Revised triple alpha reaction rate in high temperature environments

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(Dated: July 13, 2020)

The triple alpha reaction is one of the most important reactions in the nuclear astrophysics. However, its reaction rate in high temperature environments at $T_9 > 2$ was still uncertain because the radiative decay probability of the 3_1^- state in ^{12}C as a key parameter to estimate the reaction rate was unknown. In the present work, we have determined the radiative decay probability of the 3_1^- state to be $1.3^{+1.2}_{-0.8} \times 10^{-6}$ for the first time by measuring the ^{1}H(^{12}C,^{12}Cp) reaction, and found that the 3_1^- state noticeably enhances the triple alpha reaction rate. Although it had been considered that the triple alpha reaction rate at $T_9 > 2$ is significantly smaller than the estimation in NACRE, the new triple alpha reaction rate is consistent with that in NACRE within its uncertainty.

When our universe began about 13.8 billion years ago, no elements existed there. All of the elements were synthesized in the history of the universe by nuclear reactions.

Helium, the second abundant element in the universe, was synthesized by a series of proton/neutron capture or transfer reactions during the big bang nucleosynthesis in 3–20 minutes after the beginning of the universe. Since there is no bound state in the $A = 5$ isobar, this proton and neutron capture chain suspended at $A = 4$. ^{8}Be nuclei were produced in ^{4}He + ^{4}He collisions, but they decayed back to two ^{4}He nuclei with very short lifetimes. Therefore, heavy elements with $A > 4$ were rarely synthesized in the early universe.

Heavier elements than He were synthesized in stars. Stars synthesize ^{4}He in proton-proton chain reactions or the CNO cycle during they remain in the main sequence. ^{4}He becomes abundant in cores of stars when stars exhaust hydrogen and leave from the main sequence. However it is not trivial how heavy elements are synthesized from ^{4}He in stars unless the the bottlenecks at $A = 5$ and 8 are solved. This was a serious puzzle in physics until 1950s.

It is widely known that this puzzle was solved by E. E. Salpeter and F. Hoyle [1]. Salpeter proposed that ^{12}C should be synthesized by the triple alpha (3α) reaction in dense and hot environments in stars [2], and Hoyle predicted that a 3α resonance should exist at slightly above the 3α decay threshold in ^{12}C to explain the cosmic abundance ratio of He:C:O in a scenario with the 3α reaction [3]. This predicted 3α resonance was experimentally established by D. N. F. Dunbar *et al.* [4]. This state is now called the Hoyle state.

In the 3α reaction, an α particle is captured by ^{8}Be which is a 2α resonance, and consequently an excited state in ^{12}C is populated as a 3α resonance. At normal stellar temperatures $T_9 \sim 0.1$ (T_9 is the temperature in units of 10^9 K.), this process proceeds mainly via the Hoyle state at $E_x = 7.654$ MeV, but high-lying 3α resonances such as 3^-_1 at $E_x = 9.64$ MeV and 2^+_2 at $E_x = 9.87$ MeV play a significant role at higher temperatures. Most of these 3α resonances decay back to three α particles, but an extremely little faction of them is de-excited to the ground state in ^{12}C by radiative decay. The 3α reaction rate, therefore, strongly depends on the radiative decay probabilities of the 3α resonances, which are given by the ratios of the radiative decay widths $\Gamma_{\rm rad}$ to the total widths $\Gamma_{\rm tot}$. $\Gamma_{\rm rad}$ is the sum of the γ-decay width Γ_γ and the pair production decay width $\Gamma_{e^+e^-}$.

The 3α reaction is the doorway reaction that bypasses the $A = 5$ and 8 bottlenecks and allow the production of heavier elements, and thus it is one of the most important nuclear reactions in the nucleosynthesis. S. Wanajo theoretically examined the νp-process during the supernovae explosion and found that a small variation of the 3α reaction rate at $T_9 > 2$ drastically changes the heavy element production [5]. If the 3α reaction rate would increase several times at $T_9 > 2$, the production of p-nuclei with $A > 80$ would be suppressed by several orders of magnitude.

In nuclear astrophysical calculations, the 3α reaction rate esitmated in the NACRE compilation [6] has been widely used. However, large uncertainties remained in the 3α reaction rate at $T_9 > 2$ in NACRE due to the lack of the experimental information on the 3^-_1 and 2^+_2 states.

The 2^+_2 state was naturally predicted as an excited state of the relative motion of the α particles in the Hoyle state by α cluster-model (ACM) calculations [7–10], but its existence was experimentally controversial for a long time. H. O. U. Fynbo *et al.* reported that the 2^+_2 state was not observed in the β decay of ^{12}N and ^{12}B, and claimed its contribution to the 3α reaction is negligible [11]. Later, M. Itoh *et al.* found the 2^+_2 state [12], and W. Zimmerman *et al.* experimentally determined its energy, total width, and radiative decay width [13]. On the other hand, the radiative decay width of the 3^-_1 state is still unknown.

The 3^-_1 state decays to the ground state by either a direct decay or a sequential decay via the 2^+_1 state at $E_x = 4.440$ MeV. The direct decay is an $E3$ transition, and its width is already known as 0.31 ± 0.04 meV from the (e, e') measurement [14]. Since the total width of the 3^-_1 state is 46 ± 3 keV [15], the direct-decay probability is $(6.7 \pm 1.0) \times 10^{-9}$. This is the lower limit of the radiative decay probability of the 3^-_1 state. On the other hand, the $E1$ and $E3$ transitions are actually allowed in the sequential decay, however the isospin symmetry suppresses the $E1$ transition since both the 3^-_1 and 2^+_1 states are isoscalar states. Nevertheless, the $E1$ transition might still have a lager width than the $E3$ transition due to the two reasons. First, the isospin symmetry is slightly broken due to the Coulomb interaction. Second, $E1$ transitions are generally much stronger than $E3$ transitions. Actually, it was reported that a typical $E1$ transition rate between the isoscalar states around $A = 12$ is $10^{-3.6}$ Weisskopf unit [16], which corresponds to $\Gamma_{\rm rad} = 15$ meV in the $3^-_1 \to 2^+_1$ transition. This is significantly larger than $\Gamma_{\rm rad} = 2$ meV adopted in NACRE. Therefore, the 3α reaction rate via the 3^-_1 state might be much larger than the estimation in NACRE.

A pioneering work to determine the radiative decay probability of the 3^-_1 state was carried out by measuring α inelastic scattering from ^{12}C back in 1970s [17]. Once the 3α resonances in ^{12}C are excited by the α inelastic scattering, these states decay either to three α particles or to the ground state in ^{12}C by emitting γ rays or e^+e^- pairs. The radiative decay events can be identified by detecting ^{12}C in the final state without detecting γ rays nor e^+e^- pairs. In Ref. [17], recoil ^{12}C nuclei after radiative decay were detected in coincidence with scattered α particles. However, small ^{13}C impurities in the isotopically enriched ^{12}C target caused serious backgrounds, and thus only the upper limit of the radiative decay probability of the 3^-_1 state was reported as 8.2×10^{-7} at a confidence level of 95%.

In the present work, proton inelastic scattering from ^{12}C was measured in order to determine the radiative decay probability of the 3^-_1 state. The measurement was carried out under the inverse kinematic condition in which a ^{12}C beam bombarded a hydrogen target. Scattered ^{12}C nuclei were detected in coincidence with recoil protons. Since no ^{13}C impurity was contained in the ^{12}C beam, the signal-to-noise ratio was much improved.

The experiment was carried out at the cyclotron facility in Research Center for Nuclear Physics (RCNP), Osaka University. A ^{12}C^{5+} beam at 262 MeV bombarded a hydrogen target in the scattering chamber of the Grand Raiden (GR) spectrometer [18]. The unreacted beam was stopped in the Faraday cup downstream of a collimator plate for GR. A solid hydrogen target (SHT) system was newly developed to improve the hydrogen-to-contaminant ratio better than the gas target [19]. Pure hydrogen gas was fully converted to the parahydrogen whose thermal conductivity is about 10 times higher than the normal hydrogen [20]. The parahydrogen gas was filled into the target cell made of copper and cooled down to 9.6 K by a Gifford-McMahon refrigerator. A very thin SHT with a thickness of 0.65 mm was made to keep the excitation-energy resolution in ^{12}C better than 0.65 MeV at the full width at half maximum. The entrance and exit windows of the target cell were 15 mm in diameter and sealed with 6-μm thick aramid films. Backgrounds due to the window films were subtracted by an empty-cell measurement.

Recoil protons were detected by using the GAGG [21] based light ion (Gion) telescope which was located at $\theta_{\rm lab} = -41^\circ$. The Gion telescope consisted of a double-sided Si strip detector (DSSD) and 24 GAGG scintillators. The particle identification was carried out with the ΔE-E correlation between the DSSD and the GAGG scintillators. The thickness of the DSSD was 650 μm, and the sensitive area was 48 mm in horizontal and 128 mm in vertical. The front and rear sides of the DSSD were divided into the 16 vertical strips and 32 horizontal strips, respectively. The GAGG crystals with a dimension of 18 mm $\times$ 18 mm $\times$ 25 mm were wrapped with enhanced specular reflector (ESR) films [22]. The thickness of the ESR film was 65 μm. The 24 GAGG crystals were mounted on avalanche photodiodes and stacked in 8 rows and 3 columns behind the DSSD. The distance between the Gion telescope and the target was 125 mm, and the the 8 rows of the GAGG crystals were arranged to arch with respect to the target arched with respect to the target.

The GR spectrometer was located at $\theta_{\rm lab} = 2.8°$ covering $\Delta\theta_{\rm lab} = \pm 0.8°$ and $\Delta\phi_{\rm lab} = \pm 30$ mr in the horizontal and vertical directions. Scattered ^{12}C nuclei or decay α particles from excited states in ^{12}C were momentum analyzed by GR and detected by the focal plane detectors. The focal plane detectors consisted of the two multiwire drift chambers (MWDCs) and two plastic scintillators (PS1 and PS2). They were tilted along the focal plane by 45° with respect to the central orbit of GR. Helium bags were installed between the detectors to suppress the multiple scattering by air. The MWDCs were operated using a detection gas of He (50%) + CH_4 (50%). The thicknesses of PS1 and PS2 were 1 mm and 10 mm so that ^{12}C nuclei stop in PS1 but α particles penetrate it. By using an anti-coincidence technique between PS1 and PS2, trigger signals for ^{12}C events were generated.

Figure 1(a) shows the excitation-energy spectrum for the ^{12}C(p,p') reaction obtained with the SHT after the backgrounds due to the window films were subtracted. In the inverse kinematic measurement using the discrete-type position detector like the DSSD, spurious peaks are observed in excitation-energy spectra near the most backward angle where recoil protons can be emitted (critical angle). Therefore, we eliminated events near the critical angle from the present analysis by reducing the effective area of Gion to 73%, 51% and 3% for the three different excitation-energy regions at $E_x < 8.5$ MeV, 8.5 MeV $\le E_x < 10.7$ MeV, and $E_x \ge 10.7$ MeV, respectively.

The excitation-energy spectrum for the radiative decay events was acquired from the coincidence events between protons and ^{12}C nuclei. Accidental coincidence events in which a ^{12}C nucleus and a proton from different events were detected at the same time caused serious backgrounds. In such events, two recoil protons must be emitted, therefore we set the angular acceptance of Gion to be large enough to detect both of these protons for rejecting most of the accidental coincidence events. In addition, the angular and energy correlations between the detected proton and ^{12}C were also employed to reject the accidental coincidence events.

The accidental coincidence events can be virtually generated by the event mixing analysis of singles events in GR and Gion. It was found that the accidental coincidence events were reduced by a factor of 100 thanks to the angular and energy correlations. The gray histogram in Fig. 1(b) presents the excitation-energy spectrum for the remaining accidental coincidence events. The excitation-energy spectrum for the true coincidence events was obtained by subtracting these accidental coincidence events as shown by the open histogram.

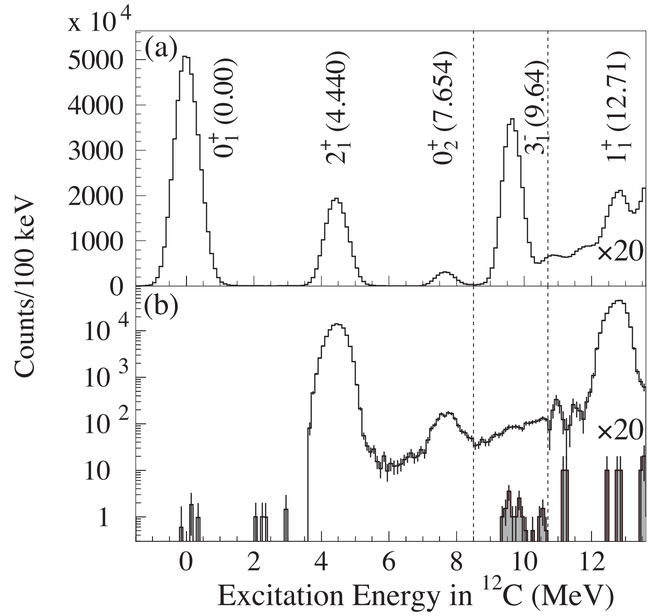

FIG. 1. Excitation-energy spectra of ^{12}C for (a) the singles events and (b) the coincidence events in the inelastic proton scattering. The gray histogram presents the accidental coincidence events. The vertical dashed lines at $E_x = 8.5$ and 10.7 MeV divide the spectra into the three excitation-energy regions measured by using different sensitive areas of Gion. The spectra at $E_x \ge 10.7$ MeV are multiplied by a factor of 20.

The yields of the singles and coincidence events were obtained to determine the radiative decay probabilities. Figures 2(a) and (b) show the excitation-energy spectra of the singles and coincidence events around the 3_1^- state, which were measured with 51% of the sensitive area of Gion optimized for $E_x = 8.5$–10.7 MeV. Both of the spectra were fitted by the two gaussian functions for the 3_1^- and 0_2^+ states and the smooth background function. The centroids and widths of the gaussian functions were determined to reproduce the singles spectrum, and the same values were used for the coincidence spectrum. The background function was assumed to be the sum of the semi-phenomenological function taken from Ref. [23] and a constant offset. It should be noted that a small peak due to the 3_1^- state was observed in the coincidence spectrum at the statistical peak significance of 91%. Finally, the singles and coincidence yields of the 3_1^- state were obtained as listed in Table I. Similarly, the 0_2^+ and

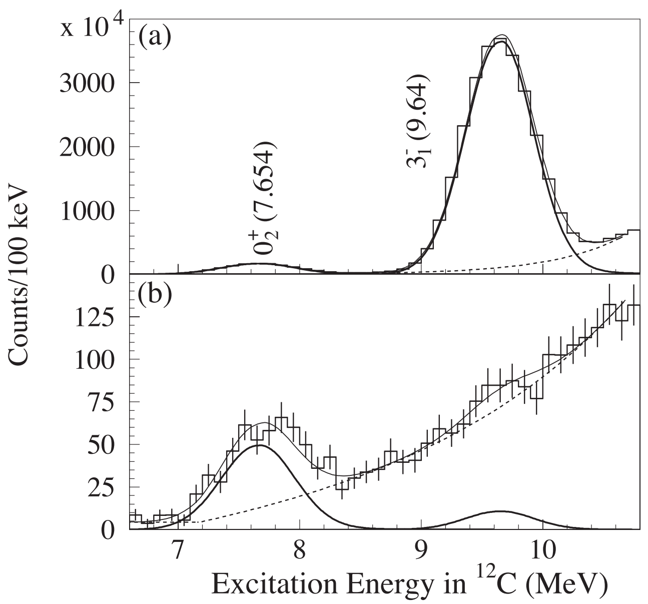

FIG. 2. Excitation-energy spectra of ^{12}C around the 3_1^- state for (a) the singles events and (b) the coincidence events in the inelastic proton scattering. Thin solid lines show the fit functions for the 0_2^+ and 3_1^- states while the dashed lines show the background. The thick solid lines present the sum of the all fit functions.

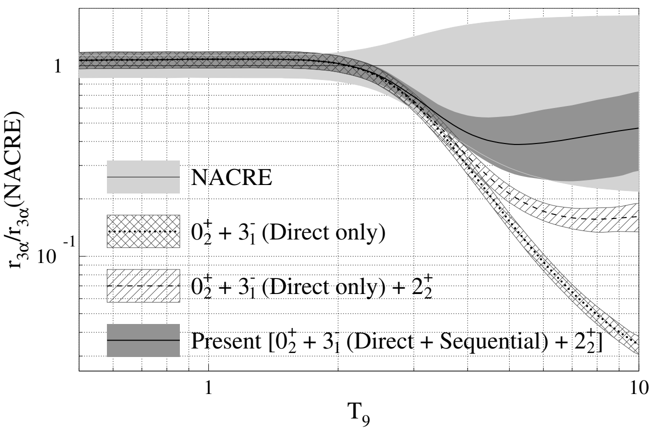

FIG. 3. Ratios of the 3α reaction rates to that from NACRE [6] and their uncertainties at T_9 = 0.5–10. The light gray band shows uncertainty in NACRE. The dotted line shows the 3α rate when the 0_2^+ state and the direct decay of the 3_1^- state are taken into account as suggested in Ref. [11]. The dashed line shows the same calculation with the dotted line but the contribution from the 2_2^+ state is also taken into account as suggested is Ref. [13]. The thick solid line presents the new calculation taking into account all the contributions from the 0_2^+, 3_1^-, and 2_2^+ states. The numerical values of the 3α rates at T_9 = 0.01–10 can be found in the Supplemental Material.

1_1^+ states were also analyzed with the excitation-energy spectra measured with 73% and 3% of the sensitive area of Gion optimized for $E_x < 8.5$ MeV and $E_x \geq 10.7$ MeV, respectively. The systematic uncertainties of the yields were estimated by changing the shape of the backgrounds. The uncertainties of the yields in Table I include the statistical and systematic uncertainties.

The radiative decay probability is given by

$$\frac{\Gamma_{\rm rad}}{\Gamma_{\rm tot}} = \frac{\text{(Yield of coincidence events)}}{\text{(Yield of singles events)}} \frac{1}{\epsilon_g \epsilon_s}.$$

ϵ_g is the geometrical efficiency for the coincidence measurement, and ϵ_s is the event-selection efficiency in the accidental-event rejection with the angular and energy correlations. These efficiencies were estimated by the Monte Carlo calculation as listed in Table I. Their uncertainties mainly stem from the ununiformity of the target thickness. Finally, the radiative decay probabilities for the 0_2^+, 3_1^-, and 1_1^+ states were obtained as listed in Table I. The present radiative decay probabilities for the 0_2^+ and 1_1^+ states are consistent with the literature values [15], and this warrants the reliability of the present analysis. Unfortunately, the present data cannot deny the null result for the radiative decay probability of the 3_1^- state at the fully high statistical confidence level, but its most likelihood value is larger than the previous upper limit [17]. We should carefully consider the possibility of the overestimation in the present measurement.

A possible reason for the overestimation is a wrong particle identification by the focal plane detector of GR. Because the magnetic rigidities of the decay α particles emitted from the 3_1^- state are almost same with that of ^{12}C, a sizable fraction of the decay α particles reached the focal plane as well as ^{12}C. If such α particles had been misidentified as ^{12}C, this event would have been recognized as a radiative decay event. However, this scenario is not plausible. We have estimated the probability of misidentifying the α particle as ^{12}C is lower than 10^{-7} from the data analysis and the Monte Carlo calculation.

In conventional ACMs, predicted wave functions are purely isoscalar because all of nuclear states are described on the basis of relative motions of isoscalar α particles. Therefore, the $E1$ decay from the 3_1^- state to the 2_1^+ state is extremely suppressed. The $\mathcal{D}_{3h}$ symmetry, which was proposed to be well conserved in ^{1}2C [24], also prohibits the $E1$ transition between the 3-1 and 2+1 states. Under the $\mathcal{D}_{3h}$ symmetry, the 3_1^- state has a $K = 3$ quantum number while the 2_1^+ state is described as a member of the ground-state $K = 0$ rotational band. The $\Delta K = 2$ transition is strictly forbidden in the $E1$ transition. Therefore, the large radiative decay probability, although its uncertainty is quite large, suggests that the $\mathcal{D}_{3h}$ symmetry breaking should be considered as well as the isospin symmetry breaking. Further experimental and theoretical studies are strongly desired.

Figure 3 presents the various 3α reaction rates $r_{3\alpha}$ calculated with the formula given in Ref. [6] divided by the

TABLE I. Summary of the experimental information for the 0_2^+, 3_1^-, and 1_1^+ states in ^{12}C.

	0_2^+	3_1^-	1_1^+
Yield of singles events	$(2.06 \pm 0.03) \times 10^7$	$(2.47 \pm 0.01) \times 10^8$	$(3.05^{+0.72}_{-0.76}) \times 10^6$
Yield of coincidence events	957^{+74}_{-79}	71^{+62}_{-42}	$(1.43 \pm 0.01) \times 10^4$
Geometrical and event-selection efficiency $\epsilon_g \times \epsilon_s$	$(0.317 \times 0.344) \pm 0.019$	$(0.703 \times 0.306) \pm 0.036$	$(0.988 \times 0.182) \pm 0.023$
$\Gamma_{\rm rad}/\Gamma_{\rm tot}$ (present)	$(4.3 \pm 0.8) \times 10^{-4}$	$1.3^{+1.2}_{-0.8} \times 10^{-6}$	$(2.6 \pm 0.7) \times 10^{-2}$
$\Gamma_{\rm rad}/\Gamma_{\rm tot}$ (previous) [15]	$(4.16 \pm 0.11) \times 10^{-4}$	$< 8.2 \times 10^{-7}$(95%C.L.)	$(2.21 \pm 0.07) \times 10^{-2}$
$\Gamma_{\rm tot}$ (eV) [15]	9.3 ± 0.9	$(46 \pm 3) \times 10^3$	0.40 ± 0.05

3α rate in NACRE. The resonance parameters except the radiative decay probability of the 3_1^- state used in the calculation were taken from Ref. [15]. The light gray band shows the uncertainty in NACRE. There remained the large uncertainty in the high temperature region at $T_9 > 2$ due to the poor experimental information on the 3_1^- and 2_2^+ states.

According to the suggestion in Ref. [11], when only the 0_2^+ state and the direct radiative decay of the 3_1^- state are taken into account, the 3α rate becomes much smaller than that in NACRE at high T_9 as shown by the dotted line. By including the 2_2^+ state as reported in Refs. [12, 13], the 3α rate restores but it is still smaller than NACRE as shown by the dashed line because the the radiative decay probability of the 2_2^+ state is much smaller than the assumption in NACRE. In the present work, we have determined that the sequential radiative decay probability of the 3_1^- state is significantly larger than the value assumed in NACRE for the first time. The 3α rate obtained by taking into account all the contribution from the 0_2^+, 3_1^-, and 2_2^+ states further restores as plotted by the thick solid line. It should be noted that the new rate is consistent with NACRE within a large uncertainty which was inevitable before, but its uncertainty is now reduced at high temperatures.

In summary, we have obtained the radiative decay probability of the 3_1^- state in ^{12}C by measuring the ^{1}H(^{12}C,^{12}Cp) reaction in order to determine the 3α reaction rate in high temperature environments. We determined that the radiative decay probability of the 3_1^- state is $\Gamma_{\rm rad}/\Gamma_{\rm tot} = 1.3^{+1.2}_{-0.8} \times 10^{-6}$ for the first time, and updated the 3α reaction rate. It had been considered that the 3α reaction rate at $T_9 > 2$ is significantly smaller than the estimation in NACRE. However, we found that the 3_1^- state noticeably enhances the 3α reaction rate. The new rate is consistent with that in NACRE within its uncertainty, but the uncertainty is now reduced.

The authors are grateful to the RCNP cyclotron crews for the stable operation of the cyclotron facilities. The authors also thank Prof. S. Kurosawa from Tohoku University for his kind support in the development of the Gion telescope with the GAGG scintillators and Prof. Y. Funaki from Kanto Gakuin University for the fruitful discussion on the α cluster structure in ^{12}C. M. T. appreciates the support of Grant-in-Aid for JSPS Research Fellow JP14J01256. This research was supported by JSPS KAKENHI, Grants No. JP15H02091 and No. JP19H05604, and Fonds de la Recherche Scientifique - FNRS Grant Number 4.45.10.08.

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