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Transport policies in polycentric cities

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ABSTRACT

This paper studies how transit lines should be developed in polycentric cities. In several growing metropolitan areas, local authorities have to decide whether to rely on existing radial lines connecting suburban areas to the city-center, or to develop new circular lines directly connecting suburban areas. An efficient transit system aims at reducing external costs of transport (congestion and pollution) by attracting private car users. We study the effect of two types of policies on the modal split. First, we compare the effect of three administration regimes (public, semi-public and private) on the external costs of transport. Second, we consider the opening of a new transit line directly linking the suburbs. We find that it reduces aggregate user cost but is not Pareto-improving unless crowding is high on existing transit lines. Our analysis is complemented by a numerical illustration based on an open source Fortran program. This tool needs a relatively small set of data and can be used by policy makers wishing to investigate the pricing reforms or the possibility of opening a new transit line for a specific case study.

1. Introduction

Urban growth generally involves the development of economic activities in the outskirts, markedly changing the structure of traffic flows. For instance, Aguilera et al. (2009), describing the evolution of traffic flows in the metropolitan region of Paris, show that the proportion of standard commuting from the suburbs to the city center has declined, while reverse commuting from the center to the suburbs, and commuting between suburbs, have both increased. This pattern is seen in many other metropolitan areas (Jun, 2004; Zhao et al., 2011).

Growing concerns about congestion, environmental issues and public health prompted several local authorities to explore reforms of urban transport to curb the use of private cars and reduce the generated external costs. With respect to this objective, road pricing is an efficient tool, but in practice, users tend to oppose it. When road pricing is unfeasible, other reforms, such as discounted fares on public transport, are frequently considered (cf. Parry and Small, 2009). Instead of centralized decision making, the privatization of some transport activities may enable different pricing schemes (cf. de Palma et al., 2007). Although, to the best of our knowledge, it has never been tested at the city level, several countries have extensively privatized their motorways. In this paper, we study the effect of road privatization at the city level as a benchmark case, to better understand to what extent it might be a solution to congestion and inefficiencies. Improving public transport provision is another tool to reduce the use of private cars. Improving service quality or investment in new lines connecting suburbs are likely to make public transport more attractive by reducing waiting time and obviating tedious transit through the city center. For instance, the metropolitan area of Paris is building a new transit line of 200 km connecting the suburbs. Anas and Timilsina (2015) develop a model for the city of Beijing and find that improving transit services in the periphery is one of the most effective policies in reducing the usage of private cars and related emissions.[1] In most

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[1] As highlighted in Table 1, Beijing has already build two peripheral transit lines.

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Table 1
Cities with circular subway line.
Source: Bono et al. (2022), Metrobits.org, urbanrail.net and wikipedia.org.

City	First metro line	First circular line	Network length (2015)	Comments
Shanghai	1995	2007	588 km	This city had the most rapidly expanding metro infrastructure in the world: 588 km in 20 years. The circle line (line 4, 33 km) was completed in 2007.
Beijing	1969	1984, 2008	551 km	The first circular metro line (line 2, 23 km) is located within the city center. The first circular line connecting SBDs was launched in 2008 (line 10, 57 km).
Seoul	1974	1978	450 km	Seoul belong to the few examples where a circular line (line 2, 54km) was planed since the origin.
Tokyo	1927	1991	430 km	The E line (41 km) has a circular shape but train cannot run continuously. They must reverse at the terminus.
London	1863	1884	402 km	The case of London is unusual. Unlike most cities, the "Circle" (27 km) was part of the initial development plan.
Moscow	1932	1950, 2016	329 km	The first circle line (line 5, 19 km) was launched in 1950. The "Moscow Central Circle" line (54 km) was in operation in 2016. Another circular line is expected in the next years (line 11/11A)
Madrid	1919	1979	293 km	The Circular line is 23 km long. It seems that there is a political consensus to build a new circular line
Singapore	1987	–	170 km	The Circle line (40 km) construction started in 2009. The last 4 km are planned to be finished by 2026. Other (larger) circular lines are under construction and expected.
Glasgow	1896	1896	10 km	Glasgow is an example where the initial (and single) subway line is circular.

cities,[2] the public transport networks were initially designed with radial lines when urban planners were dealing with commutes between suburbs and city centers. As a result, commuting by public transit between suburban areas usually requires an inconvenient transit via the city center, encouraging the use of private cars for these trips. Projects of peripheral transit lines generally starts later in the urban development of cities (Vuchic, 2005), when public transport becomes crowded, congestion in the city center is high and commutes between suburbs increase.

Table 1 reviews and comments the development of peripheral lines for a selection of public transport networks around the world. It is clear that public transport networks of most cities were initially designed with radial lines. We found three exceptions: London, Seoul and Glasgow where circular lines were planned at an early stage in the development of the city's urban transport network.[3]

In this paper, we study urban commuting in a polycentric city composed of one city center and two suburban areas.[4] This setting is motivated by the study of the potential effects of the Grand Paris network project in France. Our analysis is based on an analytical model rather than a transport simulation. As we comment at the end of the paper, this approach ensures transparency and better identifies the forces at play. We consider two transport modes: public transit linking each suburban area to the city center, and private cars. We study the modal choice made by exogenously located workers for all origin–destination (OD) pairs and address two important questions related to commuting efficiency. First, we examine the efficiency of various administration regimes for each mode. We consider unpriced equilibrium, fully public, semi-public and private regimes. In semi-public regimes, transit is managed by the public sector, and roads are managed by a private operator. Second, we consider the investment in a new circular transit line directly connecting suburbs, and evaluate its impact on average user cost. Our analysis takes some account of service quality in public transport as measured by service frequency.

We show that the unpriced equilibrium is not optimal (it does not minimize aggregate user cost) because it leads to an overuse of private cars, especially for trips between the suburbs. The optimum can be achieved in a fully public regime either by imposing a road toll or by subsidizing public transport. We show that there is always an optimal fare-toll gap that yields optimal modal split as an equilibrium. If the administration of the roads is delegated to a private operator, then this operator sets the tolls at a high level, leading to an equilibrium where public transport is overused and crowded. When the regulator can choose the level of public transport fares but the roads are administered by the private operator, we show that the optimum can be reached. We also discuss the case of a duopoly where the two transport modes are managed by competing private operators. In this case, tolls and fares are strategic complements, and we obtain an equilibrium where pricing of the two modes is set too high.

Developing a transit network directly connecting the suburbs is always of benefit to commuters traveling between those suburbs, but then service frequency is likely to be reduced on radial lines following decrease in demand. This can lengthen waiting time for some users and so can limit the overall benefit of the investment. On crowded lines, however, a moderate decrease in demand will not cause any significant drop in service. The net impact of the new line is then likely to be positive on the average user cost. The

[2] In this paper, we use the terms "cities" and "metropolitan area" interchangeably. As we make clear further, whatever the terminology, we refer to the existence of one Center Business District and several Secondary Business Districts.

[3] Paris could also be seen as an exception since two tangential lines forming a circle were built at an early stage of its network development: lines 2 and 6.

[4] In the literature, the city center is often called the central business district (CBD) and suburban areas, the suburban business districts (SBDs). In this paper, we use the terms city center or CBD and suburban areas or SBD indifferently.

new line directly connecting the suburbs will be attractive for commuters in these areas because it reduces travel time and improves the reliability of travel times. The predictability of travel time is a key driver of modal choice (Jackson and Jucker, 1982; de Jong and Bliemer, 2015). A larger variability of travel time for a given line will increase the user costs for the corresponding commuters. Our formulation does not take into account directly this stochastic structure but we explain how it can be interpreted as a certainty equivalent correspondence of the more general formulation.

Our analysis is extended by numerical illustrations of several configurations.[5] We show that under standard conditions, opening a new transit line between the suburbs increases total welfare, but the improvement is driven by the effect on suburb-to-suburb commuters. Since a decrease in the number of passengers results in lower service frequency, commuting costs increase for the other passengers. Hence if such users could vote on constructing the new line, it is likely that they would oppose it unless crowding was high. We illustrate a situation where the overall impact of the new line is negative. Kilani et al. (2017) characterized a similar outcome, though in a framework with a single transport mode.

Our main contribution is the analysis of the two-mode problem within a more general and realistic network. This allows us to discuss transport policies (administration regimes and network extension) that are relevant to several metropolitan areas. In particular, we study the desirability of directly connecting suburban areas by public transport. In the previous literature on modal choice, the network and city structure are either not explicitly specified (Mohring, 1972; David and Foucart, 2014), monocentric (Hamilton, 1982; Kilani et al., 2014) or composed of a single OD pair (Tabuchi, 1993; Verhoef and Small, 2004; de Palma et al., 2008). To our knowledge, this is the first analytical model of modal choice in a city with multiple business centers, and thus a relatively complex OD matrix. Beyond its theoretical implications, this paper is critical for policy makers to pinpoint how to articulate the pricing schemes of the different transport modes and to identify under what conditions it is desirable to open a new transit line. The companion Fortran program may also be used to test the potential of alternative transport policies for a specific city.

Our framework implicitly assumes that not all workers locate efficiently. This is out of line with the prediction of the monocentric model where each workers would choose a home location close his/her job location: the monocentric model eliminates spatial mismatch, so called wasteful commuting, by construction. In our model, users have exogenous and fixed home and work locations and choose the transport mode to travel between suburbs and/or the city center. There are various reasons for making this assumption, echoing the debate raised by Hamilton (1982), who empirically observed that a large share of home-to-work trips occurs between different business centers. We refer the interested reader to Giuliano and Small (1993) who have already discussed job-housing balance/imbalance and over-commuting, defined as the difference between actual commute and the commute required to access jobs when people are efficiently located. The issues and emergence of spatial mismatch and multicentricity are not directly addressed here, as we focus on commuting behaviors between existing business districts.

The paper is organized as follows. In the next section, we set up the theoretical model. The equilibrium and the optimality conditions are derived in Section 3 where we also consider various administrative regimes for roads and transit. Section 4 analyzes public transport provision. We examine the determinants of transit frequencies and consider a modification of the transit network: investment in a new transit line directly connecting suburban areas. To illustrate the main findings, we develop a linear version of the model and a numerical example in Section 5. Several features of the model and possible extensions are discussed in Section 6. Section 7 concludes.

2. The model

We consider a city with a single city center and two suburban areas. There are both radial and circular roads directly connecting suburban areas and the central area but there are only radial transit lines connecting suburban areas to the city center. To go from one suburban area to the other, transit users must therefore travel first into the city center and then back out to their destination. This is a simplification of realistic situations with a central main square and multiple suburban areas. A number of cities have developed a radial transit network where transit lines run out from the city center to suburban districts. The Paris region is one such case.

This paper evaluates transport costs in this context and explores policy reforms that can increase urban welfare (pricing, extension of transit network and changes in transit frequencies). These issues are of great importance for many urban areas where transit developed first through radial lines. As a result, most trips between suburban areas are made by car (see Aguilera, 2005). These are relatively long-distance trips, potentially responsible for high CO_2 emissions and congestion. The objective of many reforms in the transport sector is to reduce the use of cars in metropolitan areas through a better provision of public transport services. Some metropolitan areas, such as Paris, have ongoing projects for the development of circular transit lines.

The main district is denoted C, for “Center”, the first suburban area is denoted S for “South” and the second E for “East”. Fig. 1 depicts the geometry of this city. As indicated by the arrows, we look only at commuting from the South to the Center and to the East, and from the Center to the East, and not the reverse. We assume that the flows of commuters are symmetrical. The problem would therefore be identical if we were looking at modal choices of commuters going in the opposite direction (from East to Center and South and from Center to South). Our results are therefore not affected by this simplification because we do not consider the return journey of trains.

[5] For this purpose, we wrote a standalone Fortran program specifically adapted to our setup and make it available online in https://gogs.univ-littoral.fr/mkilani/scbd or https://github.com/mkilani-dev/scbd.

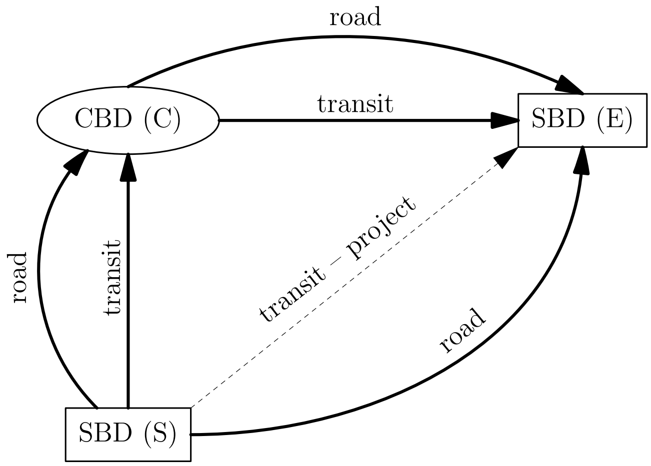

Fig. 1. The transport network. Roads connect the city center (C) and all suburban areas (south, S, and east, E), while the transit network does not connect directly suburban areas (this line is under project).

The modal choice of commuters is deciding whether to use either a car on the road (private mode, denoted R) or transit (public mode, denoted T). There are three groups of commuters, differing in their residential location (origin) and workplace (destination). The group sizes are respectively denoted N_{sc}, N_{ce} and N_{se}, where the subscripts (ij) refer to the OD pairs.[6] Thus $i = c, s$ and $j = s, e$. We define n_{ij}^M as the number of users going from i to j, using mode M, with $M = R, T$. We have $n_{ij}^R + n_{ij}^T = N_{ij}$. As shown in Fig. 1 there is a total of five links, two transit links and three road links. On each link (ij), the total number of users of mode M, is denoted u_{ij}^M. More precisely, we have $u_{ij}^R = n_{ij}^R$ for $(ij) \in \{(sc), (ce), (se)\}$ and $u_{ij}^T = n_{ij}^T + n_{se}^T$ for $(ij) \in \{(sc), (ce)\}$.

These trips conflict with the prediction of the monocentric city, whereby each worker commutes to the nearest business area. The term "wasteful commuters" was introduced by Hamilton (1982) to characterize these trips. Hamilton found that wasteful commuting was very common in the metropolitan areas he observed (Los Angeles). The empirical conclusion was first criticized by White (1988), before it was confirmed by Small and Song (1992). It is now considered as a major shortcoming of the monocentric city model (cf. Brueckner, 2011). We do not address this question, and do not consider why a household might locate "inefficiently", but accept the empirical evidence that "wasteful commuting" does exist.

Transport is costly for all modes. Road users drive directly to their destinations. It is assumed that those who drive between suburbs do not use radial roads. Congestion on roads depends on (and is increasing in) the number of users on them. Crowding in transit depends on the number of users of transit, and the cost function depends on the number of users and the service frequency. Transport cost functions are defined for each link connecting pairs of business locations i and j. They are denoted c_{ij}^M and depend on the number of users of the same link, denoted u_{ij}^M. For a given link (ij), the generalized transport cost using the road is given by

$$C_{ij}^R(u_{ij}^R) = \tau_{ij} + c_{ij}^R(u_{ij}^R) \qquad \text{for} \qquad (ij) = \{(sc), (ce) \text{ or } (se)\}, \tag{1}$$

where τ_{ij} denotes a road toll imposed on the users and $c_{ij}^R(u_{ij}^R) = F_{ij}^R + \tilde{c}_{ij}^R(u_{ij}^R)$ is the monetary value of the time spent for the commute. It is the sum of a free-flow travel cost, F_{ij}^R, and the user cost due to congestion, $\tilde{c}_{ij}^R(u_{ij}^R)$. The free-flow travel cost encompasses the monetary value of the travel time and the vehicle operating cost when roads are empty. We assume that $\tilde{c}^R(u^R)$ is twice-differentiable with $\partial \tilde{c}^R(u^R)/\partial u^R > 0$ and make no specific assumption on the second-order derivative at this stage. For public transport, the generalized transport cost on a single link (ij)[7] for the users is given by

$$C_{ij}^T(u_{ij}^T, f_{ij}) = p_{ij} + c_{ij}^T(u_{ij}^T, f_{ij}) \qquad \text{for} \qquad (ij) = \{(sc), \text{ or } (ce)\} \tag{2a}$$

where p_{ij} is the transport fares paid by the users of the public mode and $c_{ij}^T(u_{ij}^T, f_{ij}) = \frac{c_w}{2f_{ij}} + F_{ij}^T + \tilde{c}_{ij}^T(u_{ij}^T, f_{ij})$ is the monetary value of the time spent commuting by train. It is composed of the waiting time at the train station, the crowding-free travel cost, F_{ij}^T, and the monetary value of crowding when there are u_{ij}^T passengers in the train and a train frequency of f_{ij} on the line. Assuming that transit passengers arrive uniformly at the station (i.e. they do not use timetables), the waiting time is given by $c_w/2f_{ij}$ where c_w is the monetary value of the maximum waiting time between two trains. We assume that crowding costs increase with passengers and decrease with frequency, i.e.

$$\frac{\partial \tilde{c}^T(u^T, f)}{\partial f} < 0 \text{ and } \frac{\partial \tilde{c}^T(u^T, f)}{\partial u^T} > 0,$$

[6] For the notation, we adopt the convention that origin–destination pairs (ij) are typeset in lowercase letters when used as subscripts and typeset in uppercase letters elsewhere.

[7] A single link is defined as either SC or CE. When using transit, the link SE is composed of two single links.

and make no specific assumptions on the second-order partial derivatives. On the SE link, there is no direct transit line. Commuters must transit through C. We assume their travel cost is given by

$$C^T_{se}(u^T_{sc}, u^T_{ce}, f_{sc}, f_{ce}) = \beta(p_{sc} + p_{ce}) + c^T_{sc}(u^T_{sc}, f_{sc}) + c^T_{ce}(u^T_{ce}, f_{ce}) - \Gamma(\alpha). \tag{2b}$$

Transit users of the SE line are assumed to pay a fare proportional to $(p_{sc} + p_{ce})$. In the real world, fares are generally higher than $\max\{p_{sc}, p_{ce}\}$ and smaller or equal to $(p_{sc} + p_{ce})$. Values of parameter β can be set to capture these and other situations. For example, $\beta < 1$ would correspond to a discount provided to users SE. The last term in (2b) reflects the switching cost at station C that depends on the synchronization between the arrivals of trains SC and the departures of trains CE. Although transit users bear the monetary value of the time spent on each line, we add a term that captures the possibility of improving the synchronization between the two lines. We assume that the higher α, the better the coordination between the two lines. The value of α ranges between 0 and 1. It is possible that the same train starts at S, goes to C and continues to E. In this case, there is no switching cost and perfect synchronization between the two trains ($\alpha = 1$). A small value of these parameters may reflect the intermediate stop at station C. Parameter α can thus be chosen to describe a variety of situations. The operator may have limited options, in particular with respect to transfer costs. For example, the distance between platforms (or sometimes distinct stations) cannot be shortened but the transfer can be made easier by the construction of costly infrastructures such as electric escalators or moving sidewalks.

Timetables for the services are assumed to adopt uniform schedules, and train loading to be equal on all vehicles, so it is straightforward to compute both waiting time and crowding cost. All costs are expressed for a whole trip. We have to notice in the trip cost formulations given in (1), (2a) and (2b) that a stochastic structure of some terms can be adopted to account for travel time variability. Several studies in the last decades have confirmed that travel time reliability (or variability) plays an important role in users choice (Carrion and Levinson, 2012; Hjorth et al., 2015; Chakrabarti, 2015). This is particularly useful in the cost–benefit analysis for a given project. de Jong and Bliemer (2015) reviews the literature on travel time reliability and propose recommendations on how it could be included in cost–benefit analysis. These applications build on rather technical dynamic models as in Fosgerau and Engelson (2011) and Jenelius (2012) where travel cost is formulated on the basis of utility function displaying a trade-off between time spent in traveling and time spent in other activities. While we do not explicitly take into account travel time reliability in our model, our formulation can be interpreted as a certainty equivalent correspondence of the stochastic formulation. In particular, for the network we consider, the new train line SE will not only reduce travel time for a group of users, but also reduce the uncertainty related to the reliability of the provided services. For risk averse travelers and a uniform reliability of service qualities for all lines, it is less probable to have delays in a single line than in two successive lines. We further comment on this issue in Section 4.2, but notice that if we focus on the new transit line, the poor reliability of chaining transit trip through the center can be implemented in the component $\Gamma(\alpha)$ in (2b).

The transport sector is administered by one or two operators (one for each mode) who can be either private (profit-maximizing) or public (welfare-maximizing). The choice variable for the commuters is the transport mode. The operator of the roads can decide to impose a toll on a given link, and the operator of public transport decides the fares and service frequencies. For public transport, there is also the possibility of extending the transit network to make a direct connection between suburban areas S and E. We first consider that there is no cost for administering the roads or the railways. In this context, frequencies (f_{ij}) and coordination of the transit system (α) are considered exogenous. We relax these assumptions in an extension and use a cost of providing operating vehicles and of synchronizing the two lines.

3. Equilibrium, optimum and administration regimes

In this section, we use a general cost function to characterize the equilibrium and the optimum. Service frequencies (f_{ij}) and synchronization (α) are assumed to be exogenous and cost-free.[8] We compare both situations and provide the conditions for the decentralization of the optimum. We then consider various administration regimes. Under a public regime, both roads and rail transport are assumed to be managed by the social planner. Under a semi-public regime, roads are administered by a private operator, and rail transport is administered by the social planner. In a duopoly scenario, both public transit and roads are administered by private operators. We compare pricing schemes and welfare in the three scenarios.

3.1. Equilibrium

Workers are assumed to commute from their dwellings to their workplaces using either private cars or public transit. As demand is perfectly inelastic, we can rewrite n^R_{ij} as functions of n^T_{ij}:

$$n^R_{sc} = N_{sc} - n^T_{sc}, \qquad n^R_{ce} = N_{ce} - n^T_{ce}, \qquad \text{and} \qquad n^R_{se} = N_{se} - n^T_{se},$$

respectively, and we are left with three endogenous variables: n^T_{sc}, n^T_{ce}, n^T_{se}. This problem meets the Wardrop equilibrium conditions: for a given set of commuters, the user cost in the private mode is equal to the user cost in the public mode. If a mode is not used

[8] As f_{ij} is assumed to be exogenous in this section. For ease of reading, it will be removed from the notation. For instance, $C^T_{ij}(u^T_{ij}, f_{ij})$ will be denoted $C^T_{ij}(u^T_{ij})$. This assumption will be relaxed in the next section, and f_{ij} will be reintroduced.

it must be associated with higher cost. In an interior solution, when both the public and the private modes are used, we have, in equilibrium: $C^T_{ij}(u^T_{ij}) = C^R_{ij}(u^R_{ij})$ for all $(ij) = (sc), (ce)$ and (se), i.e.

$$\tau_{sc} + c^R_{sc}(u^R_{sc}) = p_{sc} + c^T_{sc}(u^T_{sc}), \tag{3a}$$

$$\tau_{ce} + c^R_{ce}(u^R_{ce}) = p_{ce} + c^T_{ce}(u^T_{ce}), \text{ and} \tag{3b}$$

$$\tau_{se} + c^R_{se}(u^R_{se}) = \beta(p_{sc} + p_{ce}) + c^T_{sc}(u^T_{sc}) + c^T_{ce}(u^T_{ce}) - \Gamma(\alpha). \tag{3c}$$

We note that in a corner solution, some modes (or links) may not be used, and the above conditions associated with these modes (or links) do not hold. We have the following result:

Proposition 1 (*Equilibrium*). *The problem of modal choice has a unique equilibrium.*

Proofs are in Appendix A. If the set of equations (3) has a feasible solution we have an interior equilibrium (where each group uses both transport modes). In all other cases, the problem has a corner solution and some groups do not use both transport modes. Given the structure of the network we consider, the computation of traffic equilibrium is the main technical difficulty encountered to solve the model. Indeed, the network equilibrium flows is characterized by a set of equations and inequalities that are not easy to solve directly. To overcome this difficulty, we follow the approach proposed by Beckman and McGuire (1956) where the problem is formulated as a nonlinear mathematical convex program whose solution is easier to compute.[9]

3.2. Optimum

The total cost is the sum of the users' and the operators' costs. An optimum is reached when the total cost is minimized. As there are no operating costs for the roads in our model, the total cost is the sum of users' costs and the cost of operating the transit system. This latter depends on the frequency of the services and the effort made to coordinate the two lines. As these values are exogenously fixed for the time being, the operator's cost is fixed. Since transit fares and road tolls are redistributed to the population, they are welfare-neutral. The objective is therefore to minimize the following social-cost function for n^T_{sc}, n^T_{ce} and n^T_{se} [10]:

$$\sum_{ij=sc,ce,se} u^R_{ij}\, c^R_{ij}(u^R_{ij}) + \sum_{ij=sc,ce} u^T_{ij} c^T_{ij}(u^T_{ij}) - n^T_{se}\Gamma(\alpha). \tag{4}$$

The endogenous variables should satisfy the usual constraints, i.e. $0 \le n^T_{sc} \le N_{sc}, 0 \le n^T_{ce} \le N_{ce}$ and $0 \le n^T_{se} \le N_{se}$. We have the following result.

Proposition 2 (*Optimum*). *The social minimization problem (4) has a solution. If there is an interior solution and if transport costs are convex for all modes then the solution is unique.*

Most standard formulations of congestion on roads (e.g. the BPR or the quadratic formulations) satisfy the conditions of propositions 2. However, the convexity of the cost function in public transport may not be satisfied if the availability of seats is taken into account. For instance, the MAS formulation (cf. de Palma et al., 2015) is not convex. In the general case we can have multiple solutions. We show in the proof that the above condition (on convexity) is a sufficient condition for unique solution.

The equilibrium is generally distinct from the optimal solution. The first-order conditions with respect to the objective function in (4) yield

$$c^T_{ij}(u^T_{ij}) + u^T_{ij}\,\frac{\partial c^T_{ij}(u^T_{ij})}{\partial n^T_{ij}} = c^R_{ij}(u^R_{ij}) - u^R_{ij}\,\frac{\partial c^R_{ij}(u^R_{ij})}{\partial n^T_{ij}} \tag{5a}$$

for groups $ij = \{sc\}, \{ce\}$, and

$$\sum_{ij=sc,ce}\left(c^T_{ij}(u^T_{ij}) + u^T_{ij}\frac{\partial c^T_{ij}(u^T_{ij})}{\partial n^T_{se}}\right) - \Gamma(\alpha) = c^R_{se}(u^R_{se}) - u^R_{se}\frac{\partial c^R_{se}(u^R_{se})}{\partial n^T_{se}} \tag{5b}$$

for $ij = \{se\}$, which is a usual statement that the optimum distribution of users is such that the social marginal costs, not the private marginal costs, are equal for all the alternatives used.

[9] With a single OD network it is possible to solve the equilibrium equation directly and check whether or not it yields an interior solution. Since the number of possible cases is limited, most authors do this to solve their simple models. However, with a slightly more complex network, the enumeration of all possible cases becomes impractical, and our procedure is both useful for analytical tractability and efficient for numerical computation.

[10] In a more general framework, we could also consider environmental costs. It would improve the social benefits of transit. We prefer not to include these costs at this stage to keep the model as simple as possible. This point is discussed in the conclusion.

3.3. Administration regimes

In most cases, urban transit systems are controlled by public authorities because of their cost structure, and because this activity is not in general profitable. To what extent the public operator should set tolls or fares to induce a decrease in transport cost is a subject of debate, and should take into account the externalities produced by the transport system.

In this section, we consider various scenarios for the management of the transport system. We start by considering a fully public administration where the social planner administers both the roads and the transit system. We then turn to the case where the administration of roads is delegated to a private operator. Finally, we look at a fully private administration where each transport system is administered by a private operator. We call this case a duopoly administration.

3.3.1. Public administration

By comparing (5) with the equilibrium conditions (3), we see that the optimum can be decentralized if

$$p_{ij}^{op} - \tau_{ij}^{op} = u_{ij}^{T} \frac{\partial c_{ij}^{T}}{\partial n_{ij}^{T}} + u_{ij}^{R} \frac{\partial c_{ij}^{R}}{\partial n_{ij}^{T}} \text{ for } (ij) = (sc) \text{ and } (ce) \tag{6a}$$

$$\beta^{op}(p_{se}^{op} + p_{ce}^{op}) - \tau_{se}^{op} = \sum_{ij=sc,ce} u_{ij}^{T} \frac{\partial c_{ij}^{T}}{\partial n_{se}^{T}} + u_{se}^{R} \frac{\partial c_{se}^{R}}{\partial n_{se}^{T}}, \tag{6b}$$

where the superscript “op” denotes the optimum. We have the following result.

Corollary 1 (*Decentralization of the Optimum*)**.** *Any pricing scheme such that the differences between the traffic fares and the road tolls correspond to the differences between the marginal social cost of crowding and of congestion ensures the optimum is reached.*

Corollary 1 implies that having control over one of the two tools (tolls or fares) is sufficient to reach the optimum so long as the social planner can set tolls or fares such that (6a) and (6b) are satisfied. A typical regime for the decentralization of the optimum is when public transport is unpriced and roads are tolled according to (6) with $p_{ij} = 0$. We note that pricing public transport only can also lead to the optimum, but it is a little more difficult to implement in practice because we have to distinguish users SE from the other two groups. Generally, flat pricing of public transport with similar fares for all groups, which is used in several cities (and is being debated for the Paris region), will not yield the optimum without road pricing. The external cost considered could also reflect environmental externalities. This would lead to greater distortions in equilibrium.

3.3.2. Semi-public administration

In this section, roads are assumed to be operated by a private operator whose objective function is to maximize profit. The transit system is operated by a public agent whose objective function is to minimize total transport cost.

Generally speaking, a private operator will increase its revenues by imposing higher tolls on road users. For the social planner, two scenarios are of interest. In the first one, the public operator sets fares at zero. If tolls are higher than the optimal ones, then compared with the first-best situation, the private mode will be underused. In the second scenario we let the public operator increase the public transport fares to return to the optimum.

Operating the roads is assumed to be costless. The private operator earns the sum of the toll revenues collected on the three roads. It is given by

$$\pi^{R}(\tau_{sc}, \tau_{ce}, \tau_{se}, p_{sc}, p_{ce}) = \sum_{ij=sc,ce,se} \tau_{ij}\, n_{ij}^{R}. \tag{7}$$

The operator is constrained by the equilibrium choice of users described in Eqs. (3). The first-order conditions with respect to the tolls yields $\lambda_{ij} = -(N_{sc} - n_{sc}^{T}) \le 0$ for $\{ij\} = \{sc\}, \{ce\}, \{se\}$, where λ_{ij} is the multiplier of constraint (3). Substituting in the first-order conditions with respect to n_{ij}^{T} shows that the tolls imposed by the private operator satisfy

$$\tau_{ij}^{sp}(\cdot) = (u_{ij}^{R} + u_{se}^{R}) \frac{\partial c_{ij}^{T}}{\partial n_{ij}^{T}} - u_{ij}^{R} \frac{\partial c_{ij}^{R}}{\partial n_{ij}^{T}} \ge 0 \tag{8a}$$

for $(ij) = (sc), (ce)$ and

$$\tau_{se}^{sp}(\cdot) = \sum_{ij=sc,ce} (u_{ij}^{R} + u_{se}^{R}) \frac{\partial c_{ij}^{T}}{\partial n_{se}^{T}} - u_{se}^{R} \frac{\partial c_{se}^{R}}{\partial n_{se}^{T}} \ge 0. \tag{8b}$$

We note that $\partial c_{ij}^{R} / \partial n_{ij}^{T} < 0$. Comparing (8) with (6), we therefore see that for the same level of transit fares, the road operator imposes tolls that are higher than the optimum level. The following proposition states that when public transport is unpriced, the private operator imposes tolls that are higher than the optimum tolls.

Proposition 3 (*Unpriced Public Transport*)**.** *If the social planner makes the public transport free ($p_{ij} = 0$):*

- *road tolls that are imposed by the private operator are higher than those that decentralize the optimum*
- *public transport is overused by all user groups (compared to the optimum).*

Tolling roads and subsidizing public transport for economic efficiency is a prevalent idea. Proposition 3 confirms that road tolls will reduce the use of roads but lead to overuse of public transport. A similar conclusion can be found in Kraus (2012) and Kilani et al. (2014). With unpriced roads it is optimal to reduce fares below the marginal social cost, but when Pigovian tolls are imposed on road users it is optimal to raise fares so that crowding costs are endogenized. The next proposition shows that it is always possible for the public operator to reach the optimum.

Proposition 4 (*Pricing Scheme in the Semi-Public Regime*)**.** *The public operator can reach the optimum by setting full fares for users SE, i.e. $\beta = 1$, and fares*

$$p_{ij}^{sp}(\cdot) = (N_{ij} + N_{se})\frac{\partial c_{ij}^T}{\partial n_{ij}^T} \geq 0 \tag{9}$$

for the other two groups $(ij) = (sc), (ce)$.

The private operator then imposes road tolls that are higher than those it would impose if public transit was free. The semi-public regime induces a strategic competition between the public and private operators. The strategic variables in this duopoly, fares (choice variable for the public operator) and road tolls (choice variable for the private operator), are strategic complements. As a result, the equilibrium is characterized by overpricing, but the public operator is able to reach the optimum, as shown in Proposition 4. In practice, this may raise the issue of acceptability, because users of the modes pay higher fares and higher tolls. The reaction function of the public operator is obtained from Eq. (9), and the reaction function for the private operator from Eq. (8).

From the expressions of the fares in Proposition 4 it is clear that user price is proportional to marginal social cost. Without crowding in public transport, the road operator will impose optimal tolls. The next proposition states this result.

Corollary 2 (*No Crowding*)**.** *If there is no crowding in public transport ($\partial c_{ij}^T/\partial n_{ij}^T = 0$) the private operator imposes socially optimum road tolls when public transport is kept unpriced.*

In this case, the optimum can be easily achieved through the privatization of roads and by charging no fares in public transit. An illustration in a simplified network and a brief discussion of this proposition is provided in the Online Appendix B. This result dates back to Knight (1924) and is usually quoted for the advocation of privatized road management. The scope of this result, however, is not general. It is well known that it is sensitive to two main assumptions (cf. Lindsey, 2012), both adopted in our framework. The first is the elastic demand in the network, and the second is the homogeneity of the users. Even if users are distinguished by their OD pairs, they have the same time values and they perceive the same magnitude of discomfort. If one of these two assumptions is not satisfied, then the private operator will impose a non-optimal road toll.

3.3.3. Private administration

In this section, we consider two distinct competing private operators free to set road prices and transit fares. The first one is operating the roads, as in the preceding section, and the second one is operating public transport. The operators do not incur any cost, and their profit is equal to their revenue. The road operator therefore maximizes objective function given in (7), as in the semi-public regime, and the public transport operator chooses transit fares to maximize

$$\pi^T = p_{sc}n_{sc}^T + p_{ce}n_{ce}^T + \beta(p_{sc} + p_{ce})n_{se}^T \tag{10}$$

In both cases, and since we assume an interior solution, equilibrium conditions in (3) constrain each operator. The first-order conditions for the road operator are still given by (8) and the first order conditions for the transit operator are (where the superscript “du” stands for “duopoly"):

$$\tau_{ij}^{du} + c_{ij}^R + n_{ij}^T\frac{\partial c_{ij}^R}{\partial n_{ij}^T} = c_{ij}^T + u_{ij}^T\frac{\partial c_{ij}^T}{\partial n_{ij}^T} \text{ for } (ij) = (sc), (ce) \text{ and} \tag{11a}$$

$$\tau_{se}^{du} + c_{se}^R + n_{se}^T\frac{\partial c_{se}^R}{\partial n_{se}^T} = c_{sc}^T + c_{ce}^T + u_{sc}^T\frac{\partial c_{sc}^T}{\partial n_{se}^T} + u_{ce}^T\frac{\partial c_{ce}^T}{\partial n_{se}^T} - \Gamma(\alpha). \tag{11b}$$

Combining these first-order conditions with the equilibrium conditions, we have an expression for the fares:

$$p_{ij}^{du} = u_{ij}^T\frac{\partial c_{ij}^T}{\partial n_{ij}^T} + n_{ij}^T\frac{\partial c_{ij}^R}{\partial n_{ij}^R} \quad \text{and} \quad \beta = \frac{u_{sc}^T\frac{\partial c_{sc}^T}{\partial n_{se}^T} + u_{ce}^T\frac{\partial c_{ce}^T}{\partial n_{se}^T} + n_{se}^T\frac{\partial c_{se}^R}{\partial n_{se}^R}}{p_{sc}^{du} + p_{ce}^{du}}. \tag{12}$$

Tolls set by the private road operator are given by Eqs. (8) and fares set by a private train operator are given by Eqs. (12). When there are no congestion on roads, i.e. $\partial c_{ij}^R/\partial n_{ij}^R = 0$, duopoly competition will implement optimal, marginal cost, pricing for public transport. The reason for this outcome is the same as that mentioned above for efficient pricing of roads when there no external costs in public transport. Eqs. (8) and (12) describe reactions functions of rival operators where the decision variables (tolls and fares) are strategic complements (Fudenberg and Tirole, 1991). Thus, equilibrium prices in this case are higher than those obtained for the other regimes.

4. Public transport provision

In the previous section, we looked at the pricing scheme for urban transport under various administration regimes. We now turn to the analysis of transport provision and address two key questions. First, we study public transport provision at given network structure. More precisely, we look at the frequencies of the service provided and the coordination between the two trains. Second, we discuss the consequences of modifying the network by building a new transit line between the two suburban areas.

For many cities whose public transport network is radial, the question of circular lines is of prime importance. Following the example of Paris, some cities have projects to add new peripheral transit lines to their urban transit networks.

4.1. Endogenous service frequency and synchronization

Consider that the operator chooses the frequency of the services and the level of the synchronization between the two trains (α) at the central station. The cost of providing operating vehicles and their synchronization is given by

$$\kappa \sum_{ij=sc,ce} f_{ij} + \upsilon(\alpha), \tag{13}$$

where $\kappa > 0$ denotes the unit cost of operating a vehicle (the summation is done over all links), and $\upsilon(\alpha)$ (with $\upsilon'(\cdot) > 0$) is the cost of deploying an effort to synchronize the two trains. The above term has to be added to the social cost function (4).

The first order conditions (5) remain unchanged, but additional conditions for frequencies and coordination emerge. At optimum, we must have:

$$\kappa = -u_{ij}^T \frac{\partial c_{ij}^T(u_{ij}^T, f_{ij})}{\partial f_{ij}} \text{ for } ij = sc \text{ and } ce, \text{ and} \tag{14a}$$

$$\upsilon'(\alpha) = n_{se}^T \Gamma'(\alpha), \tag{14b}$$

whose interpretation is straightforward. The marginal cost of increasing frequencies or improving synchronization (left hand sides) must equal their social marginal benefits (right hand sides). These expressions implicitly display positive links between service frequencies or the level of coordination and the number of transit users ($f_{ij}^{*'}(u_{ij}^T) > 0$ and $\alpha^{*'}(u_{ij}^T) > 0$). If a public operator is in charge of public transport, optimal service frequencies and coordination are easily achieved.

If a private operator is in charge of public transport, its objective function described in (10) becomes

$$\pi^T = p_{sc} n_{sc}^T + p_{ce} n_{ce}^T + \beta(p_{ce} + p_{ce}) n_{se}^T - \kappa \sum_{ij=sc,ce} f_{ij} - \upsilon(\alpha),$$

subject to the same equilibrium conditions (3).

Lemma 1 (*Decentralization of Service Frequencies and Coordination*)**.** *The decision rule of the private operator with respect to service frequencies (f_{ij}) and with respect to the level of coordination (α) between the two transit lines is optimal.*

The solutions of the Lagrangian associated with the maximization problem of the private operator leads to the same first order conditions (14) as those associated with the optimum. Note that both operators, private and public, minimize the same operating cost with respect to service frequencies and coordination. Therefore, for a given number of passengers, they would choose the same frequencies and coordination. But, since the objective of the social planner is generally to increase the number of transit users, the social optimum will lead to higher frequencies and better coordination between transit lines.

4.2. Investment in a circular transit line SE

The purpose of this section is to study the consequences of building a new, direct transit line between S and E. For simplicity, and without loss of generality, we will assume that building such a line costs nothing. Although this assumption is non-realistic, we will show that such a line can, in some cases, decrease social welfare. Such cases would be more likely to be found if we considered positive building costs. In addition, we will consider interior solutions and assume that every commuter uses the direct OD connection. In other words, we assume that any commuter going from i to j uses either the direct transit line or the direct road. Consequently, we have $u_{ij}^M = n_{ij}^M$, $\forall \{ij\} \in \{sc, ce, se\}$ and $M \in \{T, R\}$.

In this new setup, looking at the modal choice on each link is extensively discussed by David and Foucart (2014). The novelty of our approach consists in looking at the welfare consequences of a modification to the transportation network. Theoretically, the main difference from the previous sections is the equilibrium condition (3c), which becomes (we neglect road prices and transit fares, as our focus is on social welfare):

$$c_{se}^R(u_{se}^R) = c_{se}^T(u_{se}^T, f_{se})$$

The total (social) cost function can be reorganized by mode and OD pairs:

$$C = n_{sc}^T c_{sc}^T + n_{ce}^T c_{ce}^T + n_{se}^T c_{se}^T + n_{sc}^R c_{sc}^R + n_{ce}^R c_{ce}^R + n_{se}^R c_{se}^R - \kappa \sum_{ij=sc,ce} f_{ij}$$

Table 2
Impact of opening a new SE transit line for commuters.

SC and CE users	SE users	Comments
Scenario 1: Congestion, crowding and fixed frequencies		
Number of car users decreases on all roads; Number of transit user increases on all lines; Commuting costs decrease for all groups (SE commuters face smaller risk of delays).		*Always Pareto improving and, therefore, welfare enhancing.* Without the SE commuters in the SC and CE trains, fixed frequencies ensures that crowding decreases. Some commuters switch from the road and transport costs decrease for all commuters.
Scenario 2: Congestion, no crowding and endogenous frequencies		
Less users of SC and CE trains reduces their frequencies, increasing the costs of using trains. Number of car users increases on SC and CE roads; Commuting costs increase on these links.	Number of car users decreases on SE roads; Commuting costs decrease on SE group who enjoy smaller travel time and more reliable service.	*Never Pareto improving - welfare can increase or decrease.* Removing SE commuters from SC and CE transit lines reduces their frequencies. Since there is no crowding, there is no benefit from having less transit users, only lower frequency pushing SC and CE users to use the roads. User costs increase on SC and CE links and decrease on the SE link. Total welfare tends to decrease although it could also increase if the gain for the SE users offsets the loss of the SC and CE users.
Scenario 3: Congestion, crowding and endogenous frequencies		
Number of car users increases on SC and CE roads if the negative effect of lower frequency is larger than the gain due to lower crowding. In that case, commuting costs increase on these links. Opposite effects are possible if the consequence of lower crowding compensate the impact of lower frequencies.	Number of car users decreases on SE roads; Commuting costs decrease (smaller travel time and more reliable train service)	*May be Pareto improving - welfare may increase or decrease.* If lower crowding in SC and CE trains offsets the negative consequences of lower frequencies, building a SE transit line would be Pareto improving. Otherwise, it is not Pareto improving but may be welfare enhancing if the effect on frequencies is low and compensated by the gain for SE users.

where the third term replaces $n^T_{se}(c^T_{sc} + c^T_{ce}) - \Gamma(\alpha)$ and $\upsilon(\alpha)$ drops as there is no longer any coordination issue for SE commuters using the transit lines.

In all cases, the new line SE makes commuters from S to E better off. This line is attractive because it reduces travel time for commuters between suburbs but also decreases the uncertainty with respect to travel time. Let us consider a simple illustration and denote by p the probability that a train trip is canceled. Assume that services are independent between transit lines. So, if commuters from S to E transit through C one of the two trains at least has an incident with probability $p(2-p)$. When the line SE is operating and used by this group the probability of an incident is p instead. Comparing both situations, the new line decreases the possibility of an incident by a factor of 2, since $2 - p \approx 2$ when p is small. Risk averse users usually overweight small probability of extreme events. This simple example shows why the new line is particularly attractive for commuters from S to E. It does not only reduces travel time, but also significantly improves travel time reliability.

Comparison between the two networks is difficult to perform formally without defining explicit functions for the cost parameters. Instead, we discuss the welfare effects on commuters of a new transit line under three scenarios. First, we assume congestion and crowding with fixed frequencies. Second, we assume congestion, no crowding in public transport, and endogenous frequencies. Third and last, we discuss the most general case, considering together congestion, crowding and endogenous frequencies.[11] The main results for the six groups of commuters are summarized in Table 2.

Scenario 1 is easy to understand and easily identifies the forces in play. With fixed frequencies, we do not consider network externalities in transit. In this case, opening the new transit line reduces the number of commuters on the two other transit lines since SE commuters no longer use the SC or CE transit lines. Commuters SE will be better off (by revealed preferences) since they enjoy smaller travel time and benefit from an improved reliability of transit services, as we have discussed above. For this group, there will be a modal switch from private car to train with a significant magnitude if travel time reliability on road is poor (possible accident, bad weather conditions, etc.). This attractiveness of train line SE is general and remains at work in the two other scenarios. Commuters from S to C and C to E that were using the transit are also better off as there is less crowding. Some road users will therefore change their modal choice, reducing congestion on roads up to the point where the cost of using roads and the cost of using transit is equal again for each OD pair. In this scenario, opening a new transit line is Pareto-improving. Every commuter is better off.

In scenario 2, we consider endogenous service frequency and congestion on roads, but no crowding in transit. This scenario corresponds to the case of a congested city where transit is underused. With the new transit line, all SE commuters are assumed to

[11] In practice, we compare the modal split equilibrium with and without the SE transit line under three different set of assumptions. The consideration of the alternative scenarios allows to shed some light on the main mechanisms at work without the need to use explicit functions. To illustrate the procedure, let describe the analytical derivations performed for the first scenario. Remember that opening a new SE transit line implies that SE commuters using transit will shift to that line. By revealed preferences (they use that line only if it is associated to a lower cost), both sides of Eq. (3c) decrease. For radial commuters, u^T_{sc} and u^T_{ce} also decrease (since they do not share transit with SE commuters anymore). It implies a decrease of both, u^R_{sc} and u^R_{ce} to balance Eqs. (3a) and (3b).

be better off. Transit users get to E faster and without crowding, and congestion decreases because more users in this group choose public transport. On the other two lines, there are fewer transit users. Since there is no crowding, this does not increase their welfare. On the contrary, because there are fewer commuters using the transit, frequencies decrease, increasing the cost of using transit for both SC and CE commuters. Some commuters will change their modal choice and use the road. Congestion increases on roads, and frequencies decrease again. We are back at equilibrium when the costs of the two modes are equal. Compared with the original network, there are more road users and fewer users of transit on the SC and CE links. They all face higher costs. Only the SE commuters are better off. Under this scenario, the opening of a new transit line is very likely to decrease social welfare.

In scenario 3, we assume congestion, crowding and endogenous frequencies. As in the two previous scenarios, the impact on SE commuters is assumed to be positive. The impact on the SC and CE commuters depends on the relative forces described above. The new line decreases both crowding and frequencies (there are fewer commuters in these transit lines). If the former dominates on both transit lines, it would be Pareto-improving: if there is less crowding and the impact on frequencies is weak, we expect the new line to be Pareto-improving. If the latter dominates, it is not Pareto-improving and could even be welfare-decreasing. If the negative impact on SC and CE commuters is greater than the positive effect on SE commuters, welfare decreases.

Our theoretical model yields the following important result. So long as frequencies are endogenous (a reasonable assumption), opening a new transit line between suburban areas may be pareto improving only if crowding in public transport is an issue. Otherwise, if there is no crowding, radial commuters are worst off (they have less radial trains and crowding remain unchanged) and only SE commuters are better off. In the worst scenario, opening a new SE transit line may be welfare-decreasing by reducing frequencies and increasing congestion on roads.

5. Numerical illustration

In this section, we provide two numerical illustrations based on the linear formulation detailed in the Online Appendix C. In the first illustration, we focus on transport policies at constant network infrastructure (no SE transit line). In the second illustration, we investigate the impact of opening a new transit line under the three scenarios described above on the decentralized (unpriced) equilibrium.

As a starting point, we mimic the observed modal choice described in Table 6 of Kilani et al. (2014) for Paris. They consider both a small and a large periphery for Paris, and differentiate between journeys from the center to the periphery, and between suburbs. It turns out that 22% to 41% of commuters going from the center to the periphery (or the reverse) use a private car. When considering periphery-to-periphery journeys, we find that 56% to 80% of commuters use a private car.

We wrote a program to perform the computations.[12] We use a right-angled isosceles triangle for the city structure. Each line connecting the city center to the suburban areas measures 5 km and the tangent measures 7.07 km. In terms of population, although there are more commuters going from the periphery to the center than from any one peripheral area to any other, there is usually a single city center and many suburban areas (more than two). As a result, the congestion between two suburbs is caused not only by commuters traveling between them, but also by those commuting between other suburbs but having to use the same peripheral routes. To overcome this problem and because we make no *a priori* assumptions, we decided to set the number of commuters on each link at 100 individuals. Free-flow travel speed is set at 15 km/h for the user of public transport. Free-flow driving speed is set at 30 km/h when the origin or the destination is the city center, but at 50 km/h between suburban areas. Congestion parameter is set at 0.02. Crowding in transit is set at 0.05. The opportunity cost of time is 10 euros per hour and the time spent waiting or switching between two trains is set 50% higher, at 15 euros per hour. For the last term in Eq. (2b), we adopt the formulation $\Gamma(\alpha) = \alpha c_w/(2f_{ce}) - (1-\alpha) s_c$, where s_c denotes the switching cost. When $\alpha = 0$, commuters SE incur: (i) the waiting cost for train SC, (ii) the waiting cost for train CE, and (iii) the full switching cost s_c. When $\alpha = 1$, they only wait for train SE and have no switching cost. As we have noticed in Section 2, high switching cost can be used to reflect the attitude of risk averse travelers when they chain several trips leading to higher probability of an incident and possible travel time delay (Jenelius, 2012). In this application we set $\alpha = 0.1$. This value corresponds to a poor coordination between the two lines, but may also reflect the higher risk of trip chaining two subsequent trains. The operating cost of public transit is used as a scale parameter and takes the value 23.1.

We explore two sets of scenarios. In the first set, denoted A and B and displayed in Table 3, we investigate the effect of various administration regimes in the absence of SE transit line. The second set of scenarios corresponds to the cases discussed in Table 2. The focus is on the opportunity of opening a SE transit line and these scenarios are denoted 1 to 3 in Table 4.

In the base scenario (Table 3), we consider an optimal train frequency (minimizing operating costs) and no SE transit line. With these frequencies of 6 trains per hour, about 80% of the population use the transit for radial journeys, and this figure falls to 30% for commuters between suburban areas. The social transport costs (including costs for commuters and operating costs) in this city reaches 2918 (i.e. 9.73 € per commuter). We then consider two alternative sets of administration regimes in Table 3. Frequencies are held constant in A and become endogenous in B (remember that we do not consider the construction of a SE transit line yet).

In scenarios A, we can see that to reach the optimum, we need to increase the use of public transport for all user groups. This can be achieved by road tolls (as displayed in the Table) or with the equivalent subsidies for public transit users. We note that this increase is modest for radial commuters, while the users of public transit must be almost doubled among the SE commuters. We then consider (semi-)private regimes. Although we only considered the privatization of roads in our model, we also consider the

[12] This program is available online so that anyone can test the impact of a change in one of these parameters on any of the scenarios described in the paper. See footnote [5] for link.

Table 3
Numerical example — Base scenario: no SE transit line, various transport policies.

	Transit users		Transit fares		Road tolls		User cost		Frequency		Total social transport cost
	$n^T_{sc} = n^T_{ce}$	n^T_{se}	$p_{sc} = p_{ce}$	p_{se}	$\tau_{sc} = \tau_{ce}$	τ_{se}	$C^k_{sc} = C^k_{ce}$	C^k_{se}	$f_{sc} = f_{ce}$	f_{se}	
Base scenario (No SE transit line) - optimal frequencies											
Unpriced equilibrium	80.8%	30.1%	–	–	–	–	5.51	15.39	6[a]	–	2,918
Scenario A: fixed frequencies (set at the base scenario), various administration regimes											
Public administration (optimum)	86.7%	57.7%	–	–	1.46	6.06	5.79	15.95	6	–	2,734
Semi-public regimes											
Private roads	90.4%	65.1%	–	–	2.29	7.73	5.88	16.13	6	–	2,752
Private public transit	40.4%	15.06%	8.54	3.94	–	–	13.59	18.40	6	–	4,085
Private regime											
Duopoly	60.3%	43.4%	12.92	10.40	8.75	12.93	18.36	25.67	6	–	3,081
Scenario B: Endogenous frequencies, various administration regimes											
Public administration (optimum)	87.8%	59.8%	–	–	1.38	5.91	5.48	15.36	6.9	–	2,681
Semi-public regimes											
Private roads	91.2%	66.6%	–	–	2.06	7.27	5.48	15.36	7.16	–	2,685
Private public transit	38.1%	10.5%	8.22	3.33	–	–	14.06	19.31	6.92	–	4,265
Private regime											
Duopoly	60.2%	43.2%	12.92	10.41	8.81	13.04	18.45	25.82	5.79	–	3,092

[a] $N_{ij} = 100$; Distances: $SC = CE = 5$ km, $SE = 7.07$ km; free flow travel speed: $SC = CE = 30$ km/h, $SE = 50$ km/h; transit speed: 15 km/h.
Opportunity cost of time: 10 €/h; $a^T_{ij} = 5\%$ $a^R_{if} = 2\%$; $\alpha = 0.1$; $c_w =$ 15 €/h; $\phi = 23.1$.

case where transit is managed by a private operator in this numerical exercise. When roads are managed by a private operator, tolls are higher than optimum, but the social cost of mobility is close to optimum. When public transit is privatized, its use decreases dramatically and social cost of transport soars. Finally, if both transport modes are privatized (duopoly scenario), then road tolls and transit fares skyrockets (due to the strategic complementarity of operator pricing). Besides being socially undesirable (social cost is above the unpriced equilibrium), it is likely to be unacceptable to the population.

In scenarios B, we study the same administration regimes but frequencies are now set endogenously. Under these scenarios, the optimum yields lower social transport costs while as soon as the public transit is privatized, social costs increase and the share of commuters using transit decreases.

To better understand how the parameters' choice impact the modal split and the scenarios, let us briefly test some alternative values. Again we rely on the Fortran computational package to compare original output with output where some parameter values were changed. First, assume an increase of +20% of the value of time. In reaction, users' cost increase by +15% at the optimum and +18% when trains are operated by private firm. Second, if travel speed decreases on the SE road (from 50 km/h to 40 km/h), we observe a slight decrease in the car users on this link and a smaller increase of car use on the SC and CE roads. This second effect is due to the modal shift of SE users (from car to train), leading to an increase of crowding in transit. The user cost increases for all groups but the effect is larger for the group SE. But, when transit line SE is available, the impact of travel speed decrease on the user cost of group SE is very small. With respect to capital cost, it is clear that when it increases the (net) benefit of the new transit line SE decreases since the average cost (user cost and investment) increases.

In Table 4 we consider the opening of a new SE transit line. We focus on the unpriced case to avoid the reproduction of a discussion similar to the one provided above. We start reproducing the base scenario for comparison purpose. The table is organized according to the discussion provided in Table 2. For scenario 1, frequencies are held constant at 6 on every link. Of course, ignoring the building cost of the new line lowers the social transport cost. The new line is pareto improving as it reduces transport costs for all commuters. Note that it is particularly important for SE commuters. The use of the new transit line by the SE commuters reduces crowding in the original transit lines, increasing their use by the radial commuters. This result is valid whatever the frequency considered on the new transit line: irrespective of the frequency on this new line, these commuters could, at worst, continue to commute as they did without the SE line.

In Scenario 2, we allow the frequencies to adjust to demand but we ignore crowding. In that case, opening the SE transit line is never pareto improving since it always reduces the number of users on the original lines, reducing their frequencies and increasing their commuting costs. Using the initial parameters of the model, we can see that this negative effect is very small for radial users and very large for SE commuters, leading to a welfare increase (scenario 2.1). For scenario 2.2, we consider the opening of a very inefficient SE transit line such that SE commuters do not really gain from this opening. This allows us to document a case where welfare decreases with the opening of the new SE transit line. We can conclude that if there is no crowding in existing transit lines opening a new line seem not desirable because it would increase commuting cost for the majority of commuters (although their welfare loss is generally compensated by the welfare gains of the minority).

As discussed earlier, under scenario 3 (considering congestion, crowding and endogenous frequencies), we can observe a welfare decrease, a welfare increase and a pareto improvement, depending on the parameter values. For example, under scenario 3.1 (pareto improvement) crowding is important (higher values for parameters a^T_{ij}) and the new line reduces user cost. Alternatively, if travel speed of the new line is too low, the user cost increases as in scenario 3.2. Here, the modest gain of users SE does not balance the loss for the other two groups (lower frequency).

Table 4
Numerical example — Scenario 1–3: effect of opening a new SE transit line.

	Transit users		User cost		Frequency		Total social transport cost
	$n^T_{sc} = n^T_{ce}$	n^T_{se}	$C^k_{sc} = C^k_{ce}$	C^k_{se}	$f_{sc} = f_{ce}$	f_{se}	
Base scenario (reproduced for comparison)	80.8%	30.1%	5.51	15.39	6*	–	2,918
Scenario 1: Effect of opening an SE transit line with congestion, crowding and fixed frequencies							
Unpriced equilibrium	82.0%	74.16%	5.27	6.58	6	6	2,082
Scenario 2: Congestion, no crowding and endogenous frequencies[a]							
Unpriced equilibrium with no SE transit line	85.8%	40.1%	4.51	13.40	6.39	–	2,536
Scenario 2.1: case with a welfare increase*							
Unpriced equilibrium with SE transit line	84.5%	76.0%	4.77	6.22	5.24	4.97	1,932
Scenario 2.2: case with a welfare decrease[b]							
Unpriced equilibrium with SE	84.5%	42.6%	4.77	12.89	5.24	3.72	2,570
Scenario 3: Congestion, crowding and endogenous frequencies							
Scenario 3.1: pareto (and welfare) improvement case[c]							
Unpriced equilibrium with no SE transit line	76.3%	21.2%	6.40	17.17	5.63	–	3,257
Unpriced equilibrium with SE transit line	76.5%	68.3%	6.37	7.68	4.98	4.71	2,389
Scenario 3.2: welfare decrease case[d]							
Unpriced equilibrium with SE transit line	80.4%	30.8%	5.59	15.08	5.11	3.16	2,953

[a]Same parameters as base scenario but with no crowding: $a^T_{ij} = 0$.

[b]Same as scenario 2-A but we assume that, compared to the existing lines, the new one is not very efficient: we set travel speed in SE at 6.5 km/h.

[c]Same parameters as base scenario but crowding must be a larger issue to achieve pareto improvement. We use: $a^T_{ij} = 0.1$.

[d]Same parameters as base scenario but compared to the existing transport network, the new SE transit line must be of low efficiency. We set the travel speed in the SE new transit line at 5.7 km/h.

To sum up, these numerical examples illustrate that even if building a new transit line was free of charge, it is not always desirable. In addition, the desirability of a new SE transit line increases if there is crowding in the existing lines and when the new lines does not affect to much the existing frequencies. For example, it might be very important in a city where existing lines are crowded and where frequencies are already at their maximum. It such situation, frequencies would not be affected by transporting less commuters while it would reduce congestion, leading to a pareto improvement.

Finally, we have not discussed the building costs of the SE transit infrastructure in this paper. We know that these costs are very high (see Bono et al. 2019, for a discussion). In our numerical example, the difference between the social costs with and without such infrastructure can be interpreted as the opportunity cost of building the line. Nevertheless, whatever the building costs, such a line seems socially desirable, though not for all commuters. As long as there are more radial commuters (SC and CE) than SE commuters (which is generally the case), if they could vote for or against the building of such a line, the majority would oppose it, as radial commuters would be worse off.

6. Discussion

Urban transport is a complex system that should be simplified to conduct analytical analysis. We have done so, but some interesting issues, which are not directly addressed in our model, still deserve some discussion. We report here a set of these points and explain how the model and our results would be affected by taking these aspects into consideration.

6.1. Demand elasticity

The perfectly inelastic demand considered in this paper is a strong assumption. What would be the effect if the demand for transport was inelastic instead of perfectly inelastic? As long as demand is inelastic (which is observed in the real world), the effects would be qualitatively similar (same sign) but mitigated: when the total transport cost decreases on an OD pair, as it is the case when the SE line is opened, more commuters would travel on that OD pair and the transport cost decrease would be mitigated by the higher transport demand. Inelastic demand (that contrasts perfectly inelastic demand) can have two sources: relocation within the city or migration from outside the city. Generally speaking, if transport costs in a city decrease, the city becomes more attractive and urban migration may take place, increasing the city size. Addressing this question would require to add an urban migration equation *à la* Harris–Todaro, adding a layer to the already complicated theoretical model.

Instead of adding a free migration decision, we slightly modify the model so that the number of users traveling from i to j becomes $N_{ij} = N^p_{ij}(C_{ij}/F^f_{ij})^{-\mu}$, where N^p_{ij} is the potential demand (the number of users when the variable part in the user cost is zero), C_{ij} is the total user cost, F_{ij} is the free-flow user cost and $-\mu$ is the constant elasticity of demand. In this approach, the model assumes a transport demand elasticity for all OD pairs and the results can be interpreted in terms attractiveness for each OD pair for people that are currently not located in the city. Traffic equilibrium is now obtained through fixed-point iterations, and a perfectly inelastic demand can be restored with $\mu = 0$.

Table 5, in the Appendix A, reports equilibrium values for three values of demand elasticity: -0.1, -0.2 and -0.5, respectively. Theses values are in line with the values reported by Small and Verhoef (2007). As expected, compared to the perfectly inelastic scenario, the opening of a new SE transit line leads to a lower decrease in transport costs and to an increase in the number of commuters on all lines. Interestingly, these effects are much larger at the periphery, suggesting that the opening of a new SE transit line would have a large impact on the population size living in suburban areas (+11.2% when the elasticity is set at −0.2) while the impact in the city center is much smaller (+0.9%).[13] These values give an idea of the impact of opening a new SE transit line on the relative attractiveness of different urban centers. It appears that SBDs become relatively much more attractive (by a factor of 10) than the CBD. We discuss the consequences of these changes in attractiveness on agglomeration externalities in the next subsection.

6.2. Relocation and agglomeration externalities

Transport policies, such as building a new transit line, may affect people's location choices in the long run. The model describes wasteful commuting, that is commuting of inefficiently located agents. If it was not the case, all worker would live where they work and the model would be pointless. In practice, many people live in one place and work in another one, for various reasons. We are going to discuss relocation forces to have an idea of how the city structure may evolve in the long run following significant changes in transport policies. A comprehensive urban model with relocation would require to take labor, land and housing markets into consideration (Chapelle et al., 2021; Graham, 2007). We decided to avoid such complication and focus only on relocation forces.

Before entering into the discussion, remember that even if we consider commuting in a single direction (from S and C to C and E) the network is assumed to be symmetric in size and transport costs (for instance, $N_{se} = N_{es}$ and $C_{se} = C_{es}$). Therefore, if a SBD, let us say S, is more attractive, so it is for the other SBD, E in this case, by symmetry. To facilitate the reasoning, the discussion focuses on the effect of opening a new SE transit line, reducing significantly C_{se} while barely affecting the transport costs on radial lines (the transport costs on SC and CE are assumed to remain constant).

To understand the forces at play, we start by assuming that job's location are fixed and study the relocation forces for the living place of agents. For SE commuters,[14] the decrease of the transport costs from S to E makes these commuters better off. They commute at a lower cost to their working place and the value of their outside options, moving to C or E remains identical. This suggests that SE commuters have no incentive to relocate in reaction to a decrease of transport costs on that link. For CE commuters, their direct transport costs do not change, but the value of one of their outside option, moving to S and commute to E, increases. By symmetry, the same results apply for CS commuters. It turns out that the SBDs become more attractive for people living in the CBD and working in a SBD. Therefore, when job's location are fixed, opening a transit line between SBDs is likely to attract people coming from the CBD. A peri-urbanization process may be observed.

Following the same arguments, if job location was endogenous in the model, a decrease of transport costs between the SBDs would also make these SBDs more attractive for companies. In conclusion, building an SE transit line would make the SBDs more attractive for workers and companies.

Linking these forces to agglomeration externalities is not an easy task. The population of the urban agglomeration (composed of a CBD and two SBDs) remains identical. Nevertheless, the opening of a SE transit line is expected to make the SBDs more attractive for firms and workers. A peri-urbanization process may be launched and it would imply an increase of agglomeration externalities in the SBDs at the cost of a decrease of these externalities in the CBD. The net effect is impossible to predict without additional assumptions at the level of the urban agglomeration.

The scenario discussed in Section 6.1 is informative. The simulation suggested that the opening of the SE transit line would have an impact about ten times higher in the SBDs compared to the CBD. This suggests that the peri-urbanization process can be important. In line with that result, if we consider a city open to urban migration, agglomeration externalities could increase in all places since we say that a new SE transit line increases the attractiveness of all centers (CBD and SBDs).

6.3. Alternative policies and captive commuters

We briefly discuss two further issues. First, instead of setting tolls and fares, alternative policies could be considered. For instance, a popular policy consists in reducing the space devoted to cars and/or strengthening the transit infrastructure. The effect of such policies can be easily analyzed in our framework. Reducing the space for cars increases the costs associated to its use. Strengthening transit reduces the costs associated to that mode. Both policies can be directly addressed through parameters in cost functions (1) and (2). For example, reducing the road space can be obtained by making the variable part in (1) more sensitive to the number of users (by increasing values of parameters a_{ij}^{R}). The obtained impacts should be equivalent to pricing policies and, furthermore, one can establish some correspondences between several policy tools with respect to their induced impacts.

Second, some commuters may be captive of their transport mode. It may be the case for those who cannot afford a car or those who are not well connected to the public transport network. To include this feature in the model we need to consider heterogeneous population, not only with respect to their location but also with respect to revenues and/or tastes. While this can be undertaken, for example *à la* (David and Foucart, 2014), it would not produce new insights and the main messages would remain unchanged: the higher the transport costs of a mode, the lower it is used. Similarly, all the policies discussed earlier (administration regimes

[13] The computer code provided can be used to develop more complex illustrations with inelastic demand (link-specific elasticities, compare distinct administration regimes).

[14] Remember that the first letter denotes the living place and the second letter the working place (fixed in this scenario). When discussing the relocation forces of SE commuters, we study to what extent they have an incentive to change their living place in reaction to a change in transport costs.

and public transport provision) would produce similar impacts but, with heterogeneous population, some users would be more or less sensitive than others to these shocks.[15]

7. Conclusion

We have developed a model of urban transport with a new feature: the consideration of a polycentric city where commuting between suburban areas is explicitly considered. This framework allows to study the effect of building a new transit line between the suburbs. It extends earlier literature on mode and route choices (Parry and Small, 2009; de Palma et al., 2007) through the adoption of a more complex and realistic network. We show that the unpriced equilibrium is in general not optimal, and cars are overused, particularly in the outskirts. The optimum can be decentralized through road pricing, or under a semi-public administration where roads are managed by a private operator and rail by a public, welfare-maximizing operator. When the two operators compete, road tolls and public transport fares are strategic complements, and monetary transport costs are significantly high.

Metropolitan areas are expanding, and both reverse commuting and commuting between suburban areas are growing. In addition, many large cities face increasing crowding issues in public transport. Policy makers must react, either by changing the pricing schemes of transport modes or developing new transit lines, or both. Several cities whose original public transit infrastructures were radial are considering investment in circular rail lines directly connecting the suburbs. The metropolitan area of Paris is a case in point, with the "Grand Paris Express" project whose estimated cost is 35 billion euros over the next decade. We show that such network expansion is socially desirable, but some users may become worse off unless the current network is overcrowded. In practice, it seems that the Parisian underground network is overused and the Grand Paris Express is likely to improve welfare for all citizens by reducing the crowding externality without affecting frequencies. One of the objectives of the model we have developed is to provide a tool to evaluate and compare several transport reforms that can provide some insights on policy choices.

The numerical example developed in Section 5 illustrates and quantifies our main findings. It is made to allow policy makers to test the theoretical predictions associated to changes in the pricing schemes or the network of specific cities. Indeed, the Fortran program is open source and could easily ba adapted to various specific contexts. The worst scenario is obtained under the semi-public regime where roads are unpriced while public transport is administered by a private, profit-maximizing operator. The privatization of roads is quite efficient because it achieves a high use of public transit. The fully private regime reaches a social cost in-between the two semi-public regimes, but it is likely to be unacceptable to the population because it implies very high tolls and fares for commuters (we note that these tolls and fares do not enter the social cost function as they are transfers).

The impact of a network expansion that consists in opening a direct transit line between the suburbs largely depends on how service frequencies adjust. When frequencies are exogenous, providing a new rail line reduces the user costs for all commuters. This is the consequence of the lower level of crowding in the radial lines with the same service quality. With endogenous frequencies, the opening of the SE transit line reduces the number of users in the radial lines, reducing the frequencies and increasing the user cost of radial commuters accordingly.

In practice, several big cities have transit systems that are operating at their capacity limits. Service frequencies are then set at the maximum possible level and would not decrease if demand decreased slightly. In our model, this situation corresponds to the exogenous frequency scenario. In that case, building a new transit line would increase welfare and is likely to be Pareto-improving. Nevertheless, when crowding in the radial lines is moderate, commuters might vote against it if asked. Although the line increases total welfare, the effect is driven by commuters at the outskirts of the city, and the effect is negative (though relatively small) for commuters located downtown.

We emphasize that we have made some simplifying assumptions to keep the model analytically tractable. We have discussed several of these assumptions in Section 6 and showed that the model can be extended, more of less easily, in several directions. For example, while the main model was based on a perfectly inelastic demand, we have developed a numerical illustration to account for inelastic (realistic) demand. The potential evolution of the city's organization has also been discussed. Relocation forces suggest that building a SE transit line may lead to peri-urbanization in the long run by making the SBDs more attractives. A higher proportion of workers and jobs would locate in these SBDs.

Many recent studies of urban transport problems rely on large transport simulation models such as TransCAD, Emme, SimMobility, MATSim, Metropolis, etc. During the last decade these tools evolved to include several interesting features and algorithms have significantly improved. Today, software are stable and relatively accessible. Often, transport models are connected to land use models to conduct long run analysis of the interaction between transport and urban forms (LUTI models). Nevertheless, even if these tools are useful to explore the benefits of a given project or policy scenario, they remain much less transparent than analytical approach like ours. In particular, it remains difficult to link the output to the assumptions. Also, even if most simulation tools have a good representation of road transport network, they often rely on shortcuts for public transport and the other transport modes (biking, walking, etc.). This is why we believe that analytical approaches remain useful to derive and understand key features of urban transport.

It is also important to note that we did not incorporate environmental externalities in the model. This important variable was disregarded to keep the model as simple as possible. In many cases environmental constraints are not expected to change the signs of the impacts we obtain. Traffic on roads generates two types of environmental externalities: global and local. Global externalities (GHG emissions, for instance) differ from congestion and crowding in that they impact the whole population, not only

[15] Note that considering captive users would exclude corner solutions in the model.

Table 5
Numerical example — Scenario 1 with not perfectly inelastic transport demand elasticity.

	Modal split (transit users)		User cost		Commuters	
	$\frac{n^T_{sc}}{N_{sc}} = \frac{n^T_{ce}}{N_{ce}}$	$\frac{n^T_{se}}{N_{se}}$	$C^k_{sc} = C^k_{ce}$	C^k_{se}	$N_{sc} = N_{ce}$	N_{se}
Base scenario (reproduced for comparison)	80.8%	30.1%	5.51	15.39	100	100
Scenario 1: Effect of opening an SE transit line with congestion, crowding and fixed frequencies (reproduced for comparison, transport demand elasticity: 0)						
Unpriced equilibrium	82.0%	74.16%	5.27	6.58	100	100
Scenario 1a with transport demand elasticity of −0.1:						
Unpriced equilibrium	82.1%	75.3%	5.27	6.62	100.3(+.3%)	105.0(+5.0%)
Scenario 1b with transport demand elasticity of −0.2						
Unpriced equilibrium	82.1%	76.2%	5.27	6.66	100.9(+.9%)	110.3(+10.3%)
Scenario 1c with transport demand elasticity of −0.5						
Unpriced equilibrium	82.3%	78.7%	5.28	6.79	102.1(+2.1%)	126.6(+26.6%)

local commuters. It increases the social cost of driving on all roads. Local externalities (noise pollution, for instance) impact local residents. Since more commuters are working in the CBD, taking these externalities into consideration would reinforce the social cost of driving to or from the city center compared to the periphery. It would lead to relatively higher tolls in the city center. In this case, the benefits of expanding the public transport network at the periphery is mainly driven by its global impact because local externalities increase with urban density. An extension in this direction is straightforward and would be useful to obtain a more general assessment of the circular line. We leave this task for future empirical research.

Finally, the last years have been marked by the rise of a discourse that proclaims a strong orientation towards sustainable mobility, on the one hand, and the need to contain the COVID 19 pandemic, on the other hand. The model and the numerical illustration provided in this paper improve the understanding of the forces at work. For instance, it is shown that multicentered, crowded, metropolitan areas can achieved these two goals with a single policy: the building of a new SE transit line. At the same time, this line reduces crowding in the radial lines and favor, to some extent, social distancing. So, if the pandemic settles in the long run, our model could be complemented by an epidemic model, like the SIR, to study the optimal train frequencies to limit the spread of the virus.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The code is shared online. The link to the code is in a footnote of the paper.

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Appendix A. Numerical exercise with inelastic transport demand

See Table 5.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.tra.2022.09.017.

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