

## Anomalies, $\eta$ , $\eta'$ as keys to glueballs.

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Glueballs are the most straightforward prediction of QCD, yet while they have likely been produced, none has been unequivocally identified. We pursue a backdoor approach through anomalies, and singularly the  $\eta$  and  $\eta'$  which brings light to this irritating situation. In particular, we advocate to consider the full decay chain  $J/\psi \rightarrow X\gamma, X \rightarrow \eta\eta'$  (into glue-rich states followed by glue-rich decays). We also suggest new BES III searches, namely for the  $\pi_1$  into  $\eta(^{\prime})\pi^0$ , (this would be the partner of their recently observed  $\eta_1(1855)$ ). Another useful investigation would be for other channels (or semi-inclusive)  $f_0(1500)$  decays (see last section)

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## 1. Introduction

While glueballs (colour neutral bound states of 2 or more gluons) are an obvious and compelling prediction of Quantum Chromodynamics (QCD), none have this far been identified with certainty. In a way, most physicists have ceased searching for them, satisfying themselves that they have certainly been produced, but are lost in a complex forest of quark states, with which they mix, pointing in particular at the total number of  $0^{++}$  or  $0^{+-}$  states between 1 and 2 GeV. More conspicuous would however be "exotics", namely mesons with the "wrong" charge-parity assignment with respect to the naïve Quark Model as a result of the presence of an extra gluon in their composition; however here also, an ambiguity may exist with 4-quark states, a pair of quarks simulating a gluon.

When glueballs searches have thus become a backwater, it is a bit shocking that such a fundamental test of QCD (and, accessorially, lattice calculations) is neglected. Fortunately, some long-overdue systematic searches combining gluon-rich production and gluon-rich decays have now been realized, thanks on the one hand to the identification of the  $\eta, \eta'$  as possible markers, and on the other hand, to the coming in full operation of BESIII in China, allowing for a never-reached before resolution in the study of the  $J/\psi$  radiative decays.

## 2. Are glueball decays flavor blind?

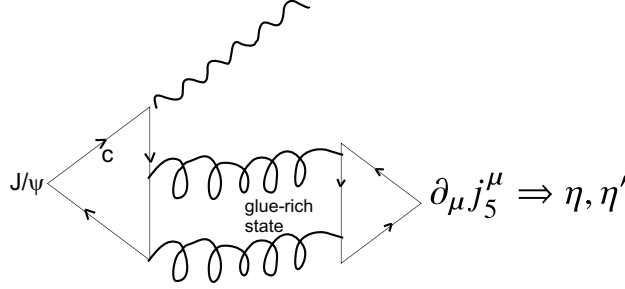
For lack of trustable calculations (we are deep in the domain of really strong forces, far away from asymptotic freedom) has led to a number of qualitative assumptions about the decays of glueballs.

- the straightforward assumption is that decays are "flavor-blind", since pure glue ignores this quantum number, with the mitigation of phase space.
- very quickly, qualifications and possible deviations were suggested: wave function overlap (a quantum mechanical approach) would suggest that heavier particles, closer to the 1-2 GeV mass of the glueballs would be preferred, say  $K$  pairs over  $\pi$ 's.
- this tendency is also advocated by the supposition that 2 gluons cannot couple to light quarks, since the decay in a pseudoscalar meson would necessitate a mass insertion
- it should however be noted that the above argument does not really apply in general cases, since it is well-known that quantum anomalies precisely relate the divergence of the axial current or light quarks to pure gluons  $\widetilde{G}^{\mu\nu}G_{\mu\nu}$ : this is precisely the term which appears in the evaluation of  $\eta, \eta'$  states. A more detailed discussion can be found in ref. [1].
- the above arguments are mostly limited to 2-body decays into pseudoscalars, but it is not absurd to assume that flavor-neutral states would decay predominantly by 2 light  $\sigma$  (the very broad partner of the  $\pi$ , only seen by partial wave interference), those particles decaying in turn in pion pairs (thus at least 4  $\pi$ )

### 3. Enter the $\eta$ and $\eta'$

Some early hints of modern glueballs searches came early (early 1980's) with the GAMS experiment [2], [3], which investigated a "gluon-rich" (central production) environment using a lead glass wall, mostly sensitive to photons. This had the effect of accessing the  $\eta$  and  $\eta'$  decay modes, and resulted in at least one glueball candidate (which they called G(1590), now the  $f_0(1500)$ ; the discrepancy in mass is most probably due to the limited phase space in the  $\eta - \eta'$  channel, which leads to severe distortion).

Interestingly, the same experiment (which was conducted in parallel at CERN and Serpukhov) led to the discovery of an exotic candidate, then called M(1406), now probably  $\pi_1(1400)$  (also sometimes called  $\tilde{\rho}$ , with exotic parity  $1^{-+}$  impossible for a quark-antiquark meson), seen decaying into  $\eta(\prime)\pi$ . In both cases, the role of the  $\eta'$  meson was key, as pointed out by Gherstein et al.[4]. A special connection between glue and  $\eta'$  was indeed known both from theoretical considerations (QCD anomaly) and the observation of the large  $J/\psi \rightarrow \eta'\gamma$  branching ratio.



**Figure 1:**  $J/\psi$  radiative decay as a glue-rich source coupled to the  $\eta(\prime)$ ; notice that no mass insertion is needed

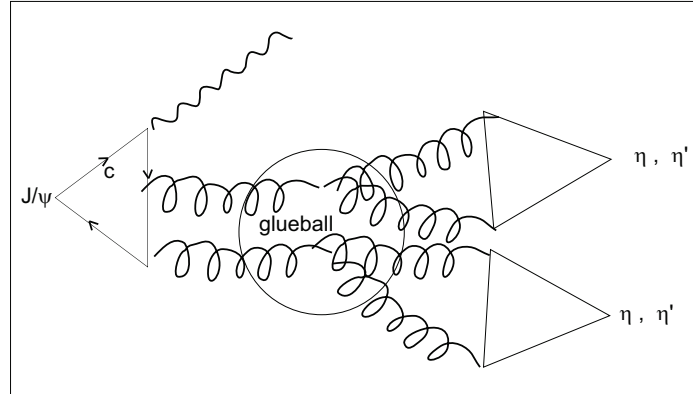
It must be noted that *no quark mass insertion is needed in this case*, due to the anomaly.

**We have developed a complete formalism to handle the radiative decays of the type  $V \rightarrow P \gamma$  or  $P \rightarrow V \gamma$  on the basis of QCD anomalies**, [6], [7], further refined in [8], which lead to a successful description of those processes without recourse to additional parameters. The larger decay of  $J/\psi$  into  $\eta'\gamma$  than  $\eta\gamma$  despite an unfavorable phase space is then put in relation to the greater admixture of the anomaly contribution in the physical  $\eta'$ .

For  $J/\psi$  decays, the emission of a photon leaves a glue-rich situation, which eventually couples to the  $\eta(\prime)$ .

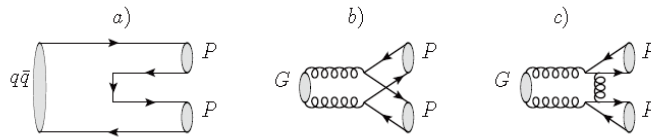
This is thus a different "glue-rich" production source, in addition to the central production. While intensity-limited by the production mechanism, it offers a much "cleaner" situation to look for glueball states.

Unfortunately, the  $J/\psi$  radiative decays into neutral mesons have proven very difficult experimentally, and the results of a search for glueballs were confusing at first. While SLAC ceased this activity, more powerful machines preferred to turn to the heavier B mesons. It was not until BES III in Beijing started taking data in recent years that difficult channels could be studied with (impressive) accuracy.



**Figure 2:**  $J/\psi$  radiative decay as a glue-rich source leading to Glueball formation and decay, here in the  $\eta \eta'$  channels

#### 4. Who are the Glueballs?



**Figure 3:** 3 graphs (in addition to figure 2 describing possible decay topologies, from ref. [1])

Many attempts have been made to identify the glueballs in the 1-2 GeV spectroscopy of mesons, with approaches including various elements (in particular, including or excluding the anomaly-mediated channels). A popular approach (see references in [1]) attempts to parametrize the various contributions according to topologies assumed from the schematic mechanisms described, for instance in fig. 3, or possibly in the left part of fig. 2. While tempting, this approach should be taken with a grain of salt. Due to strong interactions, these diagrams are just oversimplified evocations of processes, and in no way on the level of Feynman graphs!. Each of them should, be enriched with a large number of QCD extra contributions, the phases are unknown, ...

As others, we have tried this approach a few years ago, and tried to compensate the insufficient theoretical basis with a long list of processes to be fitted. On the basis of this analysis, we tried 2 fits, one without assuming the often advocated chiral suppression (against which we already listed arguments) and the more traditional one for comparison.

Both fits (in line with the then tendency) tended to present the  $f_0(1710)$  as a mostly-glueball state and the  $f_0(1500)$  mostly as a "normal" meson. As part of the fit, we had come with the

Decay ratio	Data	Fit 1	Fit 2
$\frac{\Gamma(f_0(1500) \rightarrow \eta\eta')}{\Gamma(f_0(1500) \rightarrow \eta\eta)}$	$0.38 \pm 0.16$ [25]*	$0.53^{+0.13}_{-0.11}$	$0.47^{+0.14}_{-0.12}$
$\frac{\Gamma(f_0(1710) \rightarrow \eta\eta')}{\Gamma(f_0(1710) \rightarrow \eta\eta)}$	$< 0.08$ [27]	$0.64^{+0.22}_{-0.22}$	$0.32^{+0.20}_{-0.18}$

**Figure 4:** Our previous fit to the  $f_0(1710)$  and  $f_0(1500)$  decays into  $\eta\eta'$ , allowing for the severe phase space suppression of the latter; the brackets refer to the data available at the time [1] and the whole fit is now superseded by BES III data

fit/prediction for the decay of these mesons into  $\eta\eta'$  shown in figure 4 (the reference in brackets refer to the then-available data, see the original paper).

These attempts certainly suggested that the  $f_0(1710)$  would be observed in the  $\eta\eta'$  channels when the BES III data would become available!

## 5. The new results from BES III change the perspective

We have finally come close to the holy grail!

BES III recently published a partial wave analysis of the radiative  $J/\psi$  decays complete with the  $\eta$  modes. This is precisely the tool which has been missing for tens or years (more from lack of interest than from lack of feasibility for other facilities!). [9]

And it comes with a surprise (we will deal with another surprise in the "exotics" in a later section).

Namely, the  $f_0(1710)$  is simply not seen in the full decay chain  $J/\psi \rightarrow X\gamma, X \rightarrow \eta\eta'$ , where it should dominate the  $f_0(1500)$ . Even accepting a confusion with the  $f_0(1810)$  (remember that the "mass" in the tables is not always the latest measurement, and that partial wave analysis may be affected by interferences, phase space distortions), we see that it is suppressed in the ration of 0.007/3.05.

Decay mode	Resonance	$M$ (MeV/c <sup>2</sup> )	$\Gamma$ (MeV)	$M_{\text{PDG}}$ (MeV/c <sup>2</sup> )	$\Gamma_{\text{PDG}}$ (MeV)	B.F. ( $\times 10^{-8}$ )	Sig.
$J/\psi \rightarrow \gamma X \rightarrow \gamma\eta\eta'$	$f_0(1500)$	1506	112	1506	112	$3.05 \pm 0.07$	$\gg 30\sigma$
	$f_0(1810)$	1795	95	1795	95	$0.07 \pm 0.01$	$7.6\sigma$
	$f_0(2020)$	$1935 \pm 5$	$266 \pm 9$	1992	442	$1.67 \pm 0.07$	$11.0\sigma$
	$f_0(2100)$	$2109 \pm 11$	$253 \pm 21$	2086	284	$0.33 \pm 0.03$	$5.2\sigma$
	$f_0(2330)$	$2327 \pm 4$	$44 \pm 5$	2314	144	$0.07 \pm 0.01$	$8.5\sigma$

**Figure 5:** The partial wave analysis of the full decay chain  $J/\psi \rightarrow X\gamma, X \rightarrow \eta\eta'$ , from table II of [9]

See however also a comprehensive analysis by ref [12].

**This suggests another attitude, and strongly points to the combined chain of decay as a prominent test for glueballs.**

Ironically, this most recent results brings us back to the initial observations by GAMS, with their G(1590) , now identified as the  $f_0(1500)$  as a leading candidate. Here also, the shift in estimated mass (from the very early and sparse observations) may be understood by the severe distortion of the phase space (the  $\eta\eta'$  channel being at the nominal threshold, and only possible due to the state width).

This of course prompts us to look back into the unusual properties of the  $f_0(1500)$ , but also to check what the BES III data tell us about the other "weird" observation of GAMS, namely the  $1^{-+}$  exotic. This will occupy the next 2 sections.

## 6. The very unusual decays of the $f_0(1500)$ particle

It is worth to have a new peek into the PDG tables to learn more about this state, which is back in the spotlight. In this table, we note that, as already mentioned, the  $\eta\eta'$  mode, while at threshold,

$f_0(1500)$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	Scale factor	(MeV/c)
$\pi\pi$	$(34.5 \pm 2.2) \%$	1.2	741
$\pi^+\pi^-$	seen		740
$2\pi^0$	seen		741
$4\pi$	$(48.9 \pm 3.3) \%$	1.2	692
$\eta\eta$	$(6.0 \pm 0.9) \%$	1.1	517
$\eta\eta'(958)$	$(2.2 \pm 0.8) \%$	1.4	20
$K\bar{K}$	$(8.5 \pm 1.0) \%$	1.1	569
$\gamma\gamma$	not seen		753

**Figure 6:** The main decay modes of the  $f_0(1500)$  according to Particle Data Group [? ]

is very significant (and would dominate the  $\eta\eta$  mode after phase space corrections), but also that the main observed decay mode is in  $4\pi$ , overwhelming the  $2\pi$  channel. An interesting question remains : there may very well be other, not yet observed modes (multi pions), for which the analysis have not been performed or would prove impractical. In fact, a semi-inclusive search for  $J/\psi \rightarrow \gamma X(1500)$  where  $X$  is any recoiling state with invariant mass in the range could be instructive! Remaining with the  $4\pi$  decay mode, this can certainly suggest a decay through a pair of very wide  $0^{++}$   $\sigma$  states. Given its quantum numbers, the  $\sigma$  can certainly be suspected to mix in the "glueball complex".

## 7. Return to the Exotics

Exotics is the name traditionally given to meson states which cannot be reached by the association of just one quark and an antiquark. Such basic combinations indeed have their parity and charge conjugation directly related to orbital momentum and total spin respectively, namely  $P = (-)^{L+1}$ ,  $C = (-)^{L+S}$ . As a result, a  $J^{PC} = 1^{+-}$  state is forbidden. If a gluon(or a quark-antiquark pair mimicking a gluon) is added, the selection rule fails.

Such a state was advocated by the GAMS collaboration (see above)[5], with a decay into  $\eta(\prime)\pi$ , and a mass 1406 MeV, (initially nwmaed M(1406) and probably currently known as  $\pi_1(1400)$ ).

While it was a clear sign of an exotic, the phenomenological analysis conducted then were not enthusiastic, since prejudice disfavored this precise decay mode – once again, this was an error due to disregarding axial anomalies, and the special role of the  $\eta'$ !

We advocated already at that time that the inclusion of the anomaly would on the opposite make it one leading decay mode [10].

Little work has gone on this search since, but it is interesting that BESIII has now identified a similar exotic, this time in the closely related  $\eta\eta$  channel, namely a  $\eta_1$  (1855 with  $I^G J^{PC} = O^+ 1^{-+}$ ). [9] This time, the theoretical interpretation did not miss the role of the anomaly [11], coming to a conclusion similar to ours in the case of the  $\pi_1$ .

This comforts us in believing that the *eta* system, through quantum anomalies, is a key to understanding the glueballs and exotics system!

## 8. Conclusions and Work to Do

Obviously our work in understanding the role of gluons in exotic mesons and glueballs is far from complete. Some comfort may be found in the fact that a large body of experimental evidence is now becoming available in what was for a long time a neglected field. Many theorists had probably given up in front of the complexity and the lack of fresh data, but hope is returning and results are lining up nicely.

Many channels still remain worth exploring. Obviously, the search for the exotic  $\pi_1$  in the  $\eta'\pi^0$  mode should be conducted at BES III, confirming the old GAMS result and giving extra support for a partner their  $\eta_1$  state. Further decay modes (ideally a semi-inclusive study) or the  $f_0(1500)$  would also help better understand this state.

The next step would be to use then better established glueballs by probes for heavier states (assuming that a glueball to glueball + X decay would be favored).

We may see a new sunrise on glueballs!

## 9. Acknowledgements

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