. Note that according to Wolf's Source Books (see Section 2.2 and Figure 1), Wolf started to observe with his portable telescope from 1849 onwards. At this point, we conclude that Wolf's data from his portable telescope is not a suitable reference for the SN construction due to a quality degradation over the years as explained above and in Section 3.2.1.

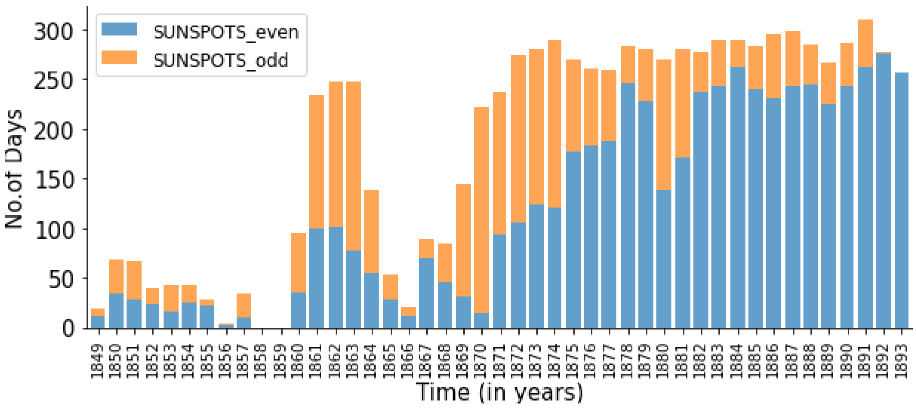

Figure 5. Number of days per year with sunspot records by Wolf. We show the number of days when Wolf reported even sunspot count in blue and those for odd counts in orange.

As an important step towards the ongoing Sunspot Number recalibration, we produced the SN Version 2.1 (1818–1848) in Bhattacharya et al. (2023), which aimed at rectifying the k-factors in the SNV1 before 1849. In Bhattacharya et al. (2023) we concluded that there was no significant impact of the 1849 jump on SNV1 after 1849. However, we were aware that a detailed analysis of the Wolf's source book data is required for the observers who demonstrate a discrepancy between the applied k-factors and our estimated k-factors in Bhattacharya et al. (2023). For instance, Emil Jenzer was scaled in the Source Books (http://www.wolfinstitute.ch/data-tables.html) using 3 different k-factors, 0.58 for 4 observations in the period October 31, 1862 - December 28, 1862, 0.49 for 7 observations in the period March 29, 1863 - December 15, 1863, and 0.75 for 6 observations from January 14, 1864 - April 29, 1864. However, his observations can be found in the *Mittheilungen* from 1861–1865 with a scaling-factor of 0.85 (Wolf, 1862). We propose that the reason for assigning multiple k-factors to a single observer could be that Wolf's observational count showed a progressive decrease until 1890, thus indicating that the k-factors are not consistent.

As depicted in the Figure 3, Wolfer's k-factors improved significantly when Wolf started using his telescope with a higher magnification (Friedli, 2020) in order to increase the resolution of his observations. It seems that Wolf was aware of the degradation of his counts around 1877 and thus, he combined the records from three observers (Wolf + Billwiller + Wolfer) together as a reference set. We can therefore conclude that Wolf's k-factors were not calculated in a statistically sound manner, and need re-evaluation.

3.3. Stability of Weber's observations

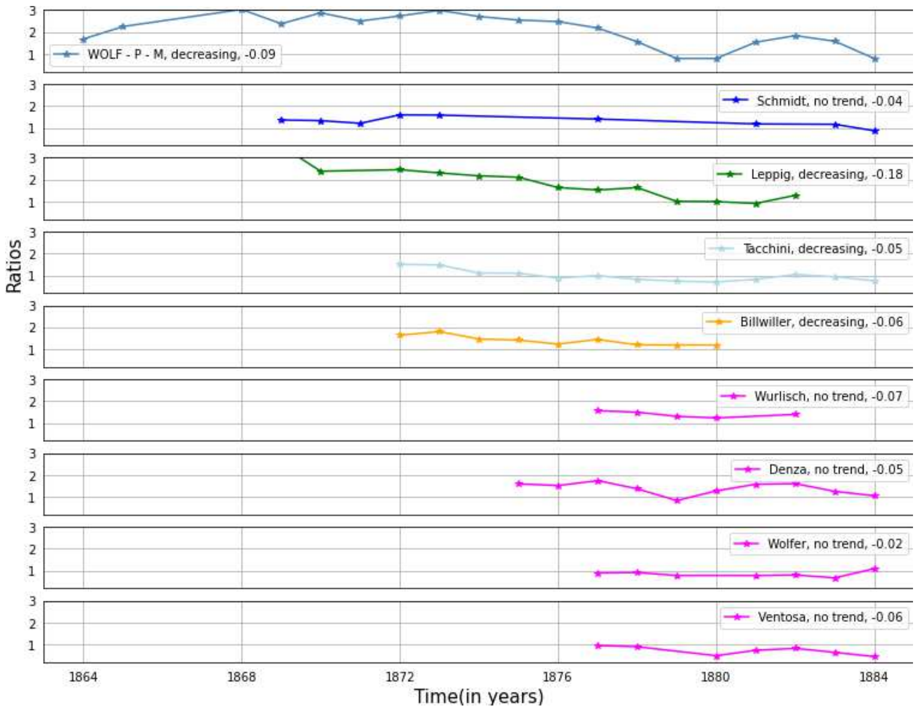

Figure 6. The panels shows the yearly ratios of the Sunspot Numbers (SN) of various observers to those of Weber. The observers' names are given as labels in each panel. The x and y-axes represent the time period and ratios, respectively. The trend along with the slope (given in yr^{-1}) indicated in each panel is determined by the Mann-Kendall test.

Heinrich Weber observed from 1863–1883 in Peckeloh (40 km east of Münster, Germany)(Wolf, 1864). We compare the ratios of SN from various observers that overlap with Weber to those from Weber and compute the trend and slope with the Mann-Kendall test. The ratios are shown in Figure 6. The analysis of Figure 6 reveals a lack of significant trends in Weber's data, except for the observations made by Wolf. However, a closer examination in Section 3.2.3 indicates that the issue lies with Wolf's data. Therefore, Weber's data presents a promising opportunity to establish a connection between Schwabe and Wolfer, in the method discussed in Section 6. Furthermore, it is worth noting the nearly linear relationship between Wolfer and Weber's ratio, suggesting the stability of Weber's observations during this particular period.

In the upper panel of Figure 7, we present the tabulated k-factors of Weber, as reported in the *Mittheilungen*. The lower panel displays our calculated annual k-factors. The difference in scale is due to our recalculation of k-factors using raw data, specifically the WOLF-P-M data. In contrast, it is likely that Wolf scaled his portable telescope's data to his standard telescope using a scaling factor of 1.5 before calculating the k-factors for Weber. Remarkably, the observed trend in the k-factors closely aligns with the trend depicted in Figure 3, confirming that the variability in the calculation of k-factors primarily arises from Wolf's data.

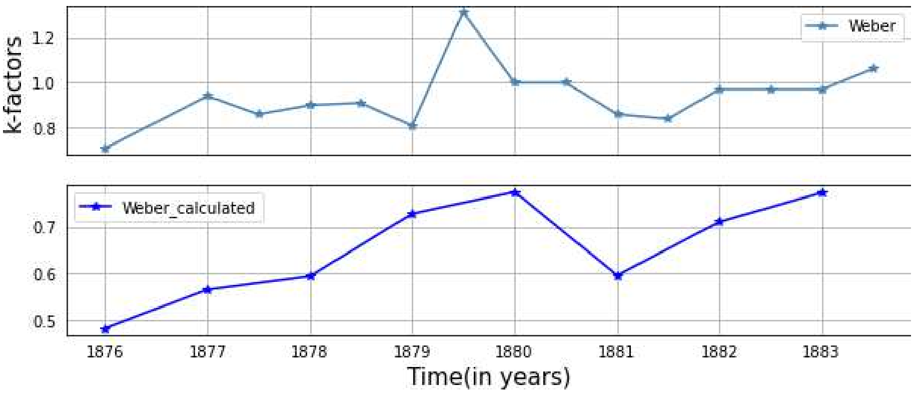

Figure 7. k-factors with respect to Wolf's records. The upper panel shows the k-factors for Weber as tabulated in the *Mittheilungen*, the lower panel shows the$k$-factors calculated by us from the raw sunspot data found in the *Mittheilungen* Note that our calculation of k-factors is done on annual basis, while those tabulated in the *Mittheilungen* are for irregular intervals. The x-axis gives the time and y-axis gives the k-factors.

4. The Backbones

In this study, we not only re-calibrate the Wolf-Wolfer transition period, but we aim to reconstruct the SN series from 1816–1944. The daily SN series began in 1818, but with the recovered data, we will be able to go as far back as 1816. Digitization of the *Mittheilungen* (digital format of Wolf's journals) allows access to the raw data of all the observers contributing to the construction of SNV1 (http://www.sidc.be/silso/versionarchive). However, the raw data in the *Mittheilungen* is available till 1944. Data from 1945 (Clette et al., 2021) has been recovered in the ETH library and are available as images in https://www.e-manuscripta.ch/, however, they have not been transcribed to a usable format yet. Hence, we limit our reconstruction till 1944. Nevertheless, we provide a robust series for 126 years, which is constructed going beyond the notion of the k-factors. This series will be presented in Sect. 6.1, while for comparison reasons in Sect. 5 we also produce a series by using the traditional k-factor approach.

For both reconstructions, to cover the entire period we chose 4 observers as backbones viz. Tevel (1816–1836), Schwabe (1825–1867), Weber (1863–1883), and Wolfer (1876–1928) backbones. They are chosen for their stability and long observation periods. The Wolf backbone in SNV1 was replaced by the Weber backbone, as the latter is more stable. It is worth mentioning that although the criteria for incorporating observers into each backbone have similarities with the description in Chatzistergos et al. (2017), there have been certain alterations. For instance, we opted for 50 overlapping observing days for sunspot count calibration, in contrast to the previously mentioned 20 in Chatzistergos et al. (2017). This decision was made to achieve an optimal fit as described in Section 6. The criteria now primarily revolve around the limiting factor, which is the reconstruction of sunspot numbers. It is important to note that criteria related to the number of overlapping years mentioned in Chatzistergos et al. (2017) with the backbone have been eliminated.

4.1. Tevel Backbone

Cornelis Tevel was a Dutch astronomer who conducted sunspot observations between 1816 and 1836. He had the misconception that sunspots were small planets orbiting the Sun, even though the true nature of sunspots was already known (Wolf, 1859; Carrasco, 2021). Meteorologist Buys Ballot rediscovered Tevel's observations and forwarded them to Rudolf Wolf for analysis of the solar cycle (Wolf, 1859; Carrasco, 2021; Bhattacharya et al., 2023). Wolf praised Tevel's astute sunspot observations and labeled him a great astronomer. Tevel documented his observations in a notebook, recording 849 sunspot drawings. Each entry included the date, month, day, hour, and sometimes more detailed sketches of interesting sunspot groups. Tevel used a projection method to create his sunspot drawings (Carrasco, 2021). The daily count of sunspot groups in Tevel's drawings was recalculated using modern classifications by Carrasco (2021), which also allowed to recover more observations than those included in the Mittheilungen (Wolf, 1859).

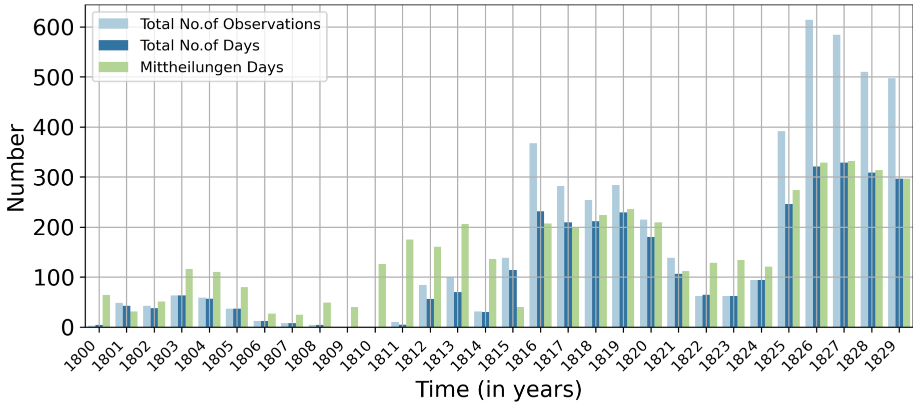

Figure 8. The figure shows number of observations available for each year (light blue) along with the number of days with at least one observation among the data used in this study (blue) and within the Mittheilungen (green) within the years 1800–1829. This provides insight into the temporal distribution and availability of data during the designated period.

It is evident from Figure 8, that there is a scarcity of data in the number of observations between 1820 and 1825, when Schwabe started observing. For the computation of SNV1 over the period between 1818–1824, the most used observers from the Mittheilungen were Tevel and Kremsmünster (Bhattacharya et al., 2023). However, Hayakawa et al. (2020) clarify that printed circles in the Mittheilungen were interpreted as 0, which led to ghost spotless days in the Kremsmünster dataset. These "ghost zeros" are located in the minimum phase of the solar cycle which is why it remained unnoticed in SNV1. The updated data we use here do not suffer from these issues, however they have a reduced coverage between 1822–1825, as depicted in Figure 8. Note that before 1816 the Mittheilungen includes only the records of Flaugergues, which we however did not include here due to the mentions of potential inaccuracies in sunspot counting (Arlt and Vaquero, 2020b), but also in group counting when compared to the original group counts by Wolf (1861), Hoyt and Schatten (1998a), and even Vaquero et al. (2016).

Among the various observers listed in Table 1, we have identified Tevel as a suitable candidate due to its temporal overlap with Schwabe's observations and also continuous data for the period 1816–1819. This overlap allows for valuable backbone-to-backbone calibrations for producing the coherent series. Consequently, we have opted to utilize the Sunspot Number recounts of Tevel, as documented in Carrasco (2021), as the backbone for the period 1816-1825 for our analysis. Figure 9 shows all the observers who are included in the Tevel backbone. The observations presented in red in figures 9, 10 and 11 have not been used in the reconstruction because of the criteria from Chatzistergos et al. (2017) mentioned at the beginning of this section.

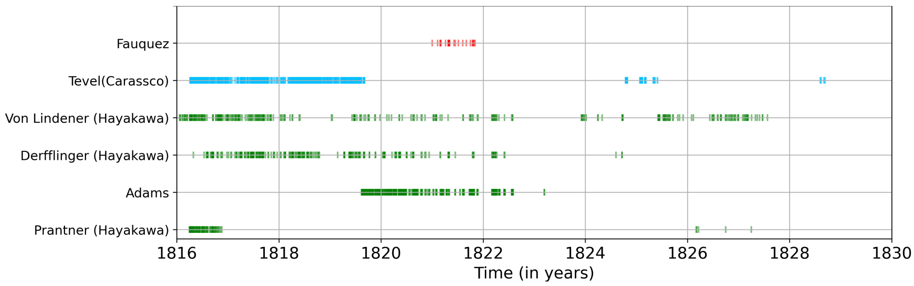

Figure 9. The time span of all the observers who overlap with Tevel. The green colored observers are included in the Tevel backbone, whereas the red ones are excluded. The backbone is indicated in blue.

4.2. Schwabe Backbone

Heinrich Schwabe was one of the most prominent observers recorded in the *Mittheilungen* between 1826 and 1868 (Arlt et al., 2013). He also serves as a backbone in the reconstructions by Clette et al. (2014); Svalgaard and Schatten (2016); Chatzistergos et al. (2017). However, his observations in SNV1 (tabulated in the *Mittheilungen*) suffered from a scale jump due to the incorrect application of the k-factors before 1849 (Bhattacharya et al., 2023). Arlt et al. (2013) analyzed 135000 sunspots based on Schwabe's sunspot drawings from November 5, 1825 to December 29, 1867, and provides a detailed record of his drawings, including sunspot positions and umbral areas. We use this reconstruction as our backbone and denote it as Schwabe (Arlt) (see Section 2.3).

Figure 10 shows all the observers who are calibrated against the Schwabe backbone. The red coloured observers are the ones who are not included in the calibration. These are observers who do not have enough overlapping days with Schwabe to fulfil the selection criteria (Chatzistergos et al., 2017, See Section 6) as well as the observers from the Kew Observatory who only reported spot areas and not counts.
Note that Wolf's observations with the standard telescope (WOLF-S-M) found in the *Mittheilungen* from 1849–1859 are in reality a combination of all observers with no way to distinguish them. Hence, we use the source book data (see section 2.2) for Wolf in this calibration.

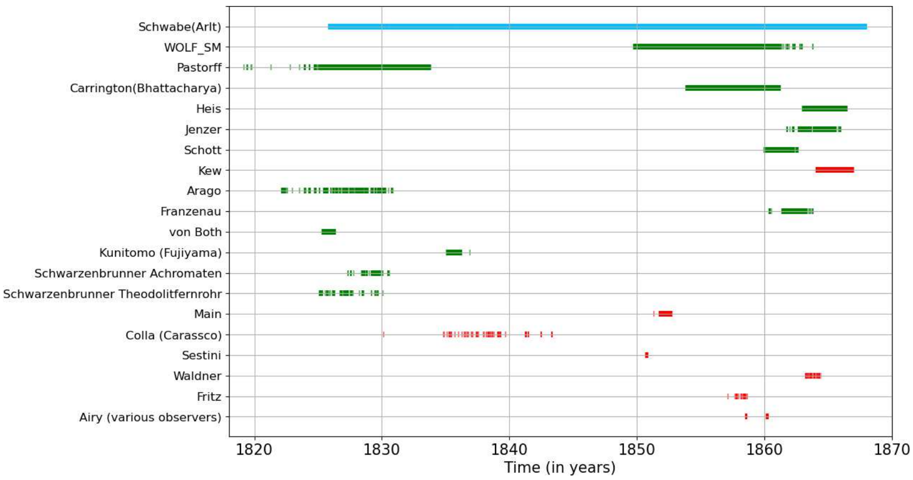

Figure 10. The time span of all the observers who overlap with Schwabe. The green colored observers are included in the Schwabe backbone, whereas the red ones are excluded. The backbone is indicated in blue.

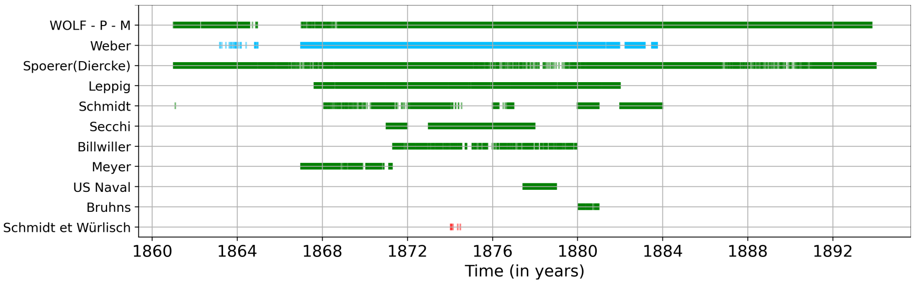

Figure 11. The time span of all the observers who overlap with Weber. The green colored observers are included in the Weber backbone, while the backbone is indicated in blue. The excluded observer is indicated in red.

4.3. Weber Backbone

Figure 11 shows all the observers that are calibrated against the Weber backbone. Note that Spoerer (Spoerer and Maunder, 1890) has only 5 observations in the *Mittheilungen* and we used the recounts by Diercke, Arlt, and Denker (2015) in the calibration and is denoted by Spoerer (Diercke) in Figure 11. Even though Spoerer has a better temporal coverage than Weber (see Figure 11), his spotless days are not digitally available in the existing databases, thus the use of the Weber data.

There is a difference in the way we treat the Weber backbone for SNV2.2 and SNV2.3. For SNV2.2 we extend the Weber backbone with the data by Wolf's portable telescope (WOLF-P-M) and use this composite instead of just Weber's records. The reason for this is the insufficient overlap between the two earlier backbones, namely Schwabe and Weber which would not allow for a meaningful cross-calibration unless Weber's data are extended in the past. In particular Schwabe's (Figure 10) and Weber's records (Figure 11) overlap completely merely for 4 years (1864–1868) and a few days in 1863. We consider that Wolf's observations with his Portable Telescope (WOLF-P-

M) are stable until at least 1877 as demonstrated in Figure 6 and can act to sufficiently accurately extend the records of Weber for this reconstruction. The composite series is calculated using the method described in Mathieu et al. (2019); Clette et al. (2007). A pictorial description of the method can be found in Figure 19 by Bhattacharya et al. (2021).

We note that this is not an issue for the calibration process used for SNV2.3, which cross-calibrates backbones using the entire backbone series, that is after being extended with the cross-calibrated records of all included observers and thus is not affected by this limited overlap between Weber and Schwabe (see Section 6). Thus for SNV2.3 we used only Weber as the backbone, without extending it with Wolf's data.

4.4. Wolfer Backbone

As mentioned in Sections 2 and 3, Wolfer is one of the most stable observers who observed over 1876–1928 and served as a backbone in the SNV1 construction, after Wolf's death. As an important outcome of the ongoing sunspot number (SN) re-calibration (Lefevre et al., 2018) efforts, SN Version 2 (Clette et al., 2016) and SNV2.1 (Bhattacharya et al., 2023) that were released in July, 2015 and January, 2023 respectively, maintains Wolfer's scale as primary scale as contrary to SNV1 which is based on Wolf reference. Here we maintain Wolfer's scale in our reconstructions as well.

Figure 12 shows 88 of the 108 observers (103 observers in the Mittheilungen and 5 recounted observer as listed in Table 1) with overlapping records to those by Wolfer. The 20 observers not shown in Fig. 12 are observers that have longer overlapping period with Weber and/or have sunspot areas listed in the Mittheilungen. Due to insufficient overlap and/or variable quality data according to the criteria mentioned in Chatzistergos et al. (2017), 40 out of these 88 observers were also excluded from the Wolfer backbone. We note that these 40 observers observed over rather short time intervals, typically covering a couple of years and the overlapping periods with the backbone were too short to have sufficient statistics for the method.

Also note that Wolfer's last observation was on 28 October, 1928 (Wolf and Brunner, 1929) and Brunner replaced him after that as the reference observer in SNV1 and SNV2. Instead of considering Brunner as a distinct backbone, our approach focused on calibrating observers over that period, such as Brunner, Broger, and Misawa, to Wolfer Backbone. That decision is partly due to the current unavailability of data in digital format after 1944. One of those observers will potentially act as a backbone in a future reconstruction when all raw data have been digitised.

Figure 12. The time span of all the observers who overlap with Wolfer. The green colored observers are included in the Wolfer backbone, whereas the red ones are excluded. The backbone is indicated in blue.

5. Reconstruction of SNV2.2 Using *k*-factors

Even though, we aim at constructing SN series by going beyond the notion of *k*-factors in this study, we first use the newly digitised raw sunspot data to produce a sunspot number series by cross-calibrating the data with the traditional *k*-factor approach. This exercise can potentially provide important insights about the data and methodology and would be important to be compared with any recalibration of sunspot number using a non-linear cross-calibration approach. We refer to this "traditional" recalibration of SN as SNV2.2 covering 1816–1944, considering it a continuation of SNV2.1 which covered only 1818–1848. In particular, the

Table 3. Summary of the observers having observations overlapping with Tevel Backbone.

ID	Name	No.of Obs	Overlapped obs	Zero obs	Start	End	k	δk	Slope (TLS)	Err
25	Adams	402	6	94	1819-08-15	1823-03-16	1.293241	0.161286	1.392945	6.02E-02
1009	Von Lindener (Hayakawa)	590	80	23	1800-06-30	1827-07-28	1.630285	0.553148	1.359993	5.82E-02
1010	Derfflinger (Hayakawa)	472	121	0	1802-09-26	1824-09-21	1.79311	0.5139	1.660874	7.28E-02
1001	Tevel(Carrasco)	743	743	16	1816-04-05	1836-05-24	1	0	1	0
1011	Prantner (Hayakawa)	215	61	1	1804-01-11	1844-08-21	1.500149	0.432191	1.330314	5.88E-02

Table 4. Summary of the observers having observations overlapping with Schwabe Backbone.

ID	Name	No.of Obs	Overlapped obs	Zero obs	Start	End	k	δk	Slope (TLS)	Err
28	Arago	90	31	6	1822-02-15	1830-04-05	2.19	1.20	1.97	0.21
29	Jenzer	575	410	15	1861-10-14	1865-12-27	0.84	0.42	0.52	0.01
31	Franzenau	279	188	3	1860-05-14	1863-10-29	1.37	0.38	1.24	0.03
34	Schott	474	331	2	1860-01-01	1862-08-26	1.06	0.19	0.93	0.01
35	Heis	1193	789	74	1863-01-01	1872-12-12	0.77	0.15	0.60	0.007
46	von Both	176	31	33	1825-05-04	1826-04-25	1.01	0.27	0.88	0.05
51	Pastorff (Low Magnification)	497	325	177	1824-05-01	1833-11-04	1.06	0.4	0.85	0.02
94	Schwarzenbrunner Theodolitfernrohr	113	63	17	1825-02-06	1830-01-26	1.10	0.42	0.78	0.04
96	Schwarzenbrunner Achromaten	135	103	1	1827-05-03	1830-07-27	0.99	0.32	0.94	0.02
151	Pastorff (High Magnification)	1063	709	2	1819-03-04	1833-10-09	0.52	0.2	0.37	0.006838
1000	Schwabe (Arlt)	10002	10002	1529	1825-11-05	1867-12-29	1	0	1	0
1003	Carrington (Bhattacharya)	1239	910	384	1853-11-09	1861-03-24	0.97	0.27	0.82	0.007
2	WOLF_SM	2678	1922	547	1849-09-30	1863-10-25	1.01	0.31	0.87	0.005
1015	Kunitomo (Fujiyama)	146	51	0	1835-02-03	1836-03-24	1.84	0.70	1.67	1.1

cross-calibration is done with k-factors given by:

$$k_i(t) = \frac{\text{backbone}(t)}{Y_i(t)} \quad (1)$$

where $Y_i(t)$ are the Wolf Counts of station i, observed at time t and backbone(t) is the value of the reference station (for more details see: Clette et al., 2007; Mathieu et al., 2019).

For this reconstruction we use the backbones described in Section 4. Wolfer is taken as the main reference for our reconstruction and thus SNV2.2 is maintained in Wolfer's scale, which is the same as the one of SNV2 and SNV2.1. The process is

Table 5. Summary of the observers having observations overlapping with Weber Backbone.

ID	Name	No.of Obs	Overlapped obs	Zero obs	Start	End	k	δk	Slope (TLS)	Err
3	WOLF - P - M	3014	2762	347	1861-01-03	1877-01-01	1.50	0.26	1.55	0.01
32	Weber	4790	4266	1250	1863-03-08	1883-09-30	0.99	0.09	0.95	0.00
42	Leppig	2533	1956	699	1867-08-20	1881-12-31	1.31	0.51	1.34	0.01
47	Billwiller	708	529	211	1871-04-27	1879-12-24	0.87	0.24	0.97	0.01
49	Secchi	710	509	208	1871-01-05	1877-12-29	0.88	0.33	0.85	0.01
60	Meyer	627	548	160	1867-01-01	1871-04-11	1.04	0.32	0.90	0.01
65	US Naval	332	167	229	1877-06-11	1878-12-31	0.78	0.26	0.63	0.03
68	Bruhns	103	89	11	1880-01-17	1880-12-31	0.93	0.13	0.88	0.03
1002	Spoerer (Diercke)	4311	3396	0	1861-01-07	1894-01-11	3.42	2.49	3.88	0.05
2000	Composite	8727	8727	1408	1861-01-03	1883-09-30	1.00	0.00	1.00	0.00

effectively to cross-calibrate the counts of all observers to Wolfer's with a daisy-chain approach going always through the backbone observers (see Clette et al., 2023, for a visualisation of the daisy chain process).

A summary of the observers used in each of the backbones are listed in Tables 3, 4, 5 and 6, respectively. In Tables 3, 4, 5 and 6, columns 1 and 2 give the ID of the observer as they appear in the digitised *Mittheilungen* and the name of the observer respectively. Column 3 gives the total number of observations made by the concerned observer. Column 4 gives the number of overlapping observations with the backbone. Column 5 gives the total number of zero counts. Columns 6 and 7 give the first and last day of observations of the concerned observer, respectively. Columns 8 and 9 give the calculated mean k-factor on daily ratios between the backbone and the corresponding observer, and the error (δk) respectively. Columns 10 and 11 give the slope of the Total Least Square (TLS) between the backbone and the corresponding observer and the associated error. The newly recovered records of observers who were not included in the *Mittheilungen* are also shown in Tables 3, 4, 5 and 6 with their Column 1 IDs in the format 100x.
As we mentioned in Section 4, for the Weber backbone for the SNV2.2 reconstruction we used an extension based on Wolf's early data. This composite backbone is also given at the end of Table 5 with a tentative ID number of 2000.

We use the Slope (TLS) values from Tables Tables 3, 4, 5 and 6 to construct the SNV2.2. In our analysis, we computed the k-factors using the same approach as Wolf indicated as k in the Tables 3, 4, 5 and 6 , which involved using simple ratios. However, we chose to employ Total Least Squares (TLS) instead because it offers greater robustness in our calibration process. When examining observers with limited overlap with the backbone, we studied their distributions and observed that the median and mean values deviated noticeably beyond the error bars. This observation prompted us to utilize the TLS slope as a more reliable alternative. Nonetheless, to facilitate comparison, we also included the traditional k-factors alongside the TLS results in the Tables 3, 4, 5 and 6.

The errors in the daily SN from 1816–1825 is given by

$$\delta \text{SN}x = \sqrt{\sum_i ((\text{Slope}_i \times \delta \text{SN}_i)^2 + (\text{Err}_i \times \text{SN}_i)^2)},$$

where Slope_i and Err_i are given in Tables 3, 4, 5, and 6, while SN_i is the sunspot number for ith observer for a given day and δ SN_i is given by $\sqrt{(\text{SN}_i)}$, considering SN_i to be Poisson distributed (Dudok de Wit, Lefèvre, and Clette, 2016). Daily errors are calculated over 1825–1944 by bootstrapping the SNV2.2. That is to eliminate one observer at a time from each backbone and reconstructing the entire series. The difference between the highest and lowest value for each day is taken as the daily error. This process is limited over 1825–1944 due to lack of sufficient observers before 1825 to meaningfully estimate the errors.

6. A New Calibration Method: Towards SNV3

As explained in the previous sections, it is of the utmost importance to identify ways to eliminate the bias introduced by Wolf. A way to solve this issue is to eliminate the root assumption that the counts of observers are linearly dependent on each other and thus to eliminate the notion of k-factors. Hence, we adopt the method of direct calibration proposed by Chatzistergos et al. (2017) in this section in an effort to eliminate the inhomogeneites in the SNV1, and produce a more robust version viz. SNV2.3.

A brief description of the method is given below. Using observations from days where both observers have recorded data, a direct calibration matrix between a secondary observer and a primary backbone observer is created. For every observation N_1 by the primary observer and $N_{2,l}$ by the secondary observer, one count is added to the corresponding cell in row N_1 and column $N_{2,l}$, where $N_{2,l}$ represents l secondary observers, that is $l = 1, 2, 3 ... n$. In order to create a matrix of probability mass function (PMF) to determine a value of G^* reported by the primary observer for each day with the given value G reported by the secondary observer, the matrix was first normalized by dividing each of its values by the total sum over the corresponding column as shown in Figure 13. The entries to be estimated are given by

$$G^*_{ij:l} = P(N_1 = i | N_{2,l} = j) \quad (2)$$

where G^*_{ij} represents the element at the i-th row and j-th column of matrix G^*, that is the conditional probability that N_1 takes the value i, given that $N_{2,l}$ takes the value j.

However, this matrix can have missing entries due to lack of observations and small overlap between the records of the two observers. Imputation of the missing entries, based on a local regression model is done by fitting the statistically significant portion of the matrix using an asymptotic exponential approach to a constant offset in the difference using the function $G^* - G = R_0 + Be^{(-aG)}$, where G^* are the mean counts of the primary observer for a given count of the secondary observer G, and R_0, B, and a are constants calculated for each pair of observers

Table 6. Summary of the observers having observations overlapping with Wolfer Backbone.

ID	Name	No.of Obs	Overlapped obs	Zero obs	Start	End	k	δk	Slope (TLS)	Err
115	Freyberg	531	511	279	1898-03-15	1903-09-27	1.28	0.61	1.25	0.035
116	Kaulbars	650	583	251	1898-04-27	1901-12-25	0.95	0.64	0.48	0.020
102	Maier	674	653	85	1895-01-03	1901-06-30	1.20	0.32	1.28	0.016
159	Batavia obs	674	648	229	1912-01-11	1921-12-31	2.02	0.58	2.12	0.031
155	Kavan	770	597	550	1911-01-04	1913-12-30	0.76	0.37	0.77	0.015
125	Boston observatory	775	645	149	1896-12-01	1906-05-31	1.45	0.52	1.48	0.022
166	Heimen	810	635	32	1915-01-03	1918-12-30	2.55	0.64	2.25	0.026
124	Subbotin	839	720	191	1900-01-16	1908-12-11	1.62	0.43	1.42	0.019
140	Baker	911	626	42	1903-05-17	1906-07-01	1.94	0.47	1.75	0.025
152	Guerrieri	937	613	85	1908-01-02	1910-12-17	1.09	0.23	1.00	0.014
135	Schatkow	1065	778	175	1902-02-04	1910-11-01	1.55	0.40	1.29	0.015
84	Lewitzky	1124	1105	225	1893-08-23	1907-12-27	1.28	0.38	1.29	0.013
154	Lucchini	1138	852	446	1908-04-22	1914-12-30	1.27	0.37	1.34	0.014
70	Janesch	1156	930	123	1882-02-08	1887-06-15	2.02	0.50	1.65	0.019
80	Schreiber	1178	940	5	1891-01-23	1895-12-30	1.55	0.29	1.57	0.016
76	Schmoll	1352	1280	510	1888-01-13	1892-09-07	1.04	0.37	0.85	0.007
72	Todd	1315	975	38	1881-01-01	1886-11-27	1.28	0.33	1.20	0.013
108	Sykora	1454	1390	282	1893-03-22	1910-11-26	0.85	0.24	0.77	0.007
144	Braak	1627	1082	522	1909-01-02	1925-12-30	1.67	0.54	1.69	0.017
157	Bodocs	1696	1242	506	1906-01-01	1916-12-28	2.18	0.69	2.40	0.020
143	Hrase	1751	1307	419	1906-01-02	1916-12-24	1.25	0.45	1.16	0.012
138	Messerschmitt	1866	1440	124	1902-10-10	1910-12-31	1.41	0.27	1.36	0.009
1006	Valencia Observatory (Carassco)	1894	1322	254	1920-01-01	1928-12-30	2.10	0.80	2.24	0.025
162	Buttlar	1918	1545	448	1914-01-03	1925-12-31	1.12	0.18	1.14	0.008
113	Kleiner	1929	1753	638	1899-02-12	1918-12-30	1.21	0.39	1.06	0.008
79	Haveford University	2006	1961	307	1888-08-14	1899-12-25	1.12	0.36	1.06	0.008
156	Bemmelen	2528	1749	727	1907-01-02	1919-12-30	2.41	0.93	2.19	0.020
139	Stempell	2732	2158	766	1903-01-05	1913-12-25	2.55	1.23	2.17	0.018
142	Williston Observatory	2810	1920	719	1909-01-06	1925-12-30	1.63	0.55	1.59	0.013
1005	Hadden (Carassco)	2490	2536	872	1890-09-01	1902-12-31	1.27	0.54	1.27	0.010
114	Woinoff	3135	2838	668	1898-04-24	1907-10-20	1.45	0.49	1.32	0.007
54	Wurlisch	2607	1536	1017	1876-01-01	1886-12-31	2.06	0.97	1.59	0.020
78	Schwab	3328	3024	705	1892-01-02	1901-12-31	1.51	0.55	1.02	0.009
107	Konkoly	3787	3771	1184	1885-01-02	1905-12-19	2.07	0.76	2.09	0.014
55	Denza	3974	2823	1175	1874-01-01	1893-12-31	1.78	0.61	1.76	0.012
67	Ricco	4306	3104	1216	1879-09-11	1917-12-30	1.07	0.38	1.10	0.008
158	Moye	4772	3516	1466	1911-01-01	1925-12-31	2.12	0.67	2.40	0.013
1004	Misawa (Hayakawa)	4030	1327	937	1921-10-03	1934-12-30	1.47	0.35	1.55	0.013
177	Brunner	4837	522	579	1926-04-01	1944-12-24	0.98	0.05	0.98	0.005
57	Ventosa	5151	3921	963	1876-01-01	1878-12-27	0.93	0.26	1.03	0.004
1013	Madrid_1 (Aparicio)	5167	3929	965	1876-01-01	1896-12-30	0.94	0.26	1.03	0.004
106	Mascari	5173	4899	974	1887-01-14	1908-12-30	1.11	0.32	1.07	0.005
69	Winkler	6265	5588	1474	1882-01-01	1910-06-03	1.59	0.44	1.38	0.006
1008	Kodaikanal Observatory	6280	2349	0	1906-01-05	1944-12-29	2.48	1.24	2.11	0.018
132	Guillaume	6349	5003	1360	1902-01-03	1925-12-29	1.32	0.31	1.45	0.006
45	Tacchini	7954	6132	1517	1871-02-22	1900-12-30	1.41	0.45	1.45	0.006
104	Broger	9734	8323	2192	1896-09-01	1935-12-31	1.02	0.27	1.05	0.003
77	Quimby	10688	8963	2467	1889-07-01	1899-12-31	1.52	0.44	1.34	0.005
56	Wolfer	14558	14558	3310	1876-08-18	1928-10-28	1.00	0.00	1.00	0.000

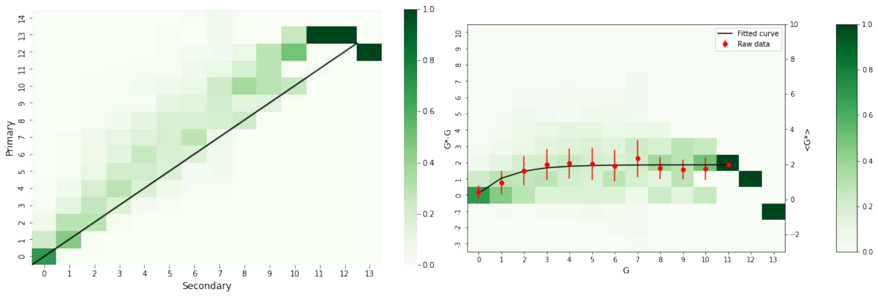

Figure 13. Example of calibration PMF matrix for Wolf (secondary observer, G) to Wolfer (primary, G^*) over 1876–1893. The left panel shows the original distribution matrix G^* vs. G, while the right panel shows the difference matrix, $G^*{=}G$ vs. G. The PMF is color coded according to the color map on the right side of each panel. The black line on the left panel has a slope of unity, while the black line on the right panel is the fitted line using the function $G^* - G = R_0 + Be^{(-aG)}$, where G^* are the mean counts of the primary observer for a given count of the secondary observer G, and R_0, B, and a are constants estimated for each pair of observers separately. The red circles depict the mean G^* values for each G column, while we also show their 1σ uncertainty.

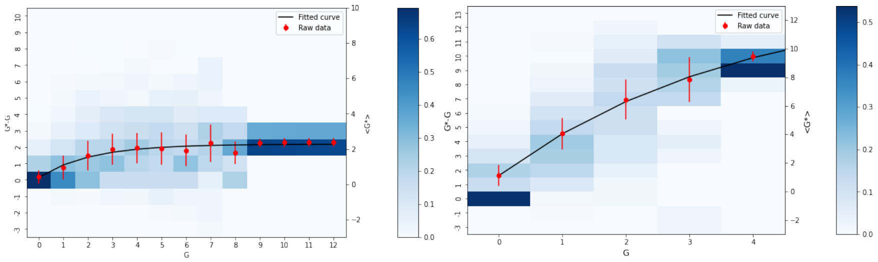

Figure 14. Calibration matrices for Wolf and Wolfer. The left figure (a) is for group counts while the right panel is for individual spot counts in bins of 10 sunspots. Both panels show the difference, $G^*{=}G$ vs. G, though we note that for the right panel we still use the same symbols even though we refer to individual sunspots. The black line is the fitted line using the function $G^* - G = R_0 + Be^{(-aG)}$, where G^* are the mean counts of the primary observer (Wolfer) for a given count of the secondary observer G (Wolf), and R_0, B, and a are constants calculated for each pair of observers separately. The red circles depict the mean G^* values for each G column along with their 1σ uncertainty. The columns with G>8 in the left panel and column for G=4 in the right panel are populated using Monte Carlo Method.

separately as shown in Figure 13. The appropriateness of the above function for group number series was demonstrated with synthetic data which had as their basis the records from the Royal Greenwich Observatory (Usoskin, Kovaltsov, and Chatzistergos, 2016; Chatzistergos, 2017). We use all the columns of the matrix in the fitting process, which used weighted least mean squares to obtain the best-fit parameters. The fit was performed after excluding columns where it varied from the true mean G^* by more than one group.

The column matrices are populated for the excluded columns using a PMF created from a bootstrap Monte Carlo (MC) simulation. In order to do this, the two observers' overlapping days are divided in half at random, rebuilt the matrix using these half-statistics, and then recalculated the matrix's fit. The outcome of this simulation was used as a PMF for the relevant column in the matrix after this process was performed 1000 times as shown in Figure 14.

By swapping out every daily count G from the matrix with the PMF of the calibrated counts G^*, each secondary observer was calibrated to the backbone observer. By doing this, we directly applied the secondary observer's observations to the backbone condition without assuming anything about the nature of the link and with a clear error estimate. In order to create a distribution based on all available observers, composite series for each backbone are created by averaging all the PMFs of all the available observations for every day. This composite of averaged PMFs clearly includes any potential inconsistencies.
The estimation of G^* can be obtained from the normalized experimental values using the following formulas:

$$\widehat{G}^*_{ij;l} = \frac{\#\{N_1 = i \text{ and } N_{2,l} = j\}}{\#\{N_{2,l} = j\}} \tag{3}$$

and

$$\widehat{G}^*_{ij;l} = \frac{\widehat{G}^*_{ij;0}}{\sum_{l=1}^{n} \widehat{G}^*_{lj;0}} \tag{4}$$

where $\widehat{G}^*_{ij;0}$ represents the estimated count of occurrences where N_1 takes the value i and $N_{2,l}$ takes the value j, divided by the count of occurrences where $N_{2,l}$ takes the value j. $\widehat{G}^*_{ij;l}$ represents the normalized value of $\widehat{G}^*_{ij;0}$, divided by the sum of all $\widehat{G}^*_{lj;0}$ values for a fixed j. #{} signifies the count or number of events satisfying the conditions within the brackets.

A more detailed description of the method can be found in Chatzistergos et al. (2017). In the following Sections we describe how we adapted the method to produce SN2.3.

6.1. Construction of SNV2.3

In this study, we present the version 2.3 of SN, which is a complete reconstruction of SN for the period 1816–1944 by using calibration matrices, and including all the datasets that are being recovered from the archives from observers all over the world (e.g. Arlt et al., 2013; Bhattacharya et al., 2021; Carrasco, 2021; Hayakawa et al., 2019, 2020, 2021a), in addition to what is available in the *Mittheilungen*. Note that we named this version 2.3 as we are only reconstructing a part of the series. We shall publish a complete version (V3) in a later publication once the recovered records from Zurich ETH library are available in digital format and additional early observers have been recovered.

One of the major adaptations of the method described in Section 6, that we propose for this reconstruction is the treatment of groups and spots separately. It has been well established that sunspot counts (f) and sunspot groups (g) have different statistical distributions (Dudok de Wit, Lefèvre, and Clette, 2016; Mathieu et al., 2019) and calibrating their combination that is the Sunspot Number (SN) ($= 10g + f$) as a single index would not capture the entire picture. We calibrate the groups on a one-to-one basis, however, we use a binning method whereby sunspots falling within a range of 1 to 10, 10-20, 20-30 and so on, are combined and represented as a single category with the value of 1, 2, 3 and so on, and then the calibration is carried out. The 0 counts are kept as it is. Examples of filled matrices after the bootstrap Monte Carlo process for groups and spot counts separately are shown in Figure 14 for secondary observer Wolf and primary observer Wolfer. It is interesting to note the deviation from linearity in both the group sunspot numbers and the sunspot numbers.

The composite series that is formed by averaging all the probability mass functions (PMFs) of the available observers for each day, allows us to create a comprehensive distribution based on the entire set of data. This composite of averaged PMFs clearly captures any inconsistency in the data that may exist over the overlapping period.

We have introduced a substantial modification to the existing method by adopting a novel approach for determining the f or g of the day. The previous method used in Chatzistergos et al. (2017) relied on a linear combination of probability mass functions (PMFs) multiplied by the corresponding f or g values. In contrast, we have opted to select the 50% quantile of the PMFs as the spot count or group number of the day. This modification effectively addresses a significant drawback of the previous method, which resulted in artificially elevated minima.

By selecting the median value as the spot count or group number, we overcome the issue of underestimating the weight assigned to zero values in the PMFs. In the previous method, the linear combination approach led to a diminished significance of zero values, thus contributing to the elevated minima observed in Chatzistergos et al. (2017). In contrast, our modified approach automatically assigns a value of zero if the PMF of 0 count for any given day exceeds 50%. This adjustment ensures that the presence of zero values is appropriately considered, rectifying the previous method's tendency to yield elevated minima.

Next, we combined the obtained f and g values in the classical way $(10g + f)$ to get the SN for the day. We estimated the daily error by using Mean Absolute Deviation (MAD) method (Konno and Koshizuka, 2005). The Wolfer backbone is taken as the reference while other backbones are calibrated by daisy chaining. We use a different procedure to cross-calibrate the backbone series compared to the individual observers since for each backbone series we have daily probability mass functions (PMFs) rather than a single G. The calibration matrix was built using a Monte Carlo simulation. The G values were chosen at random based on the PMFs and used to fill the matrix for each day with simultaneous observations from both backbone series. The final matrix was calculated as the average of all iterations after this process was done 1000 times per day. The calibration matrix correctly accounted for the composite backbone series by taking into account the PMFs' variability. Detailed information on the backbone calibration can be found in Chatzistergos et al. (2017).

The SNV2.3 series is available in daily, monthly and yearly values in https://www.sidc.be/SILSO/DATA/Schwabe-Wolf/Sunspot%20Series%20Version%202.3%20-%20SN_V2.3_daily.csv, https://www.sidc.be/SILSO/DATA/Schwabe-Wolf/

Sunspot%20Series%20Version%202.3%20-%20SN_V2.3_monthly.csv, and https://www.sidc.be/SILSO/DATA/Schwabe-Wolf/Sunspot%20Series%20Version%202.3%20-%20SN_V2.3_yearly.csv respectively.

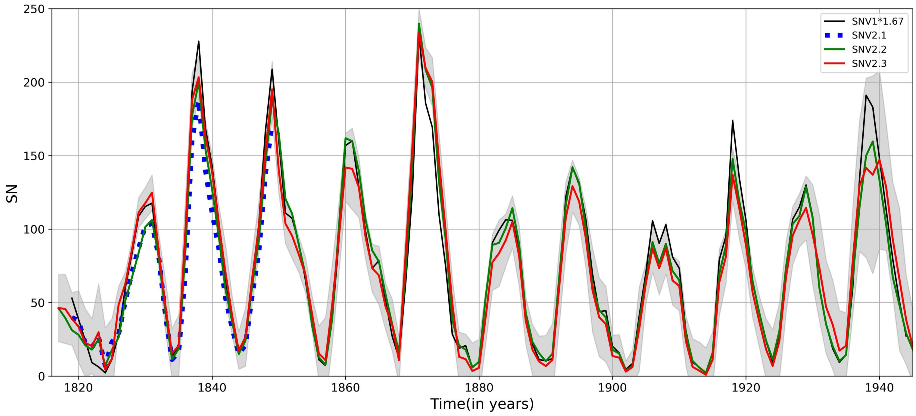

Figure 15. Annually averaged SN series. The grey shaded area represents 1σ uncertainty of SNV2.3. We note that SNV1 has been multiplied by 1.67 to bring it to the same scale as the other series.

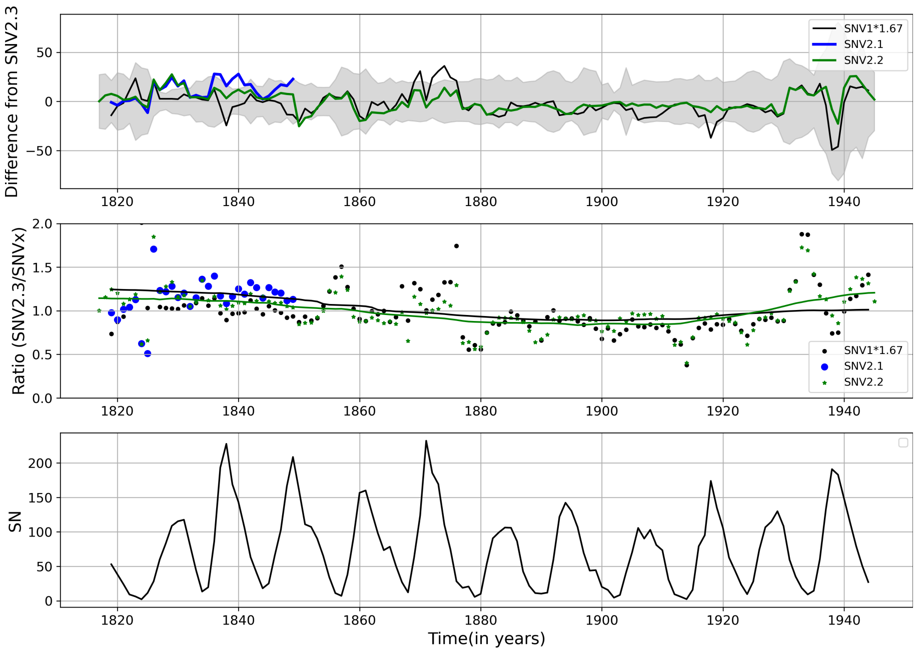

Figure 16. The first panel shows the difference between various SN series, as indicated in the legend, from SNV2.3. Shown are annually averaged values, while the grey shaded area represents the 1σ uncertainty of SNV2.3. Positive numbers imply SNV2.3 is higher that the respective SN series that it is compared to. The second panel shows the ratio between SNV2.3/SNVx. The scattered points show the actual ratio whereas the solid lines are the smoothed by moving average ratios of 10 years. The third panel shows the Sunspot Number (SNV1) for reference.

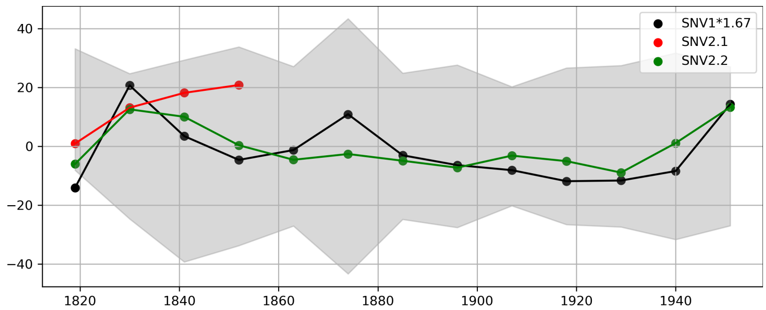

Figure 17. difference between various SN series, as indicated in the legend, from SNV2.3. Shown are solar cycle averaged values, while the grey shaded area represents the 1σ uncertainty of SNV2.3. Positive numbers imply SNV2.3 is higher that the respective SN series that it is compared to. The red dashed line marks difference of 0.

Our complete reconstruction, SNV2.3, along with most existing reconstructed SN series, except for SNV2, are shown in Figure 15. Figure 16 depicts a comparison between the newly constructed SNV2.3 series and previous SN series. It is evident from the figure that SNV2.3 exhibits a strong agreement with the existing series, but also marked differences. The minimum values of SNV2.3 align well with all versions of the SN, while the maximum values are slightly lower compared to SNV2 and SNV2.2. This can be, at least partly, attributed to the calibration method employed. We implemented significant modifications concerning data availability spanning from 1816 to 1825. We made Tevel the backbone for the period before Schwabe (1816–1825), as opposed to SNV1 and used most of the recounted data as presented in Table 1 and replaced Kremsmünster data from the Mittheilungen by Derfflinger (Hayakawa) data (Hayakawa et al., 2020) as explained in Section 4.1.
Additionally, we offer daily data starting from year 1816 instead of 1818 for SNV1. This expanded coverage became possible due to the retrieval of newly recovered data contributed by a variety of observers as listed in Table 1. It is important to note that while some of the newly recovered datasets from Stephan Prantner (Hayakawa et al., 2021c), Thaddäus Derfflinger (Hayakawa et al., 2020), and Karl von Lindener do extend back to 1800, we do not consider the data prior to 1816 for our reconstruction because the data coverage before 1816 is rather limited (See Figure 8). We plan to eventually expand this dataset further as we recover more data to encompass the earlier period.

In Figure 15 SNV2.1, SNV2.2, and SNV2.3 exhibit a strange behavior around years 1822–1823: it can be attributed to the significant lack of datapoints during those years as can be seen in Figure 8 (65 points in 1822 and 62 points in 1823, compared to 106 and 94 in 1821 and 1824 respectively) as reflected by the larger error bars. SNV2.2 exhibits higher values after 1850. However, prior to 1840, specifically from 1823 to 1849, SNV2.3 slightly surpasses SNV2.2. One possible explanation for this variation is that, before 1849, Schwabe was the sole consistent observer alongside Pastorff (See Figure 10), who utilized multiple telescopes with respective k-factors of 0.48, 1.06 and 0.80, as indicated in Table 2. When all series were used simultaneously, the k-factors of Pastorff might have underestimated the sunspot counts. Also, the stability of Schwabe's data before 1840 is still questionable as his counting method evolved since 1825. However, due to a lack of overlapping observers, as explained in Bhattacharya et al. (2023), we do not have any conclusive proof of any deviation. Since SNV2.2 uses linear k-factors, this trend remains undetected. However, SNV2.3 uses non-linear calibrations and may have solved this issue before 1849. Nevertheless, this question of instability in early Schwabe data remains open until overlapping observers such as Pastorff's data is recounted by modern conventions. In contrast to SNV2.2, in SNV2.3 the utilization of the 50% quantile of the PMFs as the spot count or group number of the day helps avoid such underestimation in the process.

The difference between SNV2.2 and SNV2.3 for the years 1860–1880 is due to the choice of backbones. We use only Weber backbone (see Section 4.3) in the construction of SNV2.3 whereas we use a combination of Wolf and Weber backbone in the construction of SNV2.2 (see Section 5).

Apart from the procedural difference, another major difference between SNV2.2 and SNV2.3 is the backbone to backbone calibration. In SNV2.3 we calibrated the whole backbone series to each other. In other words we constructed the entire backbone series and then these series are calibrated against each other. Whereas in SNV2.2 we calibrated only the backbones to each other, that is, Wolfer to Weber+Wolf-PM composite and Weber+Wolf-PM composite to Schwabe, instead of the whole series. This difference might contribute to this overall downward appearance of SNV2.3 after 1870. Also note that after 1928, the number of observers decreases significantly (see Figure 12) and few observers can bridge the Wolfer-Brunner transition (Hayakawa et al., 2023b). Nevertheless, SNV2.2 is within SNV2.3's confidence levels. From a broader perspective, as depicted in the Figure 17, SNV2.2 consistently maintains a higher scale than SNV2.3 in terms of cycle-to-cycle differences which is expected due to the overestimation of maxima when using k-factors.

The significant difference between SNV2.1 and SNV2.2 is the choice of backbone. We used the Schwabe counts from the *Mittheilungen* in SNV2.1 (Bhattacharya et al., 2023), whereas for SNV2.2 we chose the more stable Schwabe (Arlt) recounts. Since Schwabe (Arlt) is counted using modern conventions, the counts differ to those from the *Mittheilungen* even though they comes from the same observer. Also note that Pastorff used 3 different techniques to record sunspots. We included all three series with appropriate k-factors in SNV2.2, whereas only 2 series were included in SNV2.1. Hence, the difference before 1820. It is also interesting to note that SNV2.1 is scaled to Wolf's standard telescope and then adjusted to Wolfer scale. However, SNV2.2 is scaled to the composite (Wolf's portable telescope + Weber) and then scaled to Wolfer scale. Schwabe (Arlt) also straddles the Schwabe-Wolf transition period (1849) whereas Schwabe from the *Mittheilungen* stops in 1849, making SNV2.2 more robust.

SNV2.3 surpasses scaled SNV1 (SNV1*1.67) in cycle 7 (peaked in 1829), cycle 11 (peaked in 1870), and cycle 17 (peaked in 1937). These higher maximum values can be attributed to the inclusion of new observers' data in SNV2.3 that were lacking in scaled SNV1, such as Spoerer data (Diercke, Arlt, and Denker, 2015), Tevel's data (Carrasco et al., 2020), Schwabe's data (Arlt et al., 2013), Carrrington's data (Bhattacharya et al., 2021), and Misawa's data (Hayakawa et al., 2023b). Additionally, these recounts adhere to the modern convention of grouping sunspots. Furthermore, the adoption of more stable series as backbones in SNV2.3 may have contributed to resolving the underestimation issue observed in the existing series. As expected, SNV2.3 demonstrates a closer alignment with the newly constructed SNV2.2 compared to scaled SNV1, as SNV2.2 and SNV2.3 utilise the same backbones and observers.

7. Conclusion

While the application of the k-factors has been associated with the reconstruction of the series of Sunspot Numbers in the past, it needs to be re-evaluated with modern means to address the inhomogeneities that arose in the SN series due to its incorrect application. Among the major limitations of k-factors is the assumption of linearity; therefore, in order to address this problem, we implemented the method described in Section 6. Note that, the method developed by Chatzistergos et al. (2017) only addressed group counts and not the combination of groups and spots: the Sunspot Number (SN).

By applying a non-linear approach for cross-calibration of the SN series, we addressed the Wolf-Wolfer scale transition discrepancy. Studies like Friedli (2020); Usoskin, Kovaltsov, and Chatzistergos (2016) have already indicated that Wolf-Wolfer's counts relate in a non linear way and stitching them together after cross-calibrating them with a linear k-factor can lead to major overestimation/underestimation of the series. However, no particular efforts were made prior to this study to eliminate the k-factors from the SN series, unlike the GN series which saw multiple reconstruction efforts. In this study, we addressed this shortcoming of the k-factors and produced a new SN Series (SNV2.3) with a non-parametric approach and by utilizing all currently available raw data from 1816 to 1944.

As a first step we assessed the stability of long running observers, in particular of Wolfer and Wolf whose records were the basis of all previous SN reconstructions. Wolfer, who observed from 1876 to 1893, is recognized as one of the most stable observers and played a crucial role as backbone in the construction of SNV1 series following Wolf's death. Even though we find Wolf's standard telescope not showing any significant trends, we concluded that the observations with Wolf's portable telescope are not suitable as a reference for sunspot number (SN) construction due to constant degradation in quality as explained above. Observations from Wolf's portable telescope also exhibit a decline in recording odd sunspot counts over time, with a complete omission of odd spot counts in the year 1893.

For the construction of SNV2.3, four observers were chosen as the backbones based on the stability of their records and their extensive observation periods. Namely, the chosen backbone observers were: Tevel (1816–1836), Schwabe (1825–1867), Weber (1863-–1883), and Wolfer (1876—1928). This is the first SN reconstruction to replace Wolf with Weber due to its greater level of reliability.

In this study we also present SN Version 2.2 which is constructed using Wolf's traditional method of computing the k-factors for the period 1816–1944. SNV2.2 was constructed as a continuation of SNV2.1 (Bhattacharya et al., 2023) to ensure consistency and facilitate comparison. Note that for SNV2.2 we used the same backbones as for SNV2.3, with the only exception that instead of just Weber we used a composite series by stitching the early data by Wolf to those by Weber.

Throughout this study, we used all the data available in the *Mittheilungen* and further sources that have been recently recovered. However, our reconstruction is limited to before 1945 due to the unavailability of digitized raw datasets in 1945–1979. It is important to note that additional data from the ETH library (https://www.e-manuscripta.ch/search/quick?query=Tabellenbl%C3%A4tter+Sonnenbeobachtungen) has been recovered for the years after 1944, but their digitization in pdf formats is still ongoing. Once the data are digitized in usable format and become available, we plan to incorporate them into our analysis. This will allow us to extend our reconstruction and provide a comprehensive understanding of the Sunspot Number in a future version, that we will call SNV3 as it will be complete to the present with the addition of the SILSO data (https://www.sidc.be/SILSO/home).

Gathering and exploiting of all the available data sources is the goal of our new project, FARSuN: to improve access to the historical sunspot record, the primary index for understanding solar variability over the last 400 years. The project involves making the raw sunspot data from 1610 to 1980 accessible to researchers inside and outside the sunspot community. This includes data preprocessing, quality assessment, and standardization to make it Findable, Accessible, Interoperable, and Reusable (FAIR). The project involves experts from multiple fields, including statistics, historical datasets, virtual observatory, time-series, and optical character recognition. The output of the project will be a compilation of historical Sunspot Number (SN) available via standard virtual observatory tools. The project will improve the skills of the World Data Center-SILSO and the Royal Observatory of Belgium (ROB) to manage and preserve data collections, increase the visibility of this key dataset in the international community, and strengthen the activities led by the WDC-SILSO and ROB. The resulting new version of the International Sunspot Number (SN) series envisioned using the output of FARSuN will impact scientific knowledge in domains in which the SN is used as a key input (https://www.sidc.be/FARSuN).

Overall, our analysis serves as a reminder that the reconstruction of the Sunspot Number is an ongoing process, driven by the accessibility, availability and analysis of historical data. We emphasize the significance of future research endeavors to incorporate the newly recovered data and advance our understanding of solar activity throughout history.

Acknowledgements This work was supported by a PhD grant awarded by the Royal Observatory of Belgium to S. Bhattacharya. L. Lefèvre, T. Chatzistergos, and H. Hayakawa wish to acknowledge the support of ISSI https://www.issibern.ch/teams/sunspotnoser/. The authors would like to thank Dr. Frédéric Clette for his constructive comments. H. Hayakawa's research was conducted under the financial supports of

JSPS Grant-in-Aids JP20K20918, JP20H05643, JP21K13957, and JP22K02956. H. Hayakawa has been part funded by JSPS Overseas Challenge Program for Young Researchers, JSPS Overseas Challenge Program for Young Researchers, the ISEE director's leadership fund for FYs 2021–2023, Young Leader Cultivation (YLC) program of Nagoya University, Tokai Pathways to Global Excellence (Nagoya University) of the Strategic Professional Development Program for Young Researchers (MEXT), the young researcher units for the advancement of new and undeveloped fields, Institute for Advanced Research, Nagoya University of the Program for Promoting the Enhancement of Research Universities, and NIHU Multidisciplinary Collaborative Research Projects NINJAL unit "Rediscovery of Citizen Science Culture in the Regions and Today". HH thanks the Misawa Memorial Archives and Nagano Prefectural Suwa Seiryo Senior High School for letting us access, research, and publicize Misawa's sunspot drawings, Tom Teague for his recounting efforts of Carrington's book, P. Amand for letting him access the data in Kremsmünster Observatory, and Frédéric Clette for his valuable discussions for usage of historical sunspot records.

Disclosure of Potential Conflicts of Interest The authors declare that they have no conflicts of interest.

References

Aparicio, A.J., Vaquero, J.M., Carrasco, V., Gallego, M.: 2014, Sunspot numbers and areas from the Madrid Astronomical Observatory (1876–1986). *Solar Physics* **289**, 4335.

Arlt, R., Vaquero, J.M.: 2020a, Historical sunspot records. *Living Reviews in Solar Physics* **17**, 1. DOI. ADS.

Arlt, R., Vaquero, J.M.: 2020b, Historical sunspot records. *Living Reviews in Solar Physics* **17**, 1. DOI. ADS.

Arlt, R., Leussu, R., Giese, N., Mursula, K., Usoskin, I.G.: 2013, Sunspot positions and sizes for 1825-1867 from the observations by Samuel Heinrich Schwabe. *Mon. Not. Roy. Astron. Soc.* **433**, 3165. DOI. ADS.

Bhattacharya, S., Teague, E.T.H., Fay, S., Lefèvre, L., Jansen, M., Clette, F.: 2021, A Modern Reconstruction of Richard Carrington's Observations (1853-1861). *Solar Phys.* **296**, 118. DOI. ADS.

Bhattacharya, S., Lefèvre, L., Hayakawa, H., Jansen, M., Clette, F.: 2023, Scale Transfer in 1849: Heinrich Schwabe to Rudolf Wolf. *Solar Phys.* **298**, 12. DOI. ADS.

Carrasco, V.M.S.: 2021, Number of Sunspot Groups and Individual Sunspots Recorded by Tevel for the Period 1816-1836 in the Dalton Minimum. *Astrophys. J.* **922**, 58. DOI. ADS.

Carrasco, V.M.S., Vaquero, J.M., Gallego, M.C., Trigo, R.M.: 2013, Forty two years counting spots: Solar observations by D.E. Hadden during 1890-1931 revisited. **25**, 95. DOI. ADS.

Carrasco, V.M.S., Gallego, M.C., Arlt, R., Vaquero, J.M.: 2020, Revisiting the Amplitude of Solar Cycle 9: The Case of Sunspot Observations by W.C. Bond. *Solar Phys.* **295**, 127. DOI. ADS.

Carrasco, V.M.S., Gallego, M.C., Villalba Álvarez, J., Vaquero, J.M., Hayakawa, H.: 2021a, Analyses of Early Sunspot Records by Jean Tarde (1615 - 1617) and Jan Smogulecki (1621 - 1625). *Solar Phys.* **296**, 170. DOI. ADS.

Carrasco, V.M.S., Munoz-Jaramillo, A., Nogales, J.M., Gallego, M.C., Vaquero, J.M.: 2021b, Sunspot Catalog (1921-1935) and Area Series (1886-1940) from the Stonyhurst College Observatory. **256**, 38. DOI. ADS.

Carrasco, V.M.S., Muñoz-Jaramillo, A., Gallego, M.C., Vaquero, J.M.: 2022, Revisiting Christoph Scheiner's Sunspot Records: A New Perspective on Solar Activity of the Early Telescopic Era. *Astrophys. J.* **927**, 193. DOI. ADS.

Carrasco, V.M., Vaquero, J.M., Aparicio, A.J., Gallego, M.: 2014, Sunspot catalogue of the Valencia Observatory (1920–1928). *Solar Physics* **289**, 4351.

Carrasco, V., Gallego, M., Arlt, R., Vaquero, J.: 2020a, Revisiting the amplitude of solar cycle 9: the case of sunspot observations by WC Bond. *Solar Physics* **295**, 127.

Carrasco, V., Bertolin, C., Domínguez-Castro, F., de Ferri, L., Gallego, M., Vaquero, J.: 2020b, Sunspot records by Antonio Colla just after the Dalton Minimum. *Solar Physics* **295**, 112.

Chatzistergos, T.: 2017, *Analysis of historical solar observations and long-term changes in solar irradiance*, *PhD thesis*, Uni-edition. ISBN 978-3-944072-55-5. DOI. https://ui.adsabs.harvard.edu/abs/2017PhDT......259C.

Chatzistergos, T., Krivova, N.A., Yeo, K.L.: 2023, *Long-term changes in solar activity and irradiance*, arXiv. DOI. http://arxiv.org/abs/2303.03046.

Chatzistergos, T., Usoskin, I.G., Kovaltsov, G.A., Krivova, N.A., Solanki, S.K.: 2017, New reconstruction of the sunspot group numbers since 1739 using direct calibration and "backbone" methods. *Astron. Astrophys.* **602**, A69. DOI. ADS.

Clette, F., Lefèvre, L.: 2016, The New Sunspot Number: Assembling All Corrections. *Solar Phys.* **291**, 2629. DOI. ADS.

Clette, F., Berghmans, D., Vanlommel, P., Van der Linden, R.A.M., Koeckelenbergh, A., Wauters, L.: 2007,. *Advances in Space Research* **40**, 919.

Clette, F., Svalgaard, L., Vaquero, J.M., Cliver, E.W.: 2014, Revisiting the Sunspot Number. A 400-Year Perspective on the Solar Cycle. *Space Sci. Rev.* **186**, 35. DOI. ADS.

Clette, F., Cliver, E.W., Lefèvre, L., Svalgaard, L., Vaquero, J.M., Leibacher, J.W.: 2016, Preface to Topical Issue: Recalibration of the Sunspot Number. *Solar Phys.* **291**, 2479. DOI. ADS.

Clette, F., Lefèvre, L., Bechet, S., Ramelli, R., Cagnotti, M.: 2021, Reconstruction of the sunspot number source database and the 1947 Zurich discontinuity. *Solar Physics* **296**, 1.

Clette, F., Lefèvre, L., Chatzistergos, T., Hayakawa, H., Carrasco, V.M.S., Arlt, R., Cliver, E.W., Dudok de Wit, T., Friedli, T.K., Karachik, N., Kopp, G., Lockwood, M., Mathieu, S., Muñoz-Jaramillo, A., Owens, M., Pesnell, D., Pevtsov, A., Svalgaard, L., Usoskin, I.G., van Driel-Gesztelyi, L., Vaquero, J.M.: 2023, Recalibration of the Sunspot-Number: Status Report. *Solar Phys.* **298**, 44. DOI. ADS.

Cliver, E.W., Ling, A.G.: 2016, The Discontinuity Circa 1885 in the Group Sunspot Number. *Solar Phys.* **291**, 2763. DOI. ADS.

Diercke, A., Arlt, R., Denker, C.: 2015, Digitization of sunspot drawings by Spörer made in 1861–1894. *Astronomische Nachrichten* **336**, 53.

Dudok de Wit, T., Lefèvre, L., Clette, F.: 2016, Uncertainties in the sunspot numbers: estimation and implications. *Solar Physics* **291**, 2709.

Dudok de Wit, T., Lefèvre, L., Clette, F.: 2016, Uncertainties in the Sunspot Numbers: Estimation and Implications. *Solar Physics* **291**. DOI.

Ermolli, I., Chatzistergos, T., Giorgi, F., Carrasco, V.M.S., Aparicio, A.J.P.: 2023, Solar Observations by Angelo Secchi. I. Digitization of Original Documents and Analysis of Group number over 1853-1878. *The Astrophysical Journal Supplement Series* **under review**, under review. DOI.

Friedli, T.K.: 2016, Sunspot Observations of Rudolf Wolf from 1849 - 1893. *Solar Phys.* **291**, 2505. DOI. ADS.

Friedli, T.K.: 2020, Recalculation of the Wolf Series from 1877 to 1893. *Solar Phys.* **295**, 72. DOI. ADS.

Fujiyama, M., Hayakawa, H., Iju, T., Kawai, T., Toriumi, S., Otsuji, K., Kondo, K., Watanabe, Y., Nozawa, S., Imada, S.: 2019, Revisiting Kunitomo's sunspot drawings during 1835–1836 in Japan. *Solar Physics* **294**, 43.

Gray, L.J., Beer, J., Geller, M., Haigh, J.D., Lockwood, M., Matthes, K., Cubasch, U., Fleitmann, D., Harrison, G., Hood, L., Luterbacher, J., Meehl, G.A., Shindell, D., van Geel, B., White, W.: 2010, Solar Influences on Climate. *Reviews of Geophysics* **48**, 4001. DOI. http://adsabs.harvard.edu/abs/2010RvGeo..48.4001G.

Hayakawa, H., Ebihara, Y., Willis, D.M., Toriumi, S., Iju, T., Hattori, K., Wild, M.N., Oliveira, D.M., Ermolli, I., Ribeiro, J.R., Correia, A.P., Ribeiro, A.I., Knipp, D.J.: 2019, Temporal and Spatial Evolutions of a Large Sunspot Group and Great Auroral Storms Around the Carrington Event in 1859. *Space Weather* **17**, 1553. DOI. ADS.

Hayakawa, H., Besser, B.P., Iju, T., Arlt, R., Uneme, S., Imada, S., Bourdin, P.-A., Kraml, A.: 2020, Thaddäus Derfflinger's Sunspot Observations during 1802-1824: A Primary Reference to Understand the Dalton Minimum. *Astrophys. J.* **890**, 98. DOI. ADS.

Hayakawa, H., Iju, T., Kuroyanagi, C., Carrasco, V.M.S., Besser, B.P., Uneme, S., Imada, S.: 2021a, Johann Christoph Müller's Sunspot Observations in 1719 - 1720: Snapshots of the Immediate Aftermath of the Maunder Minimum. *Solar Phys.* **296**, 154. DOI. ADS.

Hayakawa, H., Uneme, S., Besser, B.P., Iju, T., Imada, S.: 2021b, Stephan Prantner's Sunspot Observations during the Dalton Minimum. *Astrophys. J.* **919**, 1. DOI. ADS.

Hayakawa, H., Uneme, S., Besser, B.P., Iju, T., Imada, S.: 2021c, Stephan Prantner's Sunspot Observations during the Dalton Minimum. *Astrophys. J.* **919**, 1. DOI. ADS.

Hayakawa, H., Arlt, R., Iju, T., Besser, B.: 2023a, Karl von Lindener's Sunspot Observations during 1800 – 1827: Another Long-Term Dataset for the Dalton Minimum. *Journal of Space Weather and Space Climate* **in press**, in press. DOI.

Hayakawa, H., Suyama, T., Clette, F., Bhattacharya, S., Lefèvre, L., Ohnishi, K.: 2023b, Katsue Misawa's Sunspot Observations in 1921 – 1934: A Primary Reference for the Wolfer-Brunner Transition. *Monthly Notices of the Royal Astronomical Society* **in press**, in press. DOI.

Hoyt, D.V., Schatten, K.H.: 1998a, Group Sunspot Numbers: A New Solar Activity Reconstruction. *Solar Phys.* **179**, 189. DOI. ADS.

Hoyt, D.V., Schatten, K.H.: 1998b, Group Sunspot Numbers: A New Solar Activity Reconstruction. *Solar Phys.* **181**, 491. DOI. ADS.

IPCC: 2021, *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA. DOI.

Izenman, A.J.: 1985, J.R. Wolf and the Zurich Sunspot Relative Numbers. *The Mathematical Intelligencer* **7**, 79.

Kendall, M.G.: 1962, Rank correlation methods. Technical report.

Konno, H., Koshizuka, T.: 2005, Mean-absolute deviation model. *Iie Transactions* **37**, 893.

Lefevre, L., Clette, F., Mathieu, S., Delouille, V., von Sachs, R.: 2018, The Sunspot Number recalibration. In: *Catalyzing Solar Connections*, 2. ADS.

Mathieu, S., von Sachs, R., Ritter, C., Delouille, V., Lefèvre, L.: 2019,. *Astrophys. J.* **886**, 7.

Owens, B.: 2013, Long-term research: Slow science. *Nature* **495**, 300. DOI. ADS.

Sivaraman, K.R., Gupta, S.S., Howard, R.F.: 1993, Measurement of Kodiakanal White-Light Images - Part One. *Solar Physics* **146**, 27. DOI. https://ui.adsabs.harvard.edu/abs/1993SoPh..146...27S.

Solanki, S.K., Krivova, N.A., Haigh, J.D.: 2013, Solar Irradiance Variability and Climate. *Annual Review of Astronomy and Astrophysics* **51**, 311. DOI. http://www.annualreviews.org/doi/abs/10.1146/annurev-astro-082812-141007.

Spoerer, F., Maunder, E.: 1890, Prof. Spoerer's researches on Sun-spots. *Monthly Notices of the Royal Astronomical Society* **50**, 251.

Svalgaard, L.: 2010, Updating the Historical Sunspot Record. In: Cranmer, S.R., Hoeksema, J.T., Kohl, J.L. (eds.) *SOHO-23: Understanding a Peculiar Solar Minimum*, *Astronomical Society of the Pacific Conference Series* **428**, 297. DOI. ADS.

Svalgaard, L., Schatten, K.H.: 2016, Reconstruction of the Sunspot Group Number: The Backbone Method. *Solar Phys.* **291**, 2653. DOI. ADS.

Usoskin, I., Kovaltsov, G., Kiviaho, W.: 2021, Robustness of Solar-Cycle Empirical Rules Across Different Series Including an Updated Active-Day Fraction (ADF) Sunspot Group Series. *Solar Phys.* **296**, 13. DOI. ADS.

Usoskin, I.G., Kovaltsov, G.A., Chatzistergos, T.: 2016, Dependence of the Sunspot-group size on the level of solar activity and its influence on the calibration of solar observers. *Solar Physics* **291**, 3793.

Usoskin, I.G., Kovaltsov, G.A., Lockwood, M., Mursula, K., Owens, M., Solanki, S.K.: 2016a, A New Calibrated Sunspot Group Series Since 1749: Statistics of Active Day Fractions. *Solar Phys.* **291**, 2685. DOI. ADS.

Usoskin, I.G., Kovaltsov, G.A., Lockwood, M., Mursula, K., Owens, M., Solanki, S.K.: 2016b, A New Calibrated Sunspot Group Series Since 1749: Statistics of Active Day Fractions. *Solar Phys.* **291**, 2685. DOI. ADS.

Vaquero, J.M., Vázquez, M.: 2009, *The Sun recorded through history* **361**, Springer Science & Business Media.

Vaquero, J.M., Svalgaard, L., Carrasco, V.M.S., Clette, F., Lefèvre, L., Gallego, M.C., Arlt, R., Aparicio, A.J.P., Richard, J.-G., Howe, R.: 2016, A Revised Collection of Sunspot Group Numbers. *Solar Phys.* **291**, 3061. DOI. ADS.

Willamo, T., Usoskin, I.G., Kovaltsov, G.A.: 2017, Updated sunspot group number reconstruction for 1749-1996 using the active day fraction method. *Astron. Astrophys.* **601**, A109. DOI. ADS.

Wolf, R.: 1848, *Nachrichten von der Sternwarte in Bern*. DOI. ADS.

Wolf, R.: 1859, Mittheilungen über die Sonnenflecken IX. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **1**, 207. ADS.

Wolf, R.: 1861, Mittheilungen über die Sonnenflecken XIII. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **2**, 83. ADS.

Wolf, R.: 1862, Mittheilungen über die Sonnenflecken XIV. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **2**, 119. ADS.

Wolf, R.: 1864, Mittheilungen über die Sonnenflecken XVI. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **2**, 163. ADS.

Wolf, R.: 1878b, Beobachtungen der Sonnenflecken, *ETH Bibliothek, Hochschularchiv*, **Hs 368:46**, 1Bd., unpublished manuscript, Zurich..

Wolf, R.: 1890, Astronomische Mittheilungen LXXVI. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **8**, 217. ADS.

Wolf, R., Brunner, W.: 1929, Astronomische Mittheilungen Nr CXVIII. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **12**, 283. ADS.

Wolf, R., Wolfer, A.: 1894, Astronomische Mittheilungen LXXXIV. *Astronomische Mitteilungen der Eidgenössischen Sternwarte Zurich* **9**, 109. ADS.