# Statistics-informed bounds for active distribution network equivalents subject to large disturbances

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***Abstract*—Distributed energy resources (DERs) are expected to provide increasingly large amounts of energy and ancillary services to the grid. However, modelling distribution-connected assets in transmission studies is a challenge due to their sheer number, and due to the low visibility in distribution grids. To tackle this challenge, one of the main approaches consists in reducing detailed models of distribution grids into equivalent models, but few researchers considered the fact that detailed models are subject to many uncertainties. In this work, we proposed to derive equivalents based on quantiles of the behaviour of detailed models. Such equivalents can then be used to obtain statistics-informed bounds on the results of any transmission studies. We demonstrate the accuracy of this approach by comparing it to probabilistic transmission and distribution (T&D) simulations in both critical clearing time computations and simulations of cascading outages.**

***Index Terms*—Load modelling, dynamic equivalents, active distribution networks, Monte Carlo methods**

## I. Introduction

Over the last fifteen years, failures to reproduce blackouts in “post-mortem” analysis has led to renewed interest for load modelling in both industry and academia [1, 2]. This has led to the development of load models of varying complexities (from ZIP loads to composite models that include motors and distributed energy sources (DERs)), the main challenge being to find accurate parameters for those load models.

Two (complementary) approaches currently exist to do so: the component-based approach and the parameter identification approach. In the first approach, surveys are used to evaluate the share of load types (e.g. residential vs industrial, mining vs. steel industry, etc.), then some rules of association are used to derive the parameters of the considered load model [3]. As this approach is somewhat qualitative, the derived models should be validated against field measurements.

In the second approach, the load model parameters are tuned such that the model behaves as closely as possible to a reference for a given set of disturbances. The reference can either come from field measurements or simulation results. However, large disturbances are rare, so using measurements only will often lead to load models that are only valid for a specific set of disturbances and operating conditions. On the other hand, simulation results can be generated at will but require more modelling efforts. In early works, researchers used fully-detailed models of distribution grids as a reference [4], but, in practice, such models are very difficult to develop. This issue has only recently been addressed in [5–7].

In those works, a complete distribution model is also used as a reference, but its parameters are given quite wide probability density functions (pdfs) due to the low visibility in distribution grids. For example, in [5], the motor share in residential loads is given by a uniform distribution on [0, 20%] and the motor stator resistances are taken in [0.03 0.13] pu. Monte Carlo (MC) simulations are then used to analyse the behaviour of this uncertain model. In [5, 6], load model parameters are tuned to match the average behaviour of the complete distribution model (load models built this way are often referred to as a dynamic equivalent of the complete distribution model). However, as the actual behaviour can vary significantly from its mean (due to high uncertainties), this can lead to overly optimistic results. On the contrary, [7] uses machine-learning-based quantile forecasting to build an interval of potential responses in addition to the mean behaviour. However, they only considered mild disturbances (load steps) and did not test their load model in transmission studies.

Based on the above, we extend the work in [5, 6] by

- using two extreme quantiles of the uncertain distribution grid behaviour to build two dynamic equivalents. Those two equivalents are then used in transmission stability studies to get statistics-informed bounds on the results.
- showing that such equivalents are accurate even for large disturbances including cascading outages.
- drawing attention to the importance of correlations between the behaviours of loads at different locations.

## II. Methodology

The methodology used in this paper is based on the one proposed in [5, 6] for deriving dynamic equivalents of active distribution networks whose behaviour is not known with

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All results can be fully reproduced using the methods and data described in this paper and references provided.

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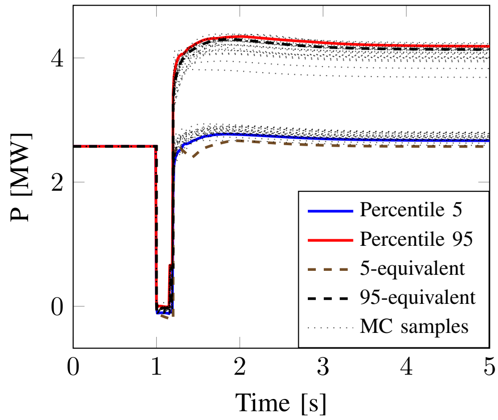



Figure 1. Comparison of the behaviour of the reduced (dashed lines) and unreduced (dotted and full lines) models for disturbance 6

certainty. The methodology consists in two main parts. In the first part, a detailed model of the considered distribution grid is built, and pdfs are defined for each of the parameters of this detailed model. Due to lack of data, such pdfs will often be relatively wide. Monte Carlo (MC) simulations are then performed on a set of $d$ disturbances to evaluate the impact of the uncertain modelling of the network on its behaviour. Of particular interest are the active and reactive power at the point of common coupling (PCC) with the transmission grid. Thus, the following values are extracted from the MC simulations: $\mathscr{P}_{P,i}(j,t)$ (resp. $\mathscr{P}_{Q,i}(j,t)$), the $i$th percentile over all MC simulations of the active (resp. reactive) during the $j$th disturbance at time $t$; and $\sigma_P(j,t)$ (resp. $\sigma_Q(j,t)$), the standard deviation of the active (resp. reactive) power for the same disturbance and at the same time.

In the second part, we build two dynamic equivalents of the distribution system model to bound the uncertain behaviour of the distribution grid: one (referred to as the 5-equivalent) that fits the 5th percentile of the active and reactive power of the distribution system, and one that fits the 95th percentile as shown in Fig. 1. It should be noted that percentiles are computed individually for each disturbance and each point in time. Those percentiles are thus synthetic and represent an envelope of the possible behaviours of the distribution grid, not a likely behaviour. Mathematically, the vector of parameters $\boldsymbol{\theta}$ of each dynamic equivalent is taken as the solution of the following weighted least-square optimisation problem:

$$\min_{\boldsymbol{\theta}} F(\boldsymbol{\theta}) = \frac{1}{d}\sum_{j=1}^{d}\left[F_P(\boldsymbol{\theta},j) + F_Q(\boldsymbol{\theta},j)\right] \tag{1}$$

$$\text{with } F_P(\boldsymbol{\theta},j) = \frac{1}{N}\sum_{t=1}^{T}\left[\delta_{i,P,j,t}\frac{P(\boldsymbol{\theta},j,t)-\mathscr{P}_{P,i}(j,t)}{\sigma_P(j,t)}\right]^2 \tag{2}$$

$$F_Q(\boldsymbol{\theta},j) = \frac{1}{N}\sum_{t=1}^{T}\left[\delta_{i,Q,j,t}\frac{Q(\boldsymbol{\theta},j,t)-\mathscr{P}_{Q,i}(j,t)}{\sigma_Q(j,t)}\right]^2 \tag{3}$$

$$\boldsymbol{\theta}^L \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}^U \tag{4}$$

with $i$ equals to 5 or 95, and where $\boldsymbol{\theta}^L$ and $\boldsymbol{\theta}^U$ are lower and upper bounds on the parameters of the dynamic equivalent. This formulation differs from the one in [5, 6] as we fit equivalents to percentiles of the distribution grid behaviour instead of its average, and by the addition of the term $\delta_{i,P,j,t}$ which is defined by

$$\delta_{5,P,j,t} = \begin{cases} 1 & \text{if } P(\boldsymbol{\theta},j,t) \leq \mathscr{P}_{P,i}(j,t) \\ \frac{1}{2} & \text{if } P(\boldsymbol{\theta},j,t) > \mathscr{P}_{P,i}(j,t) \end{cases} \tag{5}$$

$$\delta_{95,P,j,t} = \begin{cases} 1 & \text{if } P(\boldsymbol{\theta},j,t) > \mathscr{P}_{P,i}(j,t) \\ \frac{1}{2} & \text{if } P(\boldsymbol{\theta},j,t) \leq \mathscr{P}_{P,i}(j,t) \end{cases} \tag{6}$$

and the term $\delta_{i,Q,j,t}$ that has a similar definition. This factor results in a lower penalty for the equivalent that fits the 95th percentile if it consumes too much power and in a lower penalty for the 5-equivalent if it consumes too little. This leads to more conservative bounds on the behaviour of the distribution system. Different coefficients can be used if deemed necessary. A derivative-free approach is necessary to solve those optimisation problems. The Differential Evolution (DE) algorithm is chosen as it showed good performance in [5]. DE is stopped once sufficient accuracy is reached, i.e. when

$$F_P(\boldsymbol{\theta},j) \leq 1 \text{ and } F_Q(\boldsymbol{\theta},j) \leq 1 \tag{7}$$

In [5], a Least Absolute Shrinkage and Selection Operator (LASSO) was used in complement to DE in order to identify the parameters of the equivalent which are the most significant. This led to slow convergence in our case as more parameters deviate from their average value when fitting a 5th or 95th percentile than when fitting an average. If one wants to identify significant parameters, some form of sensitivity analysis might be more efficient.

It is common to "train" an equivalent by connecting it to an infinite bus to which voltage (and possibly angle and frequency) disturbances are applied [1, 4, 5]. However, we found that connecting it to a synchronous machine that has roughly the same rating as the total gross load through a line with an impedance of 0.1 pu leads to equivalents which are more accurate in transmission studies. This is because it allows for easier incorporation of fault-induced delayed voltage recovery events into the training set of the equivalent. It should be noted that the synchronous machine and line do not aim to be an equivalent of the transmission system as seen from the distribution system. As shown in section IV, our equivalents have a very good accuracy even when used at different locations with different system strengths, and during cascading outages during which the system strength at a given location can significantly vary.

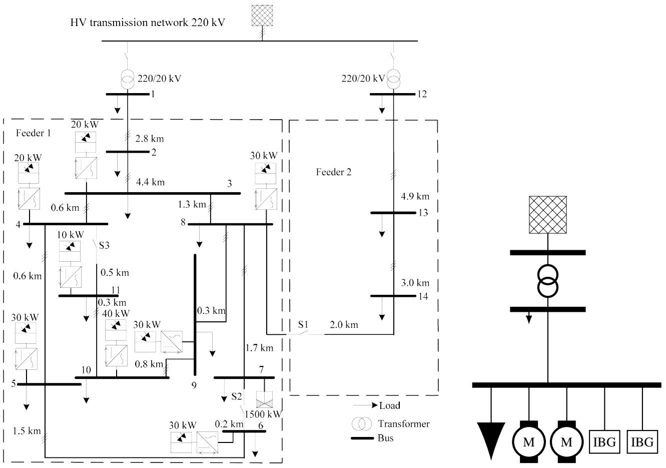
The training disturbances used in this work are impedant short-circuits of different durations applied at the PCC leading to more or less severe voltage drops as listed in Table I.

Additionally, if the equivalent is to be used in a case where the frequency goes outside of the [49, 51] Hz range (as in section IV-B), frequency ramps are added to the training disturbances to fit the frequency related parameters of the equivalent. 4 ramps are considered here, going respectively from 50 Hz to 47.5, 48.5, 51.5, and 52.5 Hz, all in 1.5s.

Table I
TRAINING DISTURBANCES

| Id | Voltage dip (pu) | Duration (ms) |
|---|---|---|
| 1 | 0.2 | 100 |
| 2 | 0.2 | 200 |
| 3 | 0.3 | 100 |
| 4 | 0.3 | 200 |
| 5 | 0.4 | 100 |
| 6 | 0.4 | 200 |
| 7 | 0.5 | 100 |

| Id | Voltage dip (pu) | Duration (ms) |
|---|---|---|
| 8 | 0.5 | 200 |
| 9 | 0.7 | 100 |
| 10 | 0.7 | 200 |
| 11 | 0.8 | 200 |
| 12 | 0.8 | 500 |
| 13 | 0.9 | 500 |
| 14 | 0.9 | 1000 |




(a) (b)

Figure 2. (a) CIGRE MV network with solar and wind generation [9], (b) Dynamic equivalent

## III. Test case

This section presents the distribution system, dynamic equivalent and transmission system used in this work. All grid models have been implemented using Dynawo [8] and are available online[1] along with the codes used in this work.

### A. Distribution system

The distribution system considered in this work is based on the CIGRE medium voltage (MV) network[2] [9] shown in Fig. 2a. Loads 1 and 12 (that account for almost 90% of the total load) have been removed as they represent separate feeders and do not include data on DERs. The distribution transformers have been downscaled to match the original voltage profile. Finally, the PV and wind installations have been upscaled to reach each 40% of the total load. Results presented in this paper were obtained at an operating point with 100% wind availability and 25% solar availability, leading to an instantaneous Inverter-Based Generation (IBG) penetration of 50%.

### B. Models of distribution grid components

IBGs are represented with a simplified model developed in [5]. This model includes grid support modules (voltage support and over-frequency support) and protections that comply with modern grid codes. The parameters of the model of each IBG (e.g. PLL response time, low-voltage ride through (LVRT) characterisation, voltage support coefficient, etc.) are sampled from a range of realistic values that are also taken from [5].

Loads are represented by an exponential load model in parallel with a first-order induction motor model. Like for IBGs, the parameters of those models (e.g. load static part exponent, motor inertia, motor stator resistance, etc.) are also randomised.

### C. Dynamic equivalent of the distribution system

The dynamic equivalent of the distribution system is a relatively simple model made of one load bus connected to the transmission grid by an equivalent impedance and a distribution transformer as shown in Fig. 2b. The load bus consists of an exponential load, two first-order inductor motor models (to allow for partial stalling of motors), and two IBG models (one that represents PV units, and one that represents wind).

As the model of IBGs in the dynamic equivalent are an aggregated of multiple units, they have to be slightly adapted in order to model the partial tripping of units. For example, instead of tripping the aggregate when the frequency drops below a threshold, the model will decrease its output power and continue to decrease it if the frequency continues to drop. When the frequency drops below a second threshold, the aggregate is completely disconnected. In this work, we use a slightly modified version of the aggregated model developed in [5]. Please refer to our source code for more information.

### D. Transmission & distribution (T&D) network

The T&D system considered is built by replacing all loads (except load 39 which represents an interconnection) of the IEEE 39-bus test system [10] by copies of the CIGRE MV network. Each of those copies is scaled such that its gross load is the same as the load it replaces in the 39-bus system. This leads to a reduction of the net load seen on the transmission side (from 6100 MW to 3600 MW) as DERs are now satisfying part of the total load. To account for this, generators 33, 34, and 36 (coal units) are disconnected and the setpoint of generator 39 is reduced from 1000 MW to 263 MW. To keep the system N-1 secure, a synchronous compensator is added in place of generator 33, and generator 38 is equipped with a faster AVR.

When simulating cascading outages (as in section IV-B), protection systems have to be modelled explicitly as they play a key role in how cascades propagate. Generators have thus been equipped with under-voltage, under- and over-speed, and loss-of-synchronism protections, transmission lines have been equipped with distance protection relays, and loads have been connected to under-frequency load shedding relays. The settings of those protections are taken from [11][3].

[1]https://fredericsabot.github.io/Publications.html

[2]Static data is available for this network. However, our methodology is very generic and could still be applied if the static data was uncertain.

[3]As opposed to [11], we do not consider protection-related uncertainties to focus on the distribution-related ones.

Table II

COMPARISON OF THE CCTS COMPUTED USING DYNAMIC EQUIVALENTS, PROBABILISTIC T&D SIMULATIONS, AND A RESTORATIVE LOAD MODEL

| Generator | CCT 5-95 bounds (ms) | | | | $a$ | $b$ |
|---|---|---|---|---|---|---|
| | Equivalents | T&D dep. | T&D ind. | Restorative | | |
| 30 | [550, 616] | [554, 631] | [583, 610] | [600, 604] | 72 | 100 |
| 31 | [240, 264] | [244, 255] | [247, 252] | [234, 234] | 100 | 100 |
| 32 | [200, 214] | [200, 210] | [204, 210] | [194, 194] | 98 | 100 |
| 35 | [214, 244] | [220, 234] | [224, 230] | [210, 222] | 100 | 100 |
| 37 | [244, 264] | [242, 260] | [250, 254] | [244, 244] | 94 | 100 |
| 38 | [180, 210] | [184, 194] | [184, 190] | [180, 180] | 100 | 100 |
| 39 | [634, stable] | [648, stable] | stable | stable | 100 | 100 |

[a] (resp. [b]) Percentage of dependent (resp. independent) T&D samples that lead to a CCT that is within the 5-95 range computed using the equivalent.

## IV. RESULTS

In this section, we compare the results obtained when loads are modelled either using a full distribution system model (T&D simulations), using our equivalents, or using simple restorative load models (netted loads). For fairness of the comparison, the parameters of the restorative load models have been tuned to match either the 5th or 95th percentile behaviour of the full distribution system model. Two types of applications are presented: critical clearing time (CCT) computation and simulation of cascading outages.

### A. Critical clearing time comparison

In this section, we compute the critical clearing times of three-phase short circuits applied on each of the generator buses. Table II compares the 90% confidence intervals of those CCTs computed with probabilistic T&D simulations to the statistics-informed bounds obtained with dynamic equivalents. The bounds obtained with the equivalents are defined as the range between the CCT computed when all loads are replaced by their "5-equivalent" and the CCT computed with the "95-equivalents". Bounds are obtained in the same way for the restorative load models.

For T&D simulations, two different confidence intervals have been computed. In both cases, 50 random samples of the uncertain distribution parameters are generated, and a CCT is computed using each of those samples. The confidence interval is then defined as the range between the 5th and the 95th percentiles of those CCTs. However, in the first case, the parameters of all distribution systems are assumed to be identical (very strong correlation between systems at different locations, referred to as the "dependent (dep.) T&D" case), and in the second case, they are considered independent (no correlation between systems at different locations, referred to as the "independent (ind.) T&D" case).

Table II shows a very good match between the intervals computed with the equivalents and those in the "dependent T&D" case. In particular, the lower bound on the CCT only differs by up to 4ms. There is however one case where the two intervals differ significantly: the upper bound on the CCT of generator 30 is 616ms with the equivalent and 631ms in the "dependent T&D" case. This difference is due to the fact that no long duration (e.g. 500 or 600ms) severe voltage drops were included in the training set of the equivalents. There is however no point in solving this issue as a generator with a CCT higher than 500ms can be considered as quite stable anyway.

The intervals are narrower in the "independent T&D" case compared to the "dependent T&D" case or the equivalent case. This is because deviations in the different loads tend to average out (central-limit theorem) which makes outliers significantly rarer. Using the equivalents thus leads to conservative intervals compared to the "independent T&D case". Actually, in this case, the latter intervals are fully included in the former.

In practice, it might be extremely difficult to estimate what part of load modelling uncertainty has strong correlation between loads at different locations (e.g. temperature dependence, devices from a same manufacturer) and what part is independent. When using 5- and 95-equivalents, one implicitly assumes a very strong (positive) correlations between all loads. This should be acceptable in most cases as it will very often lead to conservative bounds. In some specific cases, having one load that consumes more power than expected and one that consumes less could be worse than having the two loads both consuming more (or both less) than expected. If this is expected, one could perform multiple simulations by assigning randomly the 5- or 95-equivalent to each individual load and compare the results with all-5 and all-95 cases. However, we did not notice this in our simulations.

Finally, it is interesting to see that the results obtained with the simple restorative load model (with appropriately tuned parameters) do not differ that much from the "independent T&D" ones. This is because phenomena that cannot be modelled by the simple load model such as DER trips do not strongly impact transient stability. For example, DER trips induce on one hand a "dynamic braking" effect, and on the other hand a reduction of voltage support which have opposite effects on CCTs. In our system, tests indeed show that increasing the likelihood of DER trips has very low impact on CCTs (except if trips are made extremely frequent).

### B. Simulation of cascading outages

In this section, we make a similar comparison between results obtained using the equivalents, the "dependent" and "independent" T&D cases, and restorative load models, but now considering cascading outages. Here, we focus on fast cascading outages (i.e. cascading outages that are driven by electromechanical transients), simulations are thus run for 60s.

In order to trigger fast cascading outages, it is necessary to consider severe (yet realistic) initiating events [11]. In this work, we consider line three-phase faults that, due to a circuit breaker failure, are cleared in 200ms by opening the faulted line and an adjacent line.

Table III shows the intervals on the consequences (assessed in terms of percentage of load shedding) for the 10 worst (according to their average consequences in the "independent T&D" case) considered contingencies. Like in the CCT comparison, the intervals obtained with the equivalents match well those obtained in the "dependent T&D" case. There are

Table III

COMPARISON OF THE LOAD SHEDDING COMPUTED USING DYNAMIC EQUIVALENTS, PROBABILISTIC T&D SIMULATIONS, AND A RESTORATIVE LOAD MODEL. FAULTS ARE LOCATED ON LINE 1 (L1)

| Fault location (%) | Initiating event (N-2) | | Load shedding 5-95 bounds (%) | | | | | Share of dep. in 5-95 range (%)[a] | Share of ind. in 5-95 range (%)[b] | Opposite frequency collapses[c]? |
|---|---|---|---|---|---|---|---|---|---|---|
| | L1 | L2 | Equivalents | Corrected | T&D dep. | T&D ind. | Restorative | | | |
| 100 | 7-8 | 5-8 | [34, 41] | [27, 40] | [27, 44] | [27, 28] | [49, 49] | 60 | 0 | Yes |
| 0 | 5-8 | 7-8 | [34, 48] | [27, 48] | [28, 44] | [27, 28] | [100, 100] | 86 | 0 | Yes |
| 0 | 7-8 | 5-8 | [34, 63] | [27, 63] | [28, 42] | [27, 28] | [62, 62] | 88 | 0 | Yes |
| 0 | 26-29 | 26-28 | [26, 59] | | [31, 54] | [31, 36] | [24, 25] | 100 | 100 | No |
| 100 | 16-21 | 15-16 | [34, 55] | [27, 55] | [27, 42] | [27, 28] | [43, 43] | 86 | 0 | Yes |
| 0 | 16-21 | 16-24 | [26, 59] | | [31, 54] | [31, 36] | [24, 25] | 100 | 100 | No |
| 0 | 16-24 | 16-21 | [15, 55] | | [15, 54] | [17, 43] | [0, 0] | 98 | 98 | No |
| 0 | 26-27 | 26-29 | [26, 61] | | [26, 52] | [28, 42] | [19, 19] | 94 | 100 | No |
| 0 | 26-29 | 28-29 | [26, 61] | | [26, 52] | [28, 42] | [19, 19] | 94 | 100 | No |
| 0 | 26-27 | 26-29 | [21, 59] | | [22, 81] | [22, 73] | [24, 25] | 94 | 94 | No |

[a] (resp. [b]) Percentage of dependent (resp. independent) T&D samples that lead to an amount of load shedding that is within the 5-95 range computed using the equivalent.

[c] When the simulation with the 5-equivalent leads to an overfrequency collapse, but the 95-equivalent leads to under-frequency load shedding.

however significant differences for the first contingency. This is because, for this contingency, the 5- and 95-equivalents lead to opposite frequency behaviours. Indeed, the 5-equivalent leads to overfrequency causing generator 38 to trip due to overspeed which leads to a frequency drop that is halted by the activation of 3 UFLS steps (final consequences: 40% load shedding). On the other hand, the 95-equivalent directly leads to a frequency drop which is halted by a single UFLS step activation (final consequences: 34% load shedding). There are thus T&D samples for which the frequency behaviour is between those two extremes and thus stabilises without the need for generator disconnections or UFLS step activations (final consequences: 27-28% load shedding).

It is actually quite simple to correct the intervals obtained with the equivalents to account for the above issue. In case the 5- and 95-equivalents lead to opposite frequency collapses, the lower bound of the interval can be redefined as the minimum of the load shedding (between the 5- and 95-equivalents) before any frequency-related event (e.g. UFLS) occurs; and the upper bound as the maximum load shedding after frequency-related events occur. As shown in Table III, this corrected interval matches very well the "dependent T&D" one, and fully includes the "independent T&D" one.

Finally, results obtained using the simple load models are completely off which could be expected as simulation of large disturbances is very sensitive to modelling accuracy.

## V. CONCLUSION

In this paper, we build two dynamic equivalents based on high and low percentiles of the behaviour of an uncertain distribution grid model. Those equivalents were then used to build statistics-informed bounds on the results of (transmission) stability studies. Finally, we showed that those intervals match very well actual confidence intervals that could be obtained using probabilistic T&D simulations in both CCT computations and in simulations of fast cascading outages.

Perspectives for future work include considering other kinds of loads (e.g. variable-speed drives), and DERs (e.g. combined heat and power plants) in the equivalents; and assessing and improving the robustness of the equivalents to varying operating conditions.

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