On the topology patterns and symmetry breaking in two planar synthetic jets

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ABSTRACT
This article studies the flow structures and main patterns driving the flow dynamics in one and two planar synthetic jets. We perform numerical simulations at different Reynolds numbers (Re), for a similar forcing frequency, to model the synthetic jet flow and the two planar synthetic jets, which present a movement in-phase (synchronous jets) and out-of-phase (asynchronous jets). We identify two types of flow regimes as function of the Reynolds number: (i) the flow is symmetric and (ii) the symmetry is broken at Re \( \approx 110 \) and Re \( \approx 140 \) for the single and the synchronous jets, respectively. On the contrary, the flow is always asymmetric in the two asynchronous jets. We calculate the thrust produced by the several jet configurations, finding that the thrust produced by a single jet is always half of the thrust produced by the two synchronous jets; however, this quantity is much smaller in the asynchronous jets. Finally, we use higher order dynamic mode decomposition to identify the main patterns driving the flow dynamics. The solution is periodic in the single and two synchronous jets, with the forcing frequency (St) as the dominant mode. The emerging rise in amplitude of a low-frequency mode (St0 = St / 6) that sub-harmonic of the forcing frequency as the Reynolds number increases suggests a connection between this mode and the symmetry breaking. A new mode is identified in the asynchronous jets, breaking the flow periodicity.

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I. INTRODUCTION

A synthetic jet, also known as zero-net-mass flux jet, is a jet stream formed by the periodic oscillations of a membrane or a piston inside a cavity that forces the fluid to pass through a small orifice, known as the jet nozzle. The flow continuously leaves and enters the cavity by the same orifice, producing momentum while the net mass flux is zero. This characteristic property of synthetic jets presents a clear advantage, since they are formed from the working fluid of the flow system and do not require additional net mass injection to produce nonzero mean streamwise momentum flux.

On the one hand, the use of synthetic jets is nowadays very extended to increase the efficiency in several industrial applications such as fluid mixing, heat transfer enhancement, plasma actuators, jet vectoring, and also for applications in active flow control of, that is, boundary layer separation, dynamic stall in vertical axis wind turbines, or wingtip vortexes. On the other hand, synthetic jets can be used for modeling the swimming motion of several marine animals such as squids, jellyfish, or salps. The climate change has motivated several researchers to look for new alternative transportation devices reducing the environmental impact. To minimize the footprint, the search of alternative marine propulsion systems inspired or even mimicking the animal swimming motion has become a research topic of high interest in the field of marine locomotion. Several propulsion systems based on the animal swimming motion are driven by a vortex ring. Synthetic jets are devices able to artificially create vortex rings producing thrust: when the flow is ejected from the cavity and the injection phase, the flow separates and rolls up forming a vortex ring moving downstream due to self-induced velocity. On the contrary, in the suction phase, the flow topology in synthetic jet identifies a saddle point in the near field separating the flow reentering into the cavity from the flow that continues traveling downstream.

The formation and evolution of the vortex ring defines the efficiency of the propulsion system and the amount of thrust generated by the fluid. Several authors have studied in detail the formation of this type of flow, both numerically and experimentally in the last twenty years. Two nondimensional parameters, the Strouhal number, measuring the piston or membrane oscillation frequency, and the Reynolds number, which compares convective and viscous terms, define the type of vortex generated by the synthetic jet. More specifically, based on these two nondimensional parameters, Carter...
and Soria\textsuperscript{27} identified four types of flow regimes in synthetic jets, each one connected with different types of vortices. The flow regimes distinguish between the generations of: (i) laminar jets, (ii) purely vortex rings (without a continuous jet stream), (iii) transitional jets, and (iv) turbulent jets. However, if the Reynolds and Strouhal numbers are not carefully selected, the forcing intensity could be very small and the flow ejected during the injection phase reenters into the cavity during the suction phase in a reversible way, with no effect on the bulk fluid, and hence, the jet is not formed. To prevent this from happening, it is necessary to follow the formation criterion for synthetic jets established by Holman \textit{et al.}\textsuperscript{31} which relates the Strouhal and Reynolds numbers to choose the configuration that ensures the jet formation.

The generation of the optimal vortex that maximizes the system efficiency, while producing the largest thrust is the key point to generate an alternative propulsion mechanism. The swimming motion of squids or jellyfish is modeled by the vortices found in regimes (i) and (ii) from the diagram postulated by Carter and Soria\textsuperscript{27} previously mentioned. More specifically, Dahiri \textit{et al.}\textsuperscript{29} found that the most efficient jellyfish were swimming in regime (ii), while the highest thrust was produced by the jellyfish swimming in regime (i). The generation of the optimal vortex, maximizing the system efficiency and thrust, has been studied by several authors\textsuperscript{12,29} Recently, Le Clainche\textsuperscript{31} presented a simple way to predict the generation of the optimal vortex as function of the flow regime, its topological variations due to the interaction of two concentric jet were studied in Ref. 32, and, in Refs. 31 and 33, its flow structure was studied applying higher order dynamical mode decomposition (HODMD) in the case of an axisymmetric jet. Three-dimensional analysis has been also carried out in Ref. 34 showing when transitioning to turbulence. However, to the authors’ knowledge, the thrust produced by two synthetic jets operating in phase (zero phase difference) and at different phases has not been studied before in detail. Also, the origin of the flow instabilities and their possible connection to changes in the flow (i.e., symmetry breaking in planar jets) remain still unknown. Hence, the present article aims to solve these two issues.

Smith and Glezer\textsuperscript{35} and Luo and Xia\textsuperscript{36} recently studied experimentally the flow field in a pair of synthetic jets. In both cases, the authors found that, when the two jets are operated in phase, there is an enhancing effect on the overall jet due to the attraction and merging of adjacent vortex pairs. Moreover, the first authors identified that the flow rate in two jets is twice the one of a single jet, since the flow is entrained from each one of the jets. Moreover, the velocity downstream is higher and the merged vortex pair remains more coherent than in the case of a single jet. The authors also studied the effect of a phase difference between the jet working regimes, finding that the vortex pairs are deflected instead of merging, causing a vectoring effect in the combined jet stream in the direction of the jet leading in phase: the leading vortex pair attracts the second vortex pair. This vectoring effect presents a clear advantage to enhance convective heat transfer in applications with impingement cooling as presented in Refs. 37 and 38. Recently, Kim \textit{et al.}\textsuperscript{39} studied experimentally the effect of the stroke length and the distance of the two jets in the jet stream, finding that the merging point of the two jets moves upstream for short distances between the two orifices and that as the stroke length increases, the advection speed of vortices also increases.

The potential of using pairs of synthetic jet for multiple industrial applications and also as an alternative propulsion system motivates this research, which focuses in understanding the physical mechanism involved in the interaction of multiple synthetic jets. More specifically, the article studies in detail the main flow structures and frequencies leading the flow dynamics and the thrust produced by two planar synthetic jets, working with similar and different phases, providing a connection between the flow features and the thrust.

The article is organized as follows. The model description and numerical simulations are introduced in Sec. II. The methodology used to identify the main flow patterns is presented in III. The main results are presented in Secs. IV–VI. Finally, the main conclusions are presented in Sec. VII.

II. MODEL DESCRIPTION AND NUMERICAL SIMULATIONS

The incompressible, two-dimensional continuity and Navier–Stokes equations are used to model the synthetic jet flow. These equations are written in nonlinear form as

\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \frac{1}{Re} \Delta \mathbf{u}, \quad (2) \]

where \( p \) and \( \mathbf{u} \) are the nondimensional pressure and velocity vector \( \mathbf{u} = (u_x, u_z) \) with \( u_x \) and \( u_z \) as the streamwise and radial velocity components, respectively, and \( Re = \frac{UD}{\nu} \), where \( \nu \) is the kinematic viscosity of the fluid, \( U \) is the characteristic flow velocity, and \( D \) is diameter of one of the (two) jet nozzles. These equations are nondimensionalized using \( D \) and \( D/U \) as units for length and time, respectively.

The flow in synthetic jets is characterized by two nondimensional parameters, the Reynolds number, as previously introduced, and the Strouhal number, defined as \( St = fD/U \), where \( f \) is the piston oscillation frequency. The characteristic velocity \( U \) is set as the momentum velocity. In axisymmetric flows, this velocity is defined as \( U = DpV_p/(\sqrt{2}D) \), where \( Dp \) is the piston diameter and \( V_p \) is the peak amplitude of the piston velocity (see details in Le Clainche \textit{et al.}\textsuperscript{31}); however, in two-dimensional computations, the velocity is defined as \( U = 1 \), neglecting the azimuthal contribution.

The geometry modeling one and two synthetic jets is based on the setup described in Refs. 31–33, and 40, where the authors used the same configuration to model an axisymmetric jet both numerically and experimentally. Nevertheless, in this article the computational domain is two-dimensional and reflection symmetric with respect to the streamwise axis, as presented in Fig. 1. The setting parameters are \( D_p = 5D \) for the cavity diameter and depth and \( L_c = 0.2D \) for the jet stroke. As detailed below, the dimensions of the computational domain are set to minimize the influence of the boundary conditions over the solution and to avoid boundary reflections, namely, \( L = 300D \) and \( R_e = 240D \) for the streamwise and normal components, respectively.

The boundary conditions of the computational domain (see Fig. 1) are set as in Kotapati \textit{et al.}\textsuperscript{18} the inlet boundary, which defines the piston oscillation, is set as \( u_x = V_p \cdot \sin(2\pi ft) \), following a periodic movement in time \( t \) defined by the periodic phase \( \phi \in [0, \pi] \) with \( \phi \in [0, \pi] \) for the injection and suction phases, respectively, and with \( V_p = 1/5 \), and \( u_z = 0 \) for velocity and Neumann boundary conditions for the pressure; nonslip \( (u_x = u_z = 0) \) conditions are set at the wall of the jet cavity and the wall sides of the jet nozzle;
finally, the outflow boundary (top, bottom, and outlet surfaces of the computational domain) is modeled with the boundary condition proposed by Dong et al.\textsuperscript{41} as

\[-pn + vn \cdot \nabla u - \left[ \frac{1}{2} |u|^2 S_o (n \cdot u) \right] n = f_b (x, t), \quad \text{on } \partial \Omega_u, \quad (3)\]

where \( n \) is the outward-pointing unit vector normal to \( \partial \Omega_u \), the contour where \( u \) and \( p \) are still unknown, \( f_b \) is a forcing vector, and \( S_o \) is

\[ S_o (n \cdot u) = \frac{1}{2} \left( 1 - \tanh \left( \frac{n \cdot u}{U \delta} \right) \right), \quad (4) \]

where \( U \) is the characteristic velocity scale and \( \delta \) is a positive small constant that approaches to zero. In the present simulations, \( f_b = 0 \), hence, the effective stress induced will be locally balanced by the stress on the boundary \( \partial \Omega_u \) in the case of the presence of energy influx through \( \partial \Omega_u \) in Eq. (3). The term in brackets denotes the stress exerting on \( \partial \Omega_u \) and the term \(-pn + vn \cdot \nabla u\), the stress on \( \partial \Omega_u \).

Numerical simulations have been performed using the solver Nek5000,\textsuperscript{42} which is an open source code using a spatial discretization based on the spectral element method. Concerning the time discretization, an explicit third-order extrapolation scheme was used for the nonlinear terms, and an implicit second-order backward differentiation scheme for the viscous terms. The code was evolved in time for 1000 nondimensional time units, which corresponds to 30 piston oscillation cycles. Four groups of simulations are carried out according to their boundary conditions. (i) only a synthetic jet is active, two synthetic jets are active (ii) following a synchronized movement (similar inlet boundary conditions in both jets), called as synchronous jets and following a desynchronized movement, where the phase, \( \phi \), of the second jet is delayed (iii) \( \phi = \pi / 2 \) and (iv) \( \phi = \pi \) over the first jet, called as asynchronous jets. The Strouhal number is fixed in all the simulations as \( St = 0.03 \), to ensure that the flow is laminar forming a jet stream (see details in the diagram presented in Carter and Soria\textsuperscript{27}) and the Reynolds number is varied as presented in Table I.

The mesh has been generated using the software Gmsh. The mesh is composed of 3182 macro-elements, each one discretized using \( N+1 \) Gauss–Lobatto–Legendre points in the two spatial directions, and hence, the polynomial order is \( p = (N + 1) - 1 = N \).
where $\mathbf{F} \cdot \mathbf{i}$ is the projection of the force vector of the fluid in the streamwise direction; $T_{\text{conv}}$ and $T_{\text{visc}}$ are the thrust due to convective and viscous terms, respectively; $t'$ is the viscous stress tensor; $\mu$ is the kinematic viscosity; and $\Sigma_1$ to $\Sigma_3$ are the surfaces defined in Fig. 2 with their respective numbers. The viscous term is negligible [Eq. (6)] because its maximum contribution is approximately 0.8% of the total thrust if the control volume is not the one defined by width $a$ and $d = D$. This has been checked in a case where the viscosity of the fluid takes high importance, $Re = 10$, and less importance, $Re = 200$, where the two jets are synchronized. The convective term [Eq. (7)] does not include the boundaries of the solid walls because the thrust is given by the forces that the control volume applies to the external fluid in order to generate propulsion.

The mesh independence study is carried out in the simulations of the two synchronous jets ($\phi = 0$) at $Re = 200$. This is considered the most complex case, since the characteristic length of the flow field could be defined considering the interaction between the two jets by the two jet streams as $D^* = 5D$; hence, the Reynolds number is then defined as $Re^* = 5Re = 1000$ (see details of different characteristic lengths defined by the wake interaction in multi-body configurations in Ref. 43). First, the polynomial order is set to $p = 10$, being the number of points in each mesh, $N_{\text{points}}$, defined as number of elements, $N_{\text{elements}}$, as $N_{\text{points}} = N_{\text{elements}} * p^3$, and the dimensions of the computational domain, $L$ and $R_e$, are varied, generating eight different domain sizes as presented in Table II.

The test by Roache$^{44}$ is used to compare the performance of the simulations in the different computational domains. This test generalizes the Richardson extrapolation for nth-order methods using the following grid convergence index (GCI):

$$GCI_{j+1,i} = \frac{3 \left( f_i - f_{i+1} \right)}{f_i (l^m - 1)}, \quad 100,$$

where $f_i$ is the field value on the fine mesh (in this article, $f_i$ is the averaged pressure or the thrust in the biggest mesh $D8$, which is considered as the reference), $f_{i+1}$ is the same field value discretized in a rougher mesh, $n$ is the order of the method, and $l$ is the size refinement ratio (e.g., $N_{D4}/N_{D8}$, where $N_{D4}$ and $N_{D8}$ are the number of points defined...
in the meshes $D_4$ and $D_8$, respectively), which is calculated as $N_{Di} = N_{\text{elements}}/p$, with $N_{\text{elements}}$ as the number of elements of the mesh $Di$, and $p = 10$ as the polynomial order. The grid independence study is carried out comparing the pressure fields, which is an important magnitude to calculate the thrust generated by the synthetic jets, and the thrust itself, which is a critical magnitude to provide accurate curves of this quantity in the different simulations performed. The velocity fields and the flow topology patterns are also important variables to take into account, since this article shows a detailed description of the flow topology in planar synthetic jets and studies the presence of flow instabilities via modal decompositions. Nevertheless, previous research shows that these two magnitudes are quite robust; hence, even in the analysis of numerical solutions with low resolution, it is possible to identify the main topology patterns describing the flow and the main dynamic modes driving the flow motion (see details in Ref. 45, where the authors extend this discussion to the analysis of a three-dimensional hemisphere cylinder, comparing numerical and experimental results). The grid independence study is also carried out comparing the velocity fields to ensure the accuracy of the results presented following a highly restrictive criterion.

To ensure that the flow was sufficiently converged, all the simulations have been integrated in time solving 30 piston cycles (the simulations are converged after 13 piston cycles, see details in Refs. 31 and 33) except when using the domains $D_1$ and $D_3$ that the simulations were stopped before due to problems of convergence (the peak of the pressure was three orders of magnitude superior than in the remaining

### TABLE IV. Relative error in the thrust calculation as function of the different polynomial degrees $p$ in different control volumes defined for different $d$ (see previous Fig. 2) in mesh $D_8$. The reference solution is the polynomial $p = 20$. Total number of grid points $= \text{number of macro-elements} \times p^2$

<table>
<thead>
<tr>
<th>$d/D$</th>
<th>$p = 12$</th>
<th>$p = 14$</th>
<th>$p = 16$</th>
<th>$p = 18$</th>
<th>$p = 20$</th>
</tr>
</thead>
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<tr>
<td>3</td>
<td>6.474%</td>
<td>2.172%</td>
<td>9.301%</td>
<td>0.813%</td>
<td>...</td>
</tr>
<tr>
<td>5</td>
<td>6.065%</td>
<td>6.009%</td>
<td>2.847%</td>
<td>0.463%</td>
<td>...</td>
</tr>
<tr>
<td>7</td>
<td>2.551%</td>
<td>5.398%</td>
<td>3.685%</td>
<td>0.063%</td>
<td>...</td>
</tr>
<tr>
<td>11</td>
<td>0.525%</td>
<td>1.954%</td>
<td>1.552%</td>
<td>0.165%</td>
<td>...</td>
</tr>
</tbody>
</table>

![FIG. 4. Injection and suction phase in the single jet. In each sub-figure for each Reynolds number, streamlines and vorticity contours (left) and streamwise velocity contours (right). The white dot represents a saddle point in the suction phase. Snapshot extracted at $\varphi = 3\pi/4$ and $\varphi = 5\pi/4$ in the injection and suction phases, respectively. The domain shows the cavity and the near field of the jet defined in $x \in [-5.2, 10]$ (the jet exit is at $x = 0$) and $y = [-10, 5]$.](image-url)
cases). The domain $D_2$ was also discarded because the simulation presented some unexpected results. Table III compares the grid convergence index (GCI) calculated for the mean pressure field in the remaining meshes calculated in three representative points of the computational domain in cycles 20–25 (similar results are obtained for the cycles 25–30). The error comparing all the meshes with the reference mesh $D_8$ varies between $\sim 1\%$ and $6\%$ in the different points, being $D_5$ the mesh with smallest errors ($\sim 1\% - 2.5\%$). Nevertheless, to ensure the accuracy of the simulation presented in this article, we follow a highly strict criterion selecting the largest size mesh, $D_8$, to perform the numerical simulations. It is notorious that in the analysis of an axisymmetric jet at $Re = 1000$ and $St = 0.03$ carried out in Ref. 33, the authors validate the numerical solution obtained using the code Nek5000$^{42}$ with particle image velocimetry experiments. In that case, the dimensions of the computational domain were $L/D = 80D$ and $Re/D = 40$, while the dimensions of the present selected mesh $D_8$ are $L/D = 300D$ and $Re/D = 240$, which guarantees that the boundaries are sufficiently far from the jet flow (not affecting the solution) even in the case with two synthetic jets. It is relevant to mention that the meshes have been designed in order to concentrate the points near the jet nozzle, providing high accuracy in the near field, as shown in Fig. 3.

A second study is carried out to select the mesh polynomial degree. As presented in Table IV, the relative error in the thrust calculations is compared as function of the polynomial degree $p$ and the parameter $d$ defining the control volume (see previous Fig. 2), where the reference value is defined as the case with $p = 20$. The errors using the polynomial degree $p = 18$ are smaller than $1\%$ in all the cases; hence, this is the polynomial degree set for the simulations. To sum up, the mesh selected to perform the numerical simulations contains 1.030 968 grid points (see the number of macro-elements in Table II).

### III. HIGHER ORDER DYNAMIC MODE DECOMPOSITION ALGORITHM

Higher order dynamic mode decomposition (HODMD)$^{46}$ is an extension of dynamic mode decomposition (DMD)$^{47}$ introduced to identify the main patterns in complex flows and nonlinear dynamical systems. Similarly to DMD, HODMD gives an approximation of the Koopman modes$^{48}$ as an expansion of DMD modes, providing accurate and reliable results as it was tested in a wide range of complex industrial and academic applications, that is, to identify flutter in experimental data,$^{49}$ to create three-dimensional flow fields from two-dimensional data,$^{50,51}$ to identify cross-flow instabilities,$^{52}$ as a reduced...
order model for flow predictions,\textsuperscript{39} to identify flow instabilities in turbulent flows,\textsuperscript{34-38} etc. (see more details, examples and MATLAB\textsuperscript{40} codes of the method in Ref.\textsuperscript{57}).

For simplicity, the spatiotemporal data analyzed are organized in tensor form as $u_{ijk} = u^i(x_i, y_j, t_k)$, where for $i = 1$ and $2$, $u^i$ denotes the axial and normal velocity components, respectively, for $i = 1, \ldots, I$ and $j = 1, \ldots, J$, being $I$ and $J$ the number of grid points along the axial and normal components. The data are equi-distant in space with distance $\Delta x$ and $\Delta y$ for the axial and normal components, respectively, and equi-distant in time, with time step $\Delta t$. HODMD decomposes the data as an expansion of $M$ DMD modes with unitary norm, $v^n_m(x_i, y_j)$, in the following way:

$$u^i_{ijk} \approx \sum_{m=1}^{M} a_{im} v^n_m(x_i, y_j) e^{i (\delta_m + \omega_m) t_k}, \quad (9)$$

where $a_{im} \geq 0$ are the mode amplitudes, and $\delta_m$ and $\omega_m$ are the temporal growth rates and frequencies, respectively.

The multi-dimensional iterative HODMD algorithm introduced in Ref.\textsuperscript{40} is used to analyze the data presented in this article. Three main parameters are necessary to calibrate the algorithm: the index $d$, which represents the number of windows in a window shift process carried out in the original tensor of data that improves the performance of the algorithm, and the tolerances $\varepsilon$ and $\varepsilon_1$, which represent the spatial and spectral complexities. The spatial complexity $N$ is the vector space spanned by the DMD modes, and the spectral complexity is the number of $M$ DMD modes retained in the DMD expansion Eq. (9). The algorithm will filter out the flow structures with small amplitude ($a_{im} < \varepsilon_1$), retaining only the large flow structures, which are relevant to describe the flow dynamics.\textsuperscript{3} To select the dominant modes, HODMD has been applied using several tolerances, $\varepsilon = \varepsilon_1 = 5 \times 10^{-3}$, $10^{-3}$, $5 \times 10^{-4}$ and $10^{-4}$, and several values of $d = 600$, $1000$, $1350$, $1700$, and $2000$ for a number of $4369$ snapshots, representing seven piston cycles, collected after the cycle $23$ to avoid the transient dynamics. In all the results presented, the method identifies all the modes that are physical, driving the flow motion; these modes are robust and are identified using the difference calibration parameters. Some additional spurious modes are identified with some specific calibration, which are easily identifiable, since they are not present in all tests calculated.

IV. FLOW TOPOLOGY AND VORTICITY IN SYNTHETIC JETS

This section presents a detail description of the main topology patterns and vorticity fields in the results obtained modeling one and two synthetic jets. Figures 4 and 5 provide a general view of the flow in the injection and suction phases in a single and two synchronous jet test cases (phase shift between jets $\phi = 0$). In the single jet case, the flow is symmetric at $Re = 110$ and nonsymmetric at $Re = 120$ in both cases, suggesting that a flow instability emerging at $Re \in (110, 120)$ could be the mechanism triggering this pattern change. The same behavior is identified in the case of two jets, but now the symmetry breaking occurs at $Re \in (140, 150)$. The figure compares the streamlines and vorticity contours (left) and the streamwise velocity contours (right) for different Reynolds numbers. In the injection case, it is possible to identify the vortex ring producing thrust, which is characteristic of this type of flow, while the saddle point that separates the flow that reenters into the cavity and the flow that continues traveling downstream is identified in the suction phase. In the two jet case, two main jet streams are differentiated in the near field in the injection phase, suggesting that the characteristic length of the flow field is the same as in a single jet.

Figure 6 shows in detail the streamlines and vorticity contours in the single and two synchronous jet cases at $Re = 100$, when the solution is symmetric. Starting from the single jet, in the injection phase, it is possible to see the evolution of the vortex ring in the streamlines, which is formed by two counter rotating vortices with opposite vorticity sign. As reflected by the vorticity contours and the streamlines, the size of the vortex ring increases as it travels downstream. In the two synchronous jets, two vortex rings are identified emerging from the two jet nozzles each one. Their size is similar to in the single jet; nevertheless, some differences are found in the trajectories followed by each one of these vortex rings: as they travel downstream, the two vortex “rings slightly separate” to each other with respect to the symmetry

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Vorticity contours and streamlines in the single and two synchronous jets at $Re = 100$. Each sub-figure represents the injection (top) and suction (bottom) phases. The white dot represents a saddle point. The domain shows the cavity and the near field of the jet defined in $x \in [-5.2, 10]$ (the jet exit is at $x = 0$) and $y \in [-10, 5]$. Red, blue, and green colors represent $10$, $-10$, and $0$ values of vorticity contours.}
\end{figure}
plane (see the phase instant $\varphi = 20\pi/21$). Regarding the suction phase, in both the single and two jet cases, the vortex rings continue traveling downstream and the saddle point separates the flow that reenters into the cavity. However, three different behaviors are identified in the two jet cases compared to the single jet: (i) the saddle point moves more downstream, (ii) the two vortex rings continue separating to each other as they travel downstream, and (iii) two high vorticity regions are identified close to the jet nozzle during all the suction phase, which are related to the fluid reentering into the cavity. At phase instant $\varphi = 35\pi/21$, the vortex ring is separated from the jet nozzle in a single jet, while in the two jets, these high vorticity regions are still connected to the two vortex rings, although at $\varphi = 40\pi/21$, the width of these regions decreases, but their length increases following the path driven by the vortex rings. At the beginning of the injection phase, $\varphi = 5\pi/21$, when the new pair of vortex rings emerges from the two jet nozzles, it is still possible to identify these high vorticity regions, which have been definitely separated from the previous vortex rings, and are subsequently connected with the new pair of vortices at $\varphi = 10\pi/21$.

The disorganized flow behavior identified in the nonsymmetric cases (as presented in previous Figs. 4 and 5) is also reflected in the time-averaged velocity field. Figure 7 shows the evolution along the streamwise spatial direction of the time-averaged streamwise velocity profiles in the single and two synchronous jets at $\text{Re} = 100, 150$, and 200. At $\text{Re} = 100$, when the solution is symmetric, it is possible to identify and follow the evolution of the jet stream: (i) a single jet stream evolves in the case of one jet that widens and shortens downstream and (ii) two jet streams are identified in the near field of the two jet cases, which merge into a single jet stream downstream ($x/D \approx 25$), wider than the single jet. At $\text{Re} = 200$, the flow is totally disorganized, and only the near field ($x/D < 5$) is possible to identify the jet streams, in the case of one and two jets, which quickly breaks up. At $\text{Re} = 150$ in the two jets cases, the two jet streams are still identified at $x/D = 5$ and 10, but with asymmetric shape. They merge into a single jet stream at $x/D = 15$, following a meandering path downstream.

Finally, in the two asynchronous jets cases with phase shift $\phi = \pi/2$, the solution is always asymmetric even for very low Reynolds number, as presented in Fig. 8(top). The second jet interacts and breaks up the first main jet stream. Additional simulations were carried out at $\text{Re} = 10$, but at this flow conditions, the jet stream leaded by the vortex rings is not formed, and the flow is constantly reentering into the cavity. The same effect is identified in the case with two asynchronous jets with phase shift $\phi = \pi$ as seen in Fig. 8(bottom). The flow barely travels downstream, as a consequence of the injection and suction phases occurring simultaneously.

![Figure 7](image_url)
V. THRUST CHARACTERIZATION

This section introduces the thrust produced by the single, synchronous, and asynchronous jets. As previously introduced in Fig. 2, to calculate the thrust, it is necessary fixing the normal (width) and axial \( d \) dimensions defining the control volume. In principle, measuring the thrust using the distance width \( \Delta = 0 \) should be sufficient to evaluate this magnitude at the jet exit; nevertheless, regarding previous Figs. 4 and 5, although the regions of highest vorticity are located in the nearby of the jet exit (width = 0), the streamlines and vorticity contours show that the vortex ring characteristic of the synthetic jet widen as it travels downstream, justifying the interest in studying the variations of thrust for several values of width > 0.

A general overview of the thrust obtained in all the simulations carried out is presented in Fig. 9. More details about the variations of thrust with the parameters width and \( d \) are presented in the Appendix. The cases with highest thrust are the two synchronous jets, followed by the two asynchronous jets with \( \phi = \pi/2 \), the single jet and the two asynchronous jets with \( \phi = \pi \). As seen, the thrust increases with the Reynolds number and with the width distance in general, when comparing the same configuration. At width = 20, the four groups of jets are organized in four groups with similar value of thrust, although at width = 0, the distance between these four groups (of lines) is smaller.

Finally, the thrust given by a single jet is compared with half of the thrust calculated in the two synchronous jets in Fig. 10. Both thrust values are similar at width \( \leq 10 \) for the same Reynolds number. Increasing the width, the peak of thrust of a single jet is larger than half of the thrust calculated in two synchronous jets. This fact suggests that including a second propulsion mechanism (jet) duplicates the value of thrust for width \( \geq 10 \), but also duplicates the amount of power of the system. Nevertheless, this conclusion could be different varying the distance between jets, but this remains as open topic for future research. Finally, it is worth to mention that the value of the thrust calculated in each one of these cases is not exactly the same for all the values of \( d \). These differences could be connected with the

FIG. 8. Injection and suction phases in the two asynchronous jets with phase shift \( \phi = \pi/2 \) and \( \phi = \pi \). In each sub-figure, vorticity contours and streamlines (left) and streamwise velocity contours (right). Snapshot extracted at \( \varphi = 3\pi/4 \) and \( \varphi = 5\pi/4 \) in the injection and suction phases of the upper jet, respectively. The domain shows the cavity and the near field of the jet defined in \( x = -5.2, 10 \) (the jet exit is at \( x = 0 \)) and \( y = [-10, 5] \).
different flow topological patterns identified in the case with one and two jets, related to different flow scales.

VI. IDENTIFICATION OF COHERENT STRUCTURES AND FLOW PATTERNS

This section presents the main flow patterns leading the flow dynamics in the different jet configurations studied. The main frequencies and amplitudes identified in the case with a single jet, two synchronous jets, and two asynchronous jets with different phase shift are presented in Fig. 11. As expected, the dominant frequency (highest amplitude) is $St = \frac{\omega}{f} = \frac{0.03}{2\pi}$. A sub-harmonic of this forcing frequency is also identified in most of the cases when the Reynolds number is larger than a certain value, as it will be detailed below, this is

![Graph](image)

**FIG. 9.** Thrust given by all cases: one jet (“1j”), two synchronous jets (“Sync”), and two asynchronous jets with phase difference $\Phi = \pi/2$ (“Asy”) and $\Phi = \pi$ (“pi”).

![Graph](image)

**FIG. 10.** Thrust given by 1 jet and half of the thrust given by two synchronous jets.

![Graph](image)

![Graph](image)

![Graph](image)
$St_0 = St^*/6 = \frac{0.01}{\pi}$, for simplicity called as the triggering frequency. At low Reynolds number, we also find some harmonics of the triggering frequency, which also interacts with the dominant frequency, resulting in very high-order harmonics of $St_0$. Increasing the Reynolds number, the number of harmonics increases. The solution is periodic for the case with a single jet and two synchronous jets. A new frequency $St = \frac{St_0}{2} = \frac{0.005}{\pi}$ is identified in the case of two asynchronous jets with $\phi = \pi/2$. The presence of this sub-harmonic of the forcing frequency could be connected with the phase shift between the two jets. Finally, in the two asynchronous jets (with $\phi = \pi/2$ and $\pi$), an additional new frequency is identified, $St \approx 0.04$, called as quasi-periodic mode, which breaks the flow periodicity. This frequency interacts with the remaining harmonics of the forcing frequency rising the flow complexity.

Regarding the origin of the symmetry breaking identified in the single and two synchronous jets at $Re \in (110, 120)$ and $Re \in (140, 150)$, respectively, HODMD shows that in both cases the solution is periodic, suggesting that the presence of new harmonics, the changes in the amplitude of the modes, and their nonlinear interaction are connected to the mechanism triggering this flow instability. More specifically, the frequencies and amplitudes identified in Fig. 11 for the single jet are as follows: (i) at $Re = 100$ (symmetric) the forcing frequency $St^*$ and its harmonics, and (ii) at $Re \geq 200$ (nonsymmetric) the forcing frequency $St^*$, its harmonics, the triggering frequency $St_0$, and some of its harmonics (nonlinear interaction of modes). In this second case, the amplitude of the triggering frequency is larger than the amplitude of the first harmonic of the forcing frequency $St^*$, suggesting that the rise in the amplitude of the triggering frequency could be connected with the symmetry breaking.

For the two synchronous jets, these frequencies and amplitudes identified by HODMD are as follows: (i) at $Re = 10$ (symmetric) the forcing frequency $St^*$ and its harmonics and (ii) at $Re = 50$ (symmetric) the forcing frequency $St^*$, its harmonics, the triggering frequency $St_0$, and some of its harmonics (nonlinear interaction of modes). However, in this case, the mode with the largest amplitude following
the forcing frequency is the first harmonic of the forcing frequency $2St'$, and the triggering frequency is the fifth highest amplitude mode (following the second harmonic of the forcing frequency $3St'$); (iii) at $Re = 100$ (symmetric), the frequencies and amplitude of the modes follow a similar behavior as in the previous Reynolds number, with the difference that the number of harmonics of the forcing frequency $2St'/C_{3}$, and the triggering frequency is the fifth highest amplitude mode (following the second harmonic of the forcing frequency $3St'$); (iv) at $Re \geq 150$ (nonsymmetric), the amplitude of the modes is re-organized and the second highest amplitude mode that follows the forcing frequency is the triggering frequency mode $St_0$. This result suggests once more the connection of the rise in amplitude of the triggering frequency with the mechanism triggering the symmetry breaking. In other words, the flow is asymmetric in both the single jet and two synchronous jets when the second highest amplitude mode is the triggering frequency mode $St_0$, following the forcing frequency $St'$.

This suggestion is confirmed in Fig. 12, which shows the HODMD spectrum calculated in the results close to the critical Reynolds number. This is $Re \in (110, 120)$ for the single jet and $Re \in (140, 150)$ for the two synchronous jets. Again, in these two cases, the leading frequency is always the forcing frequency and the amplitude of the triggering frequency varies with the Reynolds number. In the asymmetric solutions, $Re = 120$ and $Re = 150$ for the single and two jets, this triggering mode is the second highest amplitude mode, following the forcing frequency mode. This is in contrast to the case for the symmetric solution of a single jet, where at $Re \leq 110$, the triggering mode is not even identified by HODMD. It is notorious that at $Re = 140$ the amplitude of the triggering mode is very small in contrast to the other cases, suggesting the connection of these changes with the proximity of the critical Reynolds number at which the flow bifurcation is produced. Linear stability analyses should be carried out to exactly identify such critical Reynolds number. Yet, this remains as open topic for future research. Also, it is important to take in mind that three-dimensional flow instabilities could become unstable for Reynolds numbers below the critical condition for the symmetry breaking.

Figure 13 shows the DMD modes in the single jet case. As explained, at $Re = 100$, when the flow is symmetric, only the forcing mode ($St'$) and its harmonics are identified. The highest intensity of this mode is found modeling the jet stream. The real and imaginary parts of the mode present a similar shape with a phase shift in space, suggesting that the mode is traveling (traveling mode). At $Re = 200$ and 500, this jet stream structure breaks up into smaller structures, distributed asymmetrically and irregularly widen the area of the jet stream. The triggering mode ($St_0$) is identified at these flow conditions, presenting an irregular shape with flow structures slightly bigger than the ones of the dominant mode. At $Re = 200$, it is still possible to identify the jet stream in this forcing mode, but at $Re = 500$, this shape is lost. The mode with frequency 15 $St_0 = 2 St' + 3 St_0$, also found at this flow conditions, is the result of the nonlinear interaction of modes; hence, it is possible to see the influence of both the forcing and the dominant modes. At $Re = 1000$, it is possible to identify the jet stream in the dominant mode, as in the case at $Re = 100$, but in this case, the jet stream is nonsymmetric (it is slightly inclined) and the highest intensity region reaches further downstream. Even though the flow complexity is larger at $Re = 1000$ than at $Re = 500$, the flow structures are more organized in the former case following a “jet stream” shape as in the case at lower $Re$, suggesting that at $Re = 500$ the flow regime could be transitional.

The DMD modes obtained in the two synchronous jets are presented in Fig. 14. At $Re = 10$, only the forcing mode $St'$ is found. The
The highest intensity of this mode focuses at the jet exit in both the real and imaginary part of the mode, suggesting that this mode is standing; hence, the jet stream is not formed at these flow conditions, as it was presented in the numerical simulations. This standing behavior of the leading mode is also identified at $\text{Re} = 50$, although in this flow regime, we also identify the triggering mode $\text{St}_0$ presenting a traveling character, since the flow structures, also with highest intensity focused at the two jet exits, are phase shift in the real and imaginary components of the mode. Moreover, they present some small differences in the two components, also supporting the traveling behavior of the mode. Additionally, the region of highest intensity of the mode extends further downstream. At $\text{Re} = 100$, the complexity of the flow structures increases, and also, the region of highest intensity of the mode continues extending further downstream. The traveling character of the forcing mode is still identified. At this flow condition, we identify the mode with $\text{St} = 0.45$, representing the nonlinear modal...
interaction, connected with the rise in the flow complexity. This mode is also traveling in space as reflected in the phase shift presented between its real and imaginary components, and the sizes of the flow structures are smaller than in the previous case, also in good agreement with the rise in frequency as consequence of the nonlinear interaction of modes. Finally, at Re = 150 and 200, when the flow is nonsymmetric (as explained in Sec. IV), the flow structures defining the leading mode (St) break up forming irregular shapes, while in the triggering mode, new large size structures are identified in the near field and extended further downstream. The shape of this mode suggests that the triggering mode is driving the flow motion in the far field of the jet, presenting also an extended high-intensity region along the normal component.

The DMD modes obtained in the two asynchronous jets with \( \phi = \pi/2 \) and \( \pi \) are presented in Fig. 15. In the case with \( \phi = \pi/2 \) at Re = 50, only the forcing mode St and their harmonics are found.

FIG. 14. Same as Fig. 13 for the two synchronous synthetic jets.
The shape of this dominant mode shows the interaction of the two jets forming the wake of a single jet, but nonreflection symmetric (inclined with respect to the streamwise axis). At $Re = 100$, this jet stream breaks into small structures, with their highest intensity in the near field of the jet, close to the jet nozzle. The triggering mode is also identified at these conditions, with similar shape to the dominant one. Finally, at $Re = 150$, the triggering mode is replaced by the new mode with $St = 0.04$, in charge of breaking the flow periodicity, increasing the flow complexity.

In the case of the jets with phase shift $\phi = \pi$, the two jets also interact forming a single jet stream at $Re = 50$ in the dominant mode $St^*$, but the flow structures are broken into smaller size structures compared to the case with $\phi = \pi/2$. At these conditions, the triggering mode is identified with highest activity in the near field of the flow, but this mode disappears with the rise in Reynolds number. Conversely, the quasi-periodic mode with $St = 0.04$ is identified at both $Re = 50$ and 100 with a shape similar to the dominant mode.
This quasi-periodicity that could be connected with the opposite movement of the two jets (injection-suction) could also be related to the thrust reduction associated with this test case, identified in Sec. V.

VII. CONCLUDING REMARKS

This article presents a detail study of the main flow structures driving the flow dynamics and the thrust produced in one and two planar synthetic jets. Numerical simulations have been carried out to generate several databases at different Reynolds numbers and similar piston forcing frequency in a single synthetic jet and in two synthetic jets in-phase and out-of-phase with phase difference of $\pi/2$ or $\pi$ between them, and these are called as the synchronous and asynchronous jets, respectively. Using flow topology analysis, we identify and study the evolution of the two main topology patterns characteristics of synthetic jets: the vortex rings, producing thrust, and the saddle point, separating the flow reentering into the jet cavity to the flow that continues traveling downstream. At low Reynolds number ($Re \leq 100$), the flow is symmetric in the case with a single jet, and the two synchronous jets, where the vortex rings emerge from each one of the jet nozzles and merge into a single jet in the very far field ($x/D \approx 30$). Increasing the Reynolds number, a flow instability produces a symmetry breaking ($Re > 100$), and the size of the spatial flow structures decreases with the rise in Reynolds number. On the contrary, in the two asynchronous jets, the flow is always asymmetric: in the case with phase shift $\pi/2$, the first jet stream is broken due to its interaction with the second phase-delayed jet stream; nevertheless, in the case with phase shift $\pi$, the jet stream of the jets is not formed due to the suction phase occurring in one of the jets simultaneously and with similar intensity as the injection phase in the second jet.

The thrust of the jets has been calculated and studied in detail both at the jet exit and in several control volumes defined for each one of the values of the axial component. There is a peak with highest thrust in the near field of the jet that varies as function of the control volume. Moreover, the thrust increases with the rise in the control volume. In all the cases studied, the thrust produced by a single jet is always half of the thrust produced by the two synchronous jets, but this quantity is much smaller in the asynchronous jets, especially in the jets with phase difference $\pi$.

The main patterns driving the flow motion have been identified using HODMD and connected with the variations of thrust among the different control volumes. The flow dynamics is periodic in the single and two synchronous jets, being the dominant frequency (dominant mode) the forcing frequency. At low Reynolds number, only the harmonics of the dominant frequency are identified. Increasing the Reynolds number, the method also identifies the so-called triggering frequency (triggering mode), which is the fifth sub-harmonic of the forcing mode, and its harmonics, but with low amplitude. Nevertheless, when the flow is asymmetric, the amplitude of the triggering mode rises, and it becomes the second highest harmonic, following the dominant mode. This result suggests the connection of the forcing mode with the presence of the symmetry breaking occurring in the flow. Finally, the frequencies identified in the two asynchronous jets are also the dominant and forcing frequencies, the triggering frequency and their harmonics (nonlinear modal interaction). However, a new additional frequency is identified, which is breaking the flow periodicity, in good agreement with the high spatial complexity identified in these cases.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors declare no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request. The numerical codes can be found in Ref. 57.

APPENDIX: THRUST RESULTS DETAILS

The thrust increases with the width distance, as presented in Fig. 16, which compares the variations of thrust along the streamwise direction $d$ as function of the width in the case of a single jet at $Re = 100, 200, 500, and 1000$. At $Re = 100$, when the flow is symmetric, the curves of the thrust follow the same tendency at width $\geq 10$, presenting a peak of highest thrust at $d \approx 3$. These curves increase and decrease progressively upstream and downstream this peak. Then, they are maintained almost constant (with a slightly negative slope) along the streamwise direction for $d \geq 8$. In contrast, for width $\leq 5$, this peak is identified at $d = 1$, and the value of thrust is maintained for almost all $d$ (considering small variations). At $Re = 200$ and $500$, when the flow is nonsymmetric, this regular behavior is identified for width $\geq 20$, with the highest peak identified also at $d \approx 3$ for $Re = 200$ and at $d \approx 5$ for $Re = 500$. Nevertheless, for width $\leq 10$ the peak is found at $d \approx 1$ for $Re = 500$ and at $d \approx 2$ for $Re = 200$ in all the cases except the one with width $= 0$, where the peak is identified at $d \approx 1$. Finally, at $Re = 1000$, the evolution of thrust is similar for all the values of width and the peak of highest thrust is again identified at $d \approx 3$. These tendency changes could be connected with the variations in the flow regime (transition from laminar to transitional-turbulent flow) and the different flow patterns leading the flow dynamics in each regime, as it was presented in Sec. VI.

Figure 17 compares the variations of thrust at different Reynolds numbers for width $= 0, 10, 20,$ and $45$ in the case of a single jet. As expected, the thrust increases with the Reynolds number. At width $= 0$, there is a clear difference in the value of thrust in the four cases studied, although this difference is attenuated in the solutions at lower Reynolds number, especially for width $\geq 20$, where the cases at $Re = 100$ (symmetric) and $200$ (asymmetric) present the same thrust. The solution at $Re = 1000$ presents a peak of much larger thrust than in the remaining cases, although downstream this peak, the order of magnitude of thrust is similar to the solution at $Re = 500$.

The variations of thrust as function of the width in the two synchronous jets are presented in Fig. 18. The tendency is similar to in a single jet: the thrust increases with the value of width. The peak of thrust varies from $d \in [2, 3]$ at $Re = 50$ (symmetric) and $200$ (asymmetric), and $d \in [2, 4]$ at $Re = 100$ (symmetric) and $150$...
FIG. 16. Thrust given by a single synthetic jet as function of the axial component $d$ comparing several normal dimensions of the domain width for different Reynolds numbers.

FIG. 17. Thrust given by a single synthetic jet as function the axial component $d$ comparing several Reynolds numbers for different values of the normal dimensions of the domain width.
FIG. 18. Same as Fig. 16 for the two synchronous jets.

FIG. 19. Same as Fig. 17 for the two synchronous jets.
FIG. 20. Thrust given by two asynchronous jets, $\Phi = \pi/2$, width comparison.

(a) E-d-width Asy jets Re=25  
(b) E-d-width Asy jets Re=50  
(c) E-d-width Asy jets Re=100  
(d) E-d-width Asy jet Re=150

FIG. 21. Thrust given by two asynchronous jets, $\Phi = \pi/2$, Reynolds comparison.

(a) Asynchronous jets, Re comparison , width = 0  
(b) Asynchronous jets, Re comparison , width = 10  
(c) Asynchronous jets, Re comparison , width = 20  
(d) Asynchronous jets, Re comparison , width = 35
(asymmetric). Figure 19 shows the variations of thrust for a fixed width as function of the Reynolds number. As seen, the value of thrust is similar at Re = 50 and 100 for width ≤ 10 and the case at Re = 200 always presents the highest thrust. However, this difference is attenuated with the increasing width. For width = 45, the differences in the value of thrust identified for the different Reynolds number are very small, with the largest difference identified in the peak value for Re = 150 and 200.

Finally, the thrust calculated in the two asynchronous jets with phase difference $\phi = \pi/2$ is presented in Fig. 20. As in the previous cases, the value of thrust increases with the width, except for the cases with width ≤ 10 where this value is maintained. The peak of thrust is generally found at distance d = 2 in most of the cases (Re ≥ 50). However, this peak is more attenuated than in the cases with synchronous jets or a single jet. Comparing the thrust at different Reynolds numbers for a constant width as presented in Fig. 21, it is seen that the only solution reaching a high thrust, different from the other cases, is the one at Re = 150. This fact could be connected with the presence of different type of flow patterns at this flow conditions, as detailed in Sec. VI.

REFERENCES


