



ÉCOLE  
POLYTECHNIQUE  
DE BRUXELLES  
UNIVERSITÉ LIBRE DE BRUXELLES



University of  
TEHRAN

## Robust optimization of turbomachines using efficient methods

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in fulfilment of the requirements of the PhD Degree in Sciences de l'ingénieur et technologie (ULB - "Docteur en Sciences de l'ingénieur et technologie") and in Mechanical engineering (UT)  
Année académique 2021-2022

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*Dedicated to  
my beloved parents,  
my wife, and sons.*



## **Acknowledgments**

This thesis was accomplished with the help and support of some people whom I should appreciate.

First, I would like to express my sincere gratitude to my dear professors, Dr. Nourbakhsh, Dr. Raisee, and Dr. Hendrick, who have always been my guides. Their efforts were not limited to the dissertation, and they have supported me in all fields. I thank God who gave me the honor of his discipleship. I would also like to thank Dr. Mohammad Farhat. He provided me the opportunity of conducting research and gain experience at EPFL.

I sincerely thank my dear parents who helped me during my studies as their immediate support throughout my life. I ask God to help me to repay some of their favor. I highly appreciate my devoted wife. She sacrificial provided me the opportunity to complete my doctoral studies in the best possible way and alleviated the problems of this path for my children and me. I thank my brother and sisters, who did not hesitate to help, whenever I needed supporting me with their sympathy.

I also sincerely thank all my friends who helped me complete the various sections of this dissertation, and I hope I can fulfill this right of friendship well.



## Abstract

Robust optimization has been developed to control the sensitivity of optimal design performance against deviations imposed by the system design conditions. In fact, a product designed using robust optimization techniques can withstand the uncertainties in operational conditions, physical properties, geometrical characteristics, etc. This means that robust optimization increases the probability of achieving the expected performance, even under the inevitable uncertainties. Like many industrial applications, uncertainties generally present in turbomachines. Turbomachines are used in a wide range of industries, and their performance is considerably affected by the large uncertainties in their operational and geometrical conditions. Meanwhile, the investigations on the robust optimization of turbomachines are so limited. The main challenges in achieving a robust optimization of turbomachines are the high computational cost of the procedure and methods for assessing the robustness of system performance. Accordingly, the current thesis is aimed at, firstly, representing the importance of robust optimization in turbomachines, and secondly, developing the required tools to achieve this goal. Robust optimization is carried out by coupling both optimization and uncertainty quantification (UQ) methods. Therefore, it is highly appreciated to use a combination of efficient algorithms for optimization and UQ to reduce computational costs. It should be noted that several optimization methods with acceptable performance have been developed by researchers. On the other hand, uncertainty quantification has been recently utilized in engineering problems, and it needs more attention for developing efficient methods. Therefore, to reduce the computational cost of robust optimization, this study focuses on developing efficient methods of uncertainty quantification. Simple uncertainty quantification procedures such as sampling methods have a low convergence rate. To overcome this issue, some approaches have been developed, such as full polynomials chaos expansion. However, this method lacks the required efficiency when utilized in robust optimization problems with large stochastic spaces, like engineering applications. To increase the performance of the mentioned method, a compressive sensing framework based on Bayesian theory, which has the inherent ability of adaptive and multi-fidelity sampling, has been applied for probabilistic study in the current thesis. The method is applied to challenging test cases of turbomachines. Then, its accuracy and efficiency are investigated by comparing the results with the full polynomial chaos expansion method. It is well shown that the computational gain of the presented approach is significant in the problems faced with the curse of dimensionality. Afterward, using the developed method, the physics of cavitating tip leakage vortex is investigated under uncertainties. The results clarified that the operational and geometrical uncertainties lead to undeniable

variations in the vortex flow characteristics, and ignoring such uncertainties leads to inaccurate flow analysis. As discussed, the second challenge of robust optimization is developing assessment techniques to investigate system behavior's robustness. To this end, there are some standard criteria that generally use a number of statistical characteristics of the system performance. This issue makes these criteria impotent to provide a comprehensive assessment of product performance. Accordingly, the current thesis introduces a new criterion based on the cumulative distribution function of the stochastic quantity of interest. At first, the capability of the introduced criterion in finding the robust optimal point is proved by comparing the results given by the novel and common criteria in a number of analytical test functions. This introduced technique is then used for robust optimization of marine current turbine performance as the first turbomachinery cases. Marine turbines convert free stream kinetic energy into power. Their operating conditions are associated with large uncertainties, and thus, the generated power of the deterministic optimum turbine, as expected, deals with large variations. Our study showed that the obtained robust optimum turbine has smaller variations, and the system performance will be more stable than the deterministic optimum case. Moreover, the possibility of producing greater power is higher in robust optimum turbine design. The methods developed in the current study have also been utilized for obtaining a robust optimum design of the internal cooling system of C3X, a well-known gas turbine vane. Accordingly, the blade's temperature field of robust optimum design is compared with the baseline and deterministic optimum blades. Results showed that, although the maximum temperature of the deterministic optimum blade is less than that of robust optimum design on average, the high sensitivity of the deterministic design against the presence of uncertainties leads to a considerable variation on its temperature field. Accordingly, the robust optimum design of the blade internal cooling channels is more efficient.

**Keywords:**

Robust optimization, Turbomachines, Uncertainty quantification, Polynomial chaos expansion, Robustness criterion, Efficient methods



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