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Abstract

We present a methodology for the structural empirical analysis of household consumption and time use behaviour under marital stability. Our approach is of the revealed preference type and non-parametric, meaning that it does not require a prior functional specification of individual utilities. Without making use of the transferable utility assumption, but still allowing for monetary transfers, our method can identify individuals' unobserved

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match qualities and quantify them in money metric terms. We can include both preference factors, affecting individuals' preferences over private and public goods, and match quality factors, driving differences in unobserved match quality. We demonstrate the practical usefulness of our methodology through an application to the Belgian MEqIn data. Our results reveal intuitive patterns of unobserved match quality that allow us to rationalise both the observed matches and the within-household allocations of time and money.

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1 Introduction

This is a paper about who marries whom and who gets what within formed households. Following Becker (1973) a large literature has developed on these issues. This literature is largely split into two separate strands. One strand focuses on who marries whom, while the other mainly focuses on the intrahousehold allocations of time and money within existing unions (see Browning, Chiappori, and Weiss, 2014, for a survey). Although the two strands often intersect in the theoretical literature, there is a paucity of empirical analyses that take into account the interactions between matching patterns and intrahousehold allocations. There has always been a perception that it would be desirable to develop such an overarching framework for empirical work but it is only very recently that progress has been made (see, for example, Cherchye, De Rock, Surana, and Vermeulen, 2020; Cherchye, Demuynck, De Rock, and Vermeulen, 2017; Goussé, Jacquemet, and Robin, 2017; Weber, 2018). In this paper we present a novel approach to this issue.

A simple example will motivate our approach. Suppose we have a survey of married couples with information on time use such as the market work, housework and leisure of each partner in the household. The same survey also collects information on expenditures for private goods for each individual and for intrahousehold public goods. In one of these households we observe that the woman does a lot of market work and housework relative to her partner. She also spends less on private goods than her partner. Moreover, she has a higher wage than her spouse. This is a puzzle if we consider only material welfare, since it looks like this woman could do much better by finding an alternative match which entails less work, at least as much public goods and more private expenditure for her. Clearly, there is something other than material considerations that is keeping her in the marriage. If the marriage is to be stable, her partner must have some attributes that are considered positively by his potential partners or she must have some negative attributes, or they must have attributes which are highly complementary. In short, as Becker (1973) emphasised, this requires us to consider matching and intrahousehold allocations simultaneously.

In this paper we present estimates of a structural empirical model that simultaneously takes account of detailed information on within-household allocations and the stability of the observed matching in a competitive marriage market. When we have rich data that include time use and private and public expenditures within the same household, we do see matches which seem unstable from a material point of view, albeit few are as extreme as the example in the previous paragraph (see, for example, Browning and Gørtz, 2012; Cherchye, De Rock, and Vermeulen, 2012)).[1]

Empirical analyses of within-household allocations routinely consider how outside options in the marriage market impact the intrahousehold allocation of time and expenditures, but this is usually done through "reduced form" accounting for the impact of "distribution factors" on the distribution of power within the household (for example, Pareto weights). These distribution factors include within-household variables such as the relative wages of the two partners, individual education levels, individual attitudes to family values and societal factors such as divorce settlement legislation or sex ratios in the local marriage market. See, for example, Browning, Chiappori, and Weiss (2014, table 5.1), for a listing of 17 such factors that have been used in the empirical literature. In the other strand of the literature, empirical analyses of marriage matching take limited account of observed within-household allocation of time and money (see, for example, Chiappori, 2017).

In what follows we develop a way to model matching with transfers within the household that incorporates unobserved match quality. We term this model "Additive Quantity Shifting" (AQS). Our model yields a money metric measure

[1]Notice that it is only when we have information on both time use *and* expenditures that a puzzle arises. If we do not observe expenditures we could rationalise the observation that she works more by allowing that she receives more private expenditures (for example, the data of Goussé, Jacquemet, and Robin (2017) has time use information but no information on private or public expenditures). Conversely, having only consumption information, we can rationalise any matching allocation by appealing to unobserved differences in leisure.

of unobservable match quality for a given match. It includes some preference structures that are sufficient for transferable utility (TU), such as quasi-linearity, but it is not nested within the class of Affine Conditional Indirect Utility (ACIU) preferences that are also necessary for TU (see Chiappori and Gugl, 2020). TU is an unattractive model since it imposes very strong restrictions on preferences. Under TU, households behave like a single individual, which, as many authors have remarked, makes the intrahousehold allocation literature largely redundant. By contrast, AQS allows for non-unitary households with match quality and "caring" within the household and allows for much more flexible demand systems than those allowed by ACIU. Formally our model is an imperfectly transferable utility model (ITU) (see, for example, Chiappori, 2017; Galichon, Kominers, and Weber, 2019). Compared to the transferable utility case, relatively little is known theoretically about the ITU case. In this paper we do not attempt to address issues such as the existence and uniqueness of a stable matching equilibrium for our model. Rather, we focus on the empirical problem of testing whether the matching patterns and within-household allocations that we observe for a given sample of the population is stable. If it is stable, we empirically evaluate the trade-off between the material and non-material match surpluses associated with alternative marital matchings.

Our approach requires a cross-section of households (married couples and s-ingles) and their expenditure and time use allocations. We specify observable preference factors, which affect individuals' preferences over private and public consumption and time use directly. In addition, we specify observable match quality factors which drive differences in unobserved match quality. These preference and match quality factors allow us to stratify individuals into observable "preference types" and "match quality types". Male and female individuals of the same preference type have homogeneous preferences over goods and time use. Individuals of the same marital quality type experience the same unobserved match quality.

The basic ingredient of our methodology is the revealed preference characterisation of marital stability in terms of intrahousehold allocation patterns that include both observed consumption and time use and unobserved match quality. This extends work of Cherchye, Demuynck, De Rock, and Vermeulen (2017) by including our concepts of unobserved match quality, preference types and match quality types. While our main focus is on a general specification of individual u-tilities over material consumption and match quality, we also consider the specific instance of quasi-linear utilities with additive match quality, which is often used in the existing theoretical and empirical literature on the analysis of marriage market behaviour. Our characterisation defines testable implications to check whether the observed marriage allocations can be rationalised in terms of stable marital matchings. Our testable conditions are necessary and sufficient for rationalisability: the observed consumption and time use allocations are consistent with marital stability if and only if they satisfy the conditions. A distinguishing feature of our characterisation is that it is intrinsically nonparametric in the tradition of Afriat (1967), Diewert (1973) and Varian (1982), meaning that it does not require a prior functional specification of individual utilities. The testable conditions are linear in unknowns, which makes them easy to check in practical applications. As we will explain, they also provide a productive basis for the nonparametric set identification of unobserved aspects of spouses' individual preferences and intrahousehold allocation patterns.

We focus on cross-sectional conditions for marital stability in a frictionless marriage market with within-household transfers (Becker, 1973; Shapley and Shubik, 1972). We largely abstract from intertemporal considerations and frictions that drive marital choice behaviour. Admittedly, this implies a substantial simplification of a very complex reality. However, the notion of marital stability that we consider here is a natural equilibrium concept to start from when studying marriage and consumption allocations at the level of male and female types, which is our core research question.[2] Evidently, intertemporal aspects and frictions on the marriage market do become particularly relevant when focusing on household decisions with a long-term impact (for example, related to fertility) and/or dynamic aspects of observed marriage and divorce patterns (which may rather require a search model to explain the matching allocations). Allowing for these features in our structural framework falls beyond the scope of the current study. Instead we use so-called stability indices to quantify violations of our implicit model assumptions due to frictions and unobserved preference heterogeneity. These stability indices allow us to account for deviations from exact market stability in our empirical analysis.

Our empirical application uses the Belgian MEqIn data set, which provides information on marital status for a cross-section of Belgian households. This survey also provides measures of individual's leisure, domestic work and the consumption of a Hicksian aggregate private and public commodity. We use age, education level and the presence of children as preference factors to define 12 male and fe-

[2]This also explains why this stability concept is usually considered in the related empirical literature. See, for example, Choo and Siow (2006) and more recently Galichon and Salanié (forthcoming).

male preference types, and we use marital status, spouses' education levels and BMI to define 20 match quality types.[3] In our application, we will focus on three empirical questions. First, we analyse the degree to which the observed marriages and consumption allocations satisfy our testable implications of marital stability for alternative model specifications (i.e., with general or quasi-linear utilities and with or without unobserved match quality). Second, we consider the set identification of the unobserved match quality for alternative household types. Finally, we consider singles and document the match quality of singles (i.e., unobserved "quality of singlehood") needed to rationalise male and female singlehood as a stable situation through the lens of our structural model.

Section 2 motivates our AQS model of individual utilities characterised by unobserved match quality. Section 3 presents our empirical set-up. Section 4 introduces our notion of rationalisable household consumption behaviour under the assumption of a stable marriage market. Section 5 discusses practical issues that relate to bringing our theoretical characterisation to empirical data, and introduces our concept of match quality types. Section 6 presents the set-up of our empirical application to the Belgian MEqIn data. Section 7 considers the identification of unobserved match quality for the Belgian households, and documents the corresponding intrahousehold allocation patterns. Section 8 concludes. Appendix A presents a formal argument on relation between match quality and intrahousehold allocation when using our AQS model. Appendix B discusses methodological aspects that relate to the practical application of our characterisation of marital stability with general utilities. Appendices C and D provide additional information on the sample of households that we study in our empirical application. The Online Appendix contains the proofs of our main theoretical results.

[3]See, for example, Chiappori, Oreffice, and Quintana-Domeque (2012) and Dupuy and Galichon (2014) for empirical studies on education and BMI as individual characteristics that define marital matching patterns. These authors assume transferable utility whereas we consider a more general utility specification. Moreover, Chiappori et al. (2012) specifically investigate how individuals trade off BMI and education when choosing their partners. However, they do not consider the individuals' trade-offs between immaterial match quality resulting from BMI and education and material consumption. This last question is the main focus of our empirical application.

2 Unobserved match quality and individual utility

Once a man and a woman form a couple, they consume within their household a set of n private goods, $q \in \mathbb{R}^n_+$, and a set of N (household level) public goods, $Q \in \mathbb{R}^N_+$. We denote by $q^m \in \mathbb{R}^n_+$ the private consumption of the man and by $q^w \in \mathbb{R}^n_+$ the private consumption of the woman, with $q^m + q^w = q$.

Additive quantity shifting. To capture the trade-off between match quality and consumption, we make the assumption that unobserved match quality can be quantified in terms of private consumption. This set-up guarantees that the identification of the unobserved match quality does not interfere with the economic gains generated through public consumption.

To formalise this, we express the unobserved match quality for a potential couple (i, r) by vectors $\theta^m_{i,r} \in \mathbb{R}^n$ and $\theta^w_{i,r} \in \mathbb{R}^n$, where $\theta^m_{i,r}$ represents the marital quality for man i if he matches with woman r and, similarly, $\theta^w_{i,r}$ represents the quality for woman r when matched with man i. The values of $\theta^m_{i,r}$ and $\theta^w_{i,r}$ capture the non-material benefits or cost of the match (i, r) as perceived by i and r, respectively. Conversely, if individuals value the freedom of choice when living alone, the unobserved quality of a single person captures how much they value being single as opposed to being married. Below we will often refer to the case in which the vector $\theta^m_{i,r} = 0_n$ or $\theta^w_{i,r} = 0_n$ (a vector of zeros) as the "zero match quality" case.

When matched with woman r, man i has the utility function $u^i\left(q^m, \theta^m_{i,r}, Q\right)$, and woman r has the utility function $u^r\left(q^w, \theta^w_{i,r}, Q\right)$.[4] Throughout this section we will assume that these utility functions are differentiable, strictly increasing and quasi-concave in private and public quantities.[5] We define the "conditional" utility function for man i for a given level of public quantities Q as:

$$u^i_Q(q^m, \theta^m_{i,r}) = u^i\left(q^m, \theta^m_{i,r}, Q\right),$$

and similarly for woman r. To proceed further we have to put some structure on how match quality enters preferences. To do this, we assume that the individual

[4] For technical reasons, we extend the domain of u^i and u^r such that private good vectors take values in $\mathbb{R}^n$ instead of the usual non-negative Cartesian orthant $\mathbb{R}^n_+$. As such, $u^i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^N_+ \to \mathbb{R}$ and similar for u^r.

[5] We assume differentiability in the current section for expositional convenience. To be precise, the utility functions that we construct in the sufficiency arguments of Theorems 1 and 2 below are subdifferentiable. This, however, does not affect the core of our following argument.

conditional utilities take the form:

$$u^i_Q(q^m, \theta^m_{i,r}) = u^i_Q(q^m + \theta^m_{i,r}, 0_n),$$
$$u^r_Q(q^w, \theta^w_{i,r}) = u^r_Q(q^w + \theta^w_{i,r}, 0_n),$$

which expresses differences in match quality as differences in private consumption quantities. Intuitively, the utility that r receives when consuming q^m while matched with r is the same as if he would consume $q^m + \theta^m_{i,r}$ and receive no match quality. For simplicity, we omit the 0_n match quality value and simply write $u^i_Q(q^m + \theta^m_{i,r}, 0_n)$ as $u^i_Q(q^m + \theta^m_{i,r})$ in what follows.

We term this the *"additive quantity shifting"* (AQS) structure. Given the assumed properties of the direct utility function, we have that man i's conditional utility is strictly increasing and quasi-concave in $\theta^m_{i,r}$ (for fixed levels of public goods), and similarly for woman r. In the interest of notational clarity, we will drop the superscripts m and w and subscripts i and r in our following exposition of the properties of this AQS structure.

Money metric measure of match quality. The most important feature of the AQS model is that it allows us to define a money metric measure of unobserved match quality. Let $p \in \mathbb{R}^n_{++}$ represent the price vector for private consumption and let us denote by $e_Q(p, \theta, u)$ the expenditure function (conditional on the level of public goods). Then, for an interior solution of the expenditure minimisation problem, we have (using the change of variables $\tilde{q} = q + \theta$):

$$\begin{aligned} e_Q(p, \theta, u) &= \min_q \{p'q \text{ subject to } u_Q(q + \theta) \geq u\} \\ &= \min_{\tilde{q}} \{p'(\tilde{q} - \theta) \text{ subject to } u_Q(\tilde{q}) \geq u\} \\ &= \min_{\tilde{q}} \{p'\tilde{q} \text{ subject to } u_Q(\tilde{q}) \geq u\} - p'\theta \\ &= e_Q(p, 0_n, u) - p'\theta. \end{aligned}$$

The crucial property here is that the expenditure function with match quality is additively separable in u and θ. The monetary value of a match relative to the zero match quality case is then defined by:

$$p'\theta = e_Q(p, 0_n, u) - e_Q(p, \theta, u).$$

This is a difference between two expenditure functions, which inherits linear homogeneity. With AQS, the money metric measure of match quality is bilinear in prices and the quality vector θ. The measure can be positive or negative and is zero if $\theta = 0_n$.

An increase in any component of the vector θ decreases the cost of attaining a given utility level, since $\frac{\partial e_Q(p,\theta,u)}{\partial \theta_k} = -p_k < 0$. By Shephard's lemma, the Hicksian (compensated) conditional demand for good k is:

$$\begin{aligned} h_Q^k(p,\theta,u) &= \frac{\partial e_Q(p,\theta,u)}{\partial p_k} = \frac{\partial e_Q(p,0_n,u)}{\partial p_k} - \theta_k \\ &= h_Q^k(p,0_n,u) - \theta_k, \end{aligned}$$

so that match quality shifts the compensated demands up or down relative to the zero match quality demands. Taking second order derivatives further shows that AQS requires substitution effects to be independent from match quality. As such, match quality can be seen as mainly generating income effects.

Let x denote "total expenditure". To obtain the Marshallian (uncompensated) demands, we start from the conditional indirect utility function $V_Q(p,\theta,x)$, which is obtained from the identity:

$$e_Q(p,\theta,V_Q(p,\theta,x)) = e_Q(p,0_n,V_Q(p,0_n,x)) - p'\theta = x,$$

which implies:

$$V_Q(p,\theta,x) = V_Q(p,0_n,x+p'\theta),$$

and confirms that $p'\theta$ acts like an income shifter. Using Roy's identity, the conditional Marshallian demand for good k is given by:

$$\begin{aligned} q_Q^k(p,\theta,x) &= -\frac{\frac{\partial V_Q(p,\theta,x)}{\partial p_k}}{\frac{\partial V_Q(p,\theta,x)}{\partial x}} = -\frac{\frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial p_k} + \frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial x}\theta_k}{\frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial x}} \\ &= q_Q^k(p,0_n,x+p'\theta) - \theta_k. \end{aligned}$$

This shows that match quality impacts the conditional Marshallian demand in two ways. First it shifts the demand curve up or down. This is due to the fact that match quality and private consumption act as perfect substitutes. Next, match quality also generates an income effect, as for a given match quality θ one only needs the income $x - p'\theta$ to reach the same level of utility. Consequently, ordinal preferences also depend on match quality.

Further discussion. Allowing that match quality changes ordinal preferences over private goods, conditional on public goods, is unusual.[6] The most widely used alternative formulation assumes that preferences are additive in match quality:

$$u_Q(q,\theta) = v_Q(q) + \eta(\theta),$$

where $\eta(\theta)$ is a strictly increasing index of the match quality vector. This functional form is widely used as it is very convenient: once matched, preferences no longer depend on the match quality. However, precisely this feature makes it is a strong assumption if we aspire to integrate the literatures on matching and intrahousehold allocation. Man i may be very keen to match with woman r if the match quality for him is high, but if they do match then he ceases to care for her and would not be willing to give up private goods to make her better off. By contrast, the AQS specification implies that an increase in his match quality (holding everything else constant, including woman r's match quality) will also raise her welfare if they match together and follow a collective model of household consumption (Chiappori, 1988, 1992) (see Appendix A).

Another widely used specification in theoretical and empirical analyses of matching is transferable utility. A necessary condition for transferable utility is that individual expenditure functions (conditional on public goods) take a quasi-homothetic form with the marginal cost of utility being independent of match quality and the utility level. The latter implies that the marginal cost of utility is the same across all potential matches. This is Affine Conditional Indirect Utility (ACIU) in the terminology of Chiappori and Gugl (2020):

$$e_Q(p,\theta,u) = \beta_Q(p)\,u + \alpha_Q(p,\theta).$$

Here, $\beta_Q(p)$ and $\alpha_Q(p,\theta)$ are strictly increasing, linear homogeneous and concave in prices. Although both AQS and ACIU have specifications that display additive separability between the utility level and the match quality, neither specification is nested in the other.[7] For example, the match-quality component in AQS is bilinear in prices and match quality, whereas ACIU allows the less restrictive

[6]It is similar in spirit to the widely accepted idea that preferences change when going from being single to being married or when there are children in the household.

[7]Forms that are stronger than ACIU and that are sufficient for TU are, however, nested within AQS. An example is the quasi-linear utility specification.

form $\alpha^Q(p, \theta)$. On the other hand, AQS imposes no restrictions on the utility component, whereas ACIU imposes the very strong restriction that Engel curves with zero match quality are linear in u. An important corollary of this is that our AQS specification does not impose TU, even though it admits monetary transfers. The AQS model is therefore an imperfectly transferable utility (ITU) model.

Summing up: we define the "individual" match quality for man i matched to woman r as $p'\theta^m_{i,r}$ and the individual match quality for woman r married to man i as $p'\theta^w_{i,r}$. Suppose the man has two potential partners, r and s. Holding constant public and private goods, he will strictly prefer r to s if and only if $p'\theta^m_{i,r} > p'\theta^m_{i,s}$. The AQS match quality measures are independent of Q and u and are directly operationalised if we can identify the unobserved qualities $\theta^m_{i,r}$ and $\theta^w_{i,r}$. We define the "total" match quality for the (i, r) match as the sum of the two individual qualities $p'\left(\theta^m_{i,r} + \theta^w_{i,r}\right)$. Thus, AQS provides a concept of aggregate match quality for any matched pair even though we do not assume transferable utility. In the next two sections we introduce a nonparametric revealed preference method for identifying these individual and aggregate marital qualities.

3 Empirical set-up

We consider a marriage market with a finite set of men M and a finite set of women W. Married couples are defined by a matching function $\sigma : M \cup W \to M \cup W$, such that:

- for all men $i \in M, \sigma(i) \in W$,
- for all women $r \in W, \sigma(r) \in M$,
- and $\sigma(i) = r$ if and only if $\sigma(r) = i$.

To ease the notational burden, our formal exposition will not explicitly discuss singles; we will model all observed individuals as "married" and, thus, $|M| = |W|$. Importantly, however, the analysis does implicitly include the possibility that some males or females in the data set are actually singles. Specifically, single females (males) correspond to (virtual) couples with the male (female) consuming nothing. We will include singles in our empirical application in Sections 6 and 7.

We assume that the empirical analyst observes the public consumption Q as well as the individuals' private consumption q^m and q^w for the matched couples, but not for other potential (unmatched) couples. We do observe individuals' private consumption for the married couples in our empirical application. If such information were not available, the unknown individual quantities q^m and q^w can be treated similarly to the unknown individual prices P^m and P^w in our nonparametric characterisations of marital stability in Definitions 2 and 3.[8]

As indicated in the Introduction, we define discrete preference factors to stratify male and female individuals as observable preference types, with common preferences within a type. This boils down to partitioning the male and female sets M and W into subsets, with each subset characterised by a type-specific utility function. More formally, let $\tau : M \cup W \to T_M \cup T_W$ be a type function that associates with each man i a type $\tau(i) \in T_M$ and with each women r a type $\tau(r) \in T_W$, where T_M and T_W are finite sets of men and women types. Thus, $\tau(i)$ gives the type of each man $i \in M$ and, similarly, $\tau(r)$ gives the type of each woman $r \in W$. A typical element of T_M will be denoted by ψ and a typical element of T_W will be denoted by ω.

Budget constraints are specific to both married and potential couples $(i,r) \in M \times W$. First, $p_{i,r} \in \mathbb{R}^n_{++}$ denotes the prices for private consumption and $P_{i,r} \in \mathbb{R}^N_{++}$ the prices for public consumption. Next, a potential couple (i,r) can spend the income $y_{i,r}$.[9] The couple's consumption possibilities are by the associated budget set:

$$B_{i,r} = \left\{(q^m, q^w, Q) | p'_{i,r}(q^m + q^w) + P'_{i,r} Q \leq y_{i,r}\right\}.$$

Summarising, for a given marriage market we assume the data set:

$$S = \left\{\sigma, \tau, \{q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}\}_{i \in M}, \{p_{i,r}, P_{i,r}, y_{i,r}\}_{i \in M, r \in W}\right\},$$

which consists of a matching function σ, a type function τ, observed intrahousehold allocations:

$$(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}),$$

for all married couples $(i, \sigma(i))$, and couple-specific prices $(p_{i,r}, P_{i,r})$ and incomes $y_{i,r}$ for all potential couples (i,r), such that:

$$p'_{i,\sigma(i)}(q^m_{i,\sigma(i)} + q^w_{i,\sigma(i)}) + P'_{i,\sigma(i)} Q_{i,\sigma(i)} = y_{i,\sigma(i)},$$

[8] See also Cherchye, Demuynck, De Rock, and Vermeulen (2017), who consider a closely similar empirical set-up in which only the aggregate quantities q (and not the individual quantities q^m and q^w) are observed, and Cherchye, De Rock, Surana, and Vermeulen (2020) for a set-up in which the (public or private) nature of goods is unobserved.

[9] Couple-specific budget sets are relevant, for example, when the modelled consumption includes spouses' leisure, as in our application in Sections 6 and 7. In this case, the price of an individual's leisure equals that individual's wage, and the couple's income equals full potential (labour and non-labour) income.

for every married couple $(i, \sigma(i))$. We assume that the quantities $q^m_{i,\sigma(i)}$ and $q^w_{i,\sigma(i)}$ are strictly positive for all matches.

4 Rationalisable household consumption

We begin this section by defining our concept of rationalisable household consumption behaviour, which basically states that the observed behaviour, captured by the data set S, can be represented in terms of a stable allocation on the marriage market. Subsequently, we introduce our revealed preference characterisation of rationalisable behaviour, which defines testable conditions that can be used to empirically analyse the observed behaviour under the assumption of marital stability. We first present this characterisation for general individual utilities. Next, we turn to the specific instance of quasi-linear utilities (and additive match quality). As indicated in the Introduction, quasi-linearity implies transferable utility and is popularly used in the literature on the analysis of marriage market behaviour.

Rationalisability. We say that a data set is rationalisable if there exist type-specific preferences for which the observed intrahousehold allocation is utility maximising and such that the matching is stable. Stability of the marriage market requires both "individual rationality" and "no blocking pairs". Individual rationality means that no matched individual wants to become single and, similarly, no blocking pairs means that no two currently unmatched married individuals prefer to marry each other.

In our theoretical analysis, we will solely consider the no blocking pairs condition explicitly. However, our following arguments actually also include the individual rationality condition implicitly. More specifically, the individual rationality requirement coincides with the no blocking pairs requirement when using "individuals pairing with nobody" as potentially blocking pairs. Our empirical application in Sections 6 and 7 will use both the no blocking pair and individual rationality requirements for marital stability.

For a given data set S, our rationalisability condition requires that there must exist individual utility functions and unobserved match quality vectors that make the observed household allocations consistent with marriage stability (i.e., no blocking pairs).

Definition 1. *The data set S is rationalisable by a stable matching if, for all male types $\psi \in T_M$ and female types $\omega \in T_W$, there exist, strictly monotone, continuous* *and quasi-concave utility functions* $u^\psi : \mathbb{R}^{n+N} \to \mathbb{R}$ *and* $u^\omega : \mathbb{R}^{n+N} \to \mathbb{R}$ *and, for all males* $i \in M$ *and females* $r \in W$*, there exist match quality vectors* $\theta^m_{i,r} \in \mathbb{R}^n$ *and* $\theta^w_{i,r} \in \mathbb{R}^n$ *such that, for all couples* $(i, r) \in M \times W$*, with* $\tau(i) = \psi$ *and* $\tau(r) = \omega$*, and all allocations* (q^m, q^w, Q)*, if:*

$$u^\psi(q^m + \theta^m_{i,r}, Q) \geq u^\psi(q^m_{i,\sigma(i)} + \theta^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) \text{ and}$$
$$u^\omega(q^w + \theta^w_{i,r}, Q) \geq u^\omega(q^w_{\sigma(r),r} + \theta^w_{\sigma(r),r}, Q_{\sigma(r),r}),$$

with at least one strict inequality, then $(q^m, q^w, Q) \notin B_{i,r}$*.*

In words, rationalisability imposes a separate (no blocking pair) restriction for each potential couple (i, r): any consumption allocation (q^m, q^w, Q) that gives greater utility to both individuals than in their current match, with at least one strict inequality, must be infeasible for the given budget set. If this last condition were not met, then both individuals would be better off by exiting their current marriage and remarrying each other, which would make the given matching allocation unstable.

Further, we remark that our rationalisability condition in Definition 1 automatically implies that within-household consumption allocations are Pareto efficient. In particular, for each married couple, the condition imposes that there cannot exist a consumption allocation that makes both spouses better off (and at least one spouse strictly better off) than the given allocation $(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)})$, which effectively excludes the possibility of Pareto improvements. This is a convenient implication, as the implicit assumption of Pareto efficiency fits within the collective model of household consumption (Chiappori, 1988, 1992), which has become the workhorse model in the household economics literature (see Browning, Chiappori, and Weiss (2014) for a review).

Characterisation. Our first main result shows that a data set S with observed household consumption allocations is rationalisable by a stable matching if and only if it satisfies the Axiom of Revealed Stable Matchings (ARSM). We say that an observed matching allocation that is consistent with the ARSM is *"revealed stable"*, to indicate that the associated data set does not allow us to reject stability.

Definition 2 (ARSM)**.** *A data set* S *satisfies the Axiom of Revealed Stable Matchings (ARSM) if, for all couples* $(i, r) \in M \times W$*, with* $\tau(i) = \psi$ *and* $\tau(r) = \omega$*, there exist:*

- *a utility value* $U^\psi(i)$ *for man* i *of type* ψ*,*

- *a utility value* $U^\omega(r)$ *for women* r *of type* ω,
- *price vectors* $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ *with* $P^m_{i,r} + P^w_{i,r} = P_{i,r}$,
- *match quality vectors* $\theta^m_{i,r}, \theta^w_{i,r} \in \mathbb{R}^n$,

such that, for all types $\psi \in T_m$ *and* $\omega \in T_W$*, all men* i, k *of type* ψ *and all women* r, s *of type* ω*:*

$$U^\psi(k) \geq U^\psi(i) \text{ and } U^\omega(s) \geq U^\omega(r),$$

implies:

$$\begin{aligned} y_{i,r} + p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r}) \leq\ & p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s}, \\ & + p'_{i,r}(\theta^m_{k,\sigma(k)} + \theta^w_{\sigma(s),s}), \end{aligned} \tag{BP}$$

with a strict inequality if $U^\psi(k) > U^\psi(i)$ *or* $U^\omega(s) > U^\omega(r)$.

To explain the intuition of this ARSM condition, let us first regard the simplified setting without unobserved match quality (i.e., $\theta^m_{i,r} = \theta^w_{i,r} = 0_n$). The condition first attaches a utility value $U^\psi(i)$ to every consumption bundle $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ for male i of type ψ and, similarly, a utility value $U^\omega(r)$ to every bundle $(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$ for female r of type ω. Next, it defines individual prices $P^m_{i,r}$ and $P^w_{i,r}$ reflecting the willingness-to-pay of, respectively, male i and female r for the public consumption in the allocation $(q^m_{i,r}, q^w_{i,r}, Q_{i,r})$. Pareto efficiency implies $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, that is, the individual prices $P^m_{i,r}$ and $P^w_{i,r}$ must add up to the actual price $P_{i,r}$ and can be interpreted as "Lindahl prices" associated with the efficient consumption of public goods.

The ARSM condition then imposes that there must exist at least one specification of these individual utility values $U^\psi(i), U^\omega(r)$ and individual prices $P^m_{i,r}, P^w_{i,r}$ that represents the observed data set S as a stable matching allocation. In particular, this specification must satisfy the no blocking pair requirement of Definition 1, in the following sense: if (i) male type ψ is better off with the consumption bundle of individual k than with the bundle of individual i (i.e., $U^\psi(k) \geq U^\psi(i)$) and (ii) female type ω is better off with the bundle of individual s than with the bundle of individual r (i.e., $U^\omega(s) \geq U^\omega(r)$), then we must have:

$$y_{i,r} + p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r}) \leq\ p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s}, \tag{BP'}$$

which states that the 'income' $y_{i,r}$ available to the potentially blocking pair (i, r) does not suffice to buy the "preferred" bundles $(q^m_{k,\sigma(k)}, Q_{k,\sigma(k)})$ and $(q^w_{\sigma(s),s}, Q_{\sigma(s),s})$

for the prevailing prices $p_{i,r}$, $P^m_{i,r}$ and $P^w_{i,r}$. If this inequality did not hold, then the pair (i, r) would block the observed matching allocation, which would violate marital stability.

So far, we have assumed $\theta^m_{i,r} = \theta^w_{i,r} = 0_n$. In case the unobserved match quality can be non-zero, we additionally need to correct for a potential difference in match quality. Under AQS, this difference can be expressed in money metric terms as the difference between $p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r})$ and $p'_{i,r}(\theta^m_{k,\sigma(k)} + \theta^w_{\sigma(s),s})$. Plugging this into (BP') effectively yields (BP).

Our first main result states that the ARSM condition in Definition 2 is both necessary and sufficient for an observed matching allocation to be revealed stable. In other words, it exhausts all testable implications of marital stability for the empirical setting under study.[10]

Theorem 1. *A data set S is rationalisable by a stable matching if and only if it satisfies the ARSM.*

In Appendix B we show that the ARSM condition can be reformulated in terms of inequality constraints that are linear in unknowns and characterised by (binary) integer variables. These linear inequality constraints are easily operationalised, which is convenient from an application point of view.[11] In the next section we also discuss falsifiability of the ARSM condition.

Quasi-linear utilities. We next turn to the case of quasi-linear preferences that is often considered in the literature. In this case, we can model match quality as additive and one-dimensional.[12] This yields the following utility specification:

$$v^\omega(\tilde{q}^m, Q) + \hat{q}^m + \hat{\theta}^m \text{ and } v^\psi(\tilde{q}^w, Q) + \hat{q}^w + \hat{\theta}^w,$$

with $\hat{q}^m$ and $\hat{q}^w$ the numeraire quantities, $\tilde{q}^m$ and $\tilde{q}^w$ the remaining private quantities, and the scalars $\hat{\theta}^m$ and $\hat{\theta}^w$ the individuals' match quality. We assume that the functions v^ω and v^ψ are continuous, strictly monotone and concave. For expositional simplicity, we assume that we can normalise prices such that the price of the numeraire good, $\hat{q}^m$ and $\hat{q}^w$, equals unity for all (potential) couples.

[10] See the Online Appendix for the proofs of our main Theorems 1 and 2.

[11] We used the software package IBM ILOG CPLEX Optimisation Studio for our empirical application in Sections 6 and 7. Our CPLEX codes are available upon request.

[12] Additive and single-dimensional match quality is often used in marital matching models with transferable utility. See, for example, Browning, Chiappori, and Weiss (2014) for a review and Chiappori, Iyigun, and Weiss (2015) for a specific example with quasi-linear individual utilities.

For this quasi-linear utility specification, we obtain the following modified version of the ARSM condition in Definition 2.

Definition 3 (ARSM-QL). *A data set S satisfies the Axiom of Revealed Stable Matchings with Quasi-Linear Utility (ARSM-QL) if, for all couples $(i, r) \in M \times W$, with $\tau(i) = \psi$ and $\tau(r) = \omega$, there exist:*

- *a sub-utility value $V^{\psi}(i)$ for man i of type ψ,*
- *a sub-utility value $V^{\omega}(r)$ for women r of type ω,*
- *price vectors $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ with $P^m_{i,r} + P^w_{i,r} = P_{i,r}$,*
- *match quality scalars $\hat{\theta}^m_{i,r}$ and $\hat{\theta}^w_{i,r}$,*

such that, for all types $\psi \in T_M, \omega \in T_W$ all men i, k of type ψ and all women r, s of type ω:

$$\begin{aligned} y_{i,r} + \hat{\theta}^m_{i,r} + \hat{\theta}^w_{i,r} \leq\ & p'_{i,r}(\tilde{q}^m_{k,\sigma(k)} + \tilde{q}^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s} \\ & + (V^{\psi}(i) - V^{\psi}(k) + \hat{q}^m_{i,\sigma(i)} + \hat{\theta}^m_{i,\sigma(i)}) \\ & + (V^{\omega}(r) - V^{\omega}(s) + \hat{q}^w_{\sigma(r),r} + \hat{\theta}^w_{\sigma(r),r}). \end{aligned}$$

To show the intuition of this ARSM-QL requirement for marital stability, we again start by considering the case without match quality (i.e., $\hat{\theta}^m_{i,r} = \hat{\theta}^w_{i,r} = 0$). Assume that the associated ARSM-QL condition is not met for a given data set S. This means there exists a couple (i, r) and individuals k and s such that $\tau(i) = \tau(k) = \psi$, $\tau(r) = \tau(s) = \omega$ and:

$$\begin{aligned} y_{i,r} >\ & p'_{i,r}(\tilde{q}^m_{k,\sigma(k)} + \tilde{q}^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s} \\ & + (V^{\psi}(i) - V^{\psi}(k) + \hat{q}^m_{i,\sigma(i)}) \\ & + (V^{\omega}(r) - V^{\omega}(s) + \hat{q}^w_{\sigma(r),r}). \end{aligned}$$

For the given income $y_{i,r}$ and prices $p_{i,r}, P^m_{i,r}, P^w_{i,r}$, the above inequality then shows that the couple (i, r) can buy the bundles $(\tilde{q}^m_{k,\sigma(k)}, Q_{k,\sigma(k)}, \hat{q}^m_k)$ (for the male) and $(\tilde{q}^w_{\sigma(s),s}, Q_{\sigma(s),s}, \hat{q}^w_s)$ (for the female), where:

$$\begin{aligned} & \hat{q}^m_k > V^{\psi}(i) - V^{\psi}(k) + \hat{q}^m_{i,\sigma(i)} \text{ and} \\ & \hat{q}^w_s > V^{\omega}(r) - V^{\omega}(s) + \hat{q}^w_{\sigma(r),r}. \end{aligned}$$

Under quasi-linearity, these bundles correspond to the utility values:

$$V^{\psi}(k) + \hat{q}^m_k > V^{\psi}(i) + \hat{q}^m_{i,\sigma(i)} \text{ and}$$
$$V^{\omega}(s) + \hat{q}^w_s > V^{\omega}(r) + \hat{q}^w_{\sigma(r),r}.$$

But this means that both male i and female r are better off with the (affordable) bundles $(\tilde{q}^m_{k,\sigma(k)}, Q_{k,\sigma(k)}, \hat{q}_k)$ and $(\tilde{q}^w_{\sigma(s),s}, Q_{\sigma(s),s}, \hat{q}_s)$ than with their given bundles $(\tilde{q}^m_{i,\sigma(i)}, Q_{i,\sigma(i)}, \hat{q}_{i,\sigma(i)})$ and $(\tilde{q}^w_{r,\sigma(r)}, Q_{r,\sigma(r)}, \hat{q}_{r,\sigma(r)})$. This makes the couple (i, r) a blocking pair, which entails that the observed marriage allocation is not stable.

Similar to the ARSM case that we discussed above, if $\hat{\theta}^m_{i,r}$ and $\hat{\theta}^w_{i,r}$ can be non-zero, we need to correct for potential differences in unobserved match quality (expressed in money metric terms). In this case, this boils down to adding $\hat{\theta}^m$ and $\hat{\theta}^w$ to the numeraire quantities $\hat{q}^m$ and $\hat{q}^w$, which gives the ARSM-QL requirement in Definition 3.

Our following result provides the quasi-linear utility counterpart of Theorem 1.

Theorem 2. *A data set S is rationalisable by a stable matching with individual utility functions $v^{\omega}(\tilde{q}^m, Q) + \hat{q}^m + \hat{\theta}^m$ and $v^{\psi}(\tilde{q}^w, Q) + \hat{q}^w + \hat{\theta}^w$ if and only if it satisfies the ARSM-QL.*

Interestingly, the testable implications of the ARSM-QL condition in Definition 3 are linear in continuous unknowns, which makes that they can be checked by using standard linear programming methods.

5 Match quality types, stability indices and identification

Before presenting our application to Belgian households, we discuss three methodological issues that are relevant when bringing the ARSM and ARSM-QL conditions to empirical data. First, the characterisations as such do not have empirical bite, meaning that any data set S will trivially satisfy the testable conditions. We will show that the conditions can be given empirical content by considering match quality types, which allow us to put specific structure on the match quality vectors $\theta^m_{i,r}$ and $\theta^w_{i,r}$. Second, the ARSM and ARSM-QL conditions are sharp in nature. They only allow us to conclude whether or not a data set is exactly rationalisable. In practice, it is often relevant to also study closely (instead of exactly) rationalisable behaviour. We will do so by making use of so-called stability indices. Finally, we discuss how to use the testable conditions captured by our ARSM and

ARSM-QL characterisations to (set) identify the unobserved decision structure (i.e., individual preferences and match quality) from the observed marriage and consumption allocations.

Match quality types. As a first observation, we note that the ARSM and ARSM-QL conditions as such will never reject for a given data set S: we can always find match quality vectors $\theta^m_{i,r}$ and $\theta^w_{i,r}$ that make the observed consumption allocations consistent with the rationalisability restrictions in Definitions 2 and 3. For example, rationalisability is trivially obtained by setting the values $\theta^m_{i,r}$ and $\theta^w_{i,r}$ low enough for the unmatched couples and high enough for the matched couples.

Thus, for our characterisations of marital stability to have empirical bite, we need to impose additional structure on the match quality components of the individual utilities. We do so by defining match quality factors that allow for stratifying the male and female individuals as observable marital quality types. We denote a typical male quality type by κ and a typical female quality type by λ. Following Choo and Siow (2006), the empirical literature on the analysis of marital matchings invariably assumes that the match quality for the male individuals i of type κ and female individuals r of type λ can be decomposed into a systematic components that only depend on the male and female types κ and λ and an idiosyncratic component that is specific to the individuals i and r:

$$\theta^m_{i,r} = \theta^m_{\kappa,\lambda} + \theta^m_{i,\lambda} \text{ and } \theta^w_{i,r} = \theta^w_{\kappa,\lambda} + \theta^w_{\kappa,r}.$$

Here, the idiosyncratic components $\theta^m_{i,\lambda}$ and $\theta^w_{\kappa,r}$ are allowed to depend on the type (and not the identity) of the individual's partner. These idiosyncratic components measure the departures of the individual-specific payoffs $\theta^m_{i,r}$ and $\theta^m_{i,r}$ from the type-specific payoffs $\theta^m_{\kappa,\lambda}$ and $\theta^w_{\kappa,\lambda}$.

Unobservable stochastic idiosyncratic components cannot be readily adapted to a revealed preference framework. Instead, we choose to set the idiosyncratic components $\theta^m_{i,\lambda}$ and $\theta^w_{\kappa,r}$ equal to zero, giving:

$$\theta^m_{i,r} = \theta^m_{\kappa,\lambda} \text{ and } \theta^w_{i,r} = \theta^w_{\kappa,\lambda}.$$

However, we implicitly allow for idiosyncratic match quality components by assuming that they are captured by the stability indices that we introduce in the next subsection. Our justification for this simplification is that our principal interest is in identifying type-specific match quality.

Stability indices. Above we have assumed that preferences over both consumption and match quality are type-specific with no idiosyncratic within-type variation. Moreover, we have not taken account of measurement errors in the data, frictions on the marriage market or other circumstances that may lead to deviations of observed behaviour from exactly rationalisable behaviour. Following Cherchye, Demuynck, De Rock, and Vermeulen (2017), we can evaluate the goodness-of-fit of a given model specification by introducing stability indices. These stability indices allow us to quantify how close the observed behaviour is to exactly rationalisable behaviour. This accounts for small deviations from these implicit model assumptions in our empirical analysis.

Starting from our ARSM and ARSM-QL conditions in Definitions 2 and 3, we include a stability index $s_{i,r}$ in each inequality constraint associated with a potentially blocking pair (i,r). Specifically, we replace the inequalities in Definition 2 by:

$$\begin{aligned} s_{i,r} \times y_{i,r} + p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r}) \leq p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} \\ + P^{w\prime}_{i,r} Q_{\sigma(s),s} + p'_{i,r}(\theta^m_{k,\sigma(k)} + \theta^w_{\sigma(s),s}), \end{aligned}$$

and the inequalities in Definition 3 by:

$$\begin{aligned} (s_{i,r} \times y_{i,r}) + \hat{\theta}^m_{i,r} + \hat{\theta}^w_{i,r} \leq p'_{i,r}(\tilde{q}^m_{k,\sigma(k)} + \tilde{q}^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s} \\ + (V^{\psi}(i) - V^{\psi}(k) + \hat{q}^m_{i,\sigma(i)} + \hat{\theta}^m_{i,\sigma(i)}) \\ + (V^{\omega}(r) - V^{\omega}(s) + \hat{q}^w_{\sigma(r),r} + \hat{\theta}^w_{\sigma(r),r}). \end{aligned}$$

We also add the restriction $0 \leq s_{i,r} \leq 1$. Clearly, imposing $s_{i,r} = 1$ obtains the original conditions in Definitions 2 and 3. Conversely, $s_{i,r} = 0$ means that the post-divorce income of the potentially blocking pair (i,r) is zero, which implies that the rationalisability restrictions lose any empirical bite. Intuitively, a lower stability index $s_{i,r}$ indicates that a greater income loss is to be associated with a particular option to exit marriage (and form the pair (i,r)) to rationalise the data by a stable marriage matching. In what follows we interpret this as revealing a greater violation of the strict ARSM and ARSM-QL conditions of marital stability.

For a given data set S, we measure the degree of stability by computing

$$\max \sum_i \sum_r s_{i,r},$$

subject to the above linear feasibility constraints.[13] This gives a different stability index $s_{i,r}$ for every potentially blocking pair (i, r). As explained above, these stability indices capture alternative possible explanations of observed deviations from our strict ARSM and ARSM-QL conditions.

Set identification. By using the stability indices that we presented above, we can construct new data sets that satisfy our (ARSM/ARSM-QL) rationalisability conditions. Specifically, we can use the computed values of $s_{i,r}$ to re-scale the original income levels $y_{i,r}$. This defines minimally adjusted data sets that are rationalisable by a stable matching (with general and quasi-linear utilities). For these new data sets, we can address alternative identification questions by starting from our rationalisability conditions. Particularly, we can set identify the unknowns in the ARSM and ARSM-QL conditions (such as individual utilities, individual "Lindahl" prices and unobserved match quality) in Definitions 2 and 3.

A main focus of our following empirical analysis will be on identifying the unobserved match quality of a pair (i, r). When expressed in money metric terms, this equals $p'_{i,r}\theta^m_{i,r}$ for the male and $p'_{i,r}\theta^w_{i,r}$ for the female. These last expressions are linear in the unknown match quality vectors and, thus, we can define upper/lower bounds by maximising/minimising these linear functions subject to our linear rationalisability restrictions in Definitions 2 and 3. This effectively set identifies the unobserved matching match quality based on our ARSM and ARSM-QL characterisations.

6 Belgian household data

We apply our method to a sample of Belgian households drawn from the MEqIn data set, which contains a rich set of economic and socio-demographic variables.[14] In what follows we first discuss how the MEqIn data were collected and we motivate our sample selection criteria. Next, we explain how we define our observable types

[13]Technically, following our discussion in Section 4, this is a mixed integer linear programming problem (i.e., linear objective and linear constraints with continuous as well as (binary) integer unknowns) for the general utility case, and a linear programming problem (i.e., linear objective and linear constraints with continuous unknowns) for the quasi-linear utility case.

[14]The MEqIn dataset is collected by a team of researchers from the Université catholique de Louvain, the University of Leuven, the Université libre de Bruxelles, and the University of Antwerp. The collection of the MEqIn data was enabled by the financial support of the Belgian Science Policy Office (BELSPO) through the grant BR/121/A5/MEQIN (BRAIN MEqIn). The MEqIn data is available upon request for researchers and students. For detailed information on the data set, we refer to Capéau et al. (2020) and the following website (which also includes a codebook): https://sites.google.com/view/meqin/data.

and provide summarising descriptives for the basic variables that we will use in our empirical analysis. Subsequently, we investigate the degree to which the observed consumption allocations satisfy our testable implications of marital stability for alternative model specifications, by using the stability indices that we introduced above. In particular, we will compare the results for general and quasi-linear individual utilities, with and without unobserved match quality.

Data. The MEqIn survey contains household information gathered in 2015-2016. The original data set comprises 3404 respondents, belonging to 2098 households. It provides detailed information on various aspects of the individual well-being of all adults living in the interviewed households, as well as information on the relative importance of the different life dimensions according to the respondents. For each surveyed household, some additional data on children could be sent back by the respondents through a drop-off questionnaire. In total, 371 families provided information on 618 children.

The set of households used for this study was subject to the following sample selection rules. First, because we need wage information, we only consider households with adults working at least 10 hours per week, with or without children. Next, we excluded the self-employed to avoid issues regarding the imputation of wages and the separation of consumption from work-related expenditures. After deleting the households with important missing information (mostly, incomplete information on one of the spouses), we obtained a sample containing 581 individuals: 194 males and females in couples, 124 single females and 69 single males.

We observe the privately consumed quantities of the two spouses. In our set-up, private consumption is a Hicksian good with price normalised to one. It includes individual expenditures on food (at home and outdoors), transport, tobacco, clothing, personal care and products, schooling and other personal expenditures. Further, we will assume that leisure is privately consumed. We also observe the publicly consumed quantities of the household, which is again a Hicksian good with price normalised to unity; it includes joint food consumption at home, joint transport, mortgage and rent, utilities and insurances, holidays, restaurant visits, child expenditures and other public expenditures. Finally, we will also treat time spent on domestic work (including child care) by the two individuals as public consumption.[15]

[15] In this respect, each individual's time spent on household production actually represents an input and not an output that is consumed inside the household (see Becker (1965)). Under the assumption that each individual produces a different household good by means of an efficient one-input technology characterised by constant returns-to-scale, however, the individual's input

Our method requires prices and incomes that apply to the exit options from marriage (i.e., becoming single or remarrying). For our labour supply application, prices correspond to individual wages. We assume that wages outside marriage are the same as inside marriage (i.e., exiting marriage does not affect labour productivity). This may seem to be a rather strong assumption in light of the literature on marriage premiums and penalties. However, we emphasise that, in principle, the wages and incomes in the counterfactual situations of being single or with a different partner can also be imputed. Moreover, it can be argued that the wage rate inside marriage is probably a good benchmark when individuals compare their opportunity sets inside their current marriage and outside marriage as a single or with a different partner.

For the observed couples, we use a consumption-based measure of total non-labour income, that is, non-labour income equals reported consumption expenditures minus full income. Then, we treat individual non-labour incomes as unknowns that are subject to the restriction that they must add up to the observed (consumption-based) total non-labour income.[16] As compared to the alternative that fixes the intrahousehold distribution of non-labour income (for example, at 50% for each individual), this procedure to endogenously define the individual non-labour incomes effectively puts minimal non-verifiable structure on these unobserved variables. However, to exclude unrealistic scenarios, in our application we will impose that individual non-labour incomes after divorce must lie between 40% and 60% of the total non-labour income under marriage. The same procedure was adopted by Cherchye, Demuynck, De Rock, and Vermeulen (2017).

Table 1 provides summary statistics for the couples in our sample. Wages are net hourly wages. Leisure is measured in hours per week. To compute leisure hours, we assume that an individual needs 8 hours per day for sleeping and personal care (i.e., leisure = $168-56-$hours worked in the labour market and at home). Full income and (Hicksian) consumption are measured in euros per week. Table 1 also reports on the presence of children, and the age, body mass index (BMI) and education levels of the individuals in our sample.[17] Individuals are deemed to be highly educated if they hold a degree beyond secondary education. As a BMI above 25 is universally considered overweight, we will use this cut-off level

value can serve as the output value.

[16]Because we define the individual non-labour incomes endogenously, we only use the stability indices $s_{i,r}$ to rescale potential (post-divorce) labour incomes (i.e., wages multiplied by total available time) and not non-labour incomes in our following analysis. This ensures that our rationalisability conditions remain linear in unknowns.

[17]Body mass index (BMI) is defined as the body mass (in kilograms (kg)) divided by the square of the body height (in meters (m)), and is expressed in units of kg/m^2.

to distinguish between high and low BMI individuals in what follows.

Table 1: sample summary statistics

	mean	**st.dev.**
male wage (euro/hour)	10.552	3.631
female wage (euro/hour)	10.145	3.505
full income (euro/week)	1638.845	734.051
male private consumption (euro/week)	126.213	59.271
female private consumption (euro/week)	116.139	57.308
public consumption (euro/week)	371.293	188.936
male leisure (hours/week)	50.550	15.333
female leisure (hours/week)	47.724	16.810
male domestic production (hours/week)	14.316	11.627
female domestic production (hours/week)	24.490	15.146
presence of children (1 = yes/0 = no)	0.553	0.498
number of children	0.920	0.969
male age (years)	41.684	9.776
female age (years)	39.918	9.397
male higher education (1 = yes/0 = no)	0.426	0.495
female higher education (1 = yes/0 = no)	0.513	0.501
male BMI (kg/m^2)	25.737	3.631
female BMI (kg/m^2)	24.225	4.486
dummy for couple	0.501	0.501
dummy for single male	0.178	0.383
dummy for single female	0.320	0.467

Notes: there are 194 couples, 124 female singles and 69 male singles; full income and consumption are in euros per week, wages in euros per hour, and leisure and domestic production in hours per week.

Preference types and match quality types. Our methodology uses preference and match quality factors to define observable preference and match quality types. For both type of factors we use variables that are popular in the empirical literature on consumption and time use decisions and marriage markets. We define preference types in terms of age, education and the presence of children. For each gender we consider 3 age classes (below 35, between 35 and 50 and above 50) and 2 education classes (higher educated or not), and we assume that parents of children can have different preferences than other individuals. In total, this defines 12 (= $3 \times 2 \times 2$) male and 12 female preference types. Similarly, we use the individuals' BMI and education levels (high or low) as match quality factors, thus assuming that the immaterial match quality may vary with spouses' education and BMI. In turn, this implies 20 male and 20 female match quality types. More specifically, we have four types of each gender (high/low educated and high/low BMI). Each of these four types is married to one of the four types of the other gender or to nobody (when single), which defines 20 (= $4 \times (4 + 1)$) possible matching outcomes per individual. Our specification of preference and match quality types allows us to model observable heterogeneity in preferences and immaterial match quality. As explained above, unobserved heterogeneity is captured by the stability indices, which we will discuss further on (see Table 6).

For each couple type, we will quantify the unobserved match quality in terms of privately consumed quantities (but not privately consumed leisure). This implies that $\theta^m_{\kappa,\lambda}$ and $\theta^w_{\kappa,\lambda}$ are scalars. This facilitates our comparison of the quality estimates for the general utility and quasi-linear utility specifications. Moreover, as we include private consumption as a Hicksian good in our empirical set-up, its price is normalised at unity for every household in our sample, which makes it easy to compare our (money metric) match quality estimates across different types of couples.

Next, given our relatively small sample size we need to introduce our two match quality factors in a parsimonious way. Therefore, we will assume that match quality is additively separable in education (EDU) and BMI. Formally, this means

$$\theta^m_{\kappa,\lambda} = \theta^m_{\kappa^{EDU},\lambda^{EDU}} + \theta^m_{\kappa^{BMI},\lambda^{BMI}} \text{ and } \theta^w_{\kappa,\lambda} = \theta^w_{\kappa^{EDU},\lambda^{EDU}} + \theta^w_{\kappa^{BMI},\lambda^{BMI}},$$

where $\kappa^{EDU}, \lambda^{EDU}$ and $\kappa^{BMI}, \lambda^{BMI}$ represent the BMI and EDU types of the males and females in our empirical analysis. In principle, richer (non-additive) specifications are possible, but they would require sufficient observed households of each match quality type to obtain an informative analysis. In what follows we will show that our additive structure does yield meaningful and intuitive empirical results. Moreover, we will use our stability indices to show that this specification is well supported empirically for the application at hand.

Tables 2 and 3 report on the marriage allocations for different EDU and BMI types in our sample of households.[18] Some interesting observations emerge. First, our data clearly reveal assortative matching in education: 66% of all observed couples consist of a male and a female of the same EDU type. The same applies to BMI, but to a lesser extent. In particular, there are quite a number of low BMI females married to high BMI males. More generally, even though "same-type" couples are clearly prevalent, the fraction of "mixed" couples is rather substantial. Relatedly, we see singles of every type. When compared to married individuals, single males and females are mostly lower educated. Next, single females typically

[18] Table 9 in Appendix C provides a further decomposition in terms of the different match quality types that we use in our empirical application.

have a higher BMI than married females, while the opposite applies to males.

Tables 4 and 5 document the consumption allocations for our different couple types. We report on the private and public consumption shares as well as total consumption (expressed in monetary value). Not surprisingly, total consumption is increasing with the level of education. Next, for married couples we observe quite some heterogeneity in consumption allocations across match quality types: both the individual private shares and the public shares vary considerably with the education and BMI levels. The same applies to singles. Here, a notable feature is that single males spend substantially less on public consumption compared to single females.

In general, there is a lot of cross-type and within-type heterogeneity in consumption and marriage behaviour. We also observe considerable variation in characteristics across household types; see Tables 10 and 11 in Appendix C. For example, single females have substantially more children than single males. The question is thus how we can rationalise these patterns of marriage and consumption behaviour. Part of the explanation may be heterogeneity in budget conditions (prices and incomes) and preferences (for example, related to education, age and the presence of children). Another part may be unobserved match quality that is specific to partners' EDU and BMI types.

Table 2: percentage shares of EDU types in our sample

	couples		
	low EDU female	**high EDU female**	**all**
low EDU male	31.443%	22.680%	54.124%
high EDU male	10.825%	35.052%	45.876%
all	42.268%	57.732%	
	singles		
	low EDU	**high EDU**	
males	66.667%	33.333%	
females	58.871%	41.129%	

Age-based marriage markets, subsampling and stability indices. Our revealed preference methodology requires a prior specification of the marriage markets that are relevant for the individuals under study. In this respect, it can hardly be assumed that all males and females in our base data set operate on the same marriage market. To account for this, we follow Cherchye, Demuynck, De Rock, and Vermeulen (2017) by constructing individual-specific marriage markets on the basis of age. Specifically, we will assume that a male (female) individual's marriage market contains single and married women (men) who are at most 5 (13)

Table 3: percentage shares of BMI types in our sample

	couples		
	low BMI female	**high BMI female**	**all**
low BMI male	36.082%	12.371%	48.454%
high BMI male	31.443%	20.103%	51.546%
all	67.526%	32.474%	
	singles		
	low BMI	**high BMI**	
males	55.072%	44.928%	
females	61.290%	38.710%	

Table 4: consumption shares and total consumption per match quality type - couples (mean values)

female type		**male type**					
EDU	*BMI*	*EDU*	*BMI*	private female	private male	public	total consumption
low	low	low	low	27.632%	32.013%	40.355%	1898.450
		low	high	27.978%	30.334%	41.689%	1708.224
		high	low	43.090%	26.243%	30.667%	2763.352
		high	high	25.904%	36.767%	37.328%	2229.483
low	high	low	low	27.128%	32.014%	40.858%	1978.279
		low	high	29.941%	34.497%	35.562%	1905.828
		high	low	21.365%	26.187%	52.448%	2037.552
		high	high	31.576%	27.626%	40.798%	2029.925
high	low	low	low	26.965%	26.429%	46.606%	2122.919
		low	high	24.017%	37.113%	38.870%	2020.818
		high	low	28.492%	31.016%	40.492%	2333.316
		high	high	31.456%	30.753%	37.790%	2566.700
high	high	low	low	28.324%	27.273%	44.404%	2185.731
		low	high	26.332%	28.875%	44.793%	2134.705
		high	low	21.287%	30.017%	48.697%	2561.594
		high	high	23.156%	28.444%	48.400%	2366.510

Notes: individual (male/female) private consumption includes leisure in monetary value, and total consumption equals individual (male/female) private consumption plus household public consumption (including domestic work in monetary value); consumption shares are defined as the proportion (share) of the total household consumption expenditures that is allocated to a particular (private or public consumption) category.

Table 5: consumption shares and total consumption per quality type - singles (mean values)

female type				
EDU	*BMI*	private female	public	total consumption
low	low	59.591%	40.409%	1031.884
low	high	53.792%	46.208%	1069.388
high	low	50.979%	49.021%	1222.796
high	high	54.784%	45.216%	1341.747
male type				
EDU	*BMI*	private male	public	total consumption
low	low	62.280%	37.720%	1022.856
low	high	66.103%	33.897%	1028.474
high	low	64.241%	35.759%	1184.676
high	high	64.130%	35.870%	1280.692

Notes: individual (male/female) private consumption includes leisure in monetary value, and total consumption equals individual (male/female) private consumption plus household public consumption (including domestic work in monetary value); consumption shares are defined as the proportion (share) of the total household consumption expenditures that is allocated to a particular (private or public consumption) category.

years older and 13 (5) years younger. These bounds correspond to the 2.5 and 97.5 percentiles of the age difference distribution in our sample of couples.

We proxy the marriage markets of the males and females in our sample by using the observed individuals of the other gender that satisfy this age constraint. Arguably, using this age criterion alone provides only a rough proxy of the relevant marriage markets for our sample. Taking a pragmatic approach, we mitigate the effects of sampling error and outlier behaviour by making use of subsampling. Specifically, we randomly draw 200 subsamples of 40 households (i.e., about 10 percent) from our original sample. We apply our revealed preference methodology to every subsample separately, and we will report summary results defined over these 200 subsamples in what follows. In our subsampling procedure, we draw every observed household 20.341 times on average (st. dev. 4.242), with a minimum of 10 times and a maximum of 35 times.

Recall that, in their original form, our ARSM and ARSM-QL characterisations define strict conditions for rationalisable household behaviour, which impose the assumptions of type-specific preferences, no measurement errors, and no frictions on the marriage market. As we explained in Section 5, we can account for deviations from these implicit assumptions by making use of stability indices ($s_{i,r}$) that take values between zero and one, with lower values revealing greater violations of the strict rationalisability conditions. We will interpret higher stability indices as suggesting greater empirical support for the underlying model assumptions. Starting from the general additive specification of match quality that we introduced above, we evaluate the following four selections of match quality factors:

EDU & BMI: $\theta^m_{\kappa,\lambda} = \theta^m_{\kappa^{EDU},\lambda^{EDU}} + \theta^m_{\kappa^{BMI},\lambda^{BMI}}$ and $\theta^w_{\kappa,\lambda} = \theta^w_{\kappa^{EDU},\lambda^{EDU}} + \theta^w_{\kappa^{BMI},\lambda^{BMI}}$;

EDU: $\theta^m_{\kappa,\lambda} = \theta^m_{\kappa^{EDU},\lambda^{EDU}}$ and $\theta^w_{\kappa,\lambda} = \theta^w_{\kappa^{EDU},\lambda^{EDU}}$;

BMI: $\theta^m_{\kappa,\lambda} = \theta^m_{\kappa^{BMI},\lambda^{BMI}}$ and $\theta^w_{\kappa,\lambda} = \theta^w_{\kappa^{BMI},\lambda^{BMI}}$;

None: $\theta^m_{\kappa,\lambda} = 0$ and $\theta^w_{\kappa,\lambda} = 0$.

We consider all four specifications for our ARSM characterisation of marital stability, which assumes general individual utilities. Next, to assess the empirical support for the quasi-linearity assumption, we also evaluate our most general specification of match quality (with the two quality factors EDU and BMI) for the ARSM-QL characterisation. Table 6 reports the mean stability index for each specification (defined over our 200 random subsamples), as well as the fractions of no blocking pair restrictions of which the stability index is below a given threshold value, for thresholds equal to 1 ("$<$ 100%"), 0.99 ("$<$ 99%"), 0.95 ("$<$ 95%") and 0.90 ("$<$ 90%"). These fractions give the probability that our rationalisability conditions are violated in a strict sense or when allowing some (marginal or moderate) deviations from exact stability.

Our specification with general utilities and two match quality factors (EDU & BMI) achieves the highest mean stability index. Interestingly, we also find that all four models with general utilities (with two, one or zero match quality factors) outperform the model with quasi-linear preferences (and two match quality factors). Admittedly, the differences between the mean indices for the different models are fairly small. However, the picture changes quite drastically when considering the fractions of no blocking pair restrictions that are violated. In this case, we do observe rather important differences between the alternative model specifications. For example, when focusing on our strict rationalisability conditions, the fraction of violated constraints reduces from no less than 25% to below 6% when assuming general instead of quasi-linear utilities in combination with unobserved match quality. In fact, the model with general utilities but without unobserved match quality still does markedly better then the model with quasi-linear utilities that includes unobserved quality, for any threshold value of the stability indices that we consider. For the model with general preferences, including unobserved match quality entails a further significant reduction of violations of our stability constraints. Going from one to two match quality factors yields an additional, albeit moderate, improvement of the model's empirical fit.

We conclude from these findings that the model with general preferences and two match quality factors provides the best fit of the observed marriage and consumption behaviour. We interpret this as offering empirical evidence against the assumption of quasi-linear preferences and favouring the model with general preferences and unobserved match quality for the sample under study. We take this evidence as a motivation to use this general model in our following empirical analysis. Admittedly, the patterns in Table 6 may also partially be explained by the fact that a more general model specification typically imposes less prior structure on the data at hand, thus yielding a better empirical fit by construction. Nevertheless, in the following section we will show that our very flexible specification also yields informative identification results, thus demonstrating its usefulness for a meaningful empirical analysis.

Table 6: stability indices for alternative model specifications

preferences	**quality factors**	**mean**	**< 100%**	**< 99%**	**< 95%**	**< 90%**
general	EDU & BMI	99.836%	5.642%	3.839%	1.023%	0.213%
general	EDU	99.757%	7.553%	5.431%	1.563%	0.347%
general	BMI	99.751%	7.571%	5.620%	1.627%	0.354%
general	none	99.264%	12.047%	10.538%	6.310%	2.548%
quasi-linear	EDU & BMI	98.816%	25.003%	19.962%	9.035%	2.965%

7 Unobserved match quality

We next focus on the identification of unobserved match quality for the model with general preferences. We begin by showing that our set identification method yields informative match quality bounds for the data at hand. Subsequently, we investigate in more detail the total (= female + male) match quality of the married couples under study, and how this aggregate quality is divided over males and females. Finally, we document the unobserved match quality of singles (i.e., unobserved "quality of singlehood"). As we will indicate, our findings reveal intuitive patterns that allow us to rationalise the observed marriage allocations as stable. Specifically, higher unobserved (immaterial) match quality typically compensates for less material consumption. In this respect, the gains of marriage associated with material consumption also include scale economies associated with intrahousehold public consumption. This will be particularly important when interpreting our results on the match quality of singles.

As discussed in Section 5, our method provides upper and lower bounds on the unobserved match quality, so effectively obtaining set identification. Our subsampling procedure outlined above yields multiple values of these upper and lower bounds for every household in our sample. We use the average of these values as our estimates for the upper and lower bounds of the immaterial match quality associated with a given marriage allocation.

As is clear from Definition 2, for an individual (male or female) of a given match quality type, our revealed preference methodology can only identify differences between the unobserved match quality associated with alternative marriages. From this perspective, it is useful to select a benchmark marriage, for which we normalise the individual match quality to zero (i.e., the "zero match quality" case using our terminology of Section 2). Then, the identified values of unobserved quality capture how much more (if positive) or less (if negative) private consumption the individual would need in this benchmark marriage to be equally well off as in the evaluated marriage. In our following analysis, we choose "same-type" marriages as our benchmark marriages, meaning that an individual's unobserved match quality is set to zero when married to an individual of the same quality (EDU and BMI) type.

Tables 12 to 15 in Appendix D report on the distributions of these upper and lower bound estimates over the different households in our sample. These show that these upper and lower bound distributions are mostly close to each other (in terms of both mean values and standard deviations), indicating that our method generates bounds that are informatively tight. In what follows we will therefore limit our attention to the average value of lower and upper bounds. Further, the results in Tables 12 to 15 reveal that the distributions of some lower and upper bound estimates are characterised by quite large standard deviations. For ease of exposition, we will abstract from this in what follows. However, it does mean that we should be cautious in interpreting the reported BMI and EDU effects, particularly when these (average) effects are small and correspond to lower and upper bound estimates with high standard deviations.

Between-household comparison. We first consider the match quality of married couples. To facilitate the interpretation, we express this match quality measure as a proportion of total consumption. Table 7 summarises our results. For each match quality type, it gives the mean values of the female, male and total (= female + male) match quality measures for the households in our sample. We also report the total consumption that we presented in Table 4 above. This allows for interpreting the importance of the unobserved (immaterial) match quality in terms of the observed (material) household consumption.

Table 7 reveals that the aggregate match quality is always negative for mixed couples (i.e., with different male and female types). Moreover, for each female (EDU-BMI) type we find that the total match quality (as a proportion of total consumption) is "most negative" when the male is of exactly the opposite (EDU-BMI) type. As same-type marriages generate a zero match quality (by our normalisation), this reveals complementarity of types in generating (aggregate) unobserved quality. For example, when a low EDU and high BMI female switches from a man of the same type to a high EDU and low BMI man, the average loss in match quality amounts to a bit more than 4% of the total consumption value (including time use). For a household that has a weekly full income of 2000 euros in total, this loss is worth about 80 euros of weekly private expenditures when quantified in money metric terms. In fact, these match quality losses associated with mixed marriages can be quite substantial, when considering that the average private expenditures (excluding leisure) of males and females in couples equal 126 and 116 euros per week, respectively (see Table 1).

Within-household comparison. We next investigate how the aggregate immaterial quality is shared between males and females. For mixed EDU types (i.e., a low EDU individual married to a high EDU individual), we see that the higher educated individual typically attains a (often substantially) higher match quality than the lower educated individual (when fixing the BMI levels). In fact, the individual match quality of the higher educated is positive in most cases, explaining why higher educated individuals may choose to marry lower educated individuals, even when the aggregate match quality is negative. This helps in rationalising the substantial fraction of mixed couples with high and low EDU individuals. These couples are difficult to rationalise on the basis of material consumption, as total consumption increases with education, giving high EDU individuals an incentive to choose high EDU partners.

For mixed BMI types (i.e., a low BMI individual married to a high BMI individual), we find that different BMI levels for the married partners always yield a lower female match quality (when fixing the EDU levels). The effect is mostly negative for males. There are two exceptions that are characterised by moderately positive effects on male match quality of mixed BMI levels: a low EDU and high BMI female type yields a higher match quality for low EDU males when the male has a low BMI than when the male has a high BMI; and a high EDU and high BMI female yields a higher match quality for low BMI males than for high BMI males, independent of the male education level.

Finally, it mostly depends on the specific match quality type whether the BMI effect dominates the EDU effect or vice versa. However, for low EDU females, the EDU effect always strongly dominates the BMI effect: being married to a high EDU male yields a substantially negative female match quality, which amounts to no less than 3 to 5% of the household's total consumption value.

Table 7: match quality (as proportion of total consumption) per type - couples

female type EDU	BMI	**male type** EDU	BMI	total consumption	quality female	quality male	total quality
low	low	low	low	1898.450	0.000%	0.000%	0.000%
		low	high	1708.224	-0.708%	-1.512%	-2.220%
		high	low	2763.352	-3.018%	1.041%	-1.978%
		high	high	2229.483	-3.606%	0.491%	-3.115%
low	high	low	low	1978.279	-1.555%	0.376%	-1.179%
		low	high	1905.828	0.000%	0.000%	0.000%
		high	low	2037.552	-5.351%	1.155%	-4.196%
		high	high	2029.925	-4.416%	1.953%	-2.463%
high	low	low	low	2122.919	0.851%	-2.328%	-1.478%
		low	high	2020.818	-0.270%	-3.293%	-3.563%
		high	low	2333.316	0.000%	0.000%	0.000%
		high	high	2566.700	-0.615%	-0.672%	-1.287%
high	high	low	low	2185.731	-1.075%	-1.592%	-2.667%
		low	high	2134.705	0.839%	-2.093%	-1.254%
		high	low	2561.594	-0.820%	0.365%	-0.455%
		high	high	2366.510	0.000%	0.000%	0.000%

Singles. As a final exercise, we consider the unobserved match quality of singles. Table 8 is based on our results in Tables 14 and 15 in Appendix D and has an analogous interpretation as Table 7. Because singles do not benefit from scale economies related to public consumption, our structural model requires a positive unobserved quality to rationalise singlehood as a stable situation. Through the lens of our structural model, this can be interpreted as a "quality of singlehood" or, alternatively, as a "cost of marriage". We find that this match quality of singles (as a proportion of total consumption) is rather substantial in most cases. The sole exception is for single males with high BMI and high EDU, who experience a match quality that is fairly close to zero. Furthermore, the BMI effects are generally larger in magnitude than the EDU effects for both males and females.

A notable observation is that the match quality of single females is generally (and often considerably) above that of single males. This can be related to the fact that single females have, on average, different preferences over public and private consumption than single males. In particular, single females are much more likely to have children than single males (see Table 11). We need a high quality of singlehood to compensate for the absence of scale economies associated with public consumption in marriage. Given that children's preferences are internalised in adults' preferences, it can be expected that this implies a stronger preference for public consumption by the average single female (see also the private and public consumption shares in Table 5). As public consumption is "cheaper" for individuals in couples (because of scale economies), we thus need a substantially higher unobserved quality for single females than for single males to rationalise their singlehood. Moreover, it is plausible that female singles with children are more reluctant to enter a new relationship than single men, given that it is much more likely that her preferences also contain her children's preferences. This, on its turn, is translated in a quality of singlehood that is higher for females than for males.

Finally, single males and single females face a similar EDU effect (albeit more pronounced for males than females): low EDU individuals experience a higher quality of singlehood than high EDU individuals. By contrast, males and females face opposite BMI effects: higher BMI leads to higher unobserved quality for females, while the opposite applies to males. This can be related to the different characteristics of single males and single females described in Tables 2 and 3. Specifically, we observe relatively many single males with low EDU and low BMI, and relatively many single females with low EDU and high BMI.

Table 8: quality of singlehood (as proportion of total consumption) per type

female type			
EDU	BMI	total consumption	quality female
low	low	1031.884	12.486%
low	high	1069.388	15.846%
high	low	1222.796	12.108%
high	high	1341.747	14.453%
male type			
EDU	BMI	total consumption	quality male
low	low	1022.856	8.378%
low	high	1028.474	3.067%
high	low	1184.676	5.757%
high	high	1280.692	1.211%

8 Conclusion

We have introduced a novel methodology to empirically analyse rational household consumption under the assumption of marriage market stability. Our method allows us to (set) identify individuals' matching surplus as capturing both observed (material) public consumption and unobserved (immaterial) match quality. Using our Additive Quantity Shifting (AQS) specification, we can quantify match quality in money metric terms. We consider a setting in which the empirical analysis can use preference and match quality factors to divide agents into observable preference and match quality types. Our methodology includes both general utilities and quasi-linear (i.e., transferable) utilities of the individual household members. The methodology builds on a revealed preference characterisation of rationalisable household behaviour that is intrinsically nonparametric, making it robust to functional specification error.

We have demonstrated the practical usefulness of our methodology through an application to the Belgian MEqIn data. We started by verifying that the model with general utilities (rather than quasi-linear utilities) and unobserved match quality provides a good fit of the data, indicating that it can rationalise the observed heterogeneity in consumption and marriage behaviour. Further, our application showed that our nonparametric method has substantial (set) identifying power, even when imposing little prior structure on the setting at hand. We can identify bounds on the unobserved match quality that are informatively tight, and we can meaningfully analyse the intrahousehold allocation of consumption and match quality.

For example, our results reveal that in "mixed" couples with a high educated and a low educated partner, the higher educated spouse experiences considerably more unobserved match quality than the lower educated spouse. In addition, we identify a positive match quality of singles, which is substantially higher for females than for males. These patterns provide an intuitive explanation of the observed marriage and consumption allocations through the lens of our structural decision model. In particular, higher unobserved (immaterial) match quality can compensate for (material) losses that follow from lower consumption. This holds in particular for individuals with a strong preference for public consumption in the household such as single females with children.

Our empirical application has principally concentrated on the identification of unobserved match quality, which is a main novelty of our newly proposed methodology. Importantly, however, our method is versatile in that it can also be used to identify other unobserved aspects of household consumption decisions (related to individual utilities and intrahousehold sharing). In addition, it can be usefully combined with other revealed preference methods, so further enriching the empirical analysis. For example, we can explicitly include a model of household production as in Cherchye, De Rock, Walther, and Vermeulen (2021). Next, we can empirically identify the degree of publicness of household consumption (defining the intrahousehold scale economies) by using the toolkit of Cherchye, De Rock, Surana, and Vermeulen (2020). Finally, as our method allows us to identify the individual (Lindahl) prices for public consumption, we can integrate the methodology of Cherchye, Cosaert, De Rock, Kerstens, and Vermeulen (2018) to evaluate individual welfare in money metric terms for households that consume public goods, along the lines of Chiappori and Meghir (2014) and Chiappori (2016).

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Appendix

A AQS and intrahousehold allocation

To show the link between match quality and intrahousehold allocation, we take a collective model of household consumption in which the partners m and w have scalar match quality indices θ^m and θ^w. We further consider the case with no public goods. This, however, is mainly for simplicity of notation. As in the main text, we can include public goods by using conditional utility concepts.

Following our argument in the main text, we consider indirect utility functions of m and w that are strictly increasing and concave in expenditures:

$$V(p, \theta, x) = V\left(p, 0, x + \Delta(p, \theta)\right), \tag{1}$$

with $\Delta(p, \theta) = p\theta$ expressing the willingness to pay to receive a given match quality θ for price p. We shall demonstrate that if his match quality increases from zero to some value $\theta^m > 0$, holding prices, total household income and her match quality constant (for example, at zero), then both household members are better off.

As prices p are fixed, we omit them from the notation. Let household income be y and consider the collective household utility function:

$$W(y, x^m, \theta^m) = \mu(y) V^w(y - x^m) + V^m(x^m + \Delta(\theta^m)).$$

Here $\mu(y)$ is the Pareto weight for the woman. Notice that we assume μ to be independent of the match quality θ. This means that we ignore general equilibrium effects of match quality on the Pareto weight μ. Alternatively, we may assume that the Pareto optimal allocation is interior in the sense that none of the participation constraints of the household members (determined by the marriage market) are binding.

The first order conditions for maximisation with respect to x^m yields a solution x^* that satisfies:

$$\mu(y) = \frac{\frac{\partial V^m(x^* + \Delta(\theta^m))}{\partial x}}{\frac{\partial V^w(y - x^*)}{\partial x}}.$$

Since the left hand side of the equation does not depend on match quality, we have:

$$\frac{\frac{\partial V^m(x^0)}{\partial x}}{\frac{\partial V^w(y - x^0)}{\partial x}} = \frac{\frac{\partial V^m(x^* + \Delta(\theta^m))}{\partial x}}{\frac{\partial V^w(y - x^*)}{\partial x}},$$

where x^0 is the optimal outlay for m at $\theta^m = 0$. Re-arrange to:

$$\frac{\frac{\partial V^w(y - x^*)}{\partial x}}{\frac{\partial V^w(y - x^0)}{\partial x}} = \frac{\frac{\partial V^m(x^* + \Delta(\theta^m))}{\partial x}}{\frac{\partial V^m(x^0)}{\partial x}}. \tag{2}$$

As both V^m and V^w are strictly concave, we have that:

$$y - x^* > y - x^0 \Leftrightarrow x^* + \Delta(\theta^m) > x^0.$$

Thus, either both members' "outlay" increases or both members' "outlay" decreases. In other words, either both members are better off, or both are worse off.

B Mixed integer linear programming formulation

We ease the notational burden by focusing on the specific case without unobserved match quality (i.e., all $\theta^m_{i,r}$ and $\theta^w_{i,r}$ are set to zero). The characterization below extends to the case with unobserved match quality by using the same transformations as in our proofs of Theorems 1 and 2 in the Online Appendix.

We can reformulate the ARSM condition in Definition 2 in terms of inequality constraints that are linear in unknowns and characterized by binary integer variables, which makes them easy to operationalise. For convenience, let us focus on the ARSM condition expressed in terms of weak inequalities (the argument for the case of strict inequalities is directly similar). Particularly, we use the binary variables $Z^{\psi}_{i,k} \in \{0,1\}$ and $Z^{\omega}_{r,s} \in \{0,1\}$ to represent the utility orderings of male type ψ and female type ω, in the following sense:

$$Z^{\psi}_{i,k} = 1 \text{ if } U^{\psi}(k) \geq U^{\psi}(i), \tag{3}$$

$$Z^{\omega}_{r,s} = 1 \text{ if } U^{\omega}(s) \geq U^{\omega}(r). \tag{4}$$

Then, we can state the following result (using G to denote a sufficiently large number; e.g., $G \geq y_{i,r}$ for all i and r).[19]

Proposition 1. *A data set S satisfies the ARSM if and only if, for all couples $(i,r) \in M \times W$, with $\tau(i) = \psi$ and $\tau(r) = \omega$, there exist*

- *a utility value $U^{\psi}(i) \in [0,1]$ for man i of type ψ,*
- *a utility value $U^{\omega}(r) \in [0,1]$ for women r of type ω,*
- *price vectors $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ with $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, and*
- *binary variables $Z^{\psi}_{i,k}, Z^{\omega}_{r,s} \in \{0,1\}$,*

such that:

$$U^{\psi}(k) - U^{\psi}(i) < Z^{\psi}_{i,k}, \tag{5}$$

$$U^{\omega}(s) - U^{\omega}(r) < Z^{\omega}_{r,s}, \tag{6}$$

and:

$$y_{i,r} - p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) - P^{m\prime}_{i,r} Q_{k,\sigma(k)} - P^{w\prime}_{i,r} Q_{\sigma(s),s} \leq (2 - Z^m_{i,k} - Z^w_{r,s})G. \tag{7}$$

[19] We note that the strict inequalities $U^{\psi}(k) - U^{\psi}(i) < Z^{\psi}_{i,k}$ and $U^{\omega}(s) - U^{\omega}(r) < Z^{\omega}_{r,s}$ are difficult to use in mixed integer linear programming analysis. Therefore, in practice we can replace them with $U^{\psi}(k) - U^{\psi}(i) + \epsilon \leq Z^{\psi}_{i,k}$ and $U^{\omega}(s) - U^{\omega}(r) + \epsilon \leq Z^{\omega}_{r,s}$ for ϵ (> 0) arbitrarily small.

For this result, we normalize without loss of generality the utility values $U^{\psi}(i)$ and $U^{\omega}(r)$ so that they can only take values between 0 and 1. Then, (5) implements (3), which ensures that the binary variables $Z^{\psi}_{i,k} \in \{0,1\}$ represent the utility orderings of male type ψ. Similarly, (6) implements (4), which pertains to the utility orderings of female type ω. Also (7) will only be binding if both $Z^{m}_{i,k}$ and $Z^{w}_{r,s}$ are equal to 1. Given all this, the equivalence between the ARSM specification in Definition 2 and the one in Proposition 1 follows readily.

C Belgian households: additional summary information

Table 9: percentage shares of match quality types in our sample

		couples			
		low EDU female		**high EDU female**	
		low BMI	high BMI	low BMI	high BMI
low EDU male	low BMI	10.309%	4.639%	8.247%	1.546%
	high BMI	10.825%	5.670%	6.701%	6.186%
high EDU male	low BMI	2.577%	1.546%	14.948%	4.639%
	high BMI	3.093%	3.608%	10.825%	4.639%
		singles			
		low EDU		**high EDU**	
		low BMI	high BMI	low BMI	high BMI
	males	34.783%	31.884%	20.290%	13.043%
	females	33.871%	25.000%	27.419%	13.710%

Table 10: preference factors per type - couples (mean values)

female type		**male type**					
EDU	BMI	EDU	BMI	female age	male age	presence of children (1 = yes/0 = no)	number of children
low	low	low	low	36.474	39.789	0.579	1.211
		low	high	35.545	38.091	0.636	0.909
		high	low	33.600	44.000	0.800	1.000
		high	high	37.500	42.333	0.500	0.667
low	high	low	low	37.222	40.000	0.889	1.556
		low	high	41.455	42.636	0.545	1.091
		high	low	36.000	40.667	1.000	1.333
		high	high	44.571	48.714	0.571	1.000
high	low	low	low	35.500	37.000	0.813	1.063
		low	high	40.077	43.538	0.615	0.923
		high	low	37.138	37.759	0.586	1.069
		high	high	41.714	44.190	0.619	1.143
high	high	low	low	40.667	43.000	0.667	1.333
		low	high	38.000	38.417	0.833	1.500
		high	low	40.750	41.750	0.875	1.375
		high	high	40.300	43.000	0.600	1.400

Table 11: preference factors per type - singles (mean values)

female type				
EDU	BMI	female age	number of children	presence of children (1 = yes/0 = no)
low	low	40.857	0.643	0.452
low	high	46.677	1.129	0.613
high	low	40.294	1.088	0.647
high	high	43.118	1.000	0.706
male type				
EDU	BMI	male age	number of children	presence of children (1 = yes/0 = no)
low	low	42.500	0.208	0.167
low	high	48.091	0.091	0.091
high	low	40.286	0.571	0.357
high	high	47.333	0.556	0.222

D Bounds on match quality

Table 12: Match quality bounds for married females (euros per week)

female type		**male type**		**lower bounds**		**upper bounds**	
EDU	BMI	EDU	BMI	mean	st.dev.	mean	st.dev.
low	low	low	low	0.000	0.000	0.000	0.000
		low	high	-12.507	14.137	-11.682	13.999
		high	low	-85.530	24.160	-81.290	25.163
		high	high	-82.706	20.591	-78.093	21.951
low	high	low	low	-32.646	25.098	-28.888	24.310
		low	high	0.000	0.000	0.000	0.000
		high	low	-112.746	39.962	-105.317	45.891
		high	high	-90.415	26.131	-88.877	25.306
high	low	low	low	16.751	10.615	19.370	10.543
		low	high	-8.063	22.816	-2.843	27.008
		high	low	0.000	0.000	0.000	0.000
		high	high	-16.097	14.505	-15.456	14.207
high	high	low	low	-31.294	24.695	-15.704	13.295
		low	high	15.294	19.394	20.514	16.367
		high	low	-22.743	16.658	-19.259	16.585
		high	high	0.000	0.000	0.000	0.000

Table 13: Match quality bounds for married males (euros per week)

male type		**female type**		**lower bounds**		**upper bounds**	
EDU	BMI	EDU	BMI	mean	st.dev.	mean	st.dev.
low	low	low	low	0.000	0.000	0.000	0.000
		low	high	3.655	16.952	11.218	17.805
		high	low	-52.024	13.297	-46.829	14.781
		high	high	-40.697	6.651	-28.900	14.098
low	high	low	low	-26.924	19.639	-24.727	18.901
		low	high	0.000	0.000	0.000	0.000
		high	low	-69.227	15.388	-63.879	15.743
		high	high	-47.202	8.183	-42.159	9.098
high	low	low	low	24.499	28.435	33.014	30.405
		low	high	18.977	35.805	28.082	31.121
		high	low	0.000	0.000	0.000	0.000
		high	high	6.512	15.949	12.184	15.438
high	high	low	low	6.220	15.702	15.696	14.779
		low	high	35.708	16.913	43.587	15.798
		high	low	-18.209	17.115	-16.298	16.905
		high	high	0.000	0.000	0.000	0.000

Table 14: Match quality bounds for single females (euros per week)

female type		**lower bounds**		**upper bounds**	
EDU	BMI	mean	st.dev.	mean	st.dev.
low	low	127.823	13.732	129.865	14.033
low	high	168.573	19.029	170.332	18.490
high	low	147.537	13.708	148.570	14.485
high	high	192.893	27.010	194.950	26.231

Table 15: Match quality bounds for single males (euros per week)

male type		**lower bounds**		**upper bounds**	
EDU	BMI	mean	st.dev.	mean	st.dev.
low	low	82.146	16.054	89.252	15.668
low	high	31.163	19.407	31.922	19.104
high	low	72.283	16.105	64.109	15.260
high	high	10.917	33.920	20.100	28.729