A geometry-based algorithm for TDoA-based localization in the presence of outliers

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***Abstract*—Localization of non-cooperative radio frequency transmitters in wireless networks can be performed by using Time Difference of Arrival (TDoA). However, TDoA measurements are often degraded by limited signal-to-noise ratio, multipath components, non-line-of-sight (NLOS) visibility, etc. This causes the TDoA measurements to have strong outliers, which cannot easily be modelled mathematically. This paper investigates algorithms to perform TDoA-based localization in the presence of outlier measurements. Our algorithm is based on a geometric interpretation, rather than a mathematical, noise model-based approach. We show that our algorithm outperforms existing algorithms when applied to real-world data in a vehicular scenario.**

***Index Terms*—time difference of arrival, outlier, wireless localization**

I. Introduction

Precise localization of unknown Radio Frequency (RF) transmitters is a crucial component of modern wireless systems, with applications in vehicular technology, sensor networks, smart cities and industry 4.0 [1], [2]. Localization through cellular networks is especially important when Global Navigation Satellite Systems (GNSS) are unreliable or not available, for instance, in urban or suburban environments due to severe multipath conditions or blockage of the satellites' signals [3].

TDoA-based localization is the most common method for target positioning in wireless sensor networks. In such a system, several anchor nodes (with known locations) measure the difference in the Time of Arrival (ToA) of a signal transmitted by a target node. The TDoA between each pair of anchors defines a hyperbola of possible locations of the target node, and the intersection of these hyperbolas indicates the target's location. The main advantage of TDoA-based localization systems is that the target transmitter node is not required to be synchronized with the anchor nodes, providing the possibility of low-cost target nodes. Another advantage is that the transmit time of the target node needs not be known by the anchor nodes. Several closed-form estimators exist for TDoA localization, such as [4]–[6]. One major problem of TDoA-based localization is the presence of outliers that may occur in the TDoA measurements. This may be due to multiple factors, such as multipath components (MPCs), NLOS propagations and time synchronization mismatch between the anchor nodes. Such outliers degrade the accuracy of the estimation if not properly accounted for [7], [8].

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Several algorithms have been proposed for dealing with outliers in TDoA measurements [6]–[13]. [6] proposed an algebrical solution to select a subset of accurate measurements but concerns multiple transmitter scenario only. [12] used redundant time measurements to reduce the negative impact of NLOS propagation time-based localization. However, redundant time measurements are not always available in passive localization scenario. Algorithms proposed in [8] and references therein, [10], [11] allow for some improvement over classical Maximum Likelihood (ML)-based estimation, but a closer visual inspection of our measurement results showed some poor results in situations where visual inspection could outperform said algorithms (as will be shown in this paper). The fact that TDoA-based localization relies on hyperbolic geometry causes outliers that deviate from the noise model to have disproportionate influence on the final result. Therefore, algorithms that rely on a more geometric approach, such as [7], [13], should outperform model-based algorithms. However, there were tested on simulation data and not on experimental data.

In this paper, we propose a new, geometry-based algorithm to deal with outlier measurements in TDoA-based measurements. The algorithm is evaluated on an experimental dataset of outdoor vehicular wireless TDoA measurements, and compared with existing algorithms in literature. It is shown that our algorithm outperforms existing methods and allows to have much more reliability in real-world measurements.

The paper is organized as follow: section II formulates the TDoA system and the outlier problem, section III details several multilateration algorithms (with and without outlier management), section IV describes the experimental setup used to generate the dataset and evaluates the algorithms' accuracy on estimating the target's position in the presence of outliers.

Notations: $\hat{}$ symbol indicates variable with errors (e.g. noise). Small letter b is a scalar, bold $\boldsymbol{b}$ is a vector, bold capital $\boldsymbol{B}$ is a matrix. The notation $\|\cdot\|$ represents the L2-norm, while $\|\cdot\|_p$ represents the L_p-norm.

II. Problem Statement

Consider N time-synchronized anchors with known positions $\boldsymbol{p_i}$ ($i = 0, \ldots, N-1$), and a target with unknown position $\boldsymbol{p_s}$, which transmits RF packets. The ToA of an RF packet at anchor i is given by:

$$t_i = \|\boldsymbol{p_i} - \boldsymbol{p_s}\|/c + t_0 \tag{1}$$

where c is the speed of transmission through the medium and t_0 the unknown transmit time. In presence of a small measurement error η_i in the ToA estimation at anchor i, the measured ToA becomes:

$$\hat{t}_i = \|\boldsymbol{p_i} - \boldsymbol{p_s}\|/c + t_0 + \eta_i \tag{2}$$

No prior assumption is made on the distribution of η_i, except that if errors are biased, they are biased identically across all anchors.

To retrieve the target's position $\boldsymbol{p_s}$, a TDoA (or Range Difference (RD)) system of equations is constructed:

$$\begin{aligned} d_i &= ct_i \\ t_{ij} &= t_j - t_i \\ d_{ij} &= d_j - d_i \\ &= \|\boldsymbol{p_j} - \boldsymbol{p_s}\| - \|\boldsymbol{p_i} - \boldsymbol{p_s}\| \end{aligned} \tag{3}$$

where $i = 0, \ldots, N-2, i < j < N$. This system of equations represents a set of hyperbolas which intersect at $\boldsymbol{p_s}$.

In practice, the ToA measurements can be further degraded due to multiple factors such as MPC, NLOS and time synchronization error. In such cases, the measurement is called an *outlier*:

$$\hat{d}_i = c\hat{t}_i = \|\boldsymbol{p_i} - \boldsymbol{p_s}\| + c(t_0 + \eta_i + \vartheta_i) \tag{4}$$

where ϑ_i is the additional error, $\vartheta_i \gg \sigma_i$ (with σ_i being the standard deviation of η_i). These outliers will degrade the estimation of $\boldsymbol{p_s}$, and are hard to model mathematically. This paper focuses on estimating $\boldsymbol{p_s}$ in presence of such outliers.

III. Localization Algorithms with and without Outliers

In this section, we will investigate different types of algorithm for TDoA localization (without or with outliers). We start with ML-type algorithms, we then briefly discuss closed-form estimation using linearisation, and we finally present our geometry-based algorithm.

A. Maximum Likelihood estimation

In absence of outliers in (2) and if η_i is i.i.d. Gaussian, the ML estimator is given by [7]:

$$\boldsymbol{p_s} = \arg\min_{\boldsymbol{p}} \|\boldsymbol{Q}(\hat{\boldsymbol{d}} - \boldsymbol{f}(\boldsymbol{p}))\|^2 \tag{5}$$

where

$$\begin{aligned} f(\boldsymbol{p})_i &= \|\boldsymbol{p_i} - \boldsymbol{p}\| \\ \hat{\boldsymbol{d}} &= [\hat{d}_0, \ldots, \hat{d}_{N-1}]^T \\ \boldsymbol{Q} &= \boldsymbol{I_N} - (1/N)\boldsymbol{J_N} \end{aligned} \tag{6}$$

1) Metrics: Equation (5) is not robust against outlier measurements [7]. Fig. 1 shows a snapshot of experiment results (the details of the experimental setup will be presented in section IV). It shows target and anchors' positions, hyperbolas generated from the measured TDoA and the estimated target's position from several estimators. As we can see, anchor 3 introduces a large bias in the estimated target's position.

In the following, we propose several algorithms that derive from the ML algorithm to mitigate the effect of outliers on the ML estimator. The procedure is the following:

1) Compute a first location estimate using (5).
2) Use that estimate to filter out some TDoAs or anchors that are perceived as outliers.
3) Compute a final location estimate from the filtered data using (5).

If most of TDoAs are not outliers, it is expected that the first estimation will carry enough information to detect outliers.

The considered filtering methods are the following:

Farthest Remove the anchor farthest from the initial location estimate.

Middle Cut the scene in half. Remove the anchors that are not in the same area as the target.

Biggest Cumulated Error (BCE) Compute the absolute error between the measured TDoA and the TDoA reconstructed from the location estimate, $e_{ij} = c|t_{ij} - \hat{t}_{ij}|$ for $i = 0, \ldots, N-2, i < j < N$. Remove anchor i such as:

$$\arg\max_i \sum_j e_{ij}$$

or such as:

$$\arg\max_{ij} e_{ij}$$

Biggest TDoA Error (BTE) Remove the TDoA with the largest error e_{ij}.

Threshold Remove all TDoAs for which $e_{ij} > \alpha, \alpha \in \mathbb{R}$

The relation between the TDoA error and the resulting distance between the hyperbola branch and the target non-linearly depends on the position of the target. E.g., a $1\,\mathrm{m}/c$ TDoA error does not produce the same displacement when the target is close or far from the center of the hyperbola. Hence, we also introduce variants for methods that use e_{ij} (i.e. the BCE and BTE methods): the estimated error is divided by the gradient of the hyperbola considered [14].
However, all these methods are still based on the ML algorithm. If the initial ML estimate is too far off (as is the case in Fig. 1), the filtering methods will be ineffective, as will be show in section IV. This is mainly because it is impossible to accurately model outliers in (4), as the nature of outlier measurements is often unpredictable.

B. Closed-form estimation using linearisation

One of the drawbacks of ML estimators is their high computational cost. Several closed-form solutions have been proposed in literature [4], [6], which rely on a linearisation of (3). While such algorithms have a lower computation complexity than ML estimators, they suffer from the same

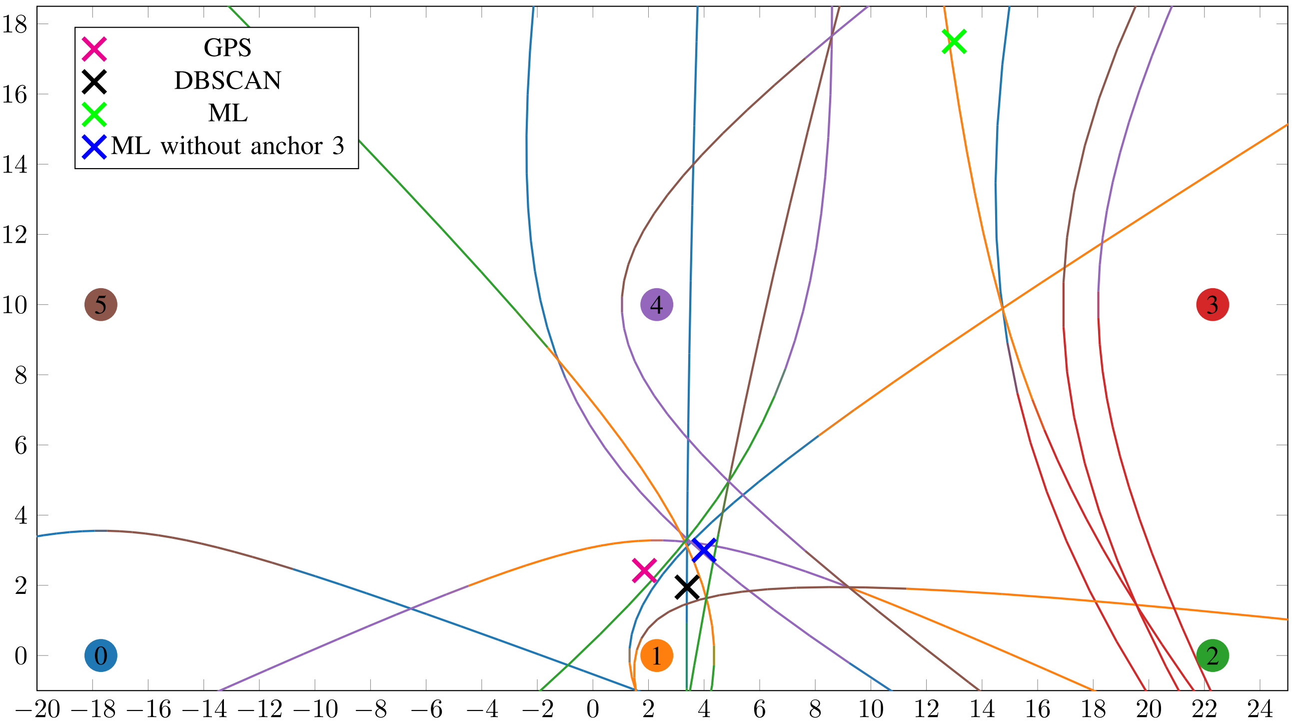


Fig. 1. Influence of outliers produced by anchor 3 on the target's location estimate. Colored dots are anchors' position. Bicolored curves correspond to hyperbolas branches generated by corresponding anchors (associated by color). GPS cross is the used ground-truth for the target's location. ML crosses are the estimation of the target's location using (5), with and without anchor 3. DBSCAN cross is the estimation using the algorithm described in subsection III-C.

fundamental flaws as ML estimators when it comes to outliers. The performance of these algorithms will be evaluated in section IV.

C. Intersection + DBSCAN

When examining Fig. 1, it is clear that most hyperbolas intersect each other close to the true target's location. However, the presence of a handful of outliers (which cannot be modelled mathematically) will draw the ML estimate away from the true target's location. A more geometric interpretation of the TDoA measurements offers an algorithm that will be more robust to the presence of outliers. This algorithm will be presented in this section.

The Intersection + DBSCAN algorithm finds the intersections of all pair of hyperbolas, then applies Density-Based Spatial Clustering of Applications with Noise (DBSCAN) on the hyperbola intersections, a clustering method robust to outlier, to deduce the target's location. It differs from most classical methods – which derive solutions through mathematical developments – by taking the geometrical aspect of the problem into account.

When TDoAs measurements are only affected by small errors, the corresponding hyperbolas and intersections will stay in a vicinity of the target's location, forming a cluster of intersections. However, when TDoAs measurements are affected by large errors (i.e. outliers), their hyperbolas and intersections will most likely exit that vicinity and fail to form a cluster.

An analytical solution for hyperbolas' intersections is presented in [15]. However, [15] only considers some anchor configurations and considers the two branches of a hyperbola. Hence, the algorithm in [15] has been extended to consider all anchor configurations, consider only the relevant branch, and avoid solving the 4th-order equation when possible.

After locating the intersections of hyperbolas, they are processed using DBSCAN to constructs clusters.

DBSCAN defines the following concepts:

core point a point p is a core point if there is at least m_p points (including p) within a ϵ distance from p

directly reachable a point q is directly reachable from p if p is a core point and q is within a ϵ distance from p.

indirectly reachable a point q is indirectly reachable from p if there is a chain $p_i, i = 1, \dots, n$, where p_{i+1} is directly reachable from p_i and q is directly reachable from p_n.

If p is a core point, then p and all points that are reachable (directly or indirectly) from p constitute a cluster. Non-core points inside a cluster are located at the border of this cluster, since only core points can reach other points. Points not belonging to a cluster are considered outliers. These concepts are depicted in Fig. 2.

The target's location is estimated by taking the median of intersections belonging to the largest cluster (as the number of members).

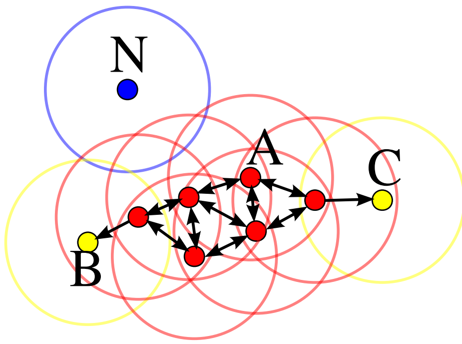
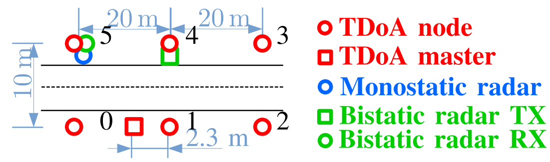
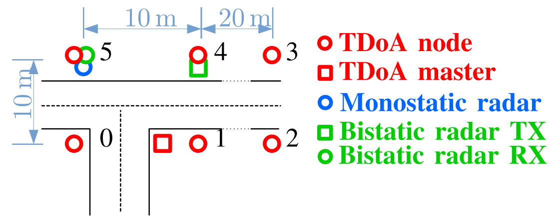


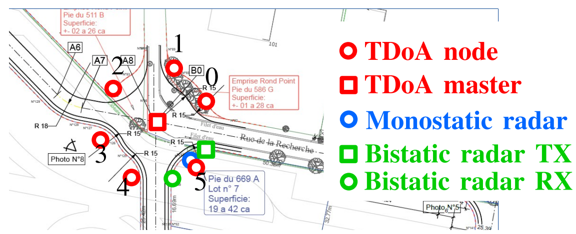
Fig. 2. Illustration of DBSCAN algorithm with $m_p = 4$ [16]. Point A and other red points are core points, since there is at least 4 points (including them) within a ϵ distance from them (red circles). Points B and C are border points, since they are reachable from a core point without having at least 4 point in their respective vicinities (yellow circles). As all red and yellow points are reachable from one another, they form a single cluster. Point N is an outlier since it is neither a core point nor reachable from one.



(a) Map for MR scenarios.



(b) Map for RJ scenarios.



(c) Map for RA scenarios.

Fig. 3. Map for the OTA-synchronized experiment. It combines 6 TDoA anchors, 1 bistatic radar, and 1 monostatic radar. The devices are placed ≈ 2 m above ground.

IV. Experiment Results

The algorithms presented in section III will be evaluated on a real-world wireless TDoA dataset to test their robustness against outliers. We first present the experimental setup, and then present the performances of the different algorithms.

A. Experimental setup

The experimental setup consists of 6 anchors (USRP-X310 Software Defined Radio (SDR)) for TDoA acquisition, 1 master (USRP-X310 SDR) for Over-The-Air (OTA) synchronization, and up to 4 targets (USRP-E310 SDR). The master and targets transmit the 802.11 non-HT training field (20 MHz) every 10 ms, at carrier frequencies of 2.55 GHz and 2.35 GHz, respectively. The anchors estimates the ToA of the received packets from the master and the targets, at a sampling rate of 100 MHz.

Targets record their location using Global Positioning System (GPS), and will be used as ground-truth[1]. Since the rates of GPS and the RF system differ, GPS locations are interpolated at estimated acquisition times of the RF packets. Furthermore, the following data preprocessing is performed:

- Discard data segments when the time interval between GPS updates is ≥ 4 s.
- Discard trajectories that are outside the smallest rectangular area containing all anchors.

The experiment considered several road scenarios: Major Road (MR), Road Junction (RJ), Roundabout (RA), depicted in Fig. 3. For each scenarios, between 30 and 40 measurements were taken, with almost 4000 snapshots per measurement and target. In total we have almost 7 millions individual ToA measurements, reduced to 2 millions after preprocessing.

The resulting dataset consists of ToA measurements, with unknown time of transmission. Those ToAs suffer from noise, but also from other phenomenons such as NLOS propagation, MPC and OTA synchronization error. Outliers are measurements where those effects inflict large errors on the ToA estimation.

From preliminary cable-connected lab experiments, we estimate that master-slave synchronization with a refresh rate of 10 ms for these devices introduces ≈ 3 ns uncertainty in TDoA estimation.

B. Performance of Maximum Likelihood and Maximum Likelihood-filtered algorithms

The ML is evaluated using a 2D-grid with 0.5 m spacing. Removing more than 1 anchor/TDoA has been evaluated (removing 2 in 1 or 2 passes), but did not produce finer accuracy than removing only 1. For simplicity, they are not shown in this paper.

The impact of the metrics (presented in subsubsection III-A1) on the ML accuracy is shown in Fig. 4. From all metrics, the *BCE* and the *threshold ($\alpha = 5$ m)* methods achieves the most accurate estimates. Their improvements at key values are summarised in Table I[2]. Nevertheless, half of tested methods performs worse than without any anchor filtering. Furthermore, several methods are very sensitive to the scenarios considered. For example, the *middle* method performs in a similar fashion than the unfiltered version when excluding roundabout scenarios, but worse when only considering the

[1] Note that while the environment setup was very good (no close building, open sky), GPS is most likely 1 m to 2 m-accurate.

[2] DBSCAN and Picard are discussed in subsection IV-D

TABLE I
ECDF VALUES OF THE ERROR BETWEEN GROUND-TRUTH LOCATIONS AND ESTIMATED ONES USING SEVERAL METHODS, ACROSS ALL SCENARIOS.

	3 m	5 m	10 m
ML	0.62	0.85	0.92
ML + BCE	0.62	0.87	0.95
ML + threshold ($\alpha = 5\,\text{m}$)	0.64	0.87	0.94
Picard ($\alpha\sigma_n = 3\,\text{m}$)	0.61	0.86	0.95
Intersection + DBSCAN	0.67	0.93	0.99

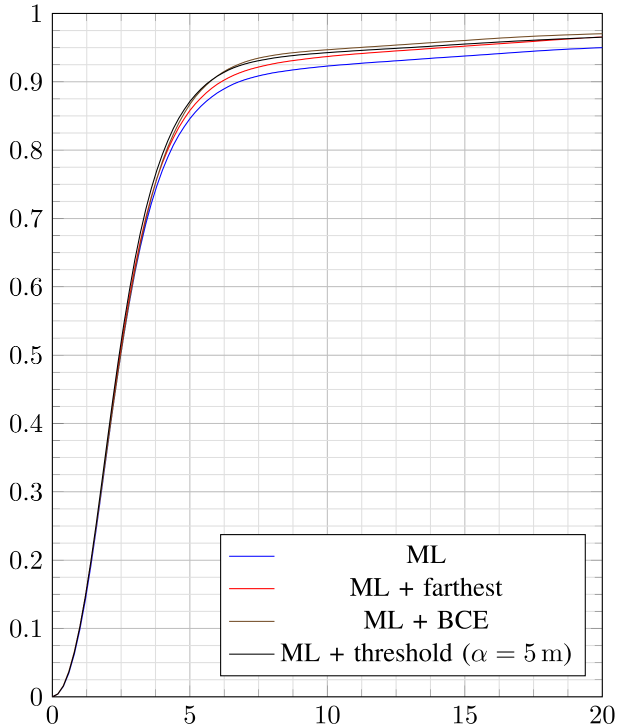


Fig. 4. eCDF values of the error between ground-truth locations and estimated ones using ML estimation, all scenarios. ML estimation coupled with metric to discard outliers exhibits better accuracy than without metric.

roundabout scenarios. This could be explained by the shape of the scenario. In the roundabout scenarios, deciding on how to split the area in half is not straightforward due to their circular geometry.

The difference between the L2-norm and the L_p-norm, $p = 0.8$ are summarized in Table II. There is no large difference when changing the norm, as expected [7].

Overall, some of these metrics help improving the accuracy of the ML estimator by a few percentage. Nonetheless, there exist major drawbacks:

TABLE II
ECDF VALUES OF THE ERROR BETWEEN GROUND-TRUTH LOCATIONS AND ESTIMATED ONES USING DIFFERENT NORMS, ACROSS ALL SCENARIOS.

	3 m	5 m	10 m
ML	0.62	0.85	0.92
ML + threshold ($\alpha = 5\,\text{m}$)	0.64	0.87	0.94
ML ($p = 0.8$)	0.60	0.84	0.94
ML ($p = 0.8$) + threshold ($\alpha = 5\,\text{m}$)	0.62	0.87	0.96

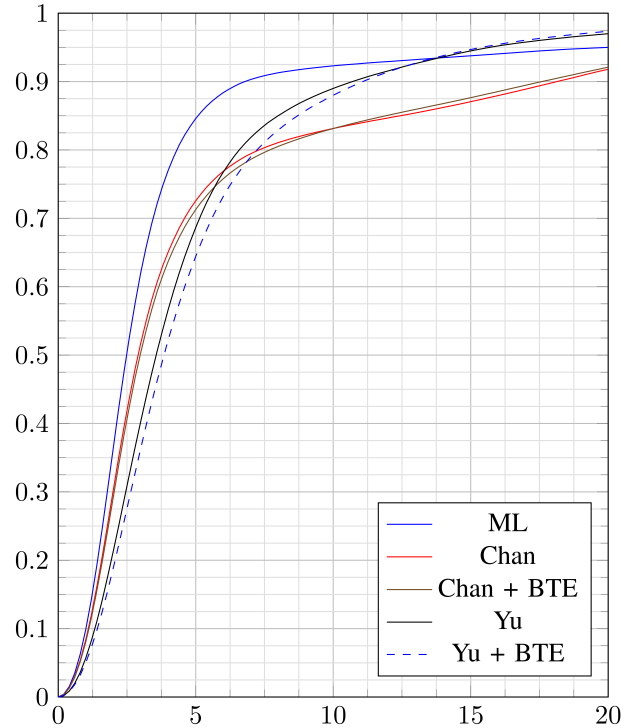


Fig. 5. eCDF values of the error between ground-truth locations and estimated ones. All evaluated closed-forms perform worst than the ML estimation.

- It is computationally heavy. It requires a 2D-grid search and must be evaluated a couple of times.
- Most of the methods are sensitive to scenarios.
- In presence of strong outliers, the first estimation can go too far away from the true location to get any information on the outliers. Fig. 1 is such an example, where no metric was capable of filtering out anchor 3, and hence recovering.

C. Performance of closed-form solutions using linearisation

The comparison between the ML estimator and closed-form estimators from Chan and Ho [4] and Yu, Gaubitch, and Heusdens [6] is depicted in Fig. 5. As expected, they all performs worse than the ML estimator.

D. Performance of Intersection + Density-Based Spatial Clustering of Applications with Noise algorithm

The Intersection + DBSCAN method has been evaluated with parameters $\epsilon = 5\,\text{m}$ and $m_p = \min(20, \#\text{point}/4)$. For comparison, algorithms proposed in [7] and the most accurate ML+metric are plotted alongside it.

Picard and Weiss [7] is able to achieve similar results than the most accurate ML+metric, with lower complexity. Intersection + DBSCAN outperforms both of them. Their improvements at key values are summarised in Table I.

While the proposed algorithm outperforms all the others in the presence of outliers, Intersection + DBSCAN comes with several drawbacks:

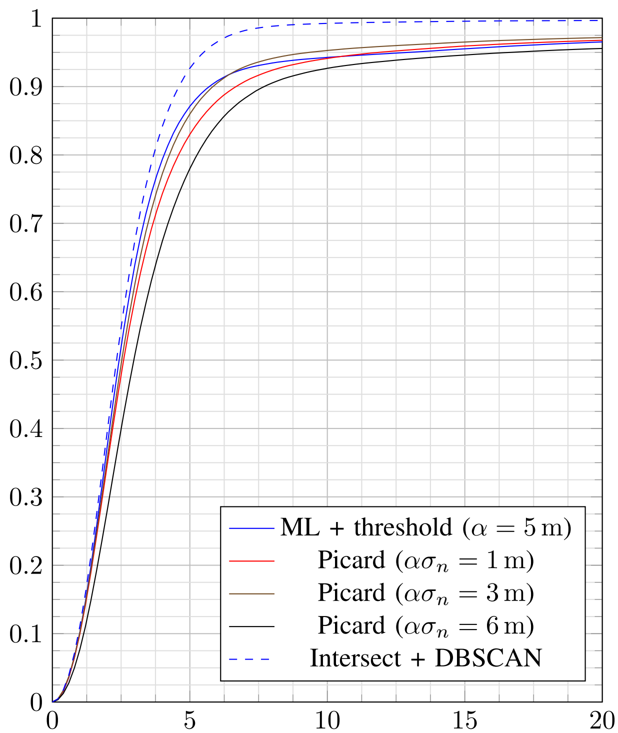


Fig. 6. eCDF values of the error between ground-truth locations and estimated ones. Picard – when using fine-tuned parameters – and ML + threshold gives similar results. Intersection + DBSCAN outperforms them.

- The intersection algorithm is not numerically stable. Due to the high non-linearity of the general solution and the finite precision of floating point arithmetic, some intersections are missing or biased.
- If there is only a small number of hyperbolas, there is a high probability of having few to no intersection. If there is not enough intersections, close to each other, then DBSCAN will not be able to form a cluster and estimate a location.

V. Conclusion

In this work, we have evaluated the robustness of several TDoA-based localization algorithms against outliers. Most conventional methods overlook the issue of outliers in measurements. They require specific noise distribution, small errors, or a priori knowledge on outliers. To achieve higher estimation accuracy, 2-pass ML estimators, combined to several metrics to detect outliers, have been evaluated. However, these estimators are computationally heavy and sensitive to the considered geometry, for little accuracy gain (less than $5\,\%$). A novel method is also presented, consisting of geometrical consensus based on hyperbolas' intersections. It achieves precise localization in presence of outliers, without requiring any prior information on them.

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