

Contents

Abstract	iii
Acknowledgements	v
1 Modelling gravitational wave emission from compact binaries	1
1.1 Introduction	1
1.2 Extreme-mass-ratio inspirals	3
1.2.1 EMRI formation channels	4
1.2.2 Scientific potential and modelling challenges	4
1.3 Gravitational self-force and multiscale expansions	5
1.3.1 Descriptions of the self-forced orbital evolution	8
1.3.2 The two-timescale expansion	9
1.3.3 Accuracy requirements	14
1.3.4 Applicability of small-mass-ratio perturbation theory	14
1.4 Quasi-circular motion in the equatorial plane	16
1.4.1 Circular equatorial geodesics in Kerr spacetime	16
1.4.2 Orbital motion in the presence of radiation reaction	19
1.4.3 Transition to plunge in the literature	21
1.4.4 Ori-Thorne-Kesden transition motion	21
1.5 Motivation and outline of this thesis	25
2 Inspiral and transition motions in Kerr spacetime	27
2.1 Introduction	27
2.2 Self-forced equatorial motion	28
2.3 Quasi-circular inspiral	30
2.3.1 Structure of the self-force	30
2.3.2 Adiabatic inspiral	31
2.3.3 First post-adiabatic inspiral	32
2.3.4 Second post-adiabatic inspiral	33
2.3.5 The n th post-adiabatic inspiral	34
2.3.6 Inspiral towards the innermost stable circular orbit	34
2.4 Transition to plunge	42
2.4.1 Leading-order transition motion	43
2.4.2 General structure of the subleading transition motion	45
2.4.3 Homogeneous solution to the linearized PT1 equations	47
2.4.4 Inhomogeneous solution to the linearized PT1 equations	49
2.5 Asymptotic inspiral-transition match	53
2.5.1 Leading-order match: 0PA-0PLT	53
2.5.2 General structure of the asymptotic match	54
2.5.3 Explicit match of the 0PA and 1PA inspirals	56
2.6 Conclusion	58

3	Inspiral, transition and plunge: waveforms in Schwarzschild spacetime	61
3.1	Introduction	61
3.1.1	Self-forced equatorial motion	62
3.1.2	Einstein field equations	63
3.2	Quasi-circular inspiral	66
3.2.1	Orbital motion: adiabatic and post-adiabatic orders	66
3.2.2	Slow-timescale expansion of the Einstein field equations	68
3.2.3	Slow-timescale expansion of the self-force	71
3.2.4	Near-ISCO solution: orbital motion	73
3.2.5	Near-ISCO solution: metric perturbations and self-force	75
3.3	Transition to plunge	76
3.3.1	Orbital motion: leading and post-leading transition to plunge	76
3.3.2	Transition-timescale expansion of the Einstein field equations	77
3.3.3	Transition-timescale expansion of the self-force	81
3.3.4	Early- and late-time solutions: orbital motion	83
3.3.5	Early- and late-time solutions: metric perturbations and self-force	83
3.4	Near-geodesic plunge	86
3.4.1	Orbital motion: geodesic and first post-geodesic orders	86
3.4.2	Field equations	91
3.5	Asymptotic match and composite solutions	98
3.5.1	Asymptotic match between inspiral and transition to plunge: orbital motion .	99
3.5.2	Asymptotic match between inspiral and transition to plunge: metric pertur- bations and self-force	102
3.5.3	Asymptotic match between transition to plunge and plunge: orbital motion and self-force	105
3.6	Waveforms	109
3.6.1	Waveform generating algorithm	109
3.6.2	Example: comparison with SXS:BBH:1108	112
3.7	Conclusion	116
A	Quasi-circular motion in Kerr spacetime	119
A.1	Expressions for the n PA auxiliary functions	119
A.2	Sources appearing in the iPLT transition equations	123
A.3	Coefficients appearing in the transition motion	126
B	Quasi-circular motion in Schwarzschild spacetime	129
B.1	Barack-Lousto-Sago basis of tensor spherical harmonics	129
B.2	The matrix operator \mathcal{M}^{ij}	130
B.3	Harmonic decomposition of the Lorenz gauge conditions	131
B.4	Equivalence of the perturbative and non-perturbative formulations	131
B.5	Slow-timescale expansion of the point-particle stress-energy tensor	132
B.6	Slow-timescale expansion of the self-force	133
B.7	Coefficients of the asymptotic near-ISCO inspiral solutions	135
B.8	Transition equations	136
B.9	Transition-timescale expansion of the point-particle stress-energy tensor	138
B.10	Coefficients of the asymptotic early- and late-time transition solutions	139
B.11	Coefficients of the asymptotic near-ISCO plunge solutions	142
B.12	Self-force matching conditions	143
B.13	Explicit values of the asymptotic coefficients	144
B.14	Vector and tensor spherical harmonics	145