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Proceeding Paper

Gapless Superfluidity and Neutron Star Cooling †

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Abstract: The presence of currents in the interior of cold neutron stars can lead to a state in which nucleons remain superfluid while the quasiparticle energy spectrum has no gap. We show within the self-consistent time-dependent nuclear energy density functional theory that the nucleon specific heat is then comparable to that in the normal phase, contrasting with the classical BCS result in the absence of super flows. This dynamical, gapless superfluid state has important implications for the cooling of neutron stars.

Keywords: neutron star; superfluidity; cooling; pairing gap; specific heat

1. Introduction

Produced during gravitational-core-collapse supernova explosions with initial temperatures as high as $\sim 10^{11} - 10^{12}$ K, neutron stars cool down to temperatures of $\sim 10^9$ K within a few days [1]. The very dense matter in their interior is expected to undergo various quantum phase transitions analogous to those observed in terrestrial laboratories [2]. Similar to electrons in conventional terrestrial superconductors, free neutrons in the inner crust and the outer core of neutron stars are predicted to form a Bardeen–Cooper–Schrieffer (BCS) condensate of $^1S_0$ Cooper pairs. Nuclear superfluidity has been supported by the rapid decline in the luminosity of the Cassiopeia A remnant [3–9] and has been corroborated by radio timing observations of frequency glitches in numerous pulsars [10], which can be interpreted as global readjustments of the rotational motions of the neutron and proton superfluids induced by the unpinning of quantized neutron superfluid vortices [11,12] (see, e.g., ref. [13] for a review).

Despite the importance of the superfluid dynamics for interpreting these latter astrophysical phenomena, most microscopic calculations of the nuclear pairing properties have been carried out thus far for static situations (see, e.g., ref. [14] for a recent review). We have recently studied the dynamics of hot neutron–proton superfluid mixtures with the self-consistent time-dependent nuclear energy density functional theory [15]. By applying it to neutron stars, we have computed $^1S_0$ neutron and proton pairing gaps in the homogeneous core in the presence of arbitrary currents and we have determined the mutual neutron–proton entrainment coupling coefficients [16]. We have also shown within the same framework that there exists a dynamical "gapless" state in which nuclear superfluidity is not destroyed, even though the energy spectrum of quasiparticle excitations exhibits no gap [17]. As will be presented in Section 2, the absence of an energy gap leads to a nucleon specific heat that is very different from that in the classical BCS state (in the absence of super flows). The implications for the cooling of neutron stars [18] will be discussed in Section 3.

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2. Gapless Superfluidity

2.1. Order Parameter

We have previously studied the nuclear superfluidity at finite temperatures and in the presence of superflows under the self-consistent time-dependent nuclear energy density functional framework [15,16]. The behavior of the order parameter Δ_q (with $q = n, p$ for neutron and proton, respectively) was found to be universal after introducing some effective superfluid velocities $\mathbb{V}_q$ and proper rescaling (see Figure 1): $\Delta_q^{(0)}$ is the order parameter at zero temperature in the absence of superflows, $T_{cq}^{(0)} = e^{\gamma}\Delta_q^{(0)}/\pi \simeq 0.567\Delta_q^{(0)}$ is the critical temperature above which superfluidity is destroyed and $\mathbb{V}_{Lq} = \Delta_q^{(0)}/(\hbar k_{Fq})$ (with k_{Fq}, the Fermi wave vector) is Landau's velocity (derived via the eponymous criterion [19,20] adapted to the context of strongly interacting nuclear superfluid mixtures).

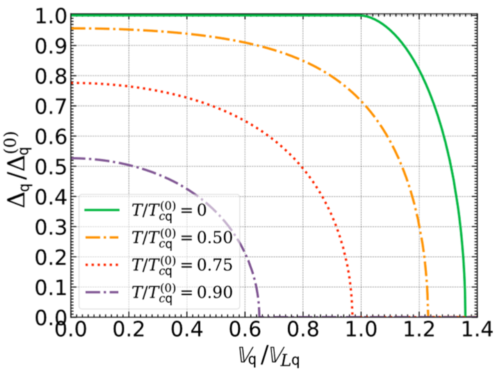

Figure 1. $^1S_0$ nucleon pairing gap Δ_q normalized to its value $\Delta_q^{(0)}$ at zero temperature in the absence of superflows as a function of the corresponding effective superfluid velocity $\mathbb{V}_q$ expressed in terms of Landau's velocity $\mathbb{V}_{Lq}$ for different temperatures ($T_{cq}^{(0)}$ being the critical temperature for the superfluid at rest). See text for details.

2.2. Gapless State and Specific Heat

Focusing on low temperatures, $T \ll T_{cq}^{(0)}$ (relevant for mature neutron stars), we have shown that the energy gap in the quasiparticle density of state $\mathcal{D}_q$ shrinks with an increasing effective superfluid velocity $\mathbb{V}_q$ and disappears at Landau's velocity $\mathbb{V}_{Lq}$ (see Figure 2). However, the order parameter Δ_q remains finite: the superfluid enters a gapless state, which persists until the critical velocity $\mathbb{V}_{cq}^{(0)} = e\mathbb{V}_{Lq}/2 \simeq 1.36\mathbb{V}_{Lq}$ is reached. Full calculations are presented in Ref. [17].

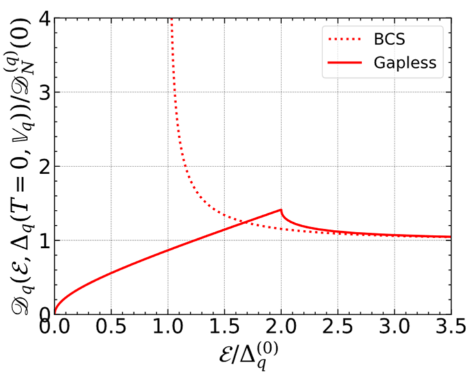

Figure 2. Quasiparticle density of states at low temperatures (normalized by the one in the normal phase $\mathcal{D}_N^{(q)}(0)$) in the BCS state ($\mathbb{V}_q = 0$) and in the gapless state ($\mathbb{V}_q = \mathbb{V}_{Lq}$).

One of the immediate consequences of gapless superfluidity is the modification of thermal properties such as the specific heat $c_V^{(q)}$, whereas the classical BCS state leads to an exponentially suppressed specific heat at low temperatures (see, e.g., ref. [21]):

$$c_V^{(q)}\left(T \ll T_{cq}^{(0)}, V_q = 0\right) \approx \frac{3\sqrt{2}}{\pi^{3/2}} \left(\frac{T_{cq}^{(0)}}{T}\frac{\pi}{e^\gamma}\right)^{5/2} \exp\left(-\frac{T_{cq}^{(0)}}{T}\frac{\pi}{e^\gamma}\right) c_N^{(q)}(T), \tag{1}$$

$c_N^{(q)}$ being the corresponding specific heat in the normal phase and $\gamma \simeq 0.577216$ denoting the Euler–Mascheroni constant; the specific heat in the gapless state is comparable to that in the normal phase, and is approximately given by [17]:

$$c_V^{(q)}\left(T \ll T_{cq}^{(0)}, \mathbb{V}_q > \mathbb{V}_{Lq}\right) \approx \sqrt{1 - \left(\frac{\Delta_q(\mathbb{V}_q)}{\Delta_q^{(0)}}\frac{\mathbb{V}_{Lq}}{\mathbb{V}_q}\right)^2}\, c_N^{(q)}(T), \tag{2}$$

and Δ_q is accurately given by the following interpolation [16]:

$$\frac{\Delta_q(\mathbb{V}_q > \mathbb{V}_{Lq})}{\Delta_q^{(0)}} = 0.5081\sqrt{1 - \frac{2\mathbb{V}_q}{e\mathbb{V}_{Lq}}}\left(2.437\frac{\mathbb{V}_q}{\mathbb{V}_{Lq}} - 4.443\sqrt{\frac{\mathbb{V}_{Lq}}{\mathbb{V}_q}} + 5.842\right). \tag{3}$$

The ratio between the specific heat in the gapless state and that in the normal phase is, therefore, an increasing universal function of $\mathbb{V}_q/\mathbb{V}_{Lq}$.

3. Astrophysical Consequences

Gapless superfluidity has important implications for the cooling of neutron stars, and in particular, for the interpretation of the thermal emission from quasipersistent soft X-ray transients, as studied in [18]. These binary systems consist of a neutron star whose crust is sporadically heated due to mass transfer from a low-mass stellar companion for a long period of time (from years to decades) before entering a cooling phase when the accretion stops. The thermal relaxation has been observed for several sources up to about 10^4 days after outbursts [22] and is governed by the diffusion of heat in the inner crust of neutron stars, which is made of ions, free electrons and superfluid neutrons (see, e.g., [23]).

Let us recall that the typical thermal timescale of a crustal layer, delimited by the radial coordinates $r_{\rm min}$ and $r_{\rm max}$, is given by [24]:

$$\tau \approx \frac{1}{4}\left(\int_{r_{\rm min}}^{r_{\rm max}} dr \sqrt{\frac{c_V}{\kappa}}\right)^2, \tag{4}$$

where κ is the thermal conductivity and c_V is the total specific heat. Thus far, the neutron contribution to the crustal specific heat has been generally thought to be negligible, assuming superfluid neutrons are in the classical BCS state. In this case, the thermal relaxation is expected to be much faster if neutrons are superfluid since τ is shorter according to Equation (4). To explain the observed late time cooling of some sources, some authors proposed that neutrons are not superfluid in the deepest region of the crust [25,26]. However, this interpretation has been recently ruled out by microscopic calculations [27,28].

Astrophysical observations and nuclear physics can be reconciled by allowing neutrons to be in the gapless superfluid state, as we show in ref. [18]. Indeed, the neutron specific heat is then strongly enhanced compared to that in the BCS state and can now dominate the electronic and ionic contributions even when considering realistic nuclear pairing properties [27,28]. The end result is a delayed thermal relaxation of the neutron star crust, as observed.

4. Conclusions

Considering the dynamics of nuclear superfluidity in the framework of the self-consistent time-dependent nuclear energy-density functional theory [15,16], we have shown that the energy spectrum of quasiparticle excitations exhibits no gap while the order parameter Δ_q remains finite at low temperatures for effective superfluid velocities $\mathbb{V}_q$ exceeding Landau's velocity $\mathbb{V}_{Lq}$, but lower than the critical velocity $\mathbb{V}_{cq}^{(0)}$.

Contrary to the classical BCS state, the gapless state is characterized by a very large specific heat comparable to that in the normal phase. This specific heat at low temperatures is shown to be a universal function of $\mathbb{V}_q/\mathbb{V}_{Lq}$. Full details are given in Ref. [17].

Focusing on the inner crust of neutron stars, we have shown in Ref. [18] that the drastic increase in the neutron specific heat can solve the apparent contradiction between the observed late time cooling of quasi persistent soft X-ray transients and microscopic nuclear pairing calculations.

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References

1. Potekhin, A.Y.; Pons, J.A.; Page, D. Neutron Stars—Cooling and Transport. *Space Sci. Rev.* **2015**, *191*, 239–291. [CrossRef]
2. Chamel, N. Superfluidity and Superconductivity in Neutron Stars. *J. Astrophys. Astron.* **2017**, *38*, 43. [CrossRef]
3. Page, D.; Prakash, M.; Lattimer, J.M.; Steiner, A.W. Rapid Cooling of the Neutron Star in Cassiopeia A Triggered by Neutron Superfluidity in Dense Matter. *Phys. Rev. Lett.* **2011**, *106*, 081101. [CrossRef] [PubMed]
4. Shternin, P.S.; Yakovlev, D.G.; Heinke, C.O.; Ho, W.C.G.; Patnaude, D.J. Cooling Neutron Star in the Cassiopeia A Supernova Remnant: Evidence for Superfluidity in the Core: Cooling Cas A Neutron Star. *Mon. Not. R. Astron. Soc. Lett.* **2011**, *412*, L108–L112. [CrossRef]
5. Elshamouty, K.G.; Heinke, C.O.; Sivakoff, G.R.; Ho, W.C.G.; Shternin, P.S.; Yakovlev, D.G.; Patnaude, D.J.; David, L. Measuring the Cooling of the Neutron Star in Cassiopeia A with all Chandra X-ray Observatory Detectors. *Astrophys. J.* **2013**, *777*, 22. [CrossRef]
6. Posselt, B.; Pavlov, G.G. Upper Limits on the Rapid Cooling of the Central Compact Object in Cas A. *Astrophys. J.* **2018**, *864*, 135. [CrossRef]
7. Wijngaarden, M.J.P.; Ho, W.C.G.; Chang, P.; Heinke, C.O.; Page, D.; Beznogov, M.; Patnaude, D.J. Diffusive Nuclear Burning in Cooling Simulations and Application to New Temperature Data of the Cassiopeia A Neutron Star. *Mon. Not. R. Astron. Soc.* **2019**, *484*, 974–988. [CrossRef]
8. Ho, W.C.G.; Zhao, Y.; Heinke, C.O.; Kaplan, D.L.; Shternin, P.S.; Wijngaarden, M.J.P. X-Ray Bounds on Cooling, Composition, and Magnetic Field of the Cassiopeia A Neutron Star and Young Central Compact Objects. *Mon. Not. R. Astron. Soc.* **2021**, *506*, 5015–5029. [CrossRef]
9. Posselt, B.; Pavlov, G.G. The Cooling of the Central Compact Object in Cas A from 2006 to 2020. *Astrophys. J.* **2022**, *932*, 83. [CrossRef]
10. Manchester, R.N. Pulsar Glitches. *Proc. IAU* **2017**, *13*, 197–202. [CrossRef]
11. Anderson, P.W.; Itoh, N. Pulsar Glitches and Restlessness as a Hard Superfluidity Phenomenon. *Nature* **1975**, *256*, 25–27. [CrossRef]
12. Pines, D.; Alpar, M.A. Superfluidity in Neutron Stars. *Nature* **1985**, *316*, 27–32. [CrossRef]
13. Haskell, B.; Melatos, A. Models of Pulsar Glitches. *Int. J. Mod. Phys. D* **2015**, *24*, 1530008. [CrossRef]
14. Sedrakian, A.; Clark, J.W. Superfluidity in Nuclear Systems and Neutron Stars. *Eur. Phys. J. A* **2019**, *55*, 167. [CrossRef]
15. Allard, V.; Chamel, N. Entrainment Effects in Neutron-Proton Mixtures within the Nuclear Energy-Density Functional Theory. II. Finite Temperatures and Arbitrary Currents. *Phys. Rev. C* **2021**, *103*, 025804. [CrossRef]
16. Allard, V.; Chamel, N. $^1S_0$ Pairing Gaps, Chemical Potentials and Entrainment Matrix in Superfluid Neutron-Star Cores for the Brussels–Montreal Functionals. *Universe* **2021**, *7*, 470. [CrossRef]
17. Allard, V.; Chamel, N. Gapless superfluidity in neutron stars—I. thermal properties. *Phys. Rev. C* **2022**, Submitted.

18. Allard, V.; Chamel, N. Evidence of gapless neutron superfluidity from the late time cooling of transiently accreting neutron stars. *Phys. Rev. Lett.* **2022**, Submitted.
19. Landau, L. Theory of the Superfluidity of Helium II. *Phys. Rev.* **1941**, *60*, 356–358. [CrossRef]
20. Lifshitz, E.M.; Pitaevskii, L. *Course of Theoretical Physics, v.9: Statistical Physics, Pt.2*; Pergamon: Oxford, UK, 1980; pp. 88–94.
21. Abrikosov, A.A. *Fundamentals of the Theory of Metals*; North Holland Publishing Company: Amsterdam, The Netherlands, 1988; p. 350.
22. Wijnands, R.; Degenaar, N.; Page, D. Cooling of Accretion-Heated Neutron Stars. *J. Astrophys. Astron.* **2017**, *38*, 49. [CrossRef]
23. Chamel, N.; Haensel, P. Physics of Neutron Star Crusts. *Living Rev. Relativ.* **2008**, *11*, 10. [CrossRef] [PubMed]
24. Henyey, L.; L'Ecuyer, J. Studies in Stellar Evolution. VIII. The Time Scale for the Diffusion of Energy in the Stellar Interior. *Astrophys. J.* **1969**, *156*, 549. [CrossRef]
25. Turlione, A.; Aguilera, D.N.; Pons, J.A. Quiescent Thermal Emission from Neutron Stars in Low-Mass X-Ray Binaries. *Astron. Astrophys.* **2015**, *577*, A5. [CrossRef]
26. Deibel, A.; Cumming, A.; Brown, E.F.; Reddy, S. Late-Time Cooling of Neutron Star Transients and the Physics of the Inner Crust. *Astrophys. J.* **2017**, *839*, 95. [CrossRef]
27. Gandolfi, S.; Palkanoglou, G.; Carlson, J.; Gezerlis, A.; Schmidt, K.E. The $^1S_0$ Pairing Gap in Neutron Matter. *Condens. Matter* **2022**, *7*, 19. [CrossRef]
28. Drissi, M.; Rios, A. Many-Body Approximations to the Superfluid Gap and Critical Temperature in Pure Neutron Matter. *Eur. Phys. J. A* **2022**, *58*, 90. [CrossRef]