ECARES

Identifying the Distribution of Welfare from Discrete Choice

Bart Capeau
ECARES, Université libre de Bruxelles

Liebrecht De Sadeleer
Department of Economics, KU Leuven

Sebastiaan Maes
Department of Economics, KU Leuven

January 2022

ECARES working paper 2022-03

ECARES
ULB - CP 114/04
50, F.D. Roosevelt Ave., B-1050 Brussels BELGIUM
www.ecares.org

Identifying the Distribution of Welfare from Discrete Choice

Bart Capéau [†,‡], Liebrecht De Sadeleer [‡], and Sebastiaan Maes[‡]

[‡]Department of Economics, KU Leuven[§]

[†]ECARES, ULB[¶]

January 29, 2022

Abstract

Empirical welfare analyses often impose stringent parametric assumptions on individuals' preferences and neglect unobserved preference heterogeneity. In this paper, we develop a framework to conduct individual and social welfare analysis for discrete choice that does not suffer from these drawbacks. We first adapt the class of individual welfare measures introduced by Fleurbaey (2009) to settings where individual choice is discrete. Allowing for unrestricted, unobserved preference heterogeneity, these measures become random variables. We then show that the distribution of these objects can be derived from choice probabilities, which can be estimated nonparametrically from cross-sectional data. In addition, we derive nonparametric results for the joint distribution of welfare and welfare differences, as well as for social welfare. The former is an important tool in determining whether the winners of a price change belong disproportionately to those groups who were initially well-off. An empirical illustration demonstrates the relevance of the methods and the importance of considering welfare instead of income.

Keywords: discrete choice, nonparametric welfare analysis, individual welfare, social welfare, money metric utility, compensating variation, equivalent variation

JEL codes: C14, C35, D12, D63, H22, I31

[§]Naamsestraat 69, 3000 Leuven, Belgium; E–mail: liebrecht.desadeleer@kuleuven.be; sebastiaan.maes@kuleuven.be

[¶]Avenue Franklin Roosevelt 50, 1050 Brussels, Belgium; E–mail: bart.capeau@ulb.be

We are most grateful to André Decoster for countless stimulating discussions and his enduring encouragement. We also thank Debopam Bhattacharya, Laurens Cherchye, Benoît Decerf, Bram De Rock, Geert Dhaene, Karim Kilani, Erwin Ooghe, Erik Schokkaert, and Frederic Vermeulen, as well as conference and workshop participants at KU Leuven, IAAE, IIPF, LAGV, LISER, and NESG for useful comments on previous versions of this paper. We are grateful to Peter Haan for providing us with a tax-benefit calculator for Germany. Bart Capéau acknowledges support from the FWO and the F.R.S.-FNRS (project number EOS 30544469). Liebrecht De Sadeleer acknowledges support from the FWO (project number G073020N). Sebastiaan Maes benefited from a PhD Fellowship of the FWO. The results and their interpretation are the authors' sole responsibilities.

1 Introduction

Discrete choice random utility models (DC-RUMs) have a long tradition in both theoretical and empirical microeconometric research. They have been applied to a wide range of problems in education, health care, industrial organisation, labour, marketing, public finance, and transportation.[1] The success of DC-RUMs can be explained by their ability to model individual demand among a discrete set of alternatives in a flexible way, allowing for the presence of unobserved heterogeneity in individual preferences. As econometric models typically explain only a small part of the variation in choice data, unobserved heterogeneity is thought to be an important driver of individual demand in empirical applications. Neglect or misspecification of this heterogeneity might introduce substantial biases into the analysis.

In this paper, we develop a framework to conduct individual and social welfare analysis in DC-RUMs that allows for unrestricted, unobserved heterogeneity in individuals' preferences. Our approach is entirely nonparametric and, therefore, does not suffer from misspecification in the econometric model. The framework is sufficiently general to study both *levels* and *differences* of individual welfare, where the latter measure individuals' gains or losses induced by an exogenous price change. Characterising these concepts is of first-order importance to applied welfare analysis for at least three reasons. Firstly, knowledge on levels of welfare enables researchers to rank individuals according to their well-being in any given situation, distinguishing between those who are well-off and those who are poor. In aggregating these levels across individuals, overall social welfare can be calculated and compared between two situations. Secondly, knowledge on differences of welfare allows to assess individuals' welfare gains or losses from a price change, distinguishing between winners and losers. Thirdly, joint knowledge on levels and differences of welfare reveals the association between individuals' gains or losses from a price change and their position in terms of initial welfare. This allows for the assessment of, for example, whether the winners of a price change belong disproportionately to those groups who were initially well-off.

To operationalise our framework, we first adapt the class of individual welfare measures introduced by Fleurbaey (2009) to settings where individual choice is discrete instead of continuous. We call them nested opportunity set (NOS) measures. These welfare measures were developed in line with the growing consensus that well-being, be it measured at the individual micro- or at a nation-wide macro-level, needs to be assessed in a multi-dimensional way, which goes beyond income alone (e.g., see Stiglitz et al., 2009 and Fleurbaey and Blanchet, 2013). We show that Samuelson's (1974) money metric utilities (MMUs) are within the class of NOS measures, and use them as a leading example to illustrate our approach.[2] As a result, the well-known compensating variation (CV) and equivalent variation (EV), which are both measures of differences of individual welfare, are embedded in our framework. Our results therefore generalise the findings of Dagsvik and Karlström (2005) and de Palma and Kilani (2011) to settings where unobserved heterogeneity is essentially unrestricted.

[1]Some parametric models within this class, such as the binary and multinomial logit models, yield convenient closed-form choice probabilities, which makes them a popular choice in applied work. For a comprehensive overview, see Train (2003).

[2]The use of welfare measures based on the expenditure function, the so-called MMUs, is a well-established practice in the applied welfare literature (for seminal contributions, see Diamond and McFadden, 1974; Dixit, 1975; King, 1983).

This paper also complements and extends the recent findings of Bhattacharya (2015, 2018) in several important directions. First, we go beyond the traditional welfare metrics based on MMUs, and provide results for the entire class of NOS welfare measures. Second, we do not only consider welfare differences such as the CV or EV, but also derive nonparametric expressions for the distribution of welfare levels and for the joint distribution of welfare levels and differences. Third, we also provide these results conditional on the endogenous pre- or post-price change choices.

As the presence of unobserved heterogeneity renders these NOS measures stochastic from the point of view of the econometrician, we then show how their distributions relate to what is typically observed in cross-sectional and panel data. In particular, we prove that the marginal distribution of NOS measures can be recovered nonparametrically from cross-sectional data by evaluating the observed choice probabilities at counterfactual prices. This allows researchers to study levels of individual welfare in any given situation. Likewise, we show that the joint distribution of welfare levels and welfare differences can be recovered nonparametrically from panel data by evaluating the observed transition probabilities at counterfactual prices.[3] Building on these results, we are able to nonparametrically characterise levels and differences in aggregate welfare for any additively separable social welfare function.

Our identification results are constructive and can be implemented in empirical work using only nonparametric regression. We also demonstrate how Boole-Fréchet inequalities (Fréchet, 1935) and stochastic revealed preference restrictions can be exploited to construct sharp bounds on the transition probabilities in the common event when only cross-sectional data is available. These bounds are functionals of the choice probabilities and are, as such, straightforward to implement. They can readily be used to set-identify the concepts that are expressed in terms of transition probabilities.

As a by-product, we can not only condition our results on exogenous characteristics, but also derive results conditional on the endogenous pre- or post-price change choices. This allows researchers to take the additional information conveyed by the observed choices into account. Conditioning on observed choices restricts the admissible set of unobserved preference heterogeneity, such that individual welfare can be measured more precise. In addition, this conditioning is also relevant from a political economy perspective. It provides answers to questions like 'What is the welfare impact of a refundable tax credit on the *unemployed*?' and 'How do congestion taxes affect the welfare of *drivers*?'.[4]

To illustrate the empirical usefulness of our results, we present an application on female labour supply. For this purpose, we make use of micro-data from the 2018 wave of the German Socio-Economic Panel (GSOEP), which contains detailed information on households' demographics, labour supply, wages, and out-of-work income. Single females' labour supply is modelled as a choice between three discrete alternatives: non-working, part-time employment, and full-time employment. Using a MMU, we present nonparametric estimates of the distribution of welfare under the current tax schedule in Germany. We find that disposable

[3] These transition probabilities are derived under the assumption that unobserved individual preferences are unaltered by the price change, which implies perfect correlation in unobserved heterogeneity before and after the price change. Alternatively, Dagsvik (2002) and Delle Site and Salucci (2013) consider models where there is imperfect correlation.

[4] In the labour supply setting, these conditional results are also required to study how welfare gains vary with the level of earned income, which is endogenous in this context.

income is a bad proxy for individuals' welfare. Furthermore, rankings according to income do not coincide with rankings according to welfare. Besides hourly wages, also (unobserved) preferences play an important role in assessing the welfare level of individuals in the baseline. We also study the effects of replacing this nonlinear and progressive schedule with a basic income flat tax. Large gains are more prevalent among those that were initially poor in terms of welfare, while the largest losers can be found among the middle-class households.

Related literature The structural modelling of individual preferences in DC-RUMs renders this class of models especially suitable for the welfare analysis of price changes. Over the last fifteen years, a methodological literature has emerged that derives closed-form expressions for the distribution of the CV and EV under ever less parametric assumptions on the nature of individuals' preferences.[5] For the class of additive DC-RUMs, Dagsvik and Karlström (2005) provide expressions for the distribution of the CV based on compensated (Hicksian) choice probabilities. The authors provide analytical results for models where unobserved heterogeneity is generalised extreme value distributed. Alternatively, de Palma and Kilani (2011) advance a direct approach for this class, in which they express this distribution in terms of uncompensated (Marshallian) choice probabilities. More recently, Bhattacharya (2015, 2018) showed that the marginal distributions of the CV and EV can be written as a functional of uncompensated choice probabilities, even when unobserved heterogeneity is essentially unrestricted, and therefore possibly nonadditive.[6] This paper generalises and extends these results as described above.

Several semiparametric methods have been developed to relax functional form assumptions on either deterministic preferences or the distribution of unobserved heterogeneity in DC-RUMs (for early results see Manski, 1975; Matzkin, 1991; and Klein and Spady, 1993). Other contributions introduce entirely nonparametric methods that do not impose functional form restrictions on either of these components for this class of models, based on either shape restrictions (e.g., see Matzkin, 1993) or the availability of a large-support special regressor (e.g., see Lewbel, 2000 and Briesch et al., 2010). The approach we follow in this paper deviates from this literature as our objective is not to recover deterministic preferences and the distribution of unobserved heterogeneity, but instead to identify individual welfare measures which are functions of both these model primitives.

In recent years, a comprehensive theoretical framework for measuring individual well-being has been developed that encompasses both the classical MMUs (Samuelson, 1974), adaptations of other measures like Pazner's (1979) ray utilities, and measures like the equivalent income and wage metrics (among others, see Pencavel, 1977; Fleurbaey, 2007; Fleurbaey, 2009; Decancq et al., 2015; and Fleurbaey and Maniquet, 2017). Almost all of these measures cardinalise preferences by associating their indifference sets with members of a

[5]Before, no closed-form expressions existed, even for the expected values of the CV and EV. Therefore, researchers had to resort to approximations, except for the most simple of DC-RUMs in which individuals have constant marginal utility of income and unobserved heterogeneity is additive and generalised extreme value distributed (Small and Rosen, 1981; McFadden, 1999). These approximations are either biased (Morey et al., 1993), rather uninformative (Herriges and Kling, 1999), or computationally burdensome (McFadden, 1999).

[6]This result only holds for price changes. When quality changes occur, Bhattacharya (2018) demonstrates that functionals of the choice probabilities only set-identify the distribution of the CV (or EV).

family of nested opportunity sets; i.e. a lower ranked indifference set is associated with a smaller opportunity set. The sizes of those opportunity sets are argued to be an ethically more meaningful basis for interpersonal comparisons of well-being than income or reports on subjective satisfaction levels. Indeed, contrary to income and subjective satisfaction, such measures ensure that individuals with the same preferences and in a situation which makes them indifferent among each other are always considered to be equally well-off. We adapt this class of individual welfare measures to settings where individual choice is discrete and show how the distribution of these NOS measures relates to what is typically nonparametrically observed in cross-sectional and panel data.

Another strand of literature focuses on the nonparametric identification of counterfactual choices and welfare under unobserved heterogeneity in models where demand is continuous instead of discrete. Most results exploit the smoothness of the underlying individual demand functions to arrive at Slutsky-like restrictions on average and quantile demands (e.g., see Dette et al., 2016; Hausman and Newey, 2016; Blundell et al., 2017; and Hoderlein and Vanhems, 2018). Other results exploit the axioms of revealed preference to attain identification under the presence of unobserved heterogeneity (e.g., see Blundell et al., 2014; and Cosaert and Demuynck, 2018). In contrast to our results, however, the availability of cross-sectional and short panel data is generally not sufficient to point-identify the distribution of welfare levels and differences in settings where demand is continuous and unobserved heterogeneity is unrestricted.

Finally, this paper contributes to the literature that applies NOS measures empirically. Using GSOEP microdata, Decoster and Haan (2015) estimate a parametric DC-RUM of labour supply and construct rankings of households based on NOS measures. Carpantier and Sapata (2016) extend the approach of Decoster and Haan (2015) by integrating unobserved preference heterogeneity through a numerical procedure (comparable to the approach of Herriges and Kling (1999) for welfare differences). Our results show that the parametric assumptions imposed in these papers are not necessary to obtain identification.

Outline of the paper The remainder of this paper is organised as follows. In the next section, we introduce the class of NOS welfare measures for settings where choice is continuous. In Section 3, our conceptual framework is laid out. We specify the DC-RUM and adapt the class of NOS welfare measures to this discrete setting. In Section 4, we show that the distribution of these objects can be derived from choice and transition probabilities. In addition, we derive nonparametric results for the joint distribution of welfare and welfare differences, as well as for social welfare. In Section 5, we discuss how the choice and transition probabilities can be retrieved from cross-sectional data. In Section 6, we illustrate our results by means of an application on female labour supply, using the GSOEP microdata. Section 7 contains concluding remarks. All proofs and some additional results are collected in the Online Appendix.

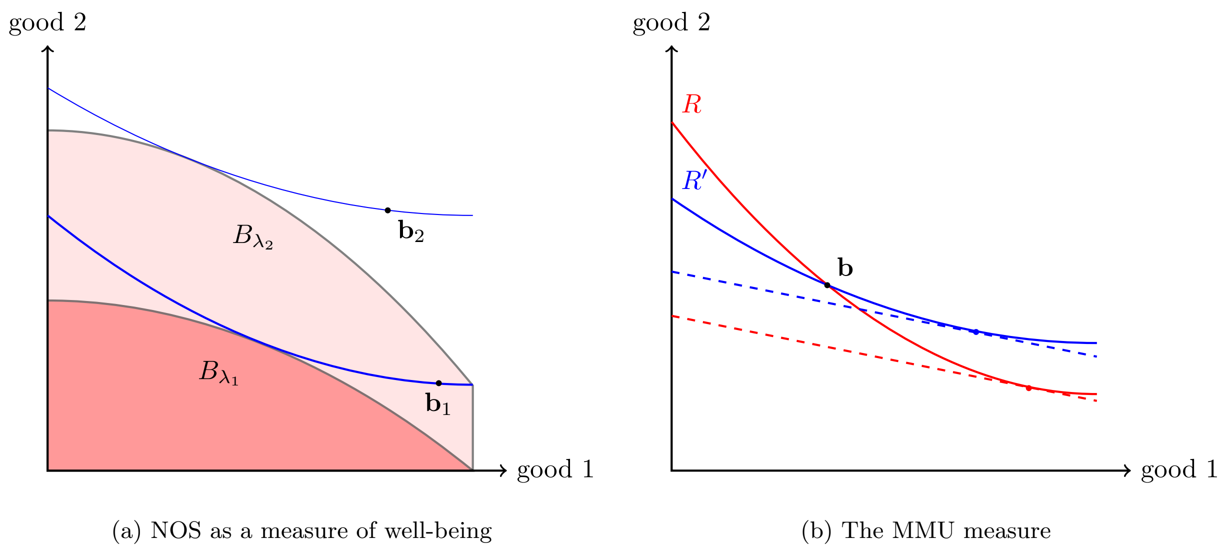


Figure 1: NOS welfare measures in a continuous choice setting

2 NOS welfare measures in a continuous setting

In this section, we briefly explain and motivate the class of NOS welfare measures, which have been introduced by Fleurbaey (2009) for settings where choice is continuous.[7] These measures cardinalise preferences by associating each indifference set with a member of a family of nested opportunity sets, which is common for all individuals. A lower ranked indifference set is associated with a smaller set from that family, such that the size of the opportunity set acts as a measure of individual well-being, respecting that individual's preferences. Formally, let $\mathrm{B} \subseteq \mathbb{R}^n_+$ be the set of all bundles $\mathbf{b}$ an agent can possibly obtain, and let $\{B_\lambda \subseteq \mathrm{B} \mid \lambda \in \Lambda \subseteq \mathbb{R}\}$ denote a family of nested opportunity sets indexed with a parameter λ such that $\lambda < \lambda'$ implies that $B_\lambda \subsetneq B_{\lambda'}$. Given a well-behaved utility function $U(\mathbf{b}) : \mathrm{B} \to \mathbb{R}$, the NOS welfare measure evaluated in a bundle $\mathbf{b} \in \mathrm{B}$ is then defined as

$$W(\mathbf{b}) = \max \left\{ \lambda \in \Lambda \mid U(\mathbf{b}) \geq \max_{\mathbf{b}' \in B_\lambda} U(\mathbf{b}') \right\}, \tag{1}$$

that is, the largest value of λ — or, equivalently, the largest opportunity set — for which the individual still weakly prefers bundle $\mathbf{b}$ above all bundles $\mathbf{b}'$ in the set B_λ.

This definition is illustrated by means of a classical trade-off between two goods in Figure 1a. Suppose first an individual obtains a bundle $\mathbf{b}_1$, and let the thick blue line denote her indifference curve passing through $\mathbf{b}_1$. This indifference curve is associated with the opportunity set B_{λ_1}, which is shaded in dark red. In accordance with the definition in Equation (1), this set is designed such that the individual could obtain, at best, a bundle equally as good as $\mathbf{b}_1$, when she would be faced with the opportunity set B_{λ_1}. Suppose now that the individual obtains a bundle $\mathbf{b}_2$, which is better than $\mathbf{b}_1$, according to her own preferences. Then the associated opportunity set B_{λ_2}, which is shaded in light red, is again such that the best bundle in this set is equally good as $\mathbf{b}_2$, and includes the set B_{λ_1}. From this illustration, it is clear that the size

[7] Note that the discussion here is informal as additional assumptions are needed to guarantee the existence and the uniqueness of the NOS welfare measure. A more rigorous treatment is provided in Section 3.2.

of these opportunity sets serves as a measure of individual well-being that respects preferences, in the sense that the well-being level of an individual in situation $\mathbf{b}_2$ is higher than her well-being level in situation $\mathbf{b}_1$, if and only if that individual prefers $\mathbf{b}_2$ to $\mathbf{b}_1$. The size of an opportunity set is measured by its indexing parameter λ.

One important set of welfare measures within the NOS class is the set of MMUs (Samuelson, 1974). In this case, the NOS are of the form

$$B_\lambda \equiv \left\{ \mathbf{b} \in \mathrm{B} \,\middle|\, \mathbf{b}'\mathbf{p}^{ref} \leq \lambda \right\}, \tag{2}$$

where $\mathbf{p}^{ref}$ is a vector of reference prices that is fixed by the researcher. In this specification, the indexing parameter λ can be interpreted as a monetary amount.

Applying Equation (1), we find that the MMU measures well-being in a bundle by the maximal monetary amount that can be granted to an individual faced with reference prices $\mathbf{p}^{ref}$, such that she would at most be equally well-off as in that bundle. This coincides with the expenditure function representation of preferences. We illustrate the opportunity sets associated with these measures in Figure 1b.

Other examples of NOS measures are the ray utilities of Pazner (1979) and the equivalent income metrics introduced in Decancq et al. (2015). We discuss these measures and how they fit in our setting in more detail in Capéau et al. (2021).

3 Conceptual framework

Our conceptual framework allows for unrestricted, unobserved heterogeneity in DC-RUMs. As this set-up does not impose any restrictions on *observed* individual characteristics, all results in this paper can be thought of as being conditional on these covariates.

3.1 Discrete choice model

Preferences Let Ω denote the universe of preference types and let $\Pr_\omega$ represent the distribution of these preference types in the population. Every preference type can be thought of as a different individual, who has idiosyncratic preferences over bundles $(y - p_c, c)$; i.e. over a finite (and common) set of alternatives $\mathcal{C}$, with $|\mathcal{C}| := n \in \mathbb{N} \setminus \{0\}$, and a numeraire. These idiosyncratic preferences are assumed to be representable by a utility function $U_c^\omega(y - p_c) := U(y - p_c, c, \omega) : \mathbb{R} \times \mathcal{C} \times \Omega \to \mathbb{R}$, in which y denotes exogenous income and p_c the price of alternative $c \in \mathcal{C}$. Prices for all alternatives, $(p_c, c \in \mathcal{C})$, are collected in a vector denoted by $\mathbf{p}$ and we will call $(\mathbf{p}, y)$ a budget set. Residual income in alternative c is defined as the amount of the numeraire left over after choosing this alternative, i.e. $y - p_c$.

Note that our formulation of preferences is very flexible as it allows them to differ arbitrarily across individuals.[8] The only economically substantial restriction we will impose on this function is that utility is continuous and strictly increasing in the numeraire.

[8]Common parametric utility specifications in additive DC-RUMs (where $U_c^\omega(y - p_c) = V_c(y - p_c) + \varepsilon_c(\omega)$), or additive DC-RUMs models with random coefficients (where $U_c^\omega(y - p_c) = V_c(y - p_c, \eta(\omega)) + \varepsilon_c(\omega)$) are encompassed by our approach.

Assumption 1. *Individual preferences are represented by a utility function $U_c^\omega(y-p_c)$ that is continuous and strictly increasing in the numeraire for every preference type $\omega \in \Omega$ and every alternative $c \in \mathcal{C}$. Moreover, preferences satisfy the following regularity conditions: (R1) For each pair of alternatives $c, c' \in \mathcal{C}$, and for each fixed y and p_c, it holds that $U_c^\omega(y-p_c) > \lim_{p_{c'}\to\infty} U_{c'}^\omega(y-p_{c'})$ and that $U_c^\omega(y-p_c) < \lim_{p_{c'}\to-\infty} U_{c'}^\omega(y-p_{c'})$. (R2) For every budget set $(\mathbf{p}, y)$, the set of types that are indifferent between two or more alternatives in the choice set $\mathcal{C}$ has probability measure zero.*

This assumption is ubiquitous in empirical work that employs (semi)parametric DC-RUMs. Monotonicity in the numeraire establishes the existence and uniqueness of our welfare measures and yields stochastic revealed preference conditions that we will exploit to obtain the identification results. Regularity condition (R1) ensures that when the price of a given alternative goes to infinity, it will never be preferred above another alternative with a finite price. Analogously, when the price of a given alternative goes to minus infinity — or equivalently residual income in that alternative goes to plus infinity — it will always be preferred above another alternative with a finite price. The negligibility of indifferences between alternatives (R2) ensures that no tie-breaking rule has to be established.

In addition, we also assume that the distribution of the preference types, denoted by $F(\omega)$, is independent of the budget set $(\mathbf{p}, y)$.

Assumption 2. *The distribution of unobserved heterogeneity $F(\omega)$ is independent of prices $\mathbf{p}$ and exogenous income y: i.e. $F(\omega \mid \mathbf{p}, y) = F(\omega)$.*

The exogeneity of budget sets is a strong, but standard, assumption in the literature on nonparametric identification of individual demand and welfare (e.g., see Hausman and Newey, 2016). Indeed, to the best of our knowledge, there are no theoretical results that allow for general forms of endogeneity in the presence of unrestricted, unobserved heterogeneity. Some forms of endogeneity, however, can be mitigated by using a control function approach (see Section 5).

Individual choice behaviour Finally, we assume that observed choice behaviour is actually generated by a DC-RUM (for a detailed technical overview on RUMs, see McFadden, 1981, 2005). This assumption entails that an individual ω chooses a given alternative i, if and only if this alternative yields the highest utility among the elements in her choice set $\mathcal{C}$.

Assumption 3. *Let $J^\omega(\mathbf{p}, y) \equiv J(\mathbf{p}, y, \omega) : \mathbb{R}^{n+1} \times \Omega \to \mathcal{C}$ denote the individual demand function. It holds that $J^\omega(\mathbf{p}, y) = i \iff U_i^\omega(y - p_i) \geq \max_{c\neq i}\{U_c^\omega(y - p_c)\}$.*

Note that individual demand is single-valued with probability one as one can neglect indifferences between alternatives by regularity condition (R2) in Assumption 1.

Choice and transition probabilities The individual choices induced by a DC-RUM are stochastic from the point of view of the econometrician, as the preferences types are non-observable. When this random variation is averaged out across types, one obtains a set $\{P_i(\mathbf{p}, y)\}_{i\in\mathcal{C}}$ of uncompensated (Marshallian) conditional choice probabilities,

$$\begin{aligned}
P_i(\mathbf{p}, y) &= \Pr_{\omega}\left[\left\{U_i^{\omega}(y - p_i) \geq \max_{c \neq i}\{U_c^{\omega}(y - p_c)\}\right\}\right] \\
&= \Pr_{\omega}\left[J^{\omega}(\mathbf{p}, y) = i\right] \\
&= \int_{\Omega} \mathbb{I}\left[J^{\omega}(\mathbf{p}, y) = i\right] dF(\omega \mid \mathbf{p}, y) \\
&= \int_{\Omega} \mathbb{I}\left[J^{\omega}(\mathbf{p}, y) = i\right] dF(\omega),
\end{aligned} \tag{3}$$

where $\mathbb{I}[\cdot]$ denotes the indicator function.[9] The last expression asymptotically coincides with the observed choice frequency for every alternative $i \in \mathcal{C}$, conditional on the budget set $(\mathbf{p}, y)$.[10] If cross-sectional data contains enough relative price and exogenous income variation, these objects are nonparametrically estimable.[11]

Another concept induced by DC-RUMs is the set $\{P_{i,j}(\mathbf{p}, \mathbf{p}', y)\}_{i,j \in \mathcal{C}}$ of uncompensated conditional transition probabilities. These probabilities are formally defined as

$$\begin{aligned}
P_{i,j}(\mathbf{p}, \mathbf{p}', y) &= \Pr_{\omega}\left[\left\{U_i^{\omega}(y - p_i) \geq \max_{c \neq i}\{U_c^{\omega}(y - p_c)\}\right\}\right. \\
&\qquad \left. \cap \left\{U_j^{\omega}(y - p_j') \geq \max_{c \neq j}\{U_c^{\omega}(y - p_c')\}\right\}\right] \\
&= \Pr_{\omega}\left[J^{\omega}(\mathbf{p}, y) = i, J^{\omega}(\mathbf{p}', y) = j\right] \\
&= \int_{\Omega} \mathbb{I}\left[J^{\omega}(\mathbf{p}, y) = i\right] \mathbb{I}\left[J^{\omega}(\mathbf{p}', y) = j\right] dF(\omega),
\end{aligned} \tag{4}$$

which asymptotically coincide with the transition frequencies from alternative i to alternative j after an exogenous price change from $\mathbf{p}$ to $\mathbf{p}'$.[12] Naturally, if there is no price change, there are no transitions between different choices. In principle, these objects are nonparametrically estimable from panel data with at least two periods. In addition, Section 5.1 shows how transition probabilities can be set-identified when only cross-sectional data are available.

Implicit in our definition of the transition probabilities is the assumption that individuals' preferences are unaffected by the price change. The perfect correlation between the preference types before and after the price change implies that transition probabilities are not simply equal to the product of their marginals: i.e. $P_{i,j}(\mathbf{p}, \mathbf{p}', y) \neq P_i(\mathbf{p}, y) P_j(\mathbf{p}', y)$.

3.2 NOS welfare measures in a discrete choice setting

In Section 2, the family of NOS welfare measures was introduced in a setting of continuously divisible goods. In this subsection, we will redefine them rigorously for settings where choice is determined by a DC-RUM

[9] These choice probabilities are designated *conditional* as they depend on a vector of prices and income. In the interest of brevity, this qualification will be dropped in the sequel.

[10] This concept is also known as the *average structural function* (e.g., see Blundell and Powell, 2004). The asymptotic equivalence follows from the law of large numbers as the choice probabilities are essentially conditional expectation functions.

[11] It is clear from Equation (3) that these probabilities are composed of both the utility function U_c^{ω} and the distribution of unobserved heterogeneity F. As such, they are not sufficiently informative to separately identify these two model primitives. Fortunately, knowledge on such primitives is not necessary for our purposes.

[12] Note, however, that transition probabilities do not impose any temporal structure. In other words, $P_{i,j}(\mathbf{p}, \mathbf{p}', y) = P_{j,i}(\mathbf{p}', \mathbf{p}, y)$. Furthermore, as shown in Section A.2 in the Online Appendix, the assumption that the exogenous income y is common to both situations with prices $\mathbf{p}$ and $\mathbf{p}'$ imposes no constraint.

that satisfies Assumptions 1–3.

Nested opportunity sets in DC-RUMs We define a family of nested opportunity sets, which is common for all preference types $\omega \in \Omega$, as follows. Let there be a closed set $\Lambda \subseteq \mathbb{R}$, and define for every $\lambda \in \Lambda$, an opportunity set $B_\lambda \subset \mathbb{R} \times \mathcal{C}$ by

$$B_\lambda := \left\{(y', c) \mid c \in \mathcal{C}, y' \in \mathbb{R}, y' \leq y_c^\lambda\right\}, \tag{5}$$

where $y_c^\lambda \in \mathbb{R}$ for all $c \in \mathcal{C}$ satisfying the following assumptions:

(a) the function

$$\Lambda \to \mathbb{R} : \lambda \mapsto y_c^\lambda \text{ is continuous for all } c \in \mathcal{C}, \tag{6}$$

(b)

$$\lambda < \lambda' \implies \begin{cases} \forall c \in \mathcal{C} : y_c^\lambda \leq y_c^{\lambda'}, \\ \exists c \in \mathcal{C} : y_c^\lambda < y_c^{\lambda'}, \end{cases} \tag{7}$$

(c) for all options c',

$$\inf_{\lambda \in \Lambda} y_{c'}^\lambda = -\infty, \tag{8}$$

(d) and for at least one option c,

$$\sup_{\lambda \in \Lambda} y_c^\lambda = +\infty. \tag{9}$$

Then $(B_\lambda)_{\lambda \in \Lambda}$ is called a family of nested opportunity sets and $\mathbf{y}^\lambda := (y_1^\lambda, \ldots, y_c^\lambda, \ldots, y_n^\lambda)$ can be seen as its upper bound.[13] Note that the family is common to all individuals.

Conditions (6) and (7) ensure that the family $(B_\lambda)_{\lambda \in \Lambda}$ is continuously increasing. Conditions (8) and (9) together with Assumption 1 imply that for every bundle $\mathbf{b}$ and preference type ω, there exists a member of the family of which all bundles are considered worse than $\mathbf{b}$ by ω, and one which contains a bundle considered to be better than $\mathbf{b}$ by ω. These properties will prove necessary to define the welfare measure.

Figure 2 provides a graphical illustration. The choice set $\mathcal{C}$ consists of three options: i,j, and k. Two members of a family of nested opportunity sets $B_{\lambda \in \Lambda}$ are shown in red. For example, all bundles in dark red belong to B_{λ_1}. For illustrative convenience, we choose $y_c^\lambda < y_c^{\lambda'}$ for all $c \in \mathcal{C}$ whenever $\lambda < \lambda'$. Finally, the upper bounds of the B_λ's, consisting of the points $y_c^{\lambda_i}$, ($i = 1, 2$ and $c = i, j, k$), are denoted by the red dots.

It is often more convenient to characterise the opportunity sets in terms of virtual prices $\widetilde{p}_c(\lambda) := y - y_c^\lambda$ instead of the upper bounds y_c^λ. In particular, we have that

$$B_\lambda := \{(y', c) \mid c \in \mathcal{C}, y' \in \mathbb{R}, y' \leq y - \widetilde{p}_c(\lambda)\}.^{14} \tag{10}$$

[13] One can prove that every family of closed nested sets of which the option-wise suprema satisfy the four conditions above, are necessarily of the form (5). Hence, the assumption that the B_λ are of this form implies no loss of generality.

[14] It might be surprising that the — individual independent — opportunity sets B_λ are linked with virtual prices $\tilde{p}_c(\lambda)$, which depend on individual incomes. However, in our discrete context, prices and incomes are only determined up to an additive constant as only $y - p_c$ enters the utility function, not y nor p_c separately. Hence, also $\tilde{p}_c(\lambda)$ must be defined such that the relevant concept $y - \tilde{p}_c(\lambda)$ is individual independent. Therefore $\tilde{p}_c(\lambda)$ itself must depend on individual income y.

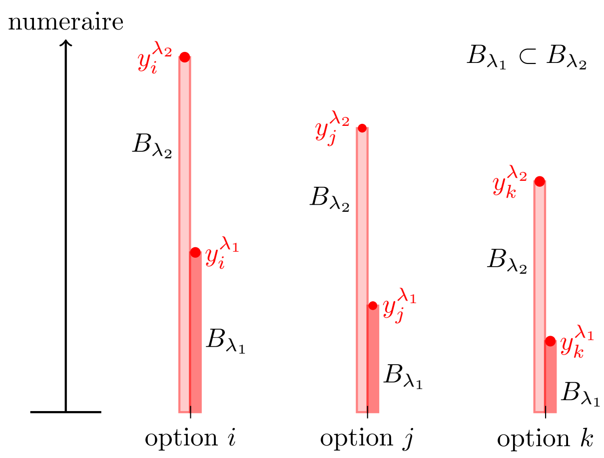


Figure 2: A graphical illustration of a family of nested opportunity sets in a discrete choice context. The opportunity set B_{λ_1} consists of the three dark red spikes. B_{λ_2} consists of the three light red spikes.

We denote the vector of virtual prices as follows: $\widetilde{\mathbf{p}}(\lambda) = \big(\widetilde{p}_1(\lambda), \ldots, \widetilde{p}_n(\lambda)\big)$. As $\mathbf{y}^\lambda$ is increasing in λ, in the sense of Equation (7), $\widetilde{\mathbf{p}}(\lambda)$ is decreasing in λ in the same way. Moreover, $\lambda \to \widetilde{\mathbf{p}}(\lambda)$ is continuous by (6), $\sup_{\lambda\in\Lambda} \widetilde{p}_{c'}(\lambda) = +\infty$ for all c' by (8) and $\inf_{\lambda\in\Lambda} \widetilde{p}_c(\lambda) = -\infty$ for at least one c by (9). The fact that those virtual prices can become negative might seem odd at first. However, in a discrete choice context, one can always redefine prices and exogenous income by increasing both by an equal amount of the numeraire. As a result, negative prices can be converted into positive prices.

Welfare measures in DC-RUMs In the continuous setting, the NOS welfare measure evaluated in a bundle was defined as the largest value of λ —or equivalently, the largest opportunity set B_λ— such that this bundle was weakly preferred over all bundles in B_λ. The same idea can be applied to a setting where choice is discrete. More precisely, we define a NOS welfare measure as

$$W^\omega(y - p_k, k) = \sup\Big\{\lambda \in \Lambda \mid U_k^\omega(y - p_k) \geq \max_{(y',c)\in B_\lambda} U_c^\omega(y')\Big\}, \tag{11}$$

that is, the largest value of λ such that option k is weakly preferred over all bundles in B_λ. Note that the dependence on the preference type ω implies that this welfare measure is a random variable. According to Assumption 1, the utility function is strictly increasing in the numeraire, which allows us to restate this definition in terms of the upper bound of the opportunity sets. Formally, we have that

$$\begin{aligned} W^\omega(y - p_k, k) &= \sup\Big\{\lambda \in \Lambda \mid U_k^\omega(y - p_k) \geq \max_{(y',c)\in B_\lambda} U_c^\omega(y')\Big\} \\ &= \sup\Big\{\lambda \in \Lambda \mid U_k^\omega(y - p_k) \geq \max_c \max_{y' \leq y_c^\lambda} U_c^\omega(y')\Big\} \\ &= \sup\Big\{\lambda \in \Lambda \mid U_k^\omega(y - p_k) \geq \max_c U_c^\omega(y_c^\lambda)\Big\}. \end{aligned} \tag{12}$$

This expression highlights that the value of the welfare measure only depends on the upper bound of the opportunity sets. Furthermore, by conditions (8) and (9) and Assumption (R1), there exists (i) a $\lambda_{\min} \in \Lambda$ such that $U_k^\omega(y - p_k) \geq \max_c U_c^\omega(y_c^{\lambda_{\min}})$, and (ii) a $\lambda_{\max} \in \Lambda$ such that $U_k^\omega(y - p_k) < \max_c U_c^\omega(y_c^{\lambda_{\max}})$. This implies that the set $\Big\{\lambda \in \Lambda \mid U_k^\omega(y - p_k) \geq \max_c U_c^\omega(y_c^\lambda)\Big\}$ is not empty by (i), and bounded by (ii). Moreover, by continuity of the utility function and of the function $\lambda \mapsto \mathbf{y}^\lambda$, $\lambda \mapsto \max_c U_c^\omega(y_c^\lambda)$ is also continuous, which implies, together with the closedness of Λ, that (iii) $\left\{\lambda \in \Lambda \mid U_k^\omega(y-p_k) \geq \max_c U_c^\omega(y_c^\lambda)\right\}$ is closed. As this set is not empty, bounded and closed, one can conclude that the suprema in Equations (11) and (12) are in fact attained and can be replaced by maxima.

Equivalently, when opportunity sets are characterised in terms of virtual prices, we can write that

$$W^\omega(y-p_k, k) = \max\left\{\lambda \in \Lambda \mid U_k^\omega(y-p_k) \geq \max_c U_c^\omega(y-\widetilde{p}_c(\lambda))\right\}. \tag{13}$$

For notational convenience, the characterisation in terms of virtual prices $\widetilde{p}_c(\lambda)$ instead of the numeraire y_c^λ will be used in the remainder of the paper.

A key insight is that the statement '$W^\omega(y-p_k, k) \geq w$' is equivalent with k (at its original price) being the optimal choice among all options with a virtual vector of prices that is welfare measure specific. This result is made precise in Lemma 1.

Lemma 1.

$$\left\{\omega \mid w \leq W^\omega(y-p_k, k)\right\} = \left\{\omega \mid U_k^\omega(y-p_k) \geq \max_c U_c^\omega(y-\widetilde{p}_c(w))\right\} \tag{14}$$

Proof. Take an arbitrary $\omega \in \Omega$ such that $w \leq W^\omega(y-p_k, k)$. Then there exists a $\lambda \geq w$ such that $U_k^\omega(y-p_k) \geq \max_c U_c^\omega(y-\widetilde{p}_c(\lambda))$. As $\lambda \geq w$, $\widetilde{p}_c(\lambda) \leq \widetilde{p}_c(w)$ for all $c \in \mathcal{C}$ and, hence, $\max_c U_c^\omega(y-\widetilde{p}_c(\lambda)) \geq \max_c U_c^\omega(y-\widetilde{p}_c(w))$, because U_c^ω is an increasing function by Assumption 1. It follows that $U_k^\omega(y-p_k) \geq \max_c U_c^\omega(y-\widetilde{p}_c(w))$. The other inclusion follows immediately from the definition of $W^\omega(y-p_k, k)$. □

This equivalence is obtained without imposing any assumption on preferences besides Assumptions 1 and 2, and is, therefore, entirely nonparametric.[15] As will be shown below, its main practical implication is that the entire distribution of objects based on NOS measures can be obtained by only evaluating choice and transition probabilities at virtual prices. This entails that these distributions can be identified from cross-sectional and panel data in a nonparametric way.

The illustration in Figure 3 builds further on Figure 2. For each option i, j, and k, the amount of the numeraire, $y-p_i$, $y-p_j$, and $y-p_k$ is shown on the vertical axis. The blue circles indicate the indifference set of the point $(y-p_k, k)$. The figure shows how to calculate the NOS welfare measure for option k. The welfare measure is defined by the nested opportunity sets $B_{\lambda\in\Lambda}$ shown in red in the figure. It is clear that λ_2 is the maximiser of Equation (11) because the red dot of $y_j^{\lambda_2}$ coincides with the blue circle at position j. This means that $U_j^\omega(y_j^{\lambda_2}) = U_k^\omega(y-p_k)$, and hence $W^\omega(y-p_k, k) = \lambda_2$.

Each member of the class of NOS measures is characterised by a different family of nested budget sets. All NOS measures agree on the welfare ranking of different bundles for a given individual, as within-individual welfare rankings coincide with the individual's preference ordering over these bundles. Given that individual preferences are respected by all NOS measures, they will also all agree whether a change in prices causes a welfare gain or loss for a given individual. They may disagree, however, on the making of interpersonal welfare comparisons.

[15] This result is reminiscent of what Bhattacharya (2018) obtains for the marginal distribution of the CV.

[16] Note that option k is not the chosen option as the bundle $(y-p_j, j)$ is situated above the indifference set of $(y-p_k, k)$ indicated by the blue circles. However, in our approach, we also allow to calculate welfare in a non-optimal bundle.

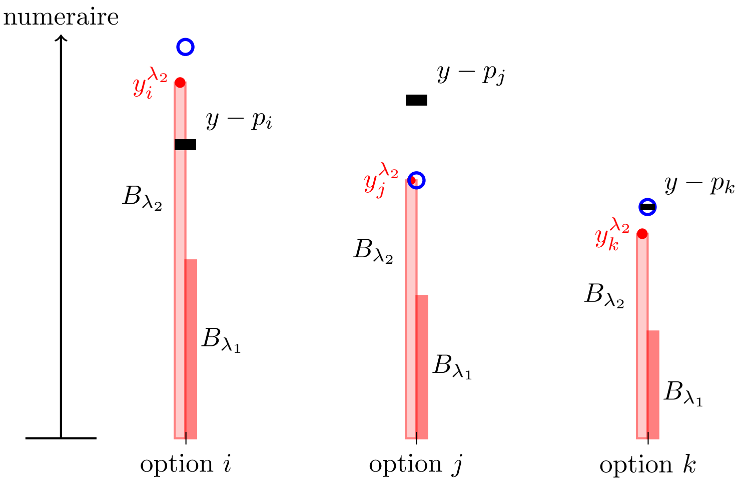


Figure 3: A graphical illustration of a NOS welfare measure in a discrete choice context.[16] The blue circles denote the indifference set of option k with residual numeraire $y - p_k$. The opportunity set B_{λ_1} consists of the three dark red spikes. B_{λ_2} consists of the three light red spikes. Black bars indicate the actual choice set.

Moreover, the size of the gain or loss is not only dependent on the particular NOS welfare measure, but also depends on the particular cardinalisation of the size of the opportunity sets associated with that measure. The way the size of the opportunity sets of a given family (and thus of a given NOS welfare measure) is measured, denoted by the parameter λ, is only unique up to a strictly positive monotone transformation. The welfare ranking and interpersonal comparison of individuals by a given measure is not affected by the particular cardinalisation of the sizes of the opportunity sets belonging to its associated family. If one is not willing to give any meaning to this cardinalisation, only the sign of welfare differences is meaningful.[17] Note that this is also true for measures of changes in welfare based on the MMU, like the CV and EV.

Examples of NOS measures: the MMU class Let $\Lambda = \mathbb{R}$, fix a set of n reference prices $\mathbf{p}^{ref}$, one for each option, and let the upper bound of the opportunity sets be $y_c^{\lambda} = \lambda - p_c^{ref}$, or equivalently, $\widetilde{p}_c(\lambda) = y - \lambda + p_c^{ref}$. The crucial property is that the upper bounds increase by the same amount for every option: i.e. $y_c^{\lambda_1} - y_c^{\lambda_2} = \lambda_1 - \lambda_2$ for all $c \in \mathcal{C}$. The MMU evaluated in option k (with price p_k) at reference prices $\mathbf{p}^{ref}$ is then defined as

$$W^{\omega}_{M(\mathbf{p}^{ref})}(y - p_k, k) \quad = \quad \max\Big\{\lambda \in \mathbb{R} \mid U_k^{\omega}(y - p_k) \geq \max_c U_c^{\omega}\left(y - (y - \lambda + p_c^{ref})\right)\Big\}. \tag{15}$$

This can also be defined implicitly as

$$U_k^{\omega}\left(y - p_k\right) = \max_c U_c^{\omega}\left(W^{\omega}_{M(\mathbf{p}^{ref})}(y - p_k, k) - p_c^{ref}\right). \tag{16}$$

Similar to the continuous case, this highlights the equivalence of the MMUs with the expenditure function representation of preferences, as each of them evaluates the expenditure function at a given set of reference prices.

[17]An exception is when two individuals are equally well-off before the price change. Who is the biggest gainer (or loser) in that case is again independent of the cardinalisation, but may differ across NOS measures.

From Equation (15), it can be seen that $W^{\omega}_{M(\mathbf{p})}(y-p_k,k)=y$ if $k=J^{\omega}(\mathbf{p},y)$. When the reference prices coincide with the actual prices, the level of well-being according to the MMU of the optimal choice in the actual situation is equal to the actual amount of the numeraire (see also Corollary 2 below).

4 Distribution of welfare levels, welfare differences, and social welfare

As discussed before, the presence of unobserved preference heterogeneity entails that NOS welfare measures are random variables from the point of view of the econometrician. This randomness can be interpreted in two distinct ways. In the first interpretation, as the econometrician does not observe an individual's preference type, they can only derive the distribution of welfare for this particular individual and not its exact realisation. That is, the distribution reflects uncertainty for the econometrician. In the second interpretation, an observed individual in the sample represents the class of individuals in the population that share the same observable characteristics. In this case, the distribution reflects inequality in welfare among the members of this class. Our theoretical results are valid for both interpretations.

For notational convenience, we will present all our expressions in terms of the complementary cumulative distribution function (CCDF) instead of the more common cumulative distribution function (i.e. $1-F(x)$ for a CDF F). The proofs are collected in the Online Appendix.

4.1 Distribution of the NOS welfare measures

In this section the marginal distribution for the NOS measures is derived in terms of choice probabilities.[18] We also provide distributional results joint with, and conditional on, the optimal observed choice.

Under Assumptions 1–3, which were introduced in Section 3, we can prove the following theorem.

Theorem 1. *The joint distribution of the NOS welfare measure W, evaluated in an option k with price p_k, and choosing j at prices $\mathbf{p}'$ and exogenous income y can be expressed in terms of transition probabilities as follows:*

$$\Pr_{\omega}\left[w\leq W^{\omega}(y-p_k,k),j=J^{\omega}(\mathbf{p}',y)\right]=P_{j,k}\Big(\mathbf{p}',\left(p_k,\widetilde{\mathbf{p}}_{-k}(w)\right),y\Big)\mathbb{I}\left[p_k\leq\widetilde{p}_k(w)\right],\tag{17}$$

where $(p_k,\widetilde{\mathbf{p}}_{-k}(w))=(\widetilde{p}_1(w),\ldots,\widetilde{p}_{k-1}(w),p_k,\widetilde{p}_{k+1}(w),\ldots,\widetilde{p}_n(w))$.

The crucial insight here is that, by Lemma 1, the event $W^{\omega}(y-p_k,k)\geq w$ is translated into a statement on k being optimal under virtual prices. The joint distribution in Equation (17) can, therefore, be expressed in terms of transition probabilities, evaluated at p_k, actual prices $\mathbf{p}'$, and virtual prices $\widetilde{\mathbf{p}}$.

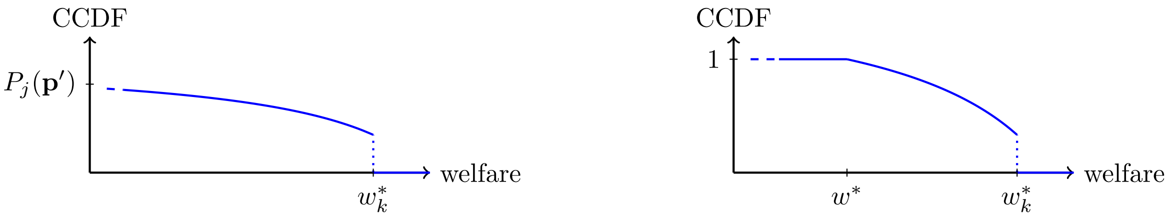
Theorem 1 is formulated in the most general form; it considers a joint distribution, and not a marginal nor a conditional, and allows the price at which the welfare in alternative k is evaluated, p_k, to be different

[18] Similar to Bhattacharya (2015), it is important to stress that identification generally fails in settings where the prices of alternatives are multiples of one another, such as in ordered choice. In such settings, there is no relative price variation in the data that identifies the effect of a price change in some alternative(s) while keeping the prices of the other alternatives fixed. Our empirical illustration, however, does not suffer from this drawback due to the non-linearities in the tax system.

from the actual prices $\mathbf{p}'$. For example, if one wants to evaluate welfare levels after a price change from $\mathbf{p}$ to $\mathbf{p}'$ when only information on choices before the price changed is available, $\mathbf{p}'$ and $\mathbf{p}$ will typically not coincide. However, in a setting where one wants to evaluate welfare at actual prices $\mathbf{p}'$, then $\mathbf{p}$ equals $\mathbf{p}'$. Usually, one wants to evaluate welfare in an optimal bundle; then k can be set equal to j in Equation (17) (and p_k equal to p'_k). In Corollary 1 below, we will derive some related distributions which are more directly relevant for applied work.

The exact specification of $\widetilde{\mathbf{p}}(w)$ depends, as explained in Section 3.2, on the specific choice of the welfare measure. Nonetheless, we can give some intuition on the role of p_k and the overall course of the distribution of welfare. We know that the lower the price p_k, the higher is the residual numeraire $y - p_k$ in option k and hence, the more the indifference set containing $(y - p_k, k)$ is shifted upwards in the numeraire dimension. A higher indifference set implies higher welfare, and therefore, the lower price p_k, the higher is the CCDF of welfare in option k and the more the distribution of welfare is shifted to the right.

Now, we examine the overall course of the CCDF in more detail by considering a typical plot of the CCDF for fixed prices p_k and $\mathbf{p}'$ in Figure 4a. When w is negative and large in absolute value, the $\tilde{p}_c(w)$ are large (and positive). Hence $p_k \leq \tilde{p}_k(w)$ and the CCDF approaches $P_j(\mathbf{p}')$ as expected. As w increases, $\widetilde{\mathbf{p}}_{-k}(w)$ decreases and the probability of choosing k at prices $(p_k, \widetilde{\mathbf{p}}_{-k}(w))$ decreases. Therefore, $\Pr_\omega [w \leq W^\omega (y - p_k, k), j = J^\omega(\mathbf{p}, y)]$ decreases until w reaches its highest value, w_k^*, where $p_k = \tilde{p}_k(w_k^*)$. There the CCDF drops to zero discontinuously, as the indicator function becomes zero. This means that w_k^* is an upper bound for welfare and that the probability distribution has a mass point.



(a) Unconditional CCDF

(b) Conditional CCDF

Figure 4: The course of the (un)conditional CCDF of welfare

We can derive some associated distributions, such as the conditional and marginal CCDFs, which are more relevant in empirical applications.

Corollary 1.

$$\Pr_\omega \left[w \leq W^\omega (y - p_k, k) \mid j = J^\omega(\mathbf{p}', y) \right] = \frac{P_{j,k}\Big(\mathbf{p}', (p_k, \widetilde{\mathbf{p}}_{-k}(w)), y\Big)}{P_j(\mathbf{p}', y)} \mathbb{I}\left[p_k \leq \tilde{p}_k(w)\right], \tag{18}$$

$$\Pr_\omega \left[w \leq W^\omega (y - p_k, k) \mid k = J^\omega(\mathbf{p}, y) \right] = \frac{P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)}{P_k(\mathbf{p}, y)} \mathbb{I}\left[p_k \leq \tilde{p}_k(w)\right], \tag{19}$$

where $\mathbf{min}\big(\mathbf{p}, \widetilde{\mathbf{p}}(w)\big) = \big(\min(p_1, \widetilde{p}_1(w)), \ldots, \min(p_n, \widetilde{p}_n(w))\big)$,

$$\Pr_\omega \left[w \leq W^\omega (y - p_k, k) \right] = P_k\Big((p_k, \widetilde{\mathbf{p}}_{-k}(w)), y\Big) \mathbb{I}\left[p_k \leq \tilde{p}_k(w)\right], \tag{20}$$

and

$$\Pr_{\omega}\left[w \leq W^{\omega}\left(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p},y)\right)\right] = \sum_k P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right]. \tag{21}$$

We find again that the different derived distributions can be expressed in terms of choice and transition probabilities. Equations (18) and (19) can be used to assess the distribution of welfare when the researcher observes which bundle is optimal and wants to take this information into account. Equation (20) describes the marginal distribution of welfare evaluated in a specific bundle, not taking into account which bundle is optimal. Finally, Equation (21) specialises this result to a setting where welfare is evaluated in the optimal bundle.

A typical example of the distribution of welfare in bundle k conditional on bundle k being optimal is plotted in Figure 4b. As before, define for an option c, w_c^* to be the highest value of w such that $p_c = \tilde{p}_c(w)$, and also define w^* to be $\min_c\{w_c^*\}$. Then we observe that for $w \leq w^*$, $p_c \leq \widetilde{p}_c(w)$ for all c, and hence, $\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)) = \mathbf{p}$. It follows that $\Pr_\omega\Big[w \leq W^\omega(y - p_k, k) \mid k = J^\omega(\mathbf{p}, y)\Big] = 1$. Hence, w^* is a lower bound for welfare in option k, conditionally on k being optimal. For $w > w^*$, $\Pr_\omega\Big[w \leq W^\omega(y - p_k, k) \mid k = J^\omega(\mathbf{p}, y)\Big]$ decreases continuously until w reaches w_k^* where $\Pr_\omega\Big[w \leq W^\omega(y - p_k, k) \mid k = J^\omega(\mathbf{p}, y)\Big]$ drops to 0, as seen in Figure 4b. Hence, w_k^* is an upper bound for welfare in option k, conditional on k being optimal, and the distribution has a mass point at w_k^*. If $w_k^* = w^*$, the distribution is thus a step function and, hence, the welfare level in bundle k, conditional on k being optimal at prices $\mathbf{p}$ and exogenous income y, is deterministic and equals w_k^*.

Examples of NOS measures: the MMU class When applying Corollary 1 to the class of MMU measures, we obtain the following result.

Corollary 2. *When using reference prices $\mathbf{p}^{ref}$, we have*

$$\begin{aligned}\Pr_{\omega}\Big[w \leq W^{\omega}_{M(\mathbf{p}^{ref})}\left(y - p_k, k\right), j = J^{\omega}(\mathbf{p}', y)\Big] = \\ P_{j,k}\big(\mathbf{p}', (p_k, y - w + \mathbf{p}^{ref}_{-k}), y\big)\mathbb{I}\left[p_k \leq y - w + p^{ref}_k\right].\end{aligned} \tag{22}$$

When $p_k = p'_k$, and the reference prices equal the actual prices $\mathbf{p}'$ and k is the optimal choice, this simplifies to

$$\Pr_{\omega}\Big[w \leq W^{\omega}_{M(\mathbf{p}')}\left(y - p'_k, k\right), k = J^{\omega}(\mathbf{p}', y)\Big] = P_k(\mathbf{p}', y)\mathbb{I}\left[w \leq y\right] \tag{23}$$

and, hence,

$$\Pr_{\omega}\Big[w \leq W^{\omega}_{M(\mathbf{p}')}\left(y - p'_k, k\right) \mid k = J^{\omega}(\mathbf{p}', y)\Big] = \mathbb{I}\left[w \leq y\right], \tag{24}$$

$$\Pr_{\omega}\Big[w \leq W^{\omega}_{M(\mathbf{p}')}\Big(y - p'_{J^{\omega}(\mathbf{p}',y)}, J^{\omega}(\mathbf{p}', y)\Big)\Big] = \mathbb{I}\left[w \leq y\right]. \tag{25}$$

Both the MMU in the optimal bundle and the MMU in bundle k, conditional on k being optimal, are, therefore, deterministic and equal the initial exogenous income y when reference equal actual prices.

4.2 Joint distribution of welfare levels and welfare differences

In this section, we derive the joint distribution of welfare levels and welfare differences. Joint knowledge on levels and differences of welfare enables investigation of the association between individuals' gains or losses from a price change and their position in terms of initial welfare. A price change is defined as an exogenous shift in prices from $\mathbf{p}$ to $\mathbf{p}'$. As discussed in Section 3.1, we will assume throughout that the unobserved preference type ω is unaltered by the price change.

4.2.1 Welfare differences in terms of NOS measures

We first study the general case in which welfare differences are defined on the basis of changes in NOS welfare measures (evaluated in optimal choices). As an intermediate step, we derive the joint distribution of welfare before and after a price change in Proposition 1.

Proposition 1. *The joint distribution of welfare in the optimal bundle i, before a price change, and welfare in the optimal bundle j, after the price change, is as follows:*

$$\begin{aligned}&\Pr_{\omega}[w \leq W_0^{\omega}(y - p_i, i), z \leq W_1^{\omega}(y - p'_j, j), i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] \\ &= P_{i,j}\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), \mathbf{min}(\mathbf{p}', \widetilde{\mathbf{p}}(z)), y\Big)\mathbb{I}\left[p_i \leq \widetilde{p}_i(w)\right]\mathbb{I}\left[p'_j \leq \widetilde{p}_j(z)\right].\end{aligned} \tag{26}$$

Proposition 1 shows that this joint distribution can be written in terms of transition probabilities, evaluated at initial, final, and virtual prices. Using this proposition, the joint distribution of welfare levels and differences can be derived.

Theorem 2. *Let the function h be defined by*

$$h_{i,j,\mathbf{p},\mathbf{p}'}(w, x, s) = P_{i,j}\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(\max(w, x))), \mathbf{min}(\mathbf{p}', \widetilde{\mathbf{p}}(s)), y\Big)\mathbb{I}\left[p'_j \leq \widetilde{p}_j(s)\right] \tag{27}$$

$$= P_{i,j}\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w), \widetilde{\mathbf{p}}(x)), \mathbf{min}(\mathbf{p}', \widetilde{\mathbf{p}}(s)), y\Big)\mathbb{I}\left[p'_j \leq \widetilde{p}_j(s)\right]. \tag{28}$$

Then, the joint distribution of the stochastic welfare measure and the difference before and after the price change of this measure becomes,

$$\begin{aligned}&\Pr_{\omega}[w \leq W_0^{\omega}(y - p_i, i), W_1^{\omega}(y - p'_j, j) - W_0^{\omega}(y - p_i, i) \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] = \\ &-\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w, x, x + z)\mathbb{I}\left[p_i \leq \min(\widetilde{p}_i(w), \widetilde{p}_i(x))\right] dx.\end{aligned} \tag{29}$$

Unfortunately, it seems that this expression cannot be simplified. However, even though expression (29) seems technically complicated, only the transition probabilities are used as input. This object is nonparametrically identified from panel data.

4.2.2 Welfare differences in terms of the CV

We now specialise our results to the joint distribution of the MMU and the CV, which is a popular choice among applied welfare economists.[19] The CV refers to the (possibly negative) amount of the numeraire an individual wants to give up after a price change to be equally well-off as before this change.

For an individual of type ω, the compensating variation CV^{ω} is implicitly defined as

$$\max_c\{U_c^{\omega}(y - p_c)\} = \max_c\{U_c^{\omega}(y - p'_c - CV^{\omega})\}, \tag{30}$$

[19]The results below in Theorems 3 and 4, and in Corollaries 3 and 4, can in fact be seen as applications of Theorem 2. The derivation for the EV is similar and can be found in the Online Appendix.

where, as before, $\mathbf{p}$ are initial prices and $\mathbf{p}'$ final prices. In fact this definition of the CV is equivalent to $MMU^{\omega}_{\mathbf{p}'}(y - p'_{J^{\omega}(\mathbf{p}',y)}, J^{\omega}(\mathbf{p}', y)) - MMU^{\omega}_{\mathbf{p}'}(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p}, y))$, i.e. the difference between the MMU with the final prices as reference price vector, in the optimal bundle after the price change, and the same MMU in the optimal bundle before the price change.[20] Note that the CV for a composition of two or more price changes cannot be calculated from the CV for each price change separately. In our more general approach of measuring a change in welfare by the difference between two valuations of a welfare metric, this problem is inherently nonexistent.

Distribution of the CV In order to derive the distribution of the CV when the choice is equal to option i under initial prices and option j under final prices, we can follow a similar strategy as Bhattacharya (2015, 2018) and de Palma and Kilani (2011). Analogously to Lemma 1, the condition $CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)$ can be translated in i being the optimal bundle when faced with a counterfactual price vector.

Lemma 2. *We have*

$$\begin{aligned} &\left\{\omega \mid CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)\right\} \\ &= \left\{\omega \mid U^{\omega}_i(y - p_i) \geq \max_c\{U^{\omega}_c(y - p'_c - z)\}, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)\right\}. \end{aligned} \tag{32}$$

With Lemma 2, we can state the following theorem.

Theorem 3. *The joint distribution of the CV and the optimal choices before and after the price change is as follows:*

$$\Pr_{\omega}[CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] = P_{i,j}(\mathbf{min}(\mathbf{p}, \mathbf{p}' + z), \mathbf{p}', y)\mathbb{I}\left[p_i \leq p'_i + z\right]. \tag{33}$$

We observe that $\Pr_{\omega}[CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)]$ is bounded from below by $p_i - p'_i$. This is as expected; if the initial optimal bundle was i and the price of p_i drops to p'_i, the numeraire must drop with at least this amount to be equally well-off as in the initial situation. This means that the minimal compensation, in terms of the joint distribution, is $p_i - p'_i$. Moreover, for $z \geq \max_k\{p_k - p'_k\}$, $\Pr_{\omega}[CV^{\omega} \leq z \mid i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] = 1$. This means that the maximal compensation, in terms of the conditional distribution, cannot be higher than the maximal price difference, which is also as expected.

The next corollary follows immediately and may again be more useful to the applied researcher.

Corollary 3.

$$\Pr_{\omega}[CV^{\omega} \leq z \mid i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] = \frac{P_{i,j}(\mathbf{min}(\mathbf{p}, \mathbf{p}' + z), \mathbf{p}', y)}{P_{i,j}(\mathbf{p}, \mathbf{p}', y)}\mathbb{I}\left[p_i \leq p'_i + z\right], \tag{34}$$

[20] Indeed, defining CV^{ω} by $MMU^{\omega}_{\mathbf{p}'}(y - p'_{J^{\omega}(\mathbf{p}',y)}, J^{\omega}(\mathbf{p}', y)) - MMU^{\omega}_{\mathbf{p}'}(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p}, y))$, we get $MMU^{\omega}_{\mathbf{p}'}(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p}, y)) = y - CV^{\omega}$ by Corollary 2. Moreover, as i is the optimal bundle before the price change, we get

$$\begin{aligned} \max_c\{U^{\omega}_c(y - p_c)\} &= U^{\omega}_{J^{\omega}(\mathbf{p},y)}(y - p_{J^{\omega}(\mathbf{p},y)}) \\ &= \max_c\{U^{\omega}_c(MMU^{\omega}_{\mathbf{p}'}(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p}, y)) - p'_c)\} \\ &= \max_c\{U^{\omega}_c(y - p'_c - CV^{\omega})\}. \end{aligned} \tag{31}$$

$$\Pr_{\omega}[CV^{\omega} \leq z \mid i = J^{\omega}(\mathbf{p}, y)] = \frac{P_i(\mathbf{min}(\mathbf{p}, \mathbf{p}' + z), y)}{P_i(\mathbf{p}, y)} \mathbb{I}\left[p_i \leq p_i' + z\right], \tag{35}$$

$$\Pr_{\omega}[CV^{\omega} \leq z \mid j = J^{\omega}(\mathbf{p}', y)] = \sum_i \frac{P_{i,j}(\mathbf{min}(\mathbf{p}, \mathbf{p}' + z), \mathbf{p}', y)}{P_j(\mathbf{p}', y)} \mathbb{I}\left[p_i \leq p_i' + z\right], \tag{36}$$

and

$$\Pr_{\omega}[CV^{\omega} \leq z] = \sum_i P_i(\mathbf{min}(\mathbf{p}, \mathbf{p}' + z), y) \mathbb{I}\left[p_i \leq p_i' + z\right].^{21} \tag{37}$$

Equation (37) gives an expression for the marginal distribution of the CV. Equations (34), (35) and (36), which present conditional distributions, can be used to calculate the distribution of the CV when the optimal bundle(s) (i) before and after price change are known; (ii) only before the price change is known; and (iii) only after the price change is known.

Joint distribution of the MMU and the CV We now apply Theorem 2 to the case where one chooses the MMU with final prices as the reference price vector, as a welfare measure. The difference in welfare before and after the price change is then equal to the CV.

Theorem 4. *The joint distribution of the MMU and the CV is as follows:*

$$\begin{aligned} &\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}(y - p_i, i), CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)] \\ &= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p}, \mathbf{p}' + \min(z, y - w)\big), \mathbf{p}', y\Big) \mathbb{I}\left[p_i \leq p_i' + \min(z, y - w)\right]. \end{aligned} \tag{38}$$

Again, Corollary 4 follows immediately.

Corollary 4.

$$\begin{aligned} &\Pr_{\omega}\left[w \leq W^{\omega}_{M(\mathbf{p}')}(y - p_i, i), CV^{\omega} \leq z \mid i = J^{\omega}(\mathbf{p}, y), j = J^{\omega}(\mathbf{p}', y)\right] \\ &\qquad = \frac{P_{i,j}\Big(\mathbf{min}\big(\mathbf{p}, \mathbf{p}' + \min(z, y - w)\big), \mathbf{p}', y\Big)}{P_{i,j}(\mathbf{p}, \mathbf{p}', y)} \mathbb{I}\left[p_i \leq p_i' + \min(z, y - w)\right], \end{aligned} \tag{39}$$

$$\begin{aligned} &\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}(y - p_i, i), CV^{\omega} \leq z \mid i = J^{\omega}(\mathbf{p}, y)] \\ &\qquad = \frac{P_i\Big(\mathbf{min}\big(\mathbf{p}, \mathbf{p}' + \min(z, y - w)\big), y\Big)}{P_i(\mathbf{p}, y)} \mathbb{I}\left[p_i \leq p_i' + \min(z, y - w)\right], \end{aligned} \tag{40}$$

$$\begin{aligned} &\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}\Big(y - p_{J^{\omega}(\mathbf{p}, y)}, J^{\omega}(\mathbf{p}, y)\Big), CV^{\omega} \leq z \mid j = J^{\omega}(\mathbf{p}', y)] \\ &\qquad = \sum_i \frac{P_{i,j}\Big(\mathbf{min}\big(\mathbf{p}, \mathbf{p}' + \min(z, y - w)\big), \mathbf{p}', y\Big)}{P_j(\mathbf{p}', y)} \mathbb{I}\left[p_i \leq p_i' + \min(z, y - w)\right], \end{aligned} \tag{41}$$

and,

$$\begin{aligned} &\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}\Big(y - p_{J^{\omega}(\mathbf{p}, y)}, J^{\omega}(\mathbf{p}, y)\Big), CV^{\omega} \leq z] \\ &\qquad = \sum_i P_i\Big(\mathbf{min}\big(\mathbf{p}, \mathbf{p}' + \min(z, y - w)\big), y\Big) \mathbb{I}\left[p_i \leq p_i' + \min(z, y - w)\right]. \end{aligned} \tag{42}$$

[21] Note that Equation (37) is the main result of Bhattacharya (2015).

The joint cumulative distribution can again be written as (a sum of) choice or transition probabilities. Each choice and transition probability is calculated using up to three price vectors: the initial price vector $\mathbf{p}$, the final price vector $\mathbf{p}'$, and a translation of the $\mathbf{p}'$ vector for the combined MMU and CV part.

4.3 Social welfare

A classical additively, separable Bergson-Samuelson social welfare function (SWF) takes the form

$$SWF = \int h(u)\, dG_U(u), \tag{43}$$

where u is the value of a utility function representing the well-being of an individual in a particular state of the world, h is a strictly increasing concave function expressing the inequality aversion, and G_U is the CDF of the well-being distribution in the population in a given state of the world.[22] For example, in the utilitarian case, we have that $h(u) = u$.

The NOS welfare measures are well suited as a representation of preferences as they are known to satisfy a set of attractive principles of interpersonal comparability (see Fleurbaey and Maniquet, 2017; 2018). We can, therefore, use these measures directly as building blocks in the SWF in Equation (43). More specifically, the equivalent to the Bergson-Samuelson SWF in our framework reads as

$$SWF = \int \int h(w)\, dF_W(w \mid \mathbf{p}, y)\, dG(\mathbf{p}, y), \tag{44}$$

where G is the CDF of the joint distribution of prices and exogenous income in the population, which can be observed from the data, and $F_W(w \mid \mathbf{p}, y)$ is the conditional CDF of the NOS measure W, and equals $\Pr_{\omega}\left[W^{\omega}\left(y - p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p}, y)\right) \leq w\right]$.[23]

Proposition 2 illustrates how the results on the distribution of individual welfare levels in Corollary 1 lead to the calculation of social welfare as defined in Equation (44), using only choice probabilities.

Proposition 2. *The conditional CDF of individual welfare in the optimal bundle can be calculated using choice probabilities:*

$$F_W(w \mid \mathbf{p}, y) = 1 - \sum_k P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right]. \tag{45}$$

Hence, social welfare can be computed from these probabilities. The joint distribution of prices and exogenous income G can be estimated separately using standard nonparametric tools.

Moreover, this expression can be used to identify if a price change, for example, due to a policy reform, has a desirable effect on social welfare. Indeed, the difference in social welfare can be calculated as follows:

$$\begin{aligned} SWF' - SWF &= \int \int h(w)\, dF_W(w \mid \mathbf{p}', y)\, dG'(\mathbf{p}', y) - \int \int h(w)\, dF_W(w \mid \mathbf{p}, y)\, dG(\mathbf{p}, y) \\ &= \int \int h(w)\, dF_W(w \mid \mathbf{p} + \Delta\mathbf{p}, y)\, dG'(\mathbf{p} + \Delta\mathbf{p}, y) \\ &\qquad - \int \int h(w)\, dF_W(w \mid \mathbf{p}, y)\, dG(\mathbf{p}, y) \\ &= \int \int h(w)\, d\Big(F_W(w \mid \mathbf{p} + \Delta\mathbf{p}, y) - F_W(w \mid \mathbf{p}, y)\Big)\, dG(\mathbf{p}, y). \end{aligned} \tag{46}$$

[22] A function f is additively separable when it can be written in the form $f(x_1, \ldots, x_n) = \sum_i f_i(x_i)$.

[23] As social welfare is a population level concept, we rely on the second interpretation of the randomness in the welfare measure (see the discussion at the beginning of Section 4).

where G (G') is the joint distribution of initial (final) prices and exogenous income, and $\Delta\mathbf{p} = \mathbf{p}' - \mathbf{p}$. With Equations (46) and (45), one can asses the desirability of a potential price change without parametric assumptions and only using choice probabilities and the initial distribution of prices and exogenous income.

Interestingly, in the spirit of Roberts (1980), we can derive conditions under which the expression for the SWF can be formulated in terms of incomes alone. In particular, when prices are equal for everyone and one uses the MMU with reference prices equal to those common prices, as individual welfare measure, one obtains a price independent SWF in terms of income.

Corollary 5. *When prices are equal for everyone and when one uses the MMU with reference prices equal to those common prices as the welfare measure, the SWF can be written solely in terms of income.*

5 Discussion on implementation

5.1 Set-identifying transition probabilities from cross-sectional data

As mentioned before, the transition probabilities are nonparametrically identifiable and estimable from panel data that contains sufficient relative price and exogenous income variation. This immediately implies that all the results from previous subsections are also nonparametrically identified in such a data setting. One simply has to evaluate the estimated transition probabilities at virtual price vectors.

In many empirical applications, however, researchers only have access to (repeated) cross-sectional data. This type of data nonparametrically identifies the choice probabilities, but not the associated transition probabilities. However, by exploiting Boole-Fréchet (Fréchet, 1935) and stochastic revealed preference inequalities, one can derive bounds on the now unobserved transition probabilities based on the observed choice probabilities.

Proposition 3. *The transition probabilities* $\{P_{i,j}(\mathbf{p},\mathbf{p}',y)\}$ *are set identified from the choice probabilities* $\{P_i\}$ *with bounds*

$$\begin{aligned} P^L_{i,i}(\mathbf{p},\mathbf{p}',y) &= \max\left\{P_i(\mathbf{p},y) + P_i(\mathbf{p}',y) - 1, P_i\Big(\big(\max\{p_i,p'_i\},\mathbf{min}\{\mathbf{p}_{-i},\mathbf{p}'_{-i}\}\big),y\Big)\right\}, \\ P^U_{i,i}(\mathbf{p},\mathbf{p}',y) &= \min\left\{P_i(\mathbf{p},y),\ P_i(\mathbf{p}',y)\right\}. \end{aligned} \tag{47}$$

For $i \neq j$, $P_{i,j}(\mathbf{p},\mathbf{p}',y) = 0$ *if* $p_i \geq p'_i$ *and* $p_j \leq p'_j$ *and*

$$\begin{aligned} P^L_{i,j}(\mathbf{p},\mathbf{p}',y) &= \max\{P_i(\mathbf{p},y) + P_j(\mathbf{p}',y) - 1,\ 0\}, \\ P^U_{i,j}(\mathbf{p},\mathbf{p}',y) &= \min\left\{P_i(\mathbf{p},y), P_j(\mathbf{p}',y)\right\}, \end{aligned} \tag{48}$$

elsewhere. These bounds are sharp.

The Boole-Fréchet inequalities ensure that the transition probabilities are weakly smaller than their associated marginal choice probabilities $P_i(\mathbf{p},y)$ and $P_j(\mathbf{p}',y)$. When $P_i(\mathbf{p},y) + P_j(\mathbf{p}',y) - 1 > 0$ they also deliver nontrivial lower bounds. The stochastic revealed preference inequalities, which stem from the strong monotonicity of the utility function (see Assumption 1), provide additional identificational power in two particular instances. Firstly, by evaluating the choice probabilities at the least-favourable price vector

$(\max\{p_i, p_i'\}, \mathbf{min}\{\mathbf{p}_{-i}, \mathbf{p}_{-i}'\})$, they yield an informative lower bound for the transition probabilities in the no-transition case where $i = j$. Secondly, when i becomes weakly less expensive and $j \neq i$ becomes weakly more expensive, the transition probability should equal zero, as it is irrational for individuals to make this transition within the context of our model.

5.2 Estimating choice probabilities

Given the exogeneity of budget sets presupposed in Assumption 2, the choice probabilities can be readily estimated using nonparametric regression, as they are essentially conditional expectation functions. Standard tools, such as kernel and series based regression, are available in most modern statistical software. One particular attractive feature of the Nadaraya-Watson kernel estimator is that the estimated choice probabilities add up to one for all price vectors when the same bandwidth is selected for every choice probability function.

In some circumstances, it might be unreasonable to assume that the budget set $(\mathbf{p}, y)$ is independent of the preference type ω. When instruments are available, however, some forms of endogeneity can be handled by using a standard control function approach (Blundell and Powell, 2004).

With samples of modest size, it might be useful to impose additional structure to mitigate the curse of dimensionality. In particular, in a setting with high-dimensional regressors, which arises when there are many alternatives or many individual-specific characteristics, a (semi)parametric estimator can be used to increase efficiency at the expense of functional form misspecification. In particular, our empirical illustration in Section 6 will make use of a semiparametric estimator that can be interpreted as a sieve approximation.

Moreover, in some applications, there might be an outside option that exhibits no independent price variation, which hinders the direct empirical implementation of our approach. However, this difficulty can be circumvented by exploiting variation in the exogenous income, as is discussed in the Online Appendix. In the latter, we also show how average welfare can be calculated directly from any of the distributional results derived above.

One point that needs further attention is that sampling noise might cause the non- or semiparametric estimates to be inconsistent with the monotonicity condition in Assumption 1 over some ranges of the data. When this condition is violated, the CDFs of our distributional results might be strictly decreasing over some sections of their support. To avoid these inconsistencies, researchers can impose shape restrictions on the estimated choice probabilities: they should be decreasing in its own price and increasing in the price of the other alternatives.[24]

6 Empirical illustration

We illustrate the empirical applicability of our results by revisiting the classical trade-off between leisure and consumption. The goal of this illustration is two-fold. Firstly, we demonstrate how the results in this paper enable researchers to assess the distribution of welfare within and across different groups in society. Secondly,

[24] Besides the case of binary choice (Bhattacharya, 2021), it is unknown whether these conditions are also sufficient. We leave this extension for future research.

we show how our results allow to evaluate the effects of an income tax reform on individual and social welfare. Thereby, we concentrate on two particular aspects: (i) a comparison of the welfare distribution before and after the reform, and (ii) the extent to which the winners and losers are (un)equally spread across the initial welfare distribution.

For this purpose, we make use of microdata from the 2018 wave of the GSOEP, which contains detailed information on households' demographics, labour supply, wages, and out-of-work income. We model the labour supply of single females (with or without children) as a choice between three discrete alternatives: non-working (NW), part-time employment (PT), and full-time employment (FT).[25] As an income tax reform, we consider the introduction of a basic income flat tax in Germany, which would replace the current nonlinear tax schedule.

We refer to the Online Appendix for more details on the estimation procedure and the implementation of our distributional results. It also contains descriptive statistics and additional empirical results.

6.1 Setting and implementation

German tax system and a basic income flat tax reform The German personal income tax system is distinctly progressive. Taxes and social security contributions are paid on earned, capital, and transfer income. After a basic tax-free allowance (8,820 euros in 2017), statutory marginal tax rates increase almost continuously from 14 to 45%. The system also has deductions for work related expenses, and allowances for lone parents and childcare expenses. There are no refundable tax credits; taxes, therefore, cannot become negative. Parents with dependent children are eligible for child benefits. For those who are not able to work, a subsistence income level is guaranteed by social assistance, which includes allowances for housing and heating costs. These benefits are means tested for income and wealth, and depend on the composition of the household.

As a policy reform, we consider the introduction of a basic income flat tax. In this exercise, a basic income of 560 euros is granted to all adults, replacing social assistance. The top-ups for dependent children range from 30% (below the age of thirteen) to 50% (above the age of thirteen) of this amount. In addition, the current nonlinear tax schedule is replaced by one where only a single rate is applied to every individual's taxable income. This rate is set to 35%, which makes the reform revenue neutral for the government, after taking labour supply responses into account.[26]

Figure 5 illustrates the impact of the reform on individuals' budget. With respect to the baseline situation, the financial situation of single females with low disposable income worsens, as the basic income is below social assistance. The largest income gains are found within the lower middle and the upper class: in between the gains are much smaller due to the flat tax rate being above the baseline average tax rate. For single females with children the losses for the lowest incomes are more pronounced, as the top-ups for dependent children in the basic income are less generous than those in the baseline social assistance.

[25] Although this might seem a case of ordered choice, the non-linearities in the tax system ensure identification. See Footnote 18.

[26] Revenue neutrality is defined with respect to the subsample of single females for which we conduct the welfare analysis.

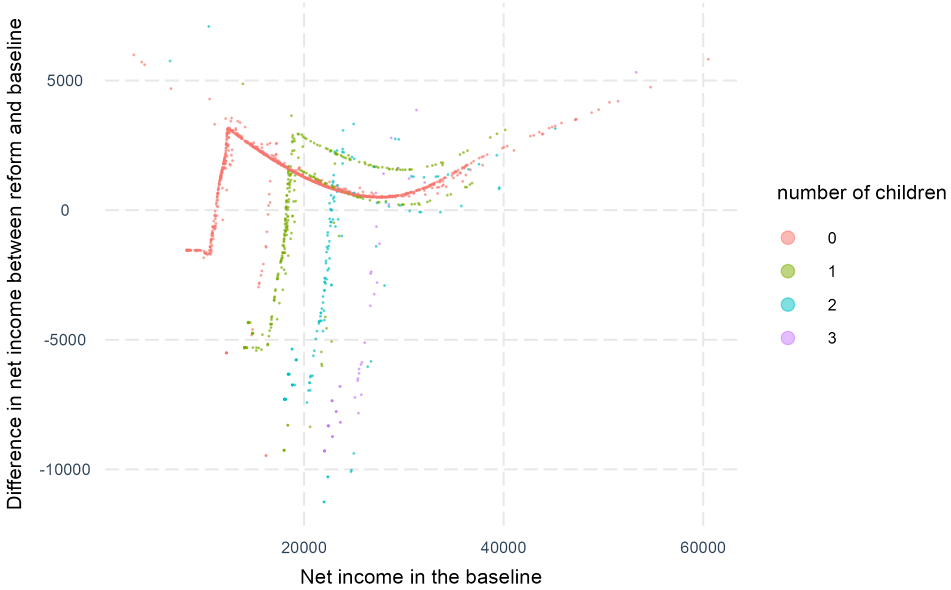


Figure 5: Differences in individuals' net income between the baseline and the reform

Data selection and estimation From the GSOEP sample, we construct a subsample with single females, with or without children, that are available to the labour market. That is, we restrict the analysis to those below 60 years old. To reduce the effect of outliers, we also drop individuals with gross hourly wages outside of 4-90 euros and gross yearly asset income above 12,000 euros. Individuals with missing working hours are also discarded. Our final subsample consists of 1,922 single females; the Online Appendix contains descriptive statistics for this subsample.

Following the literature initiated by Van Soest (1995), we model labour supply by means of a DC-RUM. This approach allows one to consider highly nonlinear budget constraints and to capture the fact that working hours typically cannot be varied continuously. To be more specific, we map observed working hours into three discrete alternatives: non-working (i.e. hours strictly lower than 5 hours/week); part-time employment (i.e. hours higher than 5 hours/week and strictly lower than 32 hours/week); and full-time employment (i.e. hours higher than 32 hours/week). For each of these alternatives, we calculate disposable income by means of a tax-benefit calculator.[27]

We model the choice probabilities for alternatives PT and FT semiparametrically, as we estimate each using a flexible regression specification that contains cubic polynomials in the disposable income d of all three alternatives and a linear index with demographic variables, such as age, years of education, number of children, and region.[28] The choice probability for NW is defined as the complement. By means of an arbitrary normalisation, we fix an individual's exogenous income to $y = d_{FT}$, that is to the amount of disposable income she would obtain when working full-time. This is convenient, as it ensures that all prices

[27]Missing wages for the non-working are imputed using a Heckman-type selection model, with variables on the number of children acting as exclusion restrictions, as they are only included in the selection equation and not in the wage equation. For each individual, monthly disposable income is calculated for 0 (i.e. NW), 20 (i.e. PT), and 40 (i.e. FT) hours of work per week.

[28]We use the logistic transformation to ensure that the dependent variable is bounded between zero and one.

are non-negative: i.e. $p_{NW} = d_{FT} - d_{NW}$, $p_{PT} = d_{FT} - d_{PT}$, and $p_{FT} = 0$. For each alternative, the shape restrictions on the choice probabilities are imposed by means of a penalty function. This penalty function also ensures that the choice probability for NW is nowhere negative in the support of the data. For more details on the estimation procedure, we refer to the Online Appendix.

Welfare measure and reference prices All our results are calculated on the basis of a MMU (see Equation (15) for a definition). We fix the reference price for each alternative at the sample median of the difference in disposable income between working full-time and that respective alternative: i.e. $p_{NW}^{ref} = \text{med}(d_{FT} - d_{NW})$, $p_{PT}^{ref} = \text{med}(d_{FT} - d_{PT})$, and $p_{FT}^{ref} = 0$.

6.2 Results

Individual and grouped welfare distributions We first study the distribution of individual welfare in the baseline, conditional on the chosen alternative (i.e. Equation (19) in Corollary 1). Figure 6 shows estimates of this distribution for all females in our subsample, partitioned in quartiles of gross hourly wages. Hourly wages reflect potential earning capacity and can be thought of as a proxy for ability.

As was noted at the outset of Section 4, each individual distribution either reflects the econometrician's uncertainty about the welfare level of an individual with such observable characteristics (i.e. choice, prices, exogenous income, and demographic variables) or it reflects the distribution of actually realised welfare levels in the population of single females with such observable characteristics. In either case, possible differences in welfare for an individual with given observable characteristics are due to unobserved preference heterogeneity. We will maintain the second interpretation in the remainder of this empirical illustration.

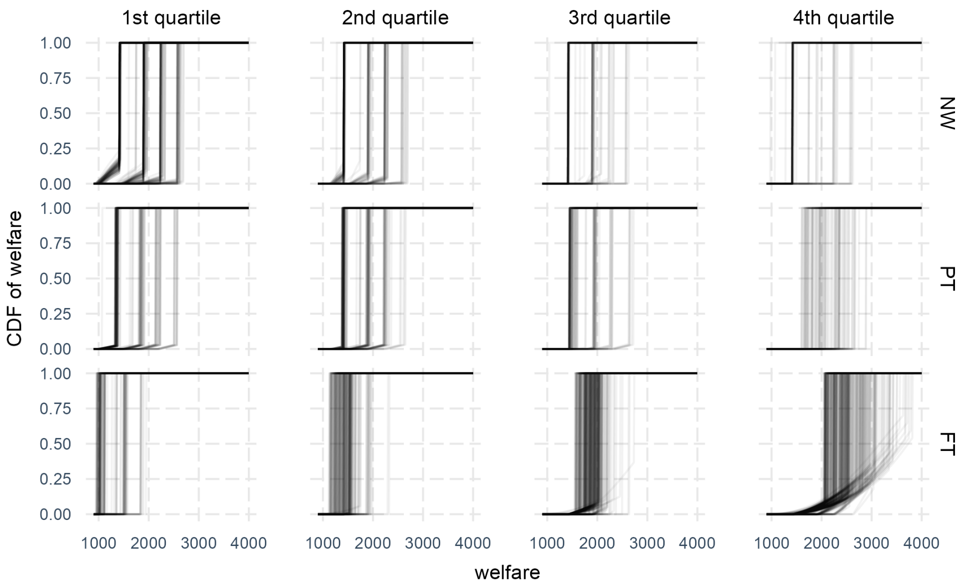


Figure 6: Individual welfare distributions for all females by alternative chosen and wage quartile

Visual inspection of Figure 6 reveals that these distribution functions have the expected shape (see

Figure 4b).[29] On the one hand, there is a critical welfare level w^* below which the virtual prices of all three alternatives are higher than the actual price, and the welfare level will surely exceed that level. On the other hand, the welfare level $w^*_{c^*}$, for which the virtual price of the actually chosen alternative c^* equals its actual price, is the highest welfare one can obtain. When $w^*_{c^*} = w^*$, the distribution degenerates to a step function, and we can determine the individual's welfare level exactly, whatever her preferences are. This happens to be the case for approximately 25% of the females in our subsample. It predominantly occurs for low-wage individuals who choose FT and high-wage individuals who choose NW.

In Figure 7, we present the welfare distribution for groups based on the alternative chosen and wage quartile. These grouped distributions are obtained by aggregating the individual distributions within each of the panels of Figure 6.[30] As a result, the distribution within these groups reflects both observed and unobserved heterogeneity.

The figure, thus, represents the welfare distribution in society for each of these twelve groups. The welfare distribution of high-wage (i.e. fourth quartile) individuals who choose FT tends to first-order dominate the welfare distribution of those who choose alternative PT, and the latter dominates that of NW. Notwithstanding some exceptions at the bottom part of the distribution, the opposite is true for the low-wage individuals (i.e. first quartile). This can be explained as follows. Low-wage individuals have a relatively low gain in disposable income from choosing PT or FT compared with NW. As a consequence, low-wage individuals who choose FT must have more intense preference for income relative to leisure than other low-wage individuals. But because their disposable income is relatively low, this implies that their welfare is relatively low. One could say that, for those individuals, their preferences are less adapted to their wages. The reverse is the case for persons with high gross hourly wages.

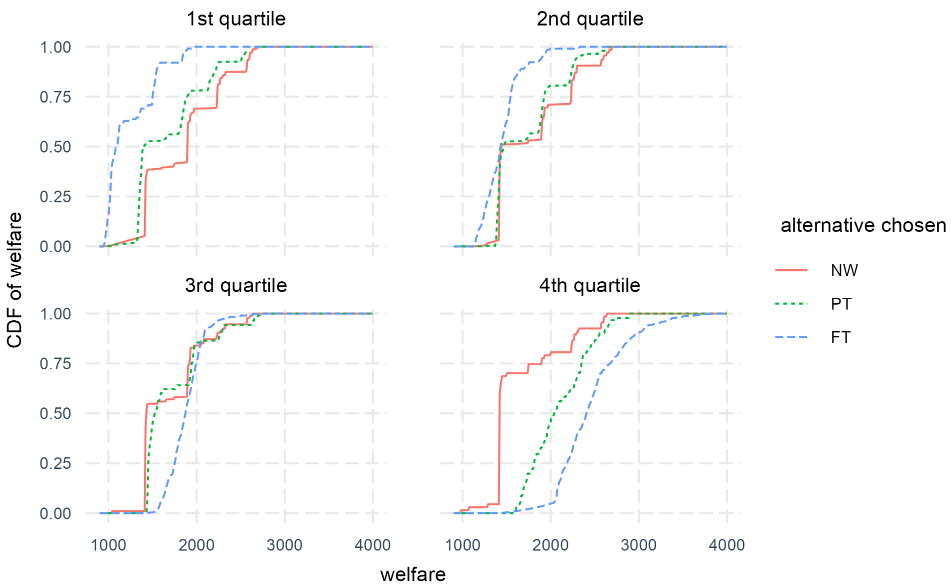


Figure 7: Grouped welfare distributions by alternative chosen and wage quartile

[29]Note that Figure 4b plots the CCDF, while the figures we present here are CDFs.

[30]In Section C.4 of the Online Appendix we show how to aggregate the individual CDFs of Figure 6.

In Figure 8, we further aggregate these distributions by integrating out the chosen alternative. The group with the highest wages tends to first-order dominate the other groups. Substantially higher wages thus lead to increased welfare, despite the large degree of unobserved preference variation we allow for. However, systematic preference differences between the populations of different wage quartiles, due to different composition of demographic variables (age, education, household size) among the wage quartiles, might play a role too in explaining the welfare dominance of the fourth quartile. In contrast, welfare levels obtained by individuals belonging to the lower three wage quartiles turn out to be more intermingled. This suggests that, besides wages, both systematic and unobserved preference differences do play an important role in assessing the welfare of an individual.

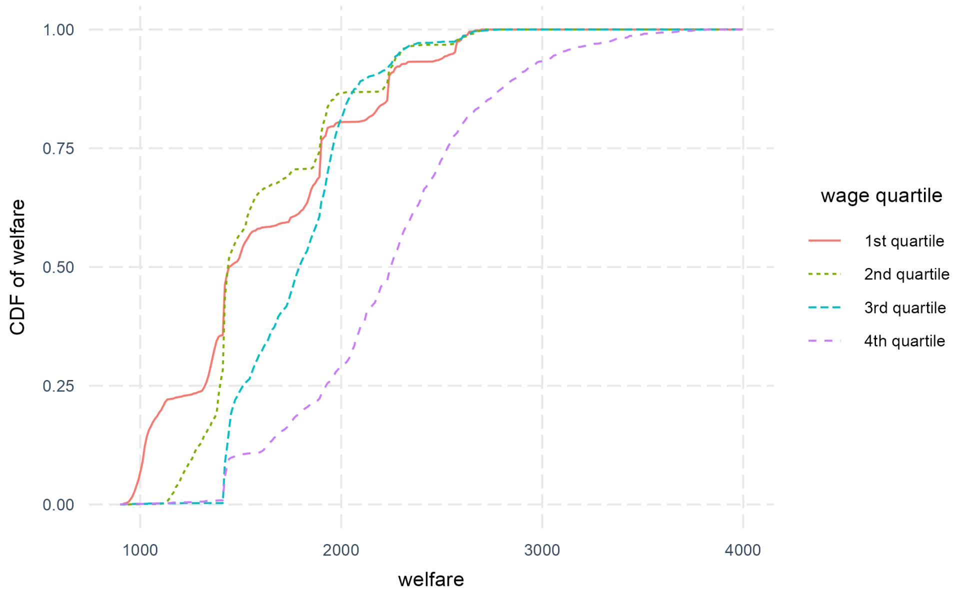


Figure 8: Grouped welfare distribution by wage quartile

Finally, in Figure 9, we depict a smoothed version of 10th, 50th, and 90th percentile of welfare in terms of disposable income. Overall, individuals' welfare increases with disposable income, but the association is not monotonic. This illustrates that income can be a bad proxy for individuals' welfare and highlights the added value of our framework. Those with low incomes are slightly better off than those who earn around 1,000 euros a month. The latter are typically those with low hourly wages, working full-time. In accordance with the large number of (almost) degenerate distributions in Figure 6, the three curves coincide for single females with monthly incomes between 1,000 and 2,000 euros. At higher levels of disposable income, the distribution of welfare exhibits much more spread.

Social welfare and reform We now discuss the effects of the simulated reform, where the existing nonlinear tax system is replaced with a basic income flat tax. Figure 10 compares the overall welfare distribution for the baseline and the reform. The welfare distribution is computed by further aggregating the distributions for the different wage groups of Figure 8 into one overall welfare distribution for the entire population of single females. There is no first-order stochastic dominance between both distributions, as

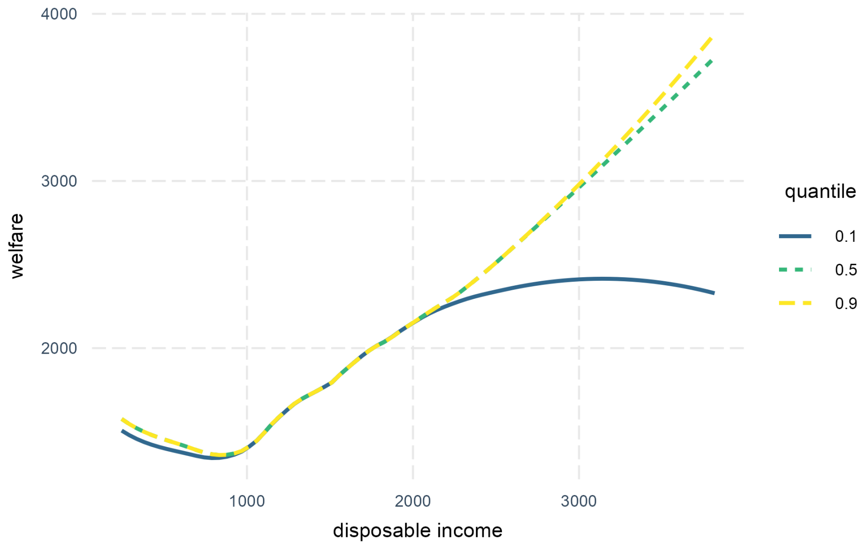


Figure 9: Welfare distribution by disposable income

their CDFs cross multiple times. Overall, the percentiles between 0 and 10, and those between 25 and 40 improve in terms of welfare, while the other percentiles loose significantly. Note, however, that the ranking of individuals between the two reforms might differ, as we only consider marginal distributions of welfare here. The distribution of gains and losses conditional on initial welfare, is discussed in the next paragraph.

There it will be seen that the extent of re-ranking is somewhat limited. It might then seem surprising that individuals with initially low welfare are gaining from the reform, while we showed in Figure 5 that those at the bottom end of the income distribution are loosing most in financial terms. However, as was shown in Figure 9, individuals having the lowest incomes do not belong to the poorest group in terms of welfare. Individuals with the lowest welfare do benefit from the tax reform.

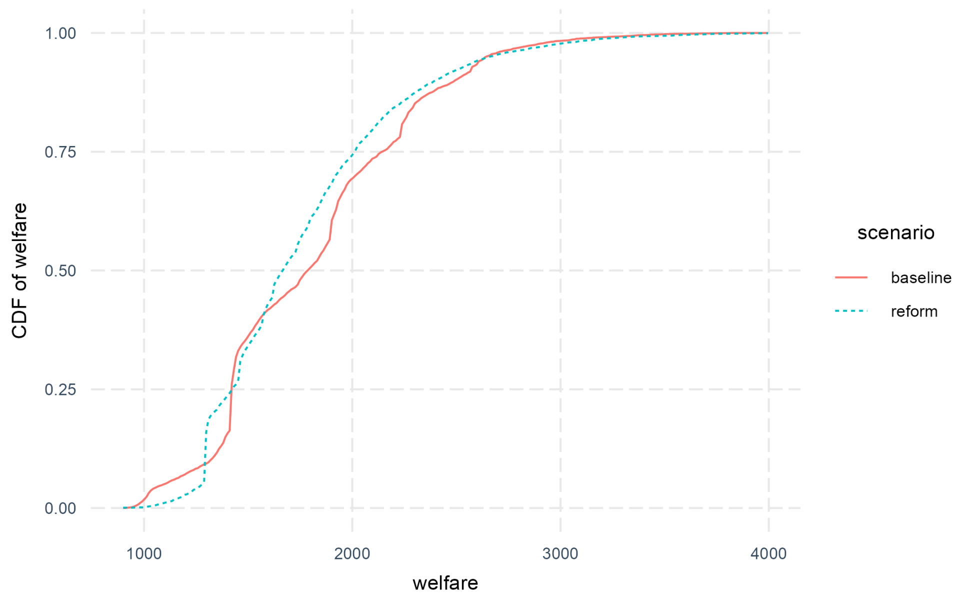


Figure 10: Welfare distribution in the baseline and reform

Winners and losers We further analyse the extent to which the winners and losers are (un)equally spread across the baseline welfare distribution. For this purpose, we consider the distribution of welfare differences conditional on the baseline welfare level, aggregated over all individuals and their baseline and reform choices.[31,32] Finally, we transform this joint distribution into the distribution of welfare differences conditional on the baseline welfare level. We refer to Appendix C.4 for more details on this procedure.

Figure 11 depicts a smoothed version of the 10th, 50th, and 90th iso-percentile contours.[33] Each point (w, z) on the qth contour, indicates the minimal welfare gain z (or loss, if z is negative) that is reached by q% of the population with baseline welfare level w. Firstly, observe that there are a considerable amount of losers. For example, welfare levels at which the 90th iso-percentile curve lies below the zero point on the vertical axis, indicate that at least 10% of the persons with this baseline welfare level exhibit a loss. This occurs in particular for welfare levels between 1,500 and 3,000 euros. Secondly, for individuals between 2,000 and 2,800 euros in the baseline, there is even a majority of losers. Thirdly, those who were poor in terms of baseline welfare (less then 1,500 euros) gained the most.

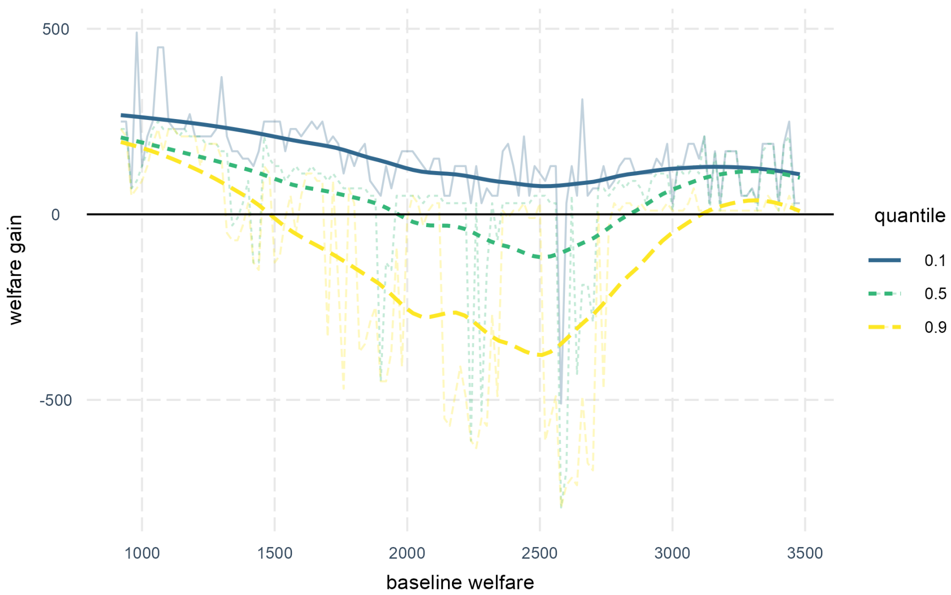


Figure 11: Distribution of welfare gains and losses conditional on baseline welfare

In Table 1, we distribute the population over three, roughly equal, groups of initial welfare levels, and four categories of winners and losers (losers, small gains, medium gains, and big gains). About 38% of the

[31]Since this concept depends on transition probabilities, it is only set-identified in our cross-sectional data. However, since the lower and upper bounds in the aggregate are not far apart (at most a 4 percentage point difference), we perform the analysis using only the upper bound. The small gap between the upper and lower bound is due to individuals' PDFs of welfare having small supports that do rarely overlap.

[32]Notice that we cannot use the simplified versions based on the CV in Theorem 4. Indeed, in our application the actual baseline and reform prices are individual specific. Using these as reference prices would imply that the resulting welfare measure does no longer comply with our definition, which requires that the nested opportunity sets are common to all individuals. An analogous argument applies to the EV.

[33]Figure 14 in the Online Appendix provides a more detailed picture.

population loses, and those losers are slightly concentrated in the initially worst-off group. Those with small gains form a majority of 48.7%; these gains are more prevalent among the initially best-off third. Moderate gainers can be mostly found within the initially worst-off third, occurring more than twice as often as for the middle group. Only a small proportion of the individuals are big winners (obtaining a gain of at least 600 euros).

Table 1: Distribution of the winners and losers in terms of baseline welfare (in %)

Baseline welfare (euros)	Welfare gain (euros)				Row sums
	(-1000; 0]	(0; 200]	(200; 600]	(600; 2,500]	
(800; 1,500]	14.4	11.7	8.3	0.1	34.5
(1,500; 2,000]	12.4	16.3	3.4	0.1	32.2
(2,000; 4,000]	11.5	20.7	1.0	0.0	33.2
Column sums	38.3	48.7	12.7	0.3	

7 Concluding remarks

In this paper, we provided a coherent framework to conduct individual and social welfare analysis for discrete choice. Allowing for unrestricted, unobserved preference heterogeneity, we argue that individual welfare measures become random variables from the point of view of the econometrician. For the class of NOS measures, we developed nonparametric methods to retrieve their distributions from observational data. In particular, we proved that all relevant marginal, conditional, and joint distributions can be expressed in terms of choice or transition probabilities, which are nonparametrically point-identified from cross-sectional and panel data, respectively. We also showed how transition probabilities can be set-identified when only cross-sectional data is available, which is important in empirical applications. To illustrate the empirical usefulness of our results, we model single female labour supply, using micro-data from the 2018 wave of the GSOEP (Germany).

There are several avenues for future research. Firstly, one could extend our results to settings where, besides prices, other attributes of the alternatives are changed. In the same strand of thinking, the welfare cost of the introduction, and removal, of some alternatives could be studied. This will likely lead to set-identification, instead of point-identification, of the distributions of interest. Secondly, another methodological innovation could be to allow for measurement and optimisation errors in the formal analysis. Depending on the specific application, a significant part of the variation in outcomes can be driven by these errors, which might bias welfare estimates. Lastly, future research is needed to assess the sensitivity of empirical welfare estimates, with respect to the choice of the welfare measure and the corresponding reference prices.

References

Debopam Bhattacharya. Nonparametric Welfare Analysis for Discrete Choice. *Econometrica*, 83(2):617–649, 2015. 3, 4, 14, 18, 19

Debopam Bhattacharya. Empirical welfare analysis for discrete choice: Some general results. *Quantitative Economics*, 9(2):571–615, 2018. 3, 4, 12, 18

Debopam Bhattacharya. The Empirical Content of Binary Choice Models. *Econometrica*, 89(1):457–474, 2021. 22

Richard Blundell and James L. Powell. Endogeneity in Semiparametric Binary Response Models. *Review of Economic Studies*, 71:655–679, 2004. 9, 22

Richard Blundell, Dennis Kristensen, and Rosa Matzkin. Bounding quantile demand functions using revealed preference inequalities. *Journal of Econometrics*, 179(2):112–127, 2014. 5

Richard Blundell, Joel Horowitz, and Matthias Parey. Nonparametric Estimation of a Nonseparable Demand Function under the Slutsky Inequality Restriction. *The Review of Economics and Statistics*, 99(2):291–304, 2017. 5

Richard A. Briesch, Pradeep K. Chintagunta, and Rosa L. Matzkin. Nonparametric Discrete Choice Models With Unobserved Heterogeneity. *Journal of Business & Economic Statistics*, 28(2):291–307, 2010. 4

Bart Capéau, Liebrecht De Sadeleer, Sebastiaan Maes, and André Decoster. Nonparametric welfare analysis for discrete choice: levels and differences of individual and social welfare. *CESifo Working Papers*, No. 9071, 2021. 7

Jean François Carpantier and Christelle Sapata. Empirical welfare analysis: when preferences matter. *Social Choice and Welfare*, 46(3):521–542, 2016. 5

Sam Cosaert and Thomas Demuynck. Nonparametric Welfare and Demand Analysis with Unobserved Individual Heterogeneity. *The Review of Economics and Statistics*, 100(2):349–361, 2018. 5

John K. Dagsvik. Discrete Choice in Continuous Time: Implications of an Intertemporal Version of the IIA Property. *Econometrica*, 70(2):817–831, 2002. 3

John K. Dagsvik and Anders Karlström. Compensating Variation and Hicksian Choice Probabilities in Random Utility Models That Are Nonlinear in Income. *The Review of Economic Studies*, 72(1):57–76, 2005. 2, 4

André de Palma and Karim Kilani. Transition choice probabilities and welfare analysis in additive random utility models. *Economic Theory*, 46(3):427–454, 2011. 2, 4, 18

Koen Decancq, Marc Fleurbaey, and Erik Schokkaert. Happiness, equivalent incomes and respect for individual preferences. *Economica*, 82:1082–1106, 2015. 4, 7

André M.J. Decoster and Peter Haan. Empirical welfare analysis with preference heterogeneity. *International Tax and Public Finance*, 22(2):224–251, 2015. 5

Paolo Delle Site and Marco Valerio Salucci. Transition choice probabilities and welfare analysis in random utility models with imperfect before–after correlation. *Transportation Research Part B: Methodological*, 58:215–242, 2013. 3

Holger Dette, Stefan Hoderlein, and Natalie Neumeyer. Testing multivariate economic restrictions using quantiles: The example of Slutsky negative semidefiniteness. *Journal of Econometrics*, 191(1):129–144, 2016. 5

Peter Diamond and Daniel McFadden. Some uses of the expenditure function in public finance. *Journal of Public Economics*, 3(1):2–21, 1974. 2

Avinash Dixit. Welfare effects of tax and price changes. *Journal of Public Economics*, 4(2):103–123, 1975. 2

Marc Fleurbaey. Two criteria for social decisions. *Journal of Economic Theory*, 134(1):421–447, 2007. 4

Marc Fleurbaey. Beyond GDP: The quest for a measure of social welfare. *Journal of Economic Literature*, 47(4):1029–1075, 2009. 1, 2, 4, 6

Marc Fleurbaey and Didier Blanchet. *Beyond GDP: Measuring welfare and assessing sustainability*. Oxford University Press, 2013. 2

Marc Fleurbaey and François Maniquet. Fairness and well-being measurement. *Mathematical Social Sciences*, 90:119–126, 2017. 4, 20

Marc Fleurbaey and François Maniquet. Optimal income taxation theory and principles of fariness. *Journal of Economic Literature*, 56(3):1029–1079, 2018. 20

Maurice Fréchet. Généralisation du théorème des probabilités totales. *Fundamenta Mathematicae*, 25(1): 379–387, 1935. 3, 21, 10

Jerry A. Hausman and Whitney K. Newey. Individual Heterogeneity and Average Welfare. *Econometrica*, 84(3):1225–1248, 2016. 5, 8

Joseph A. Herriges and Catherine L. Kling. Nonlinear Income Effects in Random Utility Models. *Review of Economics and Statistics*, 81(1):62–72, 1999. 4, 5

Stefan Hoderlein and Anne Vanhems. Estimating the distribution of welfare effects using quantiles. *Journal of Applied Econometrics*, 33(1):52–72, 2018. 5

Mervyn King. Welfare analysis of tax reforms using household data. *Journal of Public Economics*, 21(2): 183–214, 1983. 2

Roger W. Klein and Richard H. Spady. An Efficient Semiparametric Estimator for Binary Response Models. *Econometrica*, 61(2):387–421, 1993. 4

Arthur Lewbel. Semiparametric qualitative response model estimation with unknown heteroscedasticity or instrumental variables. *Journal of Econometrics*, 97(1):145–177, 2000. 4

Charles F. Manski. Maximum score estimation of the stochastic utility model of choice. *Journal of Econometrics*, 3:205–228, 1975. 4

Rosa L. Matzkin. Semiparametric Estimation of Monotone and Concave Utility Functions for Polychotomous Choice Models. *Econometrica*, 59(5):1315, 1991. 4

Rosa L. Matzkin. Nonparametric identification and estimation of polychotomous choice models. *Journal of Econometrics*, 58(1-2):137–168, 1993. 4

Daniel McFadden. Econometric models of probabilistic choice. In Charles F Manski and Daniel McFadden, editors, *Structural analysis of discrete data with econometric applications*, pages 198–272. MIT press Cambridge, MA, 1981. 8

Daniel McFadden. Computing Willingness-to-Pay in Random Utility Models. In James Melvin, Raymond Riezman, and James Moore, editors, *Trade, Theory and Econometrics*, volume 15. Routledge, 1999. 4

Daniel McFadden. Revealed Stochastic Preference: A Synthesis. *Economic Theory*, 26(2):245–264, 2005. 8

Edward R. Morey, Robert D. Rowe, and Michael Watson. A Repeated Nested-Logit Model of Atlantic Salmon Fishing. *American Journal of Agricultural Economics*, 75(3):578, 1993. 4

Elisha Pazner. Equity, nonfeasible alternatives and social choice: A reconsideration of the concept of social welfare. In JJ Laffont, editor, *Aggregation and revelation of preferences*. Elsevier, North-Holland, Amsterdam and New York, 1979. 4, 7

John H Pencavel. Constant-utility index numbers of real wages. *The American Economic Review*, 67(2): 91–100, 1977. 4

Kevin Roberts. Price-independent welfare prescriptions. *Journal of Public Economics*, 13(3):277–297, 1980. 21

Paul A Samuelson. Complementarity: An essay on the 40th anniversary of the hicks-allen revolution in demand theory. *Journal of Economic literature*, 12(4):1255–1289, 1974. 2, 4, 7

Kenneth A. Small and Harvey S. Rosen. Applied Welfare Economics with Discrete Choice Models. *Econometrica*, 49(1):105–130, 1981. 4

Joseph Stiglitz, Amartya Sen, and Jean-Paul Fitoussi. *Richesse des nations et bien-être des individus*. Odile Jacob, 2009. 2

Kenneth E Train. *Discrete Choice Methods with Simulation*. Cambridge University Press, Cambridge, 2003. 2

Arthur Van Soest. Structural models of family labor supply: a discrete choice approach. *Journal of human Resources*, pages 63–88, 1995. 24

Online Appendix

"Identifying the Distribution of Welfare from Discrete Choice"

A Additional results

In Section A.1, we derive analogue results to Theorem 3, Corollary 3, and Theorem 4, but now for the EV instead of the CV. Section A.2 discusses how differences in exogenous income and the presence of an outside option can be dealt with in the empirical implementation. In addition, we also show how average welfare can be directly derived from the distributional results derived in this paper.

A.1 Distributional results for the EV

For an individual of type ω, the equivalent variation EV^ω is defined as

$$\max_c \{U_c^\omega(y - p_c - EV^\omega)\} = \max_c \{U_c^\omega(y - p'_c)\}, \tag{49}$$

i.e, the amount of money (possibly negative) an individual has to pay before the reform to be equally well-off as after the reform.

Theorem 5. *For the distribution of the EV, we have the following results:*

$$\Pr_\omega[EV^\omega \leq z, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)] = P_{i,j}(\mathbf{p}, \mathbf{min}(\mathbf{p} + z, \mathbf{p}'), y)\mathbb{I}\left[p'_j \leq p_j + z\right], \tag{50}$$

$$\Pr_\omega[EV^\omega \leq z \mid i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)] = \frac{P_{i,j}(\mathbf{p}, \mathbf{min}(\mathbf{p} + z, \mathbf{p}'), y)}{P_{i,j}(\mathbf{p}, \mathbf{p}', y)}\mathbb{I}\left[p'_j \leq p_j + z\right], \tag{51}$$

$$\Pr_\omega[EV^\omega \leq z \mid i = J^\omega(\mathbf{p}, y)] = \sum_j \frac{P_{i,j}(\mathbf{p}, \mathbf{min}(\mathbf{p} + z, \mathbf{p}'), y)}{P_i(\mathbf{p}, y)}\mathbb{I}\left[p'_j \leq p_j + z\right], \tag{52}$$

$$\Pr_\omega[EV^\omega \leq z \mid j = J^\omega(\mathbf{p}', y)] = \frac{P_j(\mathbf{min}(\mathbf{p} + z, \mathbf{p}'), y)}{P_j(\mathbf{p}', y)}\mathbb{I}\left[p'_j \leq p_j + z\right], \tag{53}$$

$$\Pr_\omega[EV^\omega \leq z] = \sum_j P_j(\mathbf{min}(\mathbf{p} + z, \mathbf{p}'), y)\mathbb{I}\left[p'_j \leq p_j + z\right]. \tag{54}$$

Proof. We have that

$$\begin{aligned}
&\left\{EV^\omega \leq z, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)\right\} \\
&= \left\{\max_c\{U_c^\omega(y - p_c - EV^\omega)\} \geq \max_c\{U_c^\omega(y - p_c - z)\}, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)\right\} \\
&= \left\{\max_c \{U_c^\omega(y - p'_c)\} \geq \max_c\{U_c^\omega(y - p_c - z)\}, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)\right\},
\end{aligned}$$

such that,

$$
\begin{aligned}
&\Pr_\omega[EV^\omega \le z, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)] \\
&= \Pr_\omega\Big[U_i^\omega(y-p_i) \ge \max_{k\neq i} U_k^\omega(y-p_k), \quad U_j^\omega(y-p'_j) \ge \max_{l\neq j} U_l^\omega(y-p'_l), \\
&\qquad \max_c U_c^\omega(y-p'_c) \ge \max_c\{U_c^\omega(y-p_c-z)\}\Big] \\
&= \Pr_\omega\Big[U_j^\omega(y-p'_j) \ge \max_{k\neq j} U_k^\omega(y-\min(p_k+z, p'_k)), \\
&\qquad U_i^\omega(y-p_i) \ge \max_{l\neq i} U_l^\omega(y-p_l)\Big]\mathbb{I}\left[p'_j \le p_j + z\right] \\
&= P_{i,j}(\mathbf{p}, \mathbf{min}(\mathbf{p}+z, \mathbf{p}'), y)\mathbb{I}\left[p'_j \le p_j + z\right].
\end{aligned}
$$

The other equalities follow directly. □

Theorem 6. *The joint distribution of the MMU, with initial prices as reference prices, and the EV is expressed as:*

$$
\begin{aligned}
&\Pr_\omega[w \le W^\omega_{M(\mathbf{p})}(y-p_i, i), EV^\omega \le z, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)] \\
&\qquad = P_{i,j}\Big(\mathbf{p}, \mathbf{min}(\mathbf{p}', \mathbf{p}+z), y\Big)\mathbb{I}\left[p'_j \le p_j + z\right]\mathbb{I}\left[w \le y\right].
\end{aligned}
\tag{55}
$$

Proof. We have

$$
\begin{aligned}
&\Pr_\omega[w \le W^\omega_{M(\mathbf{p})}(y-p_i, i), EV^\omega \le z, i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)] \\
&= \Pr_\omega\Big[U_i^\omega(y-p_i) \ge \max_{c'} U_{c'}^\omega(y-(p_{c'}+y-w)), \quad U_i^\omega(y-p_i) \ge \max_{k\neq i} U_k^\omega(y-p_k), \\
&\qquad U_j^\omega(y-p'_j) \ge \max_{l\neq j} U_l^\omega(y-p'_l), \quad U_j^\omega(y-p'_j) \ge \max_c\{U_c^\omega(y-p_c-z)\}\Big] \\
&= \Pr_\omega\Big[U_i^\omega(y-p_i) \ge \max_{k\neq i} U_k^\omega(y-\min(p_k, p_k+y-w)), \\
&\qquad U_j^\omega(y-p'_j) \ge \max_{l\neq j} U_l^\omega(y-\min(p'_l, p_l+z))\Big]\mathbb{I}\left[p'_j \le p_j + z\right]\mathbb{I}\left[p_i \le p_i + y - w)\right] \\
&= P_{i,j}\Big(\mathbf{p}, (p'_j, \mathbf{min}(\mathbf{p}'_{-j}, \mathbf{p}_{-j}+z)), y\Big)\mathbb{I}\left[p'_j \le p_j + z\right]\mathbb{I}\left[w \le y\right] \\
&= P_{i,j}\Big(\mathbf{p}, \mathbf{min}(\mathbf{p}', \mathbf{p}+z), y\Big)\mathbb{I}\left[p'_j \le p_j + z\right]\mathbb{I}\left[w \le y\right].
\end{aligned}
$$

□

A.2 Implementation

Differences in exogenous income The assumption that the exogenous income y is common to both situations with prices $\mathbf{p}$ and $\mathbf{p}'$ imposes no constraints on the transition probabilities $P_{i,j}(\mathbf{p}, \mathbf{p}', y)$. Indeed, if exogenous incomes are different when faced with prices $\mathbf{p}$ and $\mathbf{p}'$ (denoted by y and y', respectively), we can always redefine prices and incomes in order to obtain a common exogenous income. To see this, let $\mathbf{p}'' = \mathbf{p}' - y' + y$, such that

$$
\begin{aligned}
P_{i,j}(\mathbf{p}, \mathbf{p}', y, y') &\equiv \Pr_\omega\left[i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y')\right] \\
&= \Pr_\omega\left[i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}' - y' + y, y)\right] \\
&= P_{i,j}(\mathbf{p}, \mathbf{p}'', y).
\end{aligned}
\tag{56}
$$

Presence of an outside option Moreover, in some applications, there is an outside option that exhibits no independent price variation, which also hinders the direct empirical implementation of our approach. However, this difficulty can be circumvented by exploiting variation in the exogenous income y. Suppose alternative $c_o \in \mathcal{C}$ is the outside option for which one has to evaluate the effect of a price change $\Delta p_o = p'_o - p_o$. By a change of variables, it then always holds that $P_i(\mathbf{p}', y) = P_i(\mathbf{p}' - \Delta p_o, y - \Delta p_o)$. Note that the expression at the right-hand side does not require price variation for c_o, as $p'_o - \Delta p_o = p_o$ by construction.

Average welfare A well-known implication of Fubini's theorem is that the mean of any random variable X, given that it exists, can be directly derived from its cumulative density function F_X, i.e.

$$\mathbb{E}_{F_X}(X) = \int_0^{\infty} (1 - F_X(u))du - \int_{-\infty}^{0} F_X(u)du. \tag{57}$$

This result allows us to calculate average welfare from any of the distributional results derived in this paper. Note that when only bounds on the distribution of interest are available (see Section 5.1), the expected value can be bounded by $\mathbb{E}_{F_X^U}(X) \leq \mathbb{E}_{F_X}(X) \leq \mathbb{E}_{F_X^L}(X)$, where F_X^L and F_X^U denote the CDF of the lower and upper bound respectively.

B Proofs

In this section, we provide the proofs of the results in the paper. Note that the results in Theorems 3 and 4, and in Corollaries 3 and 4, can in fact be seen as applications of Theorem 2. However, to provide more insight, we also give direct proofs below.

Theorem 1. *The joint distribution of the NOS welfare measure W, evaluated in an option k with price p_k, and choosing j at prices $\mathbf{p}'$ and exogenous income y can be expressed in terms of transition probabilities as follows:*

$$\Pr_{\omega}\left[w \leq W^{\omega}(y-p_k,k), j = J^{\omega}(\mathbf{p}',y)\right] = P_{j,k}\Big(\mathbf{p}', (p_k, \widetilde{\mathbf{p}}_{-k}(w)), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right], \tag{17}$$

where $(p_k, \widetilde{\mathbf{p}}_{-k}(w)) = (\widetilde{p}_1(w), \ldots, \widetilde{p}_{k-1}(w), p_k, \widetilde{p}_{k+1}(w), \ldots, \widetilde{p}_n(w))$.

Proof of Theorem 1. Using Lemma 1, we have that

$$\begin{aligned}
&\Pr_{\omega}\left[w \leq W^{\omega}\left(y-p_k,k\right), j = J^{\omega}(\mathbf{p}',y)\right] \\
&= \Pr_{\omega}\left[U_k^{\omega}(y-p_k) \geq \max_{c} U_c^{\omega}(y-\widetilde{p}_c(w)), U_j^{\omega}(y-p_j') \geq \max_{c'\neq j} U_{c'}^{\omega}(y-p_{c'}')\right] \\
&= \Pr_{\omega}\left[U_k^{\omega}(y-p_k) \geq \max_{c\neq k} U_c^{\omega}(y-\widetilde{p}_c(w)), U_j^{\omega}(y-p_j') \geq \max_{c'\neq j} U_{c'}^{\omega}(y-p_{c'}')\right]\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right] \\
&= P_{j,k}\Big(\mathbf{p}', (p_k, \widetilde{\mathbf{p}}_{-k}(w)), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right].
\end{aligned}$$

□

Corollary 1.

$$\Pr_{\omega}\Big[w \leq W^{\omega}\left(y-p_k,k\right) \mid j = J^{\omega}(\mathbf{p}',y)\Big] = \frac{P_{j,k}\Big(\mathbf{p}', (p_k, \widetilde{\mathbf{p}}_{-k}(w)), y\Big)}{P_j\left(\mathbf{p}',y\right)}\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right], \tag{18}$$

$$\Pr_{\omega}\Big[w \leq W^{\omega}\left(y-p_k,k\right) \mid k = J^{\omega}(\mathbf{p},y)\Big] = \frac{P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)}{P_k\left(\mathbf{p},y\right)}\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right], \tag{19}$$

where $\mathbf{min}\big(\mathbf{p}, \widetilde{\mathbf{p}}(w)\big) = \big(\min(p_1, \widetilde{p}_1(w)), \ldots, \min(p_n, \widetilde{p}_n(w))\big)$,

$$\Pr_{\omega}\left[w \leq W^{\omega}\left(y-p_k,k\right)\right] = P_k\Big(\left(p_k, \widetilde{\mathbf{p}}_{-k}(w)\right), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right], \tag{20}$$

and

$$\Pr_{\omega}\Big[w \leq W^{\omega}\left(y-p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p},y)\right)\Big] = \sum_k P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right]. \tag{21}$$

Proof of Corollary 1. Equations (18) and (20) follow directly from Theorem 1 using the definitions of conditional and marginal distributions respectively. Equation (21) follows analogously from Equation (19).

Therefore, we only prove Equation (19). We have

$$
\begin{aligned}
&\Pr_\omega\left[w \le W^\omega(y-p_k,k) \mid k = J^\omega(\mathbf{p},y)\right] \\
&= \frac{\Pr_\omega\left[w \le W^\omega(y-p_k,k), k = J^\omega(\mathbf{p},y)\right]}{P_k\left(\mathbf{p},y\right)} \\
&= \frac{P_{k,k}\left(\mathbf{p},\left(p_k,\widetilde{\mathbf{p}}_{-k}(w)\right),y\right)\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right]}{P_k\left(\mathbf{p},y\right)} \\
&= \frac{\Pr_\omega\left[U^\omega_k(y-p_k) \ge \max_{c\neq k} U^\omega_c(y-p_c), U^\omega_k(y-p_k) \ge \max_{c\neq k} U^\omega_c(y-\widetilde{p}_c(w)\right]}{P_k\left(\mathbf{p},y\right)}\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right] \\
&= \frac{\Pr_\omega\left[U^\omega_k(y-p_k) \ge \max_{c\neq k} U^\omega_c\left(y-\min(p_c,\widetilde{p}_c(w))\right)\right]}{P_k\left(\mathbf{p},y\right)}\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right] \\
&= \frac{P_k\left(\mathbf{min}\left(\mathbf{p},\left(p_k,\widetilde{\mathbf{p}}_{-k}(w)\right)\right),y\right)}{P_k\left(\mathbf{p},y\right)}\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right] \\
&= \frac{P_k\left(\mathbf{min}\left(\mathbf{p},\widetilde{\mathbf{p}}(w)\right),y\right)}{P_k\left(\mathbf{p},y\right)}\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right].
\end{aligned}
$$

□

Corollary 2. *When using reference prices* $\mathbf{p}^{ref}$*, we have*

$$
\begin{aligned}
&\Pr_\omega\left[w \le W^\omega_{M(\mathbf{p}^{ref})}\left(y-p_k,k\right), j = J^\omega(\mathbf{p}',y)\right] = \\
&\quad P_{j,k}\left(\mathbf{p}',(p_k, y-w+\mathbf{p}^{ref}_{-k}),y\right)\mathbb{I}\left[p_k \le y-w+p^{ref}_k\right].
\end{aligned} \tag{22}
$$

When $p_k = p'_k$*, and the reference prices equal the actual prices* $\mathbf{p}'$ *and* k *is the optimal choice, this simplifies to*

$$
\Pr_\omega\left[w \le W^\omega_{M(\mathbf{p}')}\left(y-p'_k,k\right), k = J^\omega(\mathbf{p}',y)\right] = P_k(\mathbf{p}',y)\mathbb{I}\left[w \le y\right] \tag{23}
$$

and, hence,

$$
\Pr_\omega\left[w \le W^\omega_{M(\mathbf{p}')}\left(y-p'_k,k\right) \mid k = J^\omega(\mathbf{p}',y)\right] = \mathbb{I}\left[w \le y\right], \tag{24}
$$

$$
\Pr_\omega\left[w \le W^\omega_{M(\mathbf{p}')}\left(y-p'_{J^\omega(\mathbf{p}',y)}, J^\omega(\mathbf{p}',y)\right)\right] = \mathbb{I}\left[w \le y\right]. \tag{25}
$$

Proof of Corollary 2. The first equation follows from plugging $\widetilde{\mathbf{p}}(w) = y - w + \mathbf{p}^{ref}(w)$ into Equation (17). Moreover, using actual prices $\mathbf{p}'$ as reference prices and taking $p_k = p'_k$, $\mathbb{I}\left[p_k \le y-w+p^{ref}_k\right]$ reduces to $\mathbb{I}\left[w \le y\right]$. Therefore,

$$
\begin{aligned}
\Pr_\omega\left[w \le W^\omega_{M(\mathbf{p}')}\left(y-p'_k,k\right), k = J^\omega(\mathbf{p}',y)\right] &= P_k\left(\mathbf{min}\left(\mathbf{p}', y-w+\mathbf{p}'\right),y\right)\mathbb{I}\left[w \le y\right] \\
&= P_k(\mathbf{p}',y)\mathbb{I}\left[w \le y\right].
\end{aligned} \tag{58}
$$

The last two equations then immediately follow from Bayes' theorem and summing over k. □

Proposition 1. *The joint distribution of welfare in the optimal bundle* i*, before a price change, and welfare in the optimal bundle* j*, after the price change, is as follows:*

$$
\begin{aligned}
&\Pr_\omega[w \le W^\omega_0(y-p_i,i), z \le W^\omega_1(y-p'_j,j), i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)] \\
&= P_{i,j}\left(\mathbf{min}\left(\mathbf{p},\widetilde{\mathbf{p}}(w)\right), \mathbf{min}\left(\mathbf{p}',\widetilde{\mathbf{p}}(z)\right),y\right)\mathbb{I}\left[p_i \le \widetilde{p}_i(w)\right]\mathbb{I}\left[p'_j \le \widetilde{p}_j(z)\right].
\end{aligned} \tag{26}
$$

Proof of Proposition 1.

$$
\begin{aligned}
&\Pr_{\omega}[w \leq W_0^{\omega}(y-p_i,i), z \leq W_1^{\omega}(y-p'_j,j), i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)]\\
&= \Pr_{\omega}\Big[U_i^{\omega}(y-p_i) \geq \max_{c'} U_{c'}^{\omega}(y-\widetilde{p}_{c'}(w)), \quad U_i^{\omega}(y-p_i) \geq \max_{k\neq i} U_k^{\omega}(y-p_k),\\
&\qquad U_j^{\omega}(y-p'_j) \geq \max_{l\neq j} U_l^{\omega}(y-p'_l), \quad U_j^{\omega}(y-p'_j) \geq \max_{c} U_c^{\omega}(y-\widetilde{p}_c(z)),\Big]\\
&= \Pr_{\omega}\Big[U_i^{\omega}(y-p_i) \geq \max_{k\neq i} U_k^{\omega}(y-\min(p_k,\widetilde{p}_k(w))),\\
&\qquad U_j^{\omega}(y-p'_j) \geq \max_{l\neq j} U_l^{\omega}(y-\min(p'_l,\widetilde{p}_l(z)))\Big]\mathbb{I}\left[p_i \leq \widetilde{p}_i(w)\right]\mathbb{I}\left[p'_j \leq \widetilde{p}_j(z)\right]\\
&= P_{i,j}\Big(\big(p_i, \mathbf{min}(\mathbf{p}_{-i},\widetilde{\mathbf{p}}_{-i}(w))\big), \big(p'_j, \mathbf{min}(\mathbf{p}'_{-j},\widetilde{\mathbf{p}}_{-j}(z)), y\Big)\mathbb{I}\left[p_i \leq \widetilde{p}_i(w)\right]\mathbb{I}\left[p'_j \leq \widetilde{p}_j(z)\right]\\
&= P_{i,j}\Big(\mathbf{min}(\mathbf{p},\widetilde{\mathbf{p}}(w)), \mathbf{min}(\mathbf{p}',\widetilde{\mathbf{p}}(z)), y\Big)\mathbb{I}\left[p_i \leq \widetilde{p}_i(w)\right]\mathbb{I}\left[p'_j \leq \widetilde{p}_j(z)\right].
\end{aligned}
$$

□

Theorem 2. *Let the function h be defined by*

$$
h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,s) = P_{i,j}\Big(\mathbf{min}(\mathbf{p},\widetilde{\mathbf{p}}(\max(w,x))), \mathbf{min}(\mathbf{p}',\widetilde{\mathbf{p}}(s)), y\Big)\mathbb{I}\left[p'_j \leq \widetilde{p}_j(s)\right] \tag{27}
$$

$$
= P_{i,j}\Big(\mathbf{min}(\mathbf{p},\widetilde{\mathbf{p}}(w),\widetilde{\mathbf{p}}(x)), \mathbf{min}(\mathbf{p}',\widetilde{\mathbf{p}}(s)), y\Big)\mathbb{I}\left[p'_j \leq \widetilde{p}_j(s)\right]. \tag{28}
$$

Then, the joint distribution of the stochastic welfare measure and the difference before and after the price change of this measure becomes,

$$
\begin{aligned}
&\Pr_{\omega}[w \leq W_0^{\omega}(y-p_i,i), W_1^{\omega}(y-p'_j,j) - W_0^{\omega}(y-p_i,i) \leq z, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)] =\\
&-\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z)\mathbb{I}\left[p_i \leq \min(\widetilde{p}_i(w),\widetilde{p}_i(x))\right] dx.
\end{aligned} \tag{29}
$$

Proof of Theorem 2. Fix i and j and define $g(w,z) = \Pr_{\omega}[w \leq W_0^{\omega}(y-p_i,i), z \leq W_1^{\omega}(y-p'_j,j), i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)]$. Then we have

$$
\begin{aligned}
&\Pr_{\omega}[w \leq W_0^{\omega}(y-p_i,i), W_1^{\omega}(y-p'_j,j) - W_0^{\omega}(y-p_i,i) \leq z, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)]\\
&= -\int_{-\infty}^{+\infty} \partial_2 g\left(\max(w,x), x+z\right) dx\\
&= -\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z)\mathbb{I}\left[p_i \leq \widetilde{p}_i(\max(w,x))\right] dx\\
&= -\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z)\mathbb{I}\left[p_i \leq \min(\widetilde{p}_i(w),\widetilde{p}_i(x))\right] dx.
\end{aligned}
$$

□

Lemma 2. *We have*

$$
\begin{aligned}
&\Big\{\omega \mid CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)\Big\}\\
&= \Big\{\omega \mid U_i^{\omega}(y-p_i) \geq \max_{c}\{U_c^{\omega}(y-p'_c-z)\}, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)\Big\}.
\end{aligned} \tag{32}
$$

Proof of Lemma 2.

$$\begin{aligned}
&\Big\{\omega \mid CV^\omega \le z, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)\Big\} \\
&= \Big\{\omega \mid \max_c\{U_c^\omega(y - p_c' - CV^\omega)\} \ge \max_c\{U_c^\omega(y - p_c' - z)\}, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)\Big\} \\
&= \Big\{\omega \mid \max_c\{U_c^\omega(y - p_c)\} \ge \max_c\{U_c^\omega(y - p_c' - z)\}, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)\Big\} \\
&= \Big\{\omega \mid U_i^\omega(y - p_i) \ge \max_c\{U_c^\omega(y - p_c' - z)\}, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)\Big\},
\end{aligned}$$

where the second equality follows from (30) and the last from $i = J^\omega(\mathbf{p},y)$. □

Theorem 3. *The joint distribution of the CV and the optimal choices before and after the price change is as follows:*

$$\Pr_\omega[CV^\omega \le z, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)] = P_{i,j}(\mathbf{min}(\mathbf{p},\mathbf{p}'+z),\mathbf{p}',y)\mathbb{I}\left[p_i \le p_i' + z\right]. \tag{33}$$

Proof of Theorem 3. We have

$$\begin{aligned}
&\Pr_\omega[CV^\omega \le z, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)] \\
&= \Pr_\omega\Big[U_i^\omega(y - p_i) \ge \max_{k\ne i} U_k^\omega(y - p_k), \quad U_j^\omega(y - p_j') \ge \max_{l\ne j} U_l^\omega(y - p_l'), \\
&\qquad\qquad U_i^\omega(y - p_i) \ge \max_c U_c^\omega(y - p_c' - z)\Big] \\
&= \Pr_\omega\Big[U_i^\omega(y - p_i) \ge \max_{k\ne i} U_k^\omega(y - \min(p_k, p_k' + z)), \quad U_j^\omega(y - p_j') \ge \max_{l\ne j} U_l^\omega(y - p_l')\Big] \\
&\qquad\qquad \mathbb{I}\left[p_i \le p_i' + z\right] \\
&= P_{i,j}(\mathbf{min}(\mathbf{p},\mathbf{p}'+z),\mathbf{p}',y)\mathbb{I}\left[p_i \le p_i' + z\right].
\end{aligned}$$

□

Theorem 4. *The joint distribution of the MMU and the CV is as follows:*

$$\begin{aligned}
&\Pr_\omega[w \le W_{M(\mathbf{p}')}^\omega(y - p_i, i), CV^\omega \le z, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)] \\
&= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}' + \min(z, y - w)\big),\mathbf{p}',y\Big)\mathbb{I}\left[p_i \le p_i' + \min(z, y - w)\right].
\end{aligned} \tag{38}$$

Proof of Theorem 4. **A direct proof of Theorem 4**

We have

$$\begin{aligned}
&\Pr_\omega[w \le W_{M(\mathbf{p}')}^\omega(y - p_i, i), CV^\omega \le z, i = J^\omega(\mathbf{p},y), j = J^\omega(\mathbf{p}',y)] \\
&= \Pr_\omega\Big[U_i^\omega(y - p_i) \ge \max_{c'} U_{c'}^\omega(y - (y - w + p_{c'}')), \quad U_i^\omega(y - p_i) \ge \max_{k\ne i} U_k^\omega(y - p_k), \\
&\qquad\qquad U_j^\omega(y - p_j') \ge \max_{l\ne j} U_l^\omega(y - p_l'), \quad U_i^\omega(y - p_i) \ge \max_c U_c^\omega(y - p_c' - z)\Big] \\
&= \Pr_\omega\Big[U_i^\omega(y - p_i) \ge \max_{k\ne i} U_k^\omega(y - \min(p_k, p_k' + y - w, p_k' + z)), \\
&\qquad\qquad U_j^\omega(y - p_j') \ge \max_{l\ne j} U_l^\omega(y - p_l')\Big]\mathbb{I}\left[p_i \le p_i' + z\right]\mathbb{I}\left[p_i \le p_i' + y - w\right] \\
&= P_{i,j}\Big(\big(p_i, \mathbf{min}\big(\mathbf{p}_{-i}, \mathbf{p}_{-i}' + \min(z, y - w)\big)\big),\mathbf{p}',y\Big)\mathbb{I}\left[p_i \le p_i' + \min(z, y - w)\right] \\
&= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}' + \min(z, y - w)\big),\mathbf{p}',y\Big)\mathbb{I}\left[p_i \le p_i' + \min(z, y - w)\right].
\end{aligned}$$

Theorem 4 as implied by Theorem 2

When choosing the MMU with the final prices as reference prices, Theorem 2 implies:

$$\begin{aligned}&\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}(y-p_i,i), CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)] \\ &= -\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z)\mathbb{I}\left[p_i \leq \min(p_i'+y-w, p_i'+y-x)\right] dx.\end{aligned} \tag{59}$$

where the function h is defined by

$$h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,s) = P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+y-\max(w,x)\big), \mathbf{min}\big(\mathbf{p}',\mathbf{p}'+y-s\big), y\Big)\mathbb{I}\left[p_j' \leq p_j'+y-s\right]. \tag{60}$$

Rewriting, (60), we obtain

$$\begin{aligned}h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,s) &= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+y-\max(w,x)\big), \mathbf{min}\big(\mathbf{p}',\mathbf{p}'+y-s\big), y\Big)\mathbb{I}\left[p_j' \leq p_j'+y-s\right] \\ &= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+y-\max(w,x)\big), \mathbf{p}', y\Big)\mathbb{I}\left[s \leq y\right],\end{aligned}$$

and hence

$$\partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z) = -P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+y-\max(w,x)\big), \mathbf{p}', y\Big)\delta(x+z-y),$$

where δ is a Dirac delta function. Plugging this in in (59), we obtain

$$\begin{aligned}&\Pr_{\omega}[w \leq W^{\omega}_{M(\mathbf{p}')}(y-p_i,i), CV^{\omega} \leq z, i = J^{\omega}(\mathbf{p},y), j = J^{\omega}(\mathbf{p}',y)] \\ &= -\int_{-\infty}^{+\infty} \partial_3 h_{i,j,\mathbf{p},\mathbf{p}'}(w,x,x+z)\mathbb{I}\left[p_i \leq \min(p_i'+y-w, p_i'+y-x)\right] dx \\ &= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+y-\max(w,y-z)\big), \mathbf{p}', y\Big)\mathbb{I}\left[p_i \leq \min(p_i'+y-w, p_i'+y-(y-z))\right] \\ &= P_{i,j}\Big(\mathbf{min}\big(\mathbf{p},\mathbf{p}'+\min(y-w,z)\big), \mathbf{p}', y\Big)\mathbb{I}\left[p_i \leq p_i'+\min(y-w,z)\right]\end{aligned}$$

as in Theorem 4. □

Proposition 2. *The conditional CDF of individual welfare in the optimal bundle can be calculated using choice probabilities:*

$$F_W(w \mid \mathbf{p},y) = 1 - \sum_k P_k\Big(\mathbf{min}\big(\mathbf{p},\widetilde{\mathbf{p}}(w)\big), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right]. \tag{45}$$

Proof of Proposition 2.

$$\begin{aligned}F_W(w \mid \mathbf{p},y) &= \Pr_{\omega}\left[W^{\omega}\left(y-p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p},y)\right) \leq w\right] \\ &= 1 - \Pr_{\omega}\left[w \leq W^{\omega}\left(y-p_{J^{\omega}(\mathbf{p},y)}, J^{\omega}(\mathbf{p},y)\right)\right] \\ &= 1 - \sum_k P_k\Big(\mathbf{min}\big(\mathbf{p},\widetilde{\mathbf{p}}(w)\big), y\Big)\mathbb{I}\left[p_k \leq \widetilde{p}_k(w)\right],\end{aligned} \tag{61}$$

where the last equality follows from Equation (21) in Corollary 1. □

Corollary 5. *When prices are equal for everyone and when one uses the MMU with reference prices equal to those common prices as the welfare measure, the SWF can be written solely in terms of income.*

Proof of Corollary 5. From Proposition 2 and the definition of the virtual prices in case of an MMU with actual prices $\mathbf{p}$ as reference prices ($\widetilde{\mathbf{p}}(w) = y - w + \mathbf{p}$), it follows that

$$\begin{aligned} F_W(w \mid \mathbf{p}, y) &= 1 - \sum_k P_k\Big(\mathbf{min}(\mathbf{p}, \widetilde{\mathbf{p}}(w)), y\Big)\mathbb{I}\left[p_k \le \widetilde{p}_k(w)\right] \\ &= 1 - \sum_k P_k\Big(\mathbf{min}(\mathbf{p}, y - w + \mathbf{p}), y\Big)\mathbb{I}\left[p_k \le y - w + p_k)\right] \\ &= 1 - \sum_k P_k\Big(\mathbf{p}, y\Big)\mathbb{I}\left[w \le y\right] \\ &= \mathbb{I}\left[y \le w\right]. \end{aligned}$$

Hence,

$$\begin{aligned} SWF &= \int\int h(w)\, dF_W(w \mid \mathbf{p}, y)\, dG(\mathbf{p}, y) \\ &= \int\int h(w) d\mathbb{I}\left[y \le w\right]\, dG(\mathbf{p}, y) \\ &= \int h(y)\, dG(\mathbf{p}, y). \end{aligned}$$

Notice that $\mathbf{p}$ in the argument of G is redundant, as prices are assumed to be identical for all persons in this case. This completes the proof. □

Proposition 3. *The transition probabilities $\{P_{i,j}(\mathbf{p}, \mathbf{p}', y)\}$ are set identified from the choice probabilities $\{P_i\}$ with bounds*

$$\begin{aligned} P^L_{i,i}(\mathbf{p}, \mathbf{p}', y) &= \max\Big\{P_i(\mathbf{p}, y) + P_i(\mathbf{p}', y) - 1, P_i\Big(\big(\max\{p_i, p'_i\}, \mathbf{min}\{\mathbf{p}_{-i}, \mathbf{p}'_{-i}\}\big), y\Big)\Big\}, \\ P^U_{i,i}(\mathbf{p}, \mathbf{p}', y) &= \min\big\{P_i(\mathbf{p}, y),\ P_i(\mathbf{p}', y)\big\}. \end{aligned} \tag{47}$$

For $i \neq j$, $P_{i,j}(\mathbf{p}, \mathbf{p}', y) = 0$ if $p_i \ge p'_i$ and $p_j \le p'_j$ and

$$\begin{aligned} P^L_{i,j}(\mathbf{p}, \mathbf{p}', y) &= \max\{P_i(\mathbf{p}, y) + P_j(\mathbf{p}', y) - 1,\ 0\}, \\ P^U_{i,j}(\mathbf{p}, \mathbf{p}', y) &= \min\left\{P_i(\mathbf{p}, y), P_j(\mathbf{p}', y)\right\}, \end{aligned} \tag{48}$$

elsewhere. These bounds are sharp.

Proof of Proposition 3. We first show that Equations (47) and (48) are valid bounds. One can immediately derive upper and lower bounds that are implied by elementary probability theory. Let A be the set $\{\omega | i = J^\omega(\mathbf{p}, y)\}$ and B the set $\{\omega | j = J^\omega(\mathbf{p}', y)\}$. We have $P(A \cap B) = P_{i,j}(\mathbf{p}, \mathbf{p}';\ y)$, $P(A) = P_i(\mathbf{p};\ y)$ and $P(B) = P_j(\mathbf{p}';\ y)$.

For the lower bound, note that

$$1 \ge P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{62}$$

and hence $P(A \cap B) \ge P(A) + P(B) - 1$ which translates into

$$P_{i,j}(\mathbf{p}, \mathbf{p}';\ y) \ge P_i(\mathbf{p};\ y) + P_j(\mathbf{p}';\ y) - 1. \tag{63}$$

For the upper bound, note that $P(A \cap B) \leq P(A)$ and $P(A \cap B) \leq P(B)$, and hence

$$P_{i,j}(\mathbf{p}, \mathbf{p}';\ y) \leq \min(P_i(\mathbf{p};\ y), P_j(\mathbf{p}';\ y)). \tag{64}$$

These inequalities coincide with those derived by Fréchet (1935).

We will now exploit the monotonicity condition imposed on the utility function U_c^ω to construct tighter bounds based on revealed preference restrictions. First consider the no-transition case. Note that if

$$U_i^\omega(y - \max\{p_i, p_i'\}) > U_k^\omega(y - \min\{p_k, p_k'\}), \tag{65}$$

then $U_i^\omega(y - p_i) > U_k^\omega(y - p_k)$ and $U_i^\omega(y - p_i') > U_k^\omega(y - p_k')$ and hence

$$P_i\big((\max\{p_i, p_i'\}, \mathbf{min}\{\mathbf{p}_{-i}, \mathbf{p}'_{-i}\}; y\big) = \Pr_\omega \left[\cap_{k \neq i} \left\{ U_i^\omega(y - \max\{p_i, p_i'\}) > U_k^\omega(y - \min\{p_k, p_k'\}) \right\} \right] \tag{66}$$

is a lower bound of $P_{i,i}(\mathbf{p}, \mathbf{p}';\ y)$.

Finally, for the transition case, some transitions are ruled out by monotonicity. Indeed, if $p_i \geq p_i'$ and $p_j \leq p_j'$, good i becomes weakly less and good j weakly more expensive after the price change. By monotonicity, it holds that $U_i^\omega(y - p_i) \leq U_i^\omega(y - p_i')$ and $U_j^\omega(y - p_j) \geq U_j^\omega(y - p_j')$, and, hence, if moreover $U_i^\omega(y - p_i) > U_k^\omega(y - p_k)$ for all $k \neq i$ and $U_j^\omega(y - p_j') > U_k^\omega(y - p_k')$ for all $k \neq j$, then

$$U_i^\omega(y - p_i') \geq U_i^\omega(y - p_i) > U_j^\omega(y - p_j) > U_i^\omega(y - p_i'), \tag{67}$$

which is a contradiction. Hence, if $p_i \geq p_i'$ and $p_j \leq p_j'$, then $P_{i,j}(\mathbf{p}, \mathbf{p}', y) = 0$.

We now show that the bounds derived above are sharp. Consider a sequence of utility functions $\{U_{i,b}^\omega : b = 1, 2, \dots\}$ for which

$$\begin{aligned} U_{i,b+1}^\omega(y - \min\{p_i, p_i'\}) &\leq U_{i,b}^\omega(y - \min\{p_i, p_i'\}), \\ U_{i,b+1}^\omega(y - \max\{p_i, p_i'\}) &= U_{i,b}^\omega(y - \max\{p_i, p_i'\}), \end{aligned} \tag{68}$$

such that $\lim_{b\to\infty} U_{i,b}^\omega(y - \min\{p_i, p_i'\}) = \lim_{b\to\infty} U_{i,b}^\omega(y - \max\{p_i, p_i'\})$, and

$$\begin{aligned} U_{c,b+1}^\omega(y - p_c) &= U_{c,b}^\omega(y - p_c), \\ U_{c,b+1}^\omega(y - p_c') &= U_{c,b}^\omega(y - p_c'), \end{aligned} \tag{69}$$

for all alternatives $c \neq i$. From the definition of the transition probabilities in (4), we have that

$$\begin{aligned} \lim_{b\to\infty} P_{i,i}^b(\mathbf{p}, \mathbf{p}', y) &:= \lim_{b\to\infty} \Pr_\omega \left[\left\{ U_{i,b}^\omega(y - p_i) \geq \max_{c \neq i}\{U_{c,b}^\omega(y - p_c)\} \right\} \cap \left\{ U_{i,b}^\omega(y - p_i') \geq \max_{c \neq i}\{U_{c,b}^\omega(y - p_c')\} \right\} \right] \\ &= \Pr_\omega \left[\lim_{b\to\infty} \left\{ U_{i,b}^\omega(y - p_i) \geq \max_{c \neq i}\{U_{c,b}^\omega(y - p_c)\} \right\} \cap \lim_{b\to\infty} \left\{ U_{i,b}^\omega(y - p_i') \geq \max_{c \neq i}\{U_{c,b}^\omega(y - p_c')\} \right\} \right] \\ &= \Pr_\omega \left[\lim_{b\to\infty} \left\{ U_{i,b}^\omega(y - \max\{p_i, p_i'\}) \geq \max_{c \neq i} \left\{ \max\{U_{c,b}^\omega(y - p_c), U_c^\omega(y - p_c')\} \right\} \right\} \right] \\ &= \Pr_\omega \left[\lim_{b\to\infty} \left\{ U_{i,b}^\omega(y - \max\{p_i, p_i'\}) \geq \max_{c \neq i} \left\{ U_{c,b}^\omega(y - \min\{p_c, p_c'\}) \right\} \right\} \right], \\ &= \lim_{b\to\infty} P_i^b \left(\max\{p_i, p_i'\}, \mathbf{min}\{\mathbf{p}_{-i}, \mathbf{p}'_{-i}\}; y \right), \end{aligned} \tag{70}$$

where the second and last equalities follow because we consider a decreasing sequence of nested events.[34]

Bivariate distributions that are on the Boole-Fréchet bounds can be constructed by using insights from copula theory. Perfect positive dependence of the choice probabilities (i.e. *co-monotonicity*) delivers the upper bound, while perfect negative dependence (i.e. *counter-monotonicity*) delivers the lower bound. Consider an additive DC-RUM, i.e. $U_c^\omega(y-p_c) := V_c(y-p_c)+\eta_c(\omega)$ for all alternatives c, for which we introduce the abbreviations $V_c = V_c(y-p_c)$ and $V_c' = V_c(y-p_c')$.

Suppose that $\eta_c(\omega) = 0$ for all c except for a $k \neq i, j$. The transition probability $P_{i,j}(\mathbf{p}, \mathbf{p}', y)$ is then equal to

$$\begin{aligned}
P_{i,j}(\mathbf{p},\mathbf{p}',y) &= \Pr_\omega\left[\{V_i - V_k \geq \eta_k(\omega)\} \cap \left\{V_j' - V_k' \geq \eta_k(\omega)\right\}\right] \mathbb{I}[V_i > V_c, \forall c \neq i,k]\mathbb{I}[V_j' > V_c', \forall c \neq j,k] \\
&= \Pr_\omega\left[\left\{\min\{V_i - V_k, V_j' - V_k'\} \geq \eta_k(\omega)\right\}\right] \mathbb{I}[V_i > V_c, \forall c \neq i,k]\mathbb{I}[V_j' > V_c', \forall c \neq j,k] \\
&= \min\left\{\Pr_\omega[V_i - V_k \geq \eta_k(\omega)], \Pr_\omega[V_j' - V_k' \geq \eta_k(\omega)]\right\} \mathbb{I}[V_i > V_c, \forall c \neq i,k]\mathbb{I}[V_j' > V_c', \forall c \neq j,k] \\
&= \min\left\{\Pr_\omega[V_i - V_k \geq \eta_k(\omega)]\mathbb{I}[V_i > V_c, \forall c \neq i,k], \Pr_\omega[V_j' - V_k' \geq \eta_k(\omega)]\mathbb{I}[V_j' > V_c', \forall c \neq j,k]\right\} \\
&= \min\{P_i(\mathbf{p},y), P_j(\mathbf{p}',y)\},
\end{aligned} \tag{71}$$

which is the Boole-Fréchet upper bound.

Now suppose that $\eta_c(\omega) = 0$ for all c except i, for which it is uniformly distributed on the unit interval, and suppose that $\mathbb{I}[V_j' > V_c', \forall c \neq j, i] = 1$ and that $0 \leq -\min_{c\neq i}\{V_i - V_c\} < V_j' - V_i' \leq 1$. In that case, the transition probability is equal to

$$\begin{aligned}
P_{i,j}(\mathbf{p},\mathbf{p}',y) &= \Pr_\omega\left[\left\{\min_{c\neq i}\{V_i - V_c\} \geq -\eta_i(\omega)\right\} \cap \left\{V_j' - V_i' \geq \eta_i(\omega)\right\}\right] \mathbb{I}[V_j' > V_c', \forall c \neq j, i] \\
&= \Pr_\omega\left[\left\{-\min_{c\neq i}\{V_i - V_c\} \leq \eta_i(\omega) \leq V_j' - V_i'\right\}\right] \mathbb{I}[V_j' > V_c', \forall c \neq j, i] \\
&= \left(\left(V_j' - V_i'\right) + \min_{c\neq i}\{V_i - V_c\}\right) \mathbb{I}[V_j' > V_c', \forall c \neq j, i] \\
&= P_j(\mathbf{p}',y) + P_i(\mathbf{p},y) - 1,
\end{aligned} \tag{72}$$

which is the Boole-Fréchet lower bound. □

[34] Recall that for a decreasing sequence of events $A_1 \supset A_2 \supset A_3 \supset \ldots$ with limit $A = \cap_{m=1}^{\infty} A_m$, it holds that $\lim_{n\to\infty} \Pr[A_n] = \Pr[A]$.

C Empirical illustration

In Section C.1, we present descriptive statistics for the estimation subsample of single females. In Section C.2, we detail our estimation procedure. Section C.3 contains the estimated parameters. In Section C.4, we provide details on the implementation of the aggregation across individuals and on the construction of the distribution of welfare differences conditional on initial welfare. Section C.5 contains additional empirical results.

C.1 Sample description

Table 2: Descriptive statistics estimation subsample

Variable	N	Min	Q_1	Median	Mean	Q_3	Max	SD	IQR
Weekly hours worked	1922	0.0	0.0	30.0	25.2	40.0	80.0	18.3	40.0
Hourly gross wage	1492	4.1	9.5	14.0	14.9	19.0	39.7	6.9	9.5
Yearly income from assets	1922	-5.0	0.0	3.0	260.8	131.5	11998.0	1085.5	131.5
Age	1922	18.0	34.0	45.0	43.3	53.0	60.0	11.1	19.0
Years education	1922	7.0	10.5	11.5	12.2	14.0	18.0	2.8	3.5
Number of children (all)	1922	0.0	0.0	0.0	0.5	1.0	3.0	0.8	1.0
Number of children (0-1)	1922	0.0	0.0	0.0	0.0	0.0	2.0	0.1	0.0
Number of children (2-4)	1922	0.0	0.0	0.0	0.0	0.0	2.0	0.2	0.0
Number of children (5-7)	1922	0.0	0.0	0.0	0.1	0.0	2.0	0.3	0.0
East Germany	1922	0.0	0.0	0.0	0.2	0.0	1.0	0.4	0.0

C.2 Estimation procedure

Choice probabilities We model the choice probabilities for alternatives PT and FT, using for each a flexible regression specification that contains cubic polynomials for the disposable income of all three alternatives, and a linear index with demographic variables, such as individual ℓ's age, years of education, number of children, and region. We use a logistic transformation to ensure that the probabilities are bounded between zero and one. Formally, we have for $c \in \{PT, FT\}$ that

$$\Pr_{\omega}[c = J^{\omega}(\mathbf{d}_\ell) \mid \mathbf{x}_\ell] := \Lambda_c(\mathbf{d}_\ell, \mathbf{x}_\ell; \boldsymbol{\theta}_c) = \left(1 + \exp\left(-\left(\alpha_c + \sum_{c'} \sum_{m=1}^{3} \beta_{c,c',m} d_{\ell,c'}^{m} + \mathbf{x}_\ell' \boldsymbol{\gamma}_c\right)\right)\right)^{-1},$$

where $\mathbf{d}_\ell := (d_{NW,\ell}, d_{PT,\ell}, d_{FT,\ell})$ is a vector with disposable incomes, $\mathbf{x}_\ell$ a vector with demographic characteristics, and $\boldsymbol{\theta}_c = (\alpha_c, \boldsymbol{\beta}_c, \boldsymbol{\gamma}_c)$ a vector with parameters. Alternative NW is defined as the complement of these two probabilities,

$$\begin{aligned} \Pr_{\omega}[NW = J^{\omega}(\mathbf{d}_\ell) \mid \mathbf{x}_\ell] &:= 1 - \sum\nolimits_{c \in \{PT,FT\}} \Pr_{\omega}[c = J^{\omega}(\mathbf{d}_\ell) \mid \mathbf{x}_\ell] \\ &= 1 - \sum\nolimits_{c \in \{PT,FT\}} \Lambda_c(\mathbf{d}_\ell, \mathbf{x}_\ell; \boldsymbol{\theta}_c), \end{aligned}$$

which ensures that for every pair $(\mathbf{d}_\ell, \mathbf{x}_\ell)$, the choice probabilities add up to one. The model is then estimated by nonlinear least squares

$$\widehat{\boldsymbol{\theta}}^{NLS} = \arg\min_{(\boldsymbol{\theta}_{PT},\boldsymbol{\theta}_{FT})} \sum_{\ell} \Bigg[\Bigg[Y_{NW,\ell} - \Big(1 - \sum\nolimits_{c\in\{PT,FT\}} \Lambda_c(\mathbf{d}_\ell, \mathbf{x}_\ell; \boldsymbol{\theta}_c)\Big) \Bigg]^2 + \sum\nolimits_{c\in\{PT,FT\}} \Big[Y_{c,\ell} - \Lambda_c(\mathbf{d}_\ell, \mathbf{x}_\ell); \boldsymbol{\theta}_c) \Big]^2 + \pi(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) \Bigg],$$

where $\{Y_{c,\ell}, c \in \{NW, PT, FT\}\}$ are dummy variables that encode individual ℓ's observed choice. The last term, i.e. $\pi(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT})$, contains a positive-valued penalty function that imposes non-negativity of $\Pr_\omega[NW = J^\omega(\mathbf{d}_\ell) \mid \mathbf{x}_\ell]$ and the shape constraints discussed in Section 5.2.

Penalty function The penalty function consists of three components. Since some of these components depend in a complex way on both the data and the parameters, we opt to impose these on a three dimensional grid $\mathcal{D}_G$ of disposable incomes: i.e. $\mathcal{D}_G \subset \mathcal{D} := \text{supp}(d_{NW}) \times \text{supp}(d_{PT}) \times \text{supp}(d_{FT})$. Let $\mathcal{X}$ denote the set of observed values of the individual demographic characteristics $\mathbf{x}_\ell$.

The first component of the penalty function ensures that the choice probability of alternative NW is non-negative for every pair $(\mathbf{d}_g, \mathbf{x}_\ell) \in \mathcal{D}_G \times \mathcal{X}$: i.e. $1 - \Lambda_{PT}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{PT}) - \Lambda_{FT}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{FT}) \geq 0$. The contribution to the penalty function then is defined as:

$$\pi_1(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) := -|\mathcal{D}_G|^{-1} \sum_{\mathbf{d}_g \in \mathcal{D}_G} \min(0, 1 - \Lambda_{PT}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{PT}) - \Lambda_{FT}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{FT})).$$

The second component of the penalty function ensures that choice probabilities for alternatives PT and FT are increasing in their own disposable income and decreasing in the disposable income of the other alternatives. To be precise, we have for every $c \in \{PT, FT\}$ and for every pair $(\mathbf{d}_g, \mathbf{x}_\ell) \in \mathcal{D}_G \times \mathcal{X}$ that

$$\frac{\partial \Lambda_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c)}{\partial d_{g,c}} = (\beta_{c,c,1} + 2\beta_{c,c,2} d_{g,c} + 3\beta_{c,c,3} d_{g,c}^2) \Gamma_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c) \geq 0$$
$$\frac{\partial \Lambda_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c)}{\partial d_{g,c'}} = (\beta_{c,c',1} + 2\beta_{c,c',2} d_{g,c'} + 3\beta_{c,c',3} d_{g,c'}^2) \Gamma_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c) \leq 0, \quad \forall c' \neq c,$$

in which $\Gamma_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c) := \Lambda_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c) \left(1 + \exp\left(\alpha_c + \sum_{c'} \sum_{m=1}^{3} \beta_{c,c',m} d_{g,c'}^m + \mathbf{x}_\ell' \boldsymbol{\gamma}_c\right)\right)^{-1}$, or equivalently that

$$(\beta_{c,c,1} + 2\beta_{c,c,2} d_{g,c} + 3\beta_{c,c,3} d_{g,c}^2) \geq 0$$
$$(\beta_{c,c',1} + 2\beta_{c,c',2} d_{g,c'} + 3\beta_{c,c',3} d_{g,c'}^2) \leq 0, \quad \forall c' \neq c.$$

The contribution to the penalty function is defined as:

$$\pi_{2,c}(\boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) := -|\mathcal{D}_G|^{-1} \sum_{\mathbf{d}_g \in \mathcal{D}_G} \min(0, \beta_{c,c,1} + 2\beta_{c,c,2} d_{g,c} + 3\beta_{c,c,3} d_{g,c}^2))$$
$$\pi_{2,c,c'}(\boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) := |\mathcal{D}_G|^{-1} \sum_{\mathbf{d}_g \in \mathcal{D}_G} \max(0, \beta_{c,c',1} + 2\beta_{c,c',2} d_{g,c'} + 3\beta_{c,c',3} d_{g,c'}^2)).$$

Finally, the third part of the penalty function ensures that the choice probability of alternative NW is decreasing in the disposable income of the other alternatives.[35] For every $c, c' \in \{PT, FT\}$ with $c \neq c'$ and

[35] Note that the second part of the penalty function also ensures that the choice probability of alternative NW is increasing in its own disposable income.

for every pair $(\mathbf{d}_g, \mathbf{x}_\ell) \in \mathcal{D}_G \times \mathcal{X}$, we have that

$$\frac{\partial \Lambda_c(\mathbf{d}_g, \mathbf{x}_\ell)}{\partial d_{g,c}} + \frac{\partial \Lambda_{c'}(\mathbf{d}_g, \mathbf{x}_\ell)}{\partial d_{g,c}} = (\beta_{c,c,1} + 2\beta_{c,c,2} d_{g,c} + 3\beta_{c,c,3} d_{g,c}^2)\Gamma_c(\mathbf{d}_c, \mathbf{x}_\ell; \boldsymbol{\theta}_c)$$
$$+ (\beta_{c',c,1} + 2\beta_{c',c,2} d_{g,c} + 3\beta_{c',c,3} d_{g,c}^2)\Gamma_{c'}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{c'}) \geq 0.$$

The contribution to the penalty function is defined as:

$$\pi_{3,c,c'}(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) := -|\mathcal{D}_G|^{-1} \sum_{\mathbf{d}_g \in \mathcal{D}_G} \min(0, (\beta_{c,c,1} + 2\beta_{c,c,2} d_{g,c} + 3\beta_{c,c,3} d_{g,c}^2)\Gamma_c(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_c)$$
$$+ (\beta_{c',c,1} + 2\beta_{c',c,2} d_{g,c} + 3\beta_{c',c,3} d_{g,c}^2)\Gamma_{c'}(\mathbf{d}_g, \mathbf{x}_\ell; \boldsymbol{\theta}_{c'})).$$

Arranging all components, the composite penalty function is then

$$\pi(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) = \pi_1(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT})$$
$$+ \sum_{c \in \{PT, FT\}} \left[\pi_{2,c}(\boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) + \sum_{c' \in \{NW, PT, FT\}, c' \neq c} \pi_{2,c,c'}(\boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) \right]$$
$$+ \pi_{3,PT,FT}(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}) + \pi_{3,FT,PT}(\mathbf{x}_\ell; \boldsymbol{\theta}_{PT}, \boldsymbol{\theta}_{FT}).$$

C.3 Estimates

Table 3 contains the estimates for the choice probabilities of alternatives PT and FT. The 90% confidence intervals are obtained by a bootstrap procedure, in which the model was re-estimated on 200 samples randomly drawn with replacement.

Table 3: Estimates choice probabilities

	PT		**FT**	
Parameter	**Estimate**	**90% CI**	**Estimate**	**90% CI**
Constant	-1.82	[-2.18, -1.25]	-2.41	[-4.00, -2.30]
$(d_{NW}/1000)$	0.00	[-0.06, 0.00]	-0.01	[-0.02, 0.00]
$(d_{NW}/1000)^2$	0.00	[-0.01, 0.08]	-0.08	[-0.20, 0.06]
$(d_{NW}/1000)^3$	0.00	[-0.08, 0.00]	-0.04	[-0.13, 0.04]
$(d_{PT}/1000)$	0.51	[0.01, 0.65]	-0.14	[-0.20, 0.00]
$(d_{PT}/1000)^2$	-0.29	[-0.27, 0.00]	0.06	[-0.02, 0.14]
$(d_{PT}/1000)^3$	0.06	[0.00, 0.07]	-0.01	[-0.06, 0.00]
$(d_{FT}/1000)$	0.00	[-0.06, 0.00]	1.09	[0.95, 2.50]
$(d_{FT}/1000)^2$	-0.05	[-0.22, 0.00]	-0.27	[-0.60, -0.16]
$(d_{FT}/1000)^3$	0.01	[0.00, 0.03]	0.03	[0.01, 0.05]
Age	0.01	[0.00, 0.02]	-0.01	[-0.01, 0.00]
Years education	-0.01	[-0.03, 0.03]	0.13	[0.10, 0.15]
Number of children (0-1)	-2.72	[-3.50, -2.29]	-2.62	[-3.42, -2.26]
Number of children (2-4)	-0.67	[-1.29, -0.42]	-1.08	[-1.18, -0.55]
Number of children (5-7)	0.26	[-0.15, 0.45]	-0.29	[-0.50, -0.01]
Number of children (all)	0.17	[0.06, 0.39]	-0.20	[-0.49, -0.13]
East Germany	-0.06	[-0.37, 0.04]	0.13	[-0.02, 0.35]

C.4 Details on the implementation

Aggregation across individuals In Figure 6, we plotted individual CDFs, i.e. for an individual ℓ with demographic characteristics $\mathbf{x}_\ell$ and a wage belonging to quartile q, $\Pr_\omega[W^\omega \leq w \mid \mathbf{p}_\ell, y_\ell, \mathbf{x}_\ell, i = J^\omega(\mathbf{p}_\ell, y_\ell, \mathbf{x}_\ell), q]$.

When aggregating these distributions to a group level, we lower the level of conditioning by, e.g., integrating out over prices, exogenous income, and demographic characteristics. Formally, we have that

$$\begin{aligned}&\Pr_\omega[W^\omega \leq w \mid i = J^\omega(\mathbf{p}, y, \mathbf{x}), q] = \\ &\qquad \int \Pr_\omega[W^\omega \leq w \mid \mathbf{p}, y, \mathbf{x}, i = J^\omega(\mathbf{p}, y, \mathbf{x}), q]\, dG(\mathbf{p}, y, \mathbf{x} \mid i = J^\omega(\mathbf{p}, y, \mathbf{x}), q).\end{aligned} \tag{73}$$

The sample analogue of Equation (73) is:

$$\Pr_\omega[W^\omega \leq w \mid i = J^\omega(\mathbf{p}, y, \mathbf{x}), q] = \sum_\ell \frac{\Pr_\omega[W^\omega \leq w \mid \mathbf{p}_\ell, y_\ell, \mathbf{x}_\ell, i = J^\omega(\mathbf{p}_\ell, y_\ell, \mathbf{x}_\ell), q = q_\ell]}{|\{\ell \mid i = J^\omega(\mathbf{p}_\ell, y_\ell, \mathbf{x}_\ell), q = q_\ell\}|}. \tag{74}$$

Distribution of welfare differences conditional on initial welfare The practical implementation of the results in Theorem 2 poses some difficulties. Firstly, the distribution depends on transition probabilities. As was noted in Section 5, with only cross-sectional data available, these transition probabilities can only be set-identified. We, therefore, calculated upper and lower bounds for the joint distribution of baseline and reform welfare levels (i.e. Equation (26) of Proposition 1). As lower and upper bounds in the aggregate are not far apart (at most a 4 percentage point difference), we continue the analysis using only the upper bound. Secondly, Equation (29) requires integration over the derivative of a transition probability, which is quite cumbersome.

We first calculate the joint distribution of baseline and reform welfare by integrating out the optimal baseline and reform choices in Equation (26) of Proposition 1. The resulting joint distribution function of initial and post reform welfare is denoted by $H_0(w, s)$, that is:

$$\begin{aligned}&H_0(w, s, \mathbf{p}, \mathbf{p}', y) = Pr\,(w \leq W_0, s \leq W_1) = \\ &\textstyle\sum_{i,j \in \{NW,PT,FT\}} \Pr_\omega\left[w \leq W_0^\omega(y - p_i, i), s \leq W_1^\omega(y - p'_j, j), i = J^\omega(\mathbf{p}, y), j = J^\omega(\mathbf{p}', y)\right].\end{aligned} \tag{75}$$

As we are interested in this distribution at the population level rather than at the individual level, we aggregate the distribution H_0 by defining

$$H(w, s) = \int H_0(w, s, \mathbf{p}, \mathbf{p}', y)\, dG(\mathbf{p}, \mathbf{p}', y), \tag{76}$$

where G is the distribution of prices and exogenous income in the population. The joint distribution of baseline welfare and the welfare gain, $Pr\,(w \leq W_0, z \geq W_1 - W_0)$, is then calculated by:

$$\begin{aligned}Pr\,(w \leq W_0, z \geq W_1 - W_0) &= \textstyle\int Pr\,(w \leq W_0, s - z \leq W_0, s = W_1)\; ds \\ &= -\textstyle\int \partial_2 H(\max(w, s - z), s)\, ds.\end{aligned} \tag{77}$$

The integral and derivative in this equation can be approximated numerically. Note that this is an approximation of Equation (29), aggregated across the population.

The distribution of gains and losses conditional on the initial welfare level can then be calculated as follows:

$$\begin{aligned} Pr\left(z \leq W_1 - W_0 \,|w = W_0\right) &= Pr\left(z + w \leq W_1 \,|w = W_0\right) \\ &= \tfrac{Pr(z+w\leq W_1, w=W_0)}{Pr(w=W_0)} \\ &= \tfrac{\partial_1 H(w, z+w)}{\partial_1 H(w, -\infty)}. \end{aligned} \tag{78}$$

Again, derivatives can be computed numerically.

C.5 Additional empirical results

Individual welfare distribution Figure 12 shows the estimates of the individual unconditional welfare distributions (i.e. Equation (21) in Corollary 1). Contrary to what was the case for the conditional individual welfare distributions in Figure 6 of the main text, we now take into account that some may have found another optimal alternative instead of that chosen by the sampled individual. While the former distributions turn out to exhibit several mass points, the deterministic cases seem to vanish. This is to be expected, as conditioning on observed choices introduces information that restricts the set of preference types. Therefore, the conditional distributions are 'less stochastic' than their associated marginal distributions.

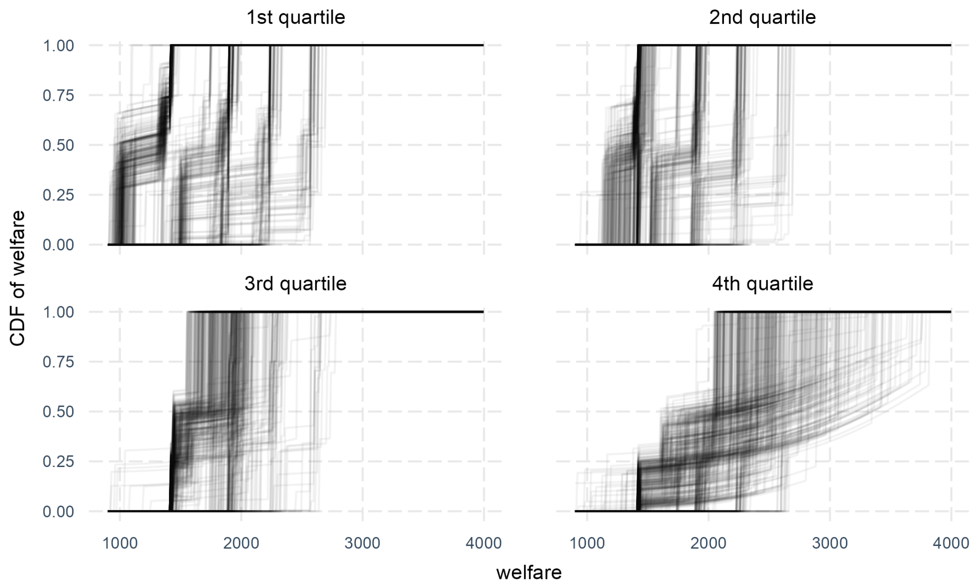
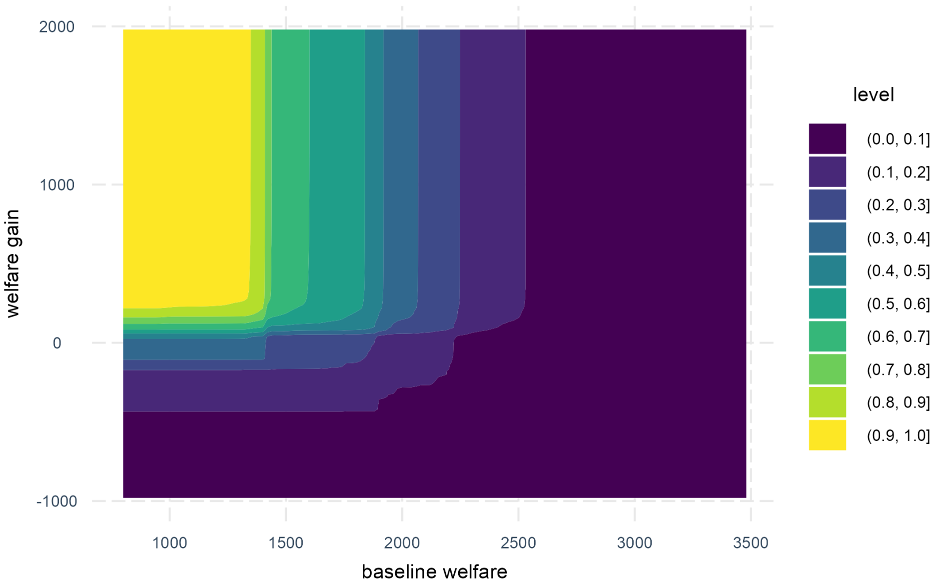


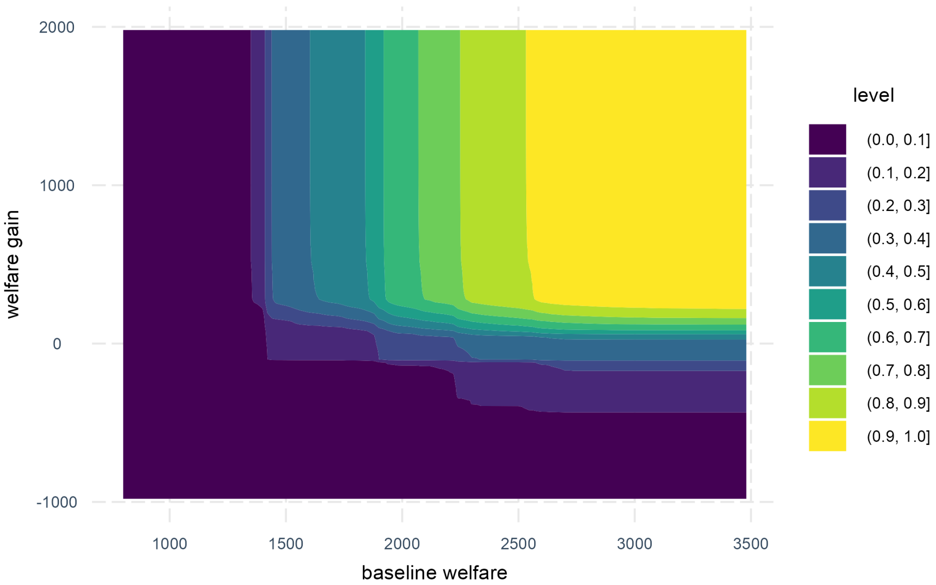
Figure 12: Unconditional individual welfare distribution by wage quartile

Winners and losers Figures 13a and 13b plot the joint distribution of baseline welfare and welfare differences (i.e. Equation (77)). In Figure 13a, the coordinates (w, z) of a point on the q-th iso-contour indicate the initial welfare level w and welfare gain z (or loss, if z is negative) such that $q\%$ of the population obtains at least that initial welfare level w and does not gain more than z. In Figure 13b, the initial welfare level w denotes the maximum level, rather than the minimum, that that number of people reach.

Figure 14 shows a more detailed, and less smoothed, version of the distribution of gains and losses conditional on baseline welfare (Figure 11 in the main text). The upper boundary of the yellow region tends to be around zero for higher welfare levels. For moderate initial welfare levels, there are several regions where the median, that is the lower bound of the blue region, falls under the zero of the vertical axis. This confirms the findings highlighted in the main text.



(a) $W_0 \geq w$



(b) $W_0 \leq w$

Figure 13: Joint distribution of baseline welfare and welfare gains/losses

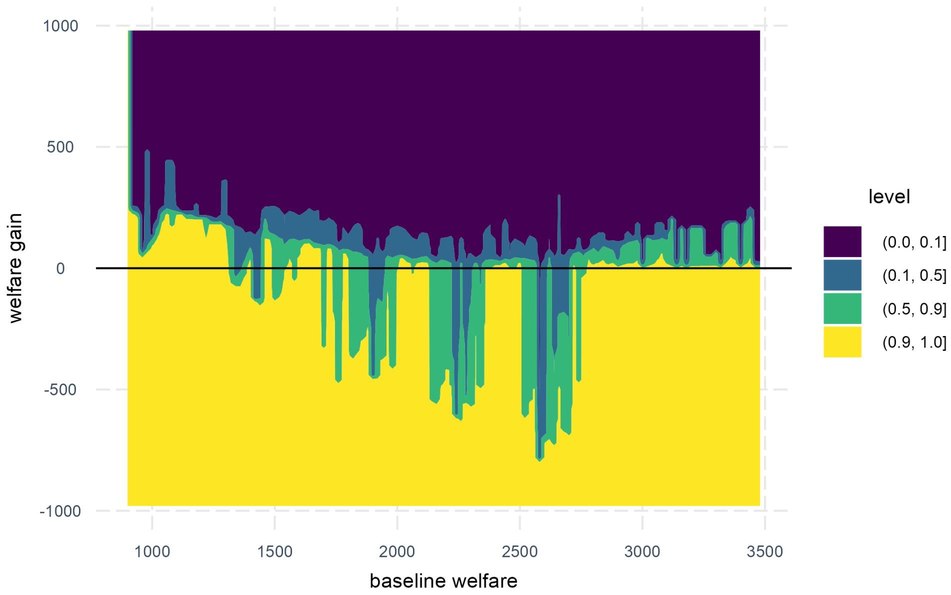


Figure 14: Distribution of welfare gains and losses, conditional on baseline welfare