



**Simplifying the Measure of Concentration
from Common Ownership:
A Note**

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June 2022

ECARES working paper 2022-19

June 6, 2022

Simplifying the measure of concentration from common ownership:

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Abstract

The note presents a simpler alternative to the Modified Herfindahl-Hirschman Index to measure the risks of market concentration in the presence of common owners such as institutional investors owning shares in multiple firms expected to compete in the same market. This new measure, the Amplified Herfindahl-Hirschman Index, delivers the same insights as the MHHI but is less data intensive and less sensitive to outliers. It thus offers a more “user friendly” and more precise alternative to competition and regulatory agencies as a decision trigger for more detailed investigations of market power risks associated with the growing presence of common ownership.

Keywords: Antitrust, Common Ownership, Institutional Investors, Market Power, Modified Herfindahl-Hirschman Index

JEL Classifications: D21, D43, G24, G28, K21, K22, L13, L21, L22, L40, L41

¹ We are grateful to Claude Crampes, Vincent Daxbek, Pierre Francotte, Renaud Foucart and Tomas Serebrisky for detailed comments on an earlier version. Any mistake is however our responsibility only.

1. Introduction

With the growing presence of institutional investors as key shareholders of firms expected to compete with each other, regulatory and competition agencies are increasingly concerned with what this evolution in the composition of ownership means for the effective degree of concentration of markets (e.g. Monopolkommission (2018)). For instance, in France, BlackRock, Vanguard and State Street all own minority shares in different pharmaceutical companies competing in at least some markets including Astrazeneca, Bristol Myers Squibb, GSK, Johnson & Johnson, Mylan, Pfizer and Sanofi.² Similar stylized facts have been observed for a number of sectors, including many regulated ones, in various countries.³

The analysis of the effects of this common ownership (CO) has become subject to still unsettled debates.⁴ Some criticisms have focused on the specific details of the concentration measure increasingly adopted to assess the market power risks: the Modified Herfindahl-Hirschman Index (MHHI).⁵ It is criticized, among other reasons, for its complexity and for the strength of some of the assumptions made to produce the actual measures needed to assess whether CO is indeed a concern or not in a specific sector.

This note argues that one way of reducing the number of disagreements while still delivering on the many necessary insights allowed by the MHHI is to rely on a simpler measure: the *Amplified Herfindahl-Hirschman Index*, or AHHI. The new index avoids some (but not all) of the criticisms addressed to the ambitious MHHI while still improving on the traditional Herfindahl-Hirschman Index (HHI) and just as importantly, while delivering results on the anti-competitive risks associated with CO similar to those delivered by the MHHI.

Section 2 briefly reviews how the MHHI came about to complement the traditional HHI in an evolving market and financial context. Section 3 describes a suggested alternative, more precise than the HHI but less assumption-intensive than the MHHI and more pragmatic under current common data availability contexts. Section 4 compares the three approaches in a case study. Section 5 concludes.

2. How did we get to the current measures of concentration under CO?

Since the 1950s, the HHI has been the indicator of reference for policy makers and regulators willing to gauge the potential threats of excessive product or service market concentration.⁶ One of its advantages is its simplicity since it only relies on data on intra-industry market shares. Formally,

$$HHI = \sum_j s_j^2 \quad (1)$$

where s_j is the market share of firm j multiplied by 100.

The index value obtained ranges from 0 in case of perfect competition to 10,000 in case of a pure monopoly. Based on their experience, the U.S. Department of Justice and the Federal Trade Commission (2010) have suggested thresholds to categorise the concentration levels of a market

² It is noteworthy that, in France, the share of the top 10 institutional investors in the three main suppliers of Covid vaccines is quite significant (29.5% for Johnson & Johnson, 34.2% for Pfizer and 39.1% for Astrazeneca, based on 2019 data). These investors represent a significant group of shareholders likely to share a common short term profit interest that needs to be balanced with the short and long term social interest likely to be driving the regulators of the sector. See Estache and Kieffer (2021) for details on the correlation between the presence of CO and the financial performance of firms in the French pharmaceutical sector.

³ See Frazzani et al. (2020) for a recent detailed overview of the evidence and of the related debates or Azar et al. (2021) for a more technical discussion of the debates.

⁴ Rock & Rubinfeld (2018), O'Brien & Waehrer (2017) and Ginsburg & Klovers (2018).

⁵ See for instance Backus et al. (2019a, 2019b, 2021), FTC (2018) or Florian and Gron (2019)

⁶ Herfindahl (1950) is the original reference.

which are commonly used to assess the risks of concentration. They imply that an HHI below 1,500 characterises an unconcentrated market; an HHI between 1,500 and 2,500 represents a moderately concentrated market, while highly concentrated markets have an HHI exceeding 2,500.

The limitations of the HHI in the current modern market and financing context had become a recurring concern as the experience was suggesting that the growing presence of institutional actors on the boards of multiple companies was de facto easing collusion and increasing market power abuse risks that had been ignored until the mid-2010s. The failure to account for the ownership structure and distribution of market shares in the computation of the HHI was the main culprit. For instance, as illustrated by Sykuta (2018), an HHI of 2,500 could result from one company with a share of roughly 49% and other companies sharing the rest more or less evenly, or it could result from four companies splitting the market evenly with 25% market share each. More conceptually, the assumption that all firms in a market are independent from each other explained a tendency for the HHI to underestimate the risk.

To account for the relevance of ownership structure effects on market concentration, scholars came up with a variety of alternative indicators. An adaptation of the existing HHI emerged as the most popular among academics and policymakers: the Modified Herfindahl-Hirschman Index (MHHI). Based on Bresnahan and Salop (1986) and further developed by Salop and O'Brien (2000), it tries to take into consideration jointly market concentration and ownership concentration linked to CO. Formally,

$$MHHI = HHI + MHHI \text{ delta}$$

$$\sum_j \sum_k s_j s_k \frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}} = \sum_j s_j^2 + \sum_j \sum_{k \neq j} s_j s_k \frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}} \quad (2)$$

where j and k index firms (firms indexed by k are competitors of j), i indexes investors, s represents the firms' market shares, β_{ij} designates investor i 's share of firm j 's equity, and γ_{ij} designates investor i 's share of firm j 's control.⁷

The main pragmatic attractiveness of the MHHI in the current policy debates is that it explicitly accounts for both the degree of product market concentration among firms captured by the HHI, and the degree of concentration exclusively linked to the presence of common ownership across firms in the market. This double monitoring option results from the fact that the MHHI formula can be broken down into two components reflecting each of the two potential sources of concentration: the standard HHI and the so-called MHHI delta, with the latter quantifying, and hence highlighting, the added concentration stemming from CO. This breakdown facilitates the interpretation of the general formula and eases the debates on the relative importance of the two main sources of market power in which competition and regulatory agencies tend to be interested. When there are no common owners, the MHHI delta is zero and the MHHI equals the basic HHI. So if there is a concentration risk, it results from the product market characteristics. On the other hand, when common owners are present, higher levels of control and/or equity in competing firms will imply a higher MHHI delta and thus a higher market concentration, ceteris paribus. This can add to an existing product market concern or become a source of concern on its own.

From a more conceptual perspective, it is useful to keep in mind that the MHHI, similar to the HHI, can be interpreted in the context of a Cournot model of competition (i.e. quantity-setting competition in a market in which firms produce a homogenous good). This can be quite useful to assess the relative importance of the two sources of concentration for the average markup in the sector. The HHI is proportional to the market share-weighted average markup in a standard Cournot

⁷ Equity shares and control shares are expressed in decimals. Here, market shares are expressed in percentage. If market shares are also expressed in decimals, then the value obtained from the equation should be multiplied by 10,000.

setting with separately owned firms. However, in the case of the presence of CO where firms maximize a weighted average of their shareholders' interests and again compete à la Cournot, the market share-weighted average markup will be proportional to the MHHI.⁸ While this conceptual background is useful, it does not apply to all cases submitted to regulatory agencies. For instance, in the case the market is better characterized by a Bertrand model (price competition), there is, however, no equivalent common interpretation of the MHHI.⁹

Other conceptual concerns include the fact that the MHHI was initially developed by Bresnahan and Salop (1986) to assess how direct horizontal shareholdings (cross-ownership) affect competition. Rock and Rubinfeld (2017), for instance, question whether the underlying logic of the indicator can then be transposed to CO issues as easily as it is done in some of the academic outputs. Moreover, some analysts raise concerns about the mathematical foundations of the measure. For instance, the MHHI delta has no mathematical upper boundary. In theory, extreme cases could see it tend towards infinity,¹⁰ and real-life cases exceeding the 10,000 threshold are not uncommon. Consequently, it is hard to give a meaningful interpretation to the MHHI values obtained, because 'how big is big'?

In addition to these conceptual limitations, the MHHI raises other very practical concerns. One of the themes regularly raised by its critics is the need to assess the share of control variable γ_{ij} . They argue that control weights are difficult to observe in practice and that assumptions are needed to be able to use the formula. A pragmatic way to circumvent this difficulty is to rely on a proportional control assumption. By assuming that an investor's share of control in a firm is equal to its equity stake in it (i.e. $\gamma_{ij} = \beta_{ij}$), the MHHI delta equation can be rewritten as follows:

$$MHHI \text{ delta} = \sum_j \sum_{k \neq j} s_j s_k \frac{\sum_i \beta_{ij} \beta_{ik}}{\sum_i \beta_{ij}^2} \quad (3)$$

This simplified expression of the MHHI delta is to be credited to Posner et al. (2017) and has proven to be popular. However, it is also subject to criticism. First, the Posner et al. (2017) suggestion is still quite data intensive and computationally burdensome. Second, and maybe more importantly, no theory or empirical evidence has yet demonstrated robustly how common shareholdings relate to control weights in practice. A key limitation of this simplification is thus that it does not account for the possibility that institutional investors may exert more influence than their passive nature might suggest. This adds to the risks associated with the fact that in practice, ownership stakes inferior to 0.5% are usually omitted. This choice makes the MHHI more sensitive to outliers. For instance, when the bulk of shares is spread over multiple (undiversified) investors with near-zero stakes while the remaining shares are concentrated in the hands of few common owners, the MHHI returns sky-high levels of market concentration if the small stakes are neglected, as Lambert and Sykuta (2019) demonstrate. But even in less extreme cases, ignoring minor shareholders can decrease the denominator of the equation and consequently inflate the degree of market power. Robustness checks are thus important to consider in this context.

A final concern expressed in the literature that deserves some discussion in this review is that the MHHI is subject to, at least, a couple of endogeneity issues. First, there is endogeneity induced by the structure of non-common owners. This is caused by the fact that the denominator in the MHHI delta equation takes into account ownership shares from *all* investors and not only the common ones. When the number of non-common owners gets larger and/or their equity shares get

⁸ This derivation of the MHHI from a Cournot competition model was initially established by Salop and O'Brien (2000) (see their Appendix section C) and can also be found in the Internet Appendix section 1 of Azar et al. (2018).

⁹ Goppelsroeder et al. (2008) provides useful implicit insights on the scope to account for common ownership issues in a Bertrand context in their discussion of complements to MHHI in the context of merger evaluations when partial ownership is present.

¹⁰ As an experiment, Lambert and Sykuta (2018) performed a constrained optimisation under reasonable and plausible restrictions which saw the MHHI delta reach a value of 26.8 billion.

smaller, the MHHI will increase (Backus et al. 2019b). Second, there is endogeneity induced by the market share component of the MHHI formula. As Boller and Morton (2020) point out, since the index moves in tandem with endogenously determined market shares, variations can be interpreted as both source and outcome of the competitive process.

In sum, the list of concerns is quite long and many deserve consideration when competition and regulatory agencies decide to quantify the risks associated with CO. Not all can be addressed in a straightforward manner but there is room for improvement and certainly for the simplification of a measure which we know will be imperfect anyway in concrete contexts. A simpler alternative to measure the risks associated with CO can still provide a reasonable trigger measure for a competition or regulatory agency to decide whether the extent to which the presence of CO or its growth deserves a more detailed investigation based on reasonably easily available data. This is what the AHHI is claiming to deliver.

3. The AHHI: a possible simpler alternative to the MHHI

The AHHI also starts from the standard HHI to identify product market concentration risks and adds a “delta” to produce a quantitative measure of the impact of ownership concentration due to CO. It thus retains the two-components approach of the MHHI. But we suggest that the second component can be simpler than the one proposed by the MHHI without giving up on the main insights on market power risks due to CO that the MHHI delivers.

Formally, this new index is expressed as follow:

$$\begin{aligned}
 & AHHI = HHI + AHHI \text{ delta} \\
 & AHHI = \sum_j s_j^2 + \sum_i \left(\sum_j \beta_{ij} * s_j^2 + \sum_{k \neq j} \beta_{ik} * s_k^2 \right) \quad (4) \\
 & \forall \beta_{ij}, \beta_{ik} \geq 0.005
 \end{aligned}$$

where j and k index firms (firms indexed by k are competitors of j), i indexes investors, s represents the firms’ market shares, β_{ij} designates investor i ’s share of firm j ’s equity, and β_{ik} designates investor i ’s share of firm k ’s equity.

Equation (4) is simpler than equation (2) and builds on similar underlying assumptions but also on a couple of differences. The similarities between AHHI and MHHI are the useful simplifications to be made when actually trying to compute the index, even if they are sometimes criticized as mentioned in section 2. The differences concern the extent to which endogeneity and sensitivity to outliers are issues that can limit the usefulness of the measure of CO related concentration. Both the similarities and the differences deserve a detailed discussion.

The two simplifying computational assumptions commonly used in the evaluation of the MHHI that have proven to be quite useful were easy to adopt for the AHHI because they do not represent a significant concern in most concrete cases. The first of these assumptions is that, in general, small owners do not have a noteworthy influence on firm decisions. This allows the omission of ownership shares of less than 0.5%. The assumption is useful in practice because tracking down all small shareholders can be difficult and can make the computation significantly more burdensome. Just as importantly, it is not crucial since, if data on small shareholders is available, a more precise picture can actually be produced just as in the case for MHHI. Its potential impact on the evaluation of risks associated with CO is however weaker for the AHHI than for the MHHI. For the MHHI, neglecting the minor stakes impacts the denominator in such a way that the risks of concentration can be significantly overestimated. In the case of the AHHI, the omission of

small stakeholders has only a negligible impact as a result of the simpler additive characteristic of the equation.

The second assumption we retain from the usual approach to the MHHI is the neglect of the corporate control variable. This is in contrast to the “original” MHHI (Bresnahan and Salop (1986), Salop and O’Brien (2000)). In other words, we do not consider voting shares a representative reflection of the influence shareholders could exert. This assumption is to address two types of uncertainties. The first is to avoid the need to assess whether shareholders are willing to act actively or prefer to remain passive. It also avoids the need to assess the potentially non-proportional weights of shareholders with respect to their voting shares. The second is linked to the difficulty of observing and thus measuring control weights from available data sources. In practice, we thus rely on a proportionate control assumption as was the case for the « simplified » MHHI calculation (Posner et al. (2017)). In a situation in which the assumptions are proving to be too strong, the limitations of the measure should emerge without much difficulty from the more detailed audit that the AHHI would have triggered. The AHHI is only a first step in the assessment of the CO related risks, as is the MHHI. The follow-up process is the one that will determine the extent to which the assumption on voting shares is excessive or not.

To ensure that the similarities between the AHHI and the MHHI do not lead to an underestimation of the potential advantages of the AHHI over the MHHI, it is necessary to dig into the significant differences between the two measures. The first concerns the way the associated endogeneity issues are dealt with. In the case of the AHHI, a *condition sine qua non* for the indicator to provide the accurate estimation of concentration it is designed for, is to only take into consideration investors holding equity stakes in *at least* two competing firms. This allows the AHHI to avoid the endogeneity issue from which the MHHI suffers, i.e., having non-common ownership stakes in the equation.

Critics might argue that the AHHI still suffers from endogeneity as it depends on market shares (as does the MHHI). However, in the context of the search for a tool trying to provide clear signals of the risks of market power abuses, we regard this characteristic as useful. The bias sends a stronger and clearer signal that there may be a case for further investigation of the risks, as the capabilities *and* incentives of horizontal shareholders to harm market competition tend to be disproportionately impacted by the market shares of their holdings in practice. The AHHI thus considers the anticompetitive effects of CO to be relatively more significant when excessive market concentration prevails. The logic is as follows: An investor – let’s say GreenRock – holding a 5% equity interest in two market-leading firms (with e.g. 40% and 30% market share respectively), will have a much greater ability to influence the market anti-competitively than if it had those same 5% stakes in two minor firms (with e.g. 2% and 1% market share). This simple example thus highlights the importance of accounting explicitly for the market shares of the firms these institutional investors decide to support.

The second difference between the AHHI and the MHHI worth mentioning is linked to the degree of influence of extreme cases (i.e. outliers) on the assessment of the risk of excessive market power. The difference in risks results from the difference in the way the MHHI delta and the AHHI delta are computed to measure the impact of CO on concentration. The simpler additive nature of the AHHI delta makes it less sensitive to outliers than the MHHI while still accounting for the fact that holdings of *at least* two competing firms within an industry is an intensifier of market power.

These two main differences lead to two significant advantages of the AHHI when compared to the MHHI. First, the equation is much less complex, which makes it easier to apprehend and less onerous to compute. Second, and more importantly, the AHHI has a well-defined mathematical boundary which makes interpreting the values obtained much more intuitive. The AHHI delta can be seen as the fragment of market concentration associated with common owners (the betas being the weights), equivalent to the MHHI delta. The AHHI delta is thus by definition always smaller than the

initial HHI value. Moreover, this implies that the theoretical upper bound of AHHI delta is 10,000 and consequently, the theoretical upper bound of AHHI is 20,000.

To make the AHHI useful in practice, it is pivotal to provide policy makers and regulators with pragmatic rules of thumb. For a holistic assessment of intra-industry concentration, it would seem reasonable to stick to the HHI's standard benchmarks (with the 1,500 and 2,500 thresholds for policy concerns) recognizing that these benchmarks can now be reached as a result of either a product or an ownership market characteristic, or both.

To make the new measure even more useful to practitioners, it is necessary to assess the relative importance of the AHHI delta, i.e. the part linked to CO, in relation to the total AHHI. We name this measure the simple Common Ownership Ratio (COR):

$$COR = \frac{AHHI \text{ delta}}{AHHI} \quad (5)$$

Since AHHI delta is de facto smaller than the HHI, this COR will never exceed 50%. To see this, remember from the AHHI formula that the betas act as weights, and that if they were equal to 1 the AHHI delta would equal the HHI. Therefore since (i) AHHI delta \leq HHI and (ii) AHHI = HHI + AHHI delta, the fraction (AHHI delta/AHHI) must be smaller than 50%.

Policymakers and regulators should be able to define COR thresholds which could indicate CO levels cause for concern, based on their expertise and review of numerous cases. This would become an alternative to the suggestion by some economists to consider MHHI increases of more than 200 to be a threshold for concern.¹¹

4. An illustration with some new insights

Consider first two countries where the same market (say electricity generation or painkilling medication) has the following characteristics. In each of these two countries, the market share of the top firm is significant but not dominant, respectively 42% and 27%. In each also, the firms have attracted institutional investors. Firms operating in Country A's market have, on average, 17% of their equity owned by common owners, while this figure is 22% for Country B. The difference in terms of CO seems not to be significant enough to hint at a determinant role of CO. The product market shares however suggest that the HHI is likely to be higher for country A and it is, 2237 vs 1417, but the level of concentration is within the benchmark suggesting there is no excessive concentration (i.e. HHI < 2,500). These market characteristics are summarized in Table 1.

Imagine, now, that we have data for each country on which proportion of a firm's equity is, on average, possessed by institutional owners (= average institutional investor ownership or avg. IIO). But we need more detail since many institutional investors present are also shareholders of competing firms within the market in each country. In order to fathom just how pervasive CO is, we report for each country which proportion of a firm is, on average, possessed by institutional owners who are simultaneously invested in competing firms of this market (= average common institutional investor ownership or avg. CIIO). To complete the description of the market, we also focus on how common institutional ownership relates to total institutional ownership. In other words, which fraction of the values of IIO stems from the values of CIIO.

All this information allows us to compute the MHHI and this allows us to show that CO matters in both countries as it stands respectively at 6389 and 1829, above the HHI in both cases. This, in turn, changes the perception of concentration risks in Country A but not in Country B. The relevant pragmatic question is whether we could have reached the same conclusion with the AHHI and the COR. The example shows that this is the case indeed. The importance of CO is confirmed by the AHHI and the COR for both countries but at lower data and assumption costs. In the Appendix

¹¹ See Elhaug (2018, 2020) and Posner et al. (2017) among others.

we provided a 6-step guide to show just how simple the computation of the AHHI and COR really are.

Table 1: How different are the insights provided by the various measures: a simple example		
	Country A	Country B
Basic market ownership characterization		
Average <i>common</i> institutional investor ownership, per market (CIIO)	17%	22%
Market share of top firm	42%	27%
Comparison of market concentration measures		
HHI	2237	1417
MHHI	6389	1829
AHHI	3754	1677
COR	40%	15%

Consider next a somewhat more complex situation in which various countries have tried to develop a joint market to increase competition and in which a supranational agency needs to assess the resulting degree of concentration in the enlarged market, observing that some of the firms are present in at least two countries, some others only in their home country. Imagine that the new market is composed of four countries. In each of the countries, the services delivered can be either quite centralized (i.e. quasi monopolistic as in Country 1) or competitive (as in Country 4). Imagine now that the market structure is not the only difference across countries and that it can also vary according to shareholder composition of firms active in each of the markets and according to the residual role of the public sector as an owner in the sector. Table 2 provides the detailed quantitative characterization of this market.

Table 2: Supranational vs national market concentration estimation across index measures					
	Overall market	Country 1	Country 2	Country 3	Country 4
Basic market ownership characterization					
Average institutional investor ownership, per market (IIO)	33%	15%	21%	53%	32%
Average <i>common</i> institutional investor ownership, per market (CIIO)	17%	10%	17%	22%	12%
Market share of top firm	16%	70%	42%	27%	19%
Level of national state ownership	mixed	None	Very high (controlling)	None	High (non-controlling)
Comparison of market concentration measures					
HHI	519	5116	2237	1417	568
MHHI	984	8494	6389	1829	660
AHHI	748	7585	3754	1677	682
COR	31%	33%	40%	15%	17%

This type of enlarged market is not unusual. The range of concentration characteristics reported in Table 2 are similar to those observed in some of the members of the European electricity

generation market for instance.¹² The Belgian power generation sector is the most concentrated one with Engie-Electrabel producing roughly 70% of net electricity output, followed by EDF-Luminus as a distant second with a 17% production share. This is the type of situation that Country 1 in our example would tend to reflect. At the other end of the European spectrum, in Italy, the incumbent player Enel accounts for a meagre 19% of generation and the next six firms each have a market share of 3% to 8%, hence depicting a much more competitive market. This would be consistent with the type of situation reflected in Country 4 of our example. Between these two extremes, we find countries such as Poland and Spain, with the former tending towards the Belgian situation while the latter displays more similarities with the Italian market. This is reflected in Countries 2 and 3 respectively.

Table 2 reports for each of these markets the average institutional investor ownership (IIO) and the average common institutional investor ownership (CIIO). In turn, this allows us to find that, on average, and across markets, common institutional investors are responsible for 56% of total institutional ownership.¹³ Now we have thus established how prevalent CO is in the markets we are interested in. The next step is to report how this information impacts the MHHI and AHHI values at the national and at the supranational level.

Relying on the usual thresholds in order to categorise the concentration levels, the enlarged overall market we are analysing is characterised by very low concentration levels as indicated by the three concentration measures, with the HHI, the MHHI and the AHHI all below 1000. From a policy perspective, the idea of enlarging the market thus seems to be a good idea in this example, including when CO is accounted for.

The insights are somewhat different when focusing only on the national markets. Some of the national markets (Countries 1 and 2) are highly concentrated, in particular when CO is accounted for as measured by the MHHI and the AHHI. Incorporating CO effects into concentration measures would end up moving Country 2's market from the moderately concentrated category to the highly concentrated category. In similar fashion, the Country 3 market would switch from the unconcentrated to the moderately concentrated category. These are indeed the insights competition and regulatory agencies would be expected to learn by moving away from the HHI and adopting the MHHI or the AHHI.

The difficulty with the similarity of insights produced by the MHHI and the AHHI is that it may seem hard to make the case to switch from the MHHI to the AHHI, besides the obviously much less demanding computational and data collection effort. In our example, the MHHI and AHHI deliver an equivalent storytelling when it comes to the U.S. Department of Justice and FTC thresholds, i.e., they consistently classify the markets in the same concentration brackets. For Countries 3 and 4, the MHHI and AHHI are virtually identical.

So, ultimately, what is the value added of the AHHI besides the obvious simplification of the measurement process? To see this, it is useful to use *all* the information reported in Table 2 and in particular the extent to which the public authorities are also present in the market as shareholders. A closer look highlights that the MHHI provides a somewhat different picture of the situation in Countries 1 and 2 compared to the AHHI. The MHHI is respectively 8494 and 6389 while the AHHI stands at 7585 and 3754 respectively, hinting at a difference in the assessment of the impact of CO in the two countries depending on the measure used.

There are two possible country-specific characteristics that could explain the differences in the assessment of the impact of CO. The first is that the IIO and the CIIO are both higher in Country 2. However, this should imply that AHHI is somewhat more of a problem for Country 2. Yet the opposite is true and hence this is not a realistic explanation. A second difference is the degree of

¹² The illustrative data reported in Table 1 is largely based on a simplified version of the data from this market.

¹³ Avg. (CIIO/IIO) = [(17/33)+(10/15)+(17/21)+(22/53)+(12/32)]/5 = 56%

residual *public* ownership in the two countries. Analytically, the MHHI puts more emphasis on this residual public role via its cross-MHHI deltas (the fraction in the MHHI equation) than the AHHI does through its additive equation. One interpretation could be that, for sectors in which the state is a large owner, the AHHI may, in some cases, underestimate this role as driver of market power. An alternative interpretation, which we favour, is that public sector ownership raises a different type of concern for the competition and regulatory authorities and hence should not be picked up by the measure of concentration. We argue that public authorities as common owners cannot thus be subjected to the same theory of harm rationale as the one developed for private institutional investors. Public owners are expected to take into account societal objectives (e.g. price affordability, security of supply, environmental concerns,...) rather than focusing only on a pure profit motive in their decision-making process. If we uphold this reasoning, it would seem reasonable to argue that the AHHI better depicts potential CO issues.

The desirability of including or excluding public sector ownership is not a settled matter. Nevertheless, the differences between the MHHI and the AHHI noted in this example make a case for a much more precise debate on whether the role of CO by public authorities should be treated differently by the market regulators instead of using tools developed for abuses of market power by private owners. Table 2 shows that the level of national ownership can make a significant difference regarding the assessment of concentration risks. The issue is thus to determine which is the best way to treat it. If national ownership matters to the risks of abuses that lead to higher profits in the sector, the AHHI would underestimate these risks. If it does not matter because public authorities have other objectives as owners, the MHHI would overestimate the risks.

Regardless of this specific debate on the relevance of public vs private ownership, there is a second insight from Table 2 to consider when comparing the informational power of MHHI and AHHI. This insight stems from the information provided by the COR, keeping in mind that it simply delivers a normalization of the AHHI delta (i.e. the increase of market power due to CO). In our example, it is the only measure that warns of a possible risk of market power at the supranational level with a value of 31%. Any of the other three measures fails to provide this warning. This implies that it is important to look at the risks in relative terms as well as in absolute terms.

Table 2 shows that when both the MHHI and the AHHI imply a cause for concern, so will the COR, as in the case of the national markets in Countries 1 and 2. But when these measures fail to raise concerns, the COR leads to a more conservative or risk averse approach to the assessment of the market power risks, as in the case of the supranational market. It is indeed the only measure that suggests there may be a reason for the competition and regulatory agencies to take a closer look at the role of CO in an enlarged market. Its value increases significantly despite all the indications provided by the three other indexes that enlarging the market is mostly good news.

The case to rely on the COR is also strengthened by the fact that it can somehow help reconcile the differences in perspectives on the need to include or exclude public sector ownership. In Table 2, the comparison of the COR for Countries 1 and 2 points to an overall higher CO related risk for the country with a strong residual public sector role despite a lower product market concentration (i.e. a lower HHI). In other words, the COR also hints at the fact that an increase in the degree of predominance of government CO is not correlated with a decrease in the risks associated with CO in general. This specific result would reinforce the assumption built in the MHHI on the need to account for public sector CO via the cross-MHHI deltas but it is derived from normalization of a measure of the simpler AHHI computed in absolute terms. In sum, overall, this message is once again delivered at a lower informational and computational cost.

5. Concluding comments

The note suggests that in general, the simplification payoffs of the AHHI when compared to the MHHI may be strong enough for policymakers and regulators to consider it as a reliable pragmatic improvement over the standard HHI avoiding many of the criticisms addressed to the MHHI. The case is even stronger to rely on the COR rather than on the AHHI alone as a way of settling the debate on the (ir-)relevance of public sector ownership in many regulated industries such as electricity, rail or water companies.

The illustration showed that the simpler index and its normalization in the form of the COR can provide at least the same information as the MHHI could provide and in some cases add information. This is a preliminary result of course. A lot more evidence should be collected for markets with very different characteristics to test the robustness of the approach in terms of the policy insights it provides, in particular when the public sector continues to be an important actor in the sector. But the illustration presented here provides significant hope for real value added from a simpler approach to the assessment of the relevance of CO.

The AHHI and the COR thus could be used to re-focus debates on the essence of the competitive implications of CO, rather than curtail any initiative in that way based on methodological criticism. This is important in a context in which there is increasing concern for what the growing presence of institutional investors on the board of utilities or of retirement residences or hospitals in various European countries implies for the management and the performance of these providers of basic services. But more generally, it should help simplify the evaluation of market power risks more systematically, at a lower data and assumption cost than the MHHI does.

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Appendix: A 6-Step Guide to Calculating the AHHI for a Market

Equation: $AHHI = HHI + AHHI \text{ delta}$

$$AHHI = \sum_j s_j^2 + \sum_i \left(\sum_j \beta_{ij} * s_j^2 + \sum_{k \neq j} \beta_{ik} * s_k^2 \right)$$

$\forall \beta_{ij}, \beta_{ik} \geq 0.005$

where j and k index firms (firms indexed by k are competitors of j), i indexes investors, s represents the firms' market shares, β_{ij} designates investor i 's share of firm j 's equity, and β_{ik} designates investor i 's share of firm k 's equity.

Context: Suppose 5 firms which are competitors by nature in a particular market. 5 investors hold shares in these firms, most of them in more than one at a time.

Step 1: List the firms participating in the market along with their respective market share.

1. Firm A – 30% market share
2. Firm B – 25% market share
3. Firm C – 20% market share
4. Firm D – 15% market share
5. Firm E – 10% market share

Step 2: Compute the HHI-component for each firm and for the market as a whole.

<i>Firm A</i>	<i>900</i>
<i>Firm B</i>	<i>625</i>
<i>Firm C</i>	<i>400</i>
<i>Firm D</i>	<i>225</i>
<i>Firm E</i>	<i>100</i>
Market HHI	2250

When solely considering the traditional HHI, the level of market concentration indicates a **moderately concentrated marketplace**.

Step 3: List investor's ownership percentages in each firm (omitting stakes inferior to 0.5%).

	<i>Firm A</i>	<i>Firm B</i>	<i>Firm C</i>	<i>Firm D</i>	<i>Firm E</i>
<i>Investor 1</i>	10%	10%	10%	10%	10%
<i>Investor 2</i>	8%	8%	-	8%	8%
<i>Investor 3</i>	5%	5%	-	5%	-
<i>Investor 4</i>	-	-	3%	-	3%
<i>Investor 5</i>	-	-	15%	-	-

Step 4: Compute the AHHI delta for each investor holding shares in *at least* 2 firms competing within the same market.

	<i>Firm A</i>	<i>Firm B</i>	<i>Firm C</i>	<i>Firm D</i>	<i>Firm E</i>	SUM
<i>Investor 1</i>	90	62,5	40	22,5	10	225
<i>Investor 2</i>	72	50	-	18	8	148
<i>Investor 3</i>	45	31,25	-	11,25	-	87,5
<i>Investor 4</i>	-	-	12	-	3	15
<i>Investor 5</i>	-	-	-	-	-	-

Accordingly, the **AHHI delta** is equal to $225+148+87,5+15 = 475,5$ and acts as an amplifier of market concentration.

Step 5: Combine the HHI and the AHHI delta to get the AHHI.

The AHHI is equal to $2250 + 475,5 = 2725,5$.

In this manner, the AHHI takes into account the effects of common ownership and subsequently indicates a **highly concentrated marketplace**.

Step 6: Compute the Common Ownership Ratio (COR).

The COR is equal to $475,5/2725,5 = 17,45\%$.

This means that in this case, ownership overlap accounts for roughly 17% of the overall market concentration.