Inversionless amplification and propagation in an electronuclear level-mixing scheme

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We study the propagation of a drive and probe bichromatic field in a compound electronuclear system [Opt. Commun. **179**, 525 (2000)] from the viewpoint of inversionless amplification. In the regimes of adiabatic pulse and steady-state propagation, part of the ground-state population is trapped in a dark state thereby reducing the absorption of a probe field. This is reinforced by spontaneous emission for the steady-state case, for which the optimal amplification length is calculated. Beyond this length, the medium becomes absorbent for the probe field. In the complementary limit of ultrashort pulses, the medium reduces to a *V* scheme in which the drive field can self-induce transparency.

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I. INTRODUCTION

The hope to realize laser amplification on γ transitions is at the origin of much theoretical and experimental research (see the recent reviews [1,2]). The main difficulty that arises in this context is the impossibility of creating an inversion of population between adequate pairs of levels. In principle, this obstacle can be overcome by exploiting quantum interference. Amplification without population inversion (AWI) was predicted [3–5] and demonstrated experimentally [6–13] on atomic transitions. For application of these ideas to γ optics, it was proposed to use a radio-frequent field to couple the hyperfine levels of a nuclear ground state to create the necessary coherence [14]. However, due to the equal population of all hyperfine levels for a sample at room temperature, this coherence cannot be realized for the ensemble of all nuclei [15], so that the extension of the principles of atomic AWI to nuclear transitions remains problematic. In response to this problem, ways to couple atomic and nuclear transitions were proposed in [16,17]. The investigated schemes involve Mössbauer nuclei, i.e., nuclei that emit or absorb electromagnetic radiation without recoil [18]. In [17], an electronuclear system was considered where the nuclei are embedded in a noncubic crystal and immersed into a dc magnetic field of critical strength and orientation. It has been shown that, under certain conditions, the ground state of the electronuclear system can be moved into a nonabsorbing—or ''dark''—state by an intense optical field. AWI for the nuclear transitions of this scheme is thus theoretically possible. However, as the field propagates in the crystal, part of the optical intensity is lost, being used for the creation of the dark state, which progressively alters the quality of AWI. We address this issue in this paper. In [19], a similar study was done for a *V* scheme in rubidium, with special attention to Doppler broadening in vapor cells.

The present analysis is linear in the γ field (hereafter referred to as the ''probe'' field) and nonlinear in the optical field (hereafter referred to as the ''drive'' field). Our aim is to estimate the optimal length of amplification as a function of the input drive field intensity and the rate of incoherent excitation. Beyond this length, the amplifying medium turns into an absorbing medium. The knowledge of this length is therefore crucial to set up an experiment. We analyze the propagation of both continuous and pulsed beams. We shall use the terms ''adiabatic pulse'' if the polarization can be eliminated on the pulse time scale and ''short pulse'' otherwise. Two mechanisms of inversionless amplification are identified. If the drive field is a continuous beam or an adiabatic pulse, the ground-state population is irreversibly driven into the dark state. Conversely, if the drive field is a short pulse, it can create a temporal window of inversion through the mechanism of Rabi oscillations. Therefore, the amplification of a probe field in this second situation rests on the exploitation of this temporal window of inversion.

This paper is organized as follows. In Sec. II we introduce the electronuclear model and present the semiclassical equations for the light-matter interaction. In Sec. III we study the propagation of adiabatic pulses and continuous waves. More briefly, we discuss the propagation of short pulses in Sec. IV, and then we conclude in Sec. V.

II. THE MODEL

We consider Mössbauer nuclei distributed in a noncubic crystal with density *N*. A dc magnetic field is applied almost parallel to the *c* axis. The total static Hamiltonian describing the Zeeman effect and the nucleus–electron-shell hyperfine interaction near the level crossing [17] possesses the following eigenstates (see Fig. 1):

$$|1\rangle=(S_1|g,m=-3/2\rangle_n+S_2|g,m=-1/2\rangle_n)\otimes|g\rangle_s\,,$$

$$|2\rangle=(S_2|g,m=-3/2\rangle_n-S_1|g,m=-1/2\rangle_n)\otimes|g\rangle_s\,,$$

$$|3\rangle=|g,m=-3/2\rangle_n\otimes|e\rangle_s\,,$$

$$|4\rangle=|e,m=-1/2\rangle_n\otimes|g\rangle_s\,,$$

where the subscripts *n* and *s* refer, respectively, to the nucleus and electron-shell parts of the state vector, while the letters *g* and *e* indicate, for each of these parts, the ground and excited states, respectively. S_1 and S_2 are parameters adjustable via the misalignment of the dc magnetic field with

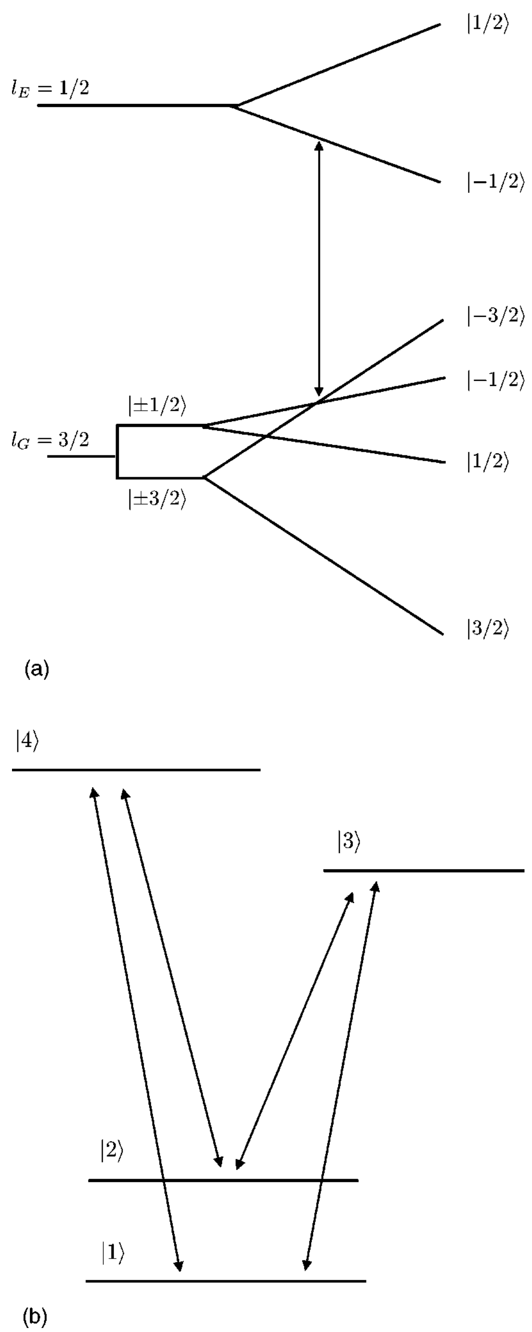

FIG. 1. Energy diagram of the nucleus interacting with the γ probe and laser pump. (a) Level-crossings scheme of the spin sublevels of the ground-state nucleus. Nuclear spin is 1/2 in the excited state and 3/2 in the ground state. The arrow indicates the γ transition. (b) States $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ as defined in the text and allowed transitions.

respect to the c axis of the crystal and m is the projection of the nuclear spin on this axis. For γ quanta propagating along this c axis, only transitions with $\Delta m = \pm 1$ are allowed, so that the state $|g, m=-1/2\rangle_n \otimes |g\rangle_s = S_2|1\rangle - S_1|2\rangle$ is nonabsorbing for that γ radiation. We let this system interact with a bichromatic field

$$E(z,t) = \frac{\hbar\Omega_d}{\mu_d} e^{i(\omega_d t - k_d z)} + \frac{\hbar\Omega_p}{\mu_p} e^{i(\omega_p t - k_p z)} + \text{c.c.},$$

where c.c. means complex conjugate. Ω_d and Ω_p are the Rabi frequencies associated with the driving and probe fields, respectively. The driving field is nearly resonant with the $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ transitions, which are characterized by the same dipole matrix element μ_d. Conversely, the probe field is coupled to the nuclear transitions $|1\rangle \rightarrow |4\rangle$ and $|2\rangle \rightarrow |4\rangle$ via the same dipole matrix element μ_p. In the rotating wave and slowly varying envelope approximations, the spatiotemporal evolution of the two fields is governed by the wave equations

$$\left(\partial_z + \frac{n_p}{c}\partial_t\right)\Omega_p = -i\frac{\kappa_p'}{2}P_p, \tag{1}$$

$$\left(\partial_z + \frac{n_d}{c}\partial_t\right)\Omega_d = -i\frac{\kappa_d'}{2}P_d. \tag{2}$$

$P_{d,p}$ are the microscopic polarizations rescaled to the dipole matrix elements, $n_{p,d}$ are the refractive indices at the two frequencies ω_p and ω_d while $\kappa_{d,p}'$ are the propagation constants:

$$\kappa_d' = \frac{N\omega_d|\mu_d|^2}{\varepsilon_0\hbar n_d c}, \quad \kappa_p' = \frac{N\omega_p|\mu_p|^2}{\varepsilon_0\hbar n_p c}.$$

Equations (1) and (2) are completed by the semiclassical density-matrix equations [17]

$$P_d = S_1\sigma_{13} + S_2\sigma_{23}, \quad P_p = S_1\sigma_{14} + S_2\sigma_{24}, \tag{3}$$

$$\frac{d\rho_{\eta\eta}}{dt} = -R\rho_{\eta\eta} + \gamma_{\|p}\rho_{44} + \gamma_{\|d}\rho_{33} + iS_\eta(\Omega_p\sigma_{4\eta} + \Omega_d\sigma_{3\eta} - \text{c.c.}), \tag{4}$$

$$\frac{d\rho_{33}}{dt} = -2\gamma_{\|d}\rho_{33} - i(\Omega_d P_d^* - \text{c.c.}), \tag{5}$$

$$\frac{d\rho_{44}}{dt} = -2\gamma_{\|p}\rho_{44} + R\sum_{\eta=1,2}\rho_{\eta\eta} - i(\Omega_p P_p^* - \text{c.c.}), \tag{6}$$

$$\frac{d\rho_{\eta\eta'}}{dt} = -(\gamma_{12} + i\omega_{\eta\eta'})\rho_{\eta\eta'} + iS_\eta(\Omega_p\sigma_{4\eta'} + \Omega_d\sigma_{3\eta'}) - iS_{\eta'}(\Omega_p\sigma_{4\eta} + \Omega_d\sigma_{3\eta})^*, \tag{7}$$

$$\frac{d\sigma_{3\eta}}{dt} = -(\gamma_{\perp d} + i\delta_\eta)\sigma_{3\eta} + i\Omega_d^*(S_\eta n_{\eta 3} + S_{\eta'}\rho_{\eta'\eta}) - iS_\eta\Omega_p^*\sigma_{34}, \tag{8}$$

$$\frac{d\sigma_{4\eta}}{dt} = -(\gamma_{\perp p} + i\Delta_\eta)\sigma_{4\eta} + i\Omega_p^*(S_\eta n_{\eta 4} + S_{\eta'}\rho_{\eta'\eta}) - iS_\eta\Omega_d^*\sigma_{43}, \tag{9}$$

$$\frac{d\sigma_{34}}{dt} = -[\gamma_{43} + i(\delta_1 - \Delta_1)]\sigma_{34} + i(\Omega_d^* P_p - \Omega_p P_d^*). \tag{10}$$

TABLE I. List of parameter values.

Parameter	Symbol	Value
Relaxation rate of P_d	$\gamma_{\perp d}$	3.0×10^9 s^{-1}
Relaxation rate of P_p	$\gamma_{\perp p}$	10^6 s^{-1}
Relaxation rate of ρ_{33}	$2\gamma_{\parallel d}$	10^3 s^{-1}
Relaxation rate of ρ_{44}	$2\gamma_{\parallel p}$	10^3 s^{-1}
Incoherent excitation of ρ_{44}	$2R$	$<10^3$ s^{-1}
Relaxation rate of ρ_{12}	γ_{12}	150 s^{-1}
Frequency separation between $\|1\rangle$ and $\|2\rangle$	ω_{21}	10^3 s^{-1}
Relaxation rate of σ_{34}	γ_{34}	3.0×10^9 s^{-1}
Drive-field Rabi frequency	Ω_d	$\sim 10^7$ s^{-1}

In these equations, we have noted $n_{ij}=\rho_{ii}-\rho_{jj}$. The indices η and η' designate the ground state 1 or 2 with $\eta\neq\eta'$. R quantifies the incoherent pumping rate of state $|4\rangle$ starting from the ground states $|1\rangle$ and $|2\rangle$. Conversely, $\gamma_{\parallel d}$ and $\gamma_{\parallel p}$ are spontaneous decay rates of the excited levels. An important parameter in the present analysis is the ratio $r=R/\gamma_{\parallel p}$: $r<1$ means that there is no population inversion between level $|4\rangle$ and the ground levels. The dephasing rates of the optical and nuclear transitions are denoted $\gamma_{\perp d}$ and $\gamma_{\perp p}$, respectively; γ_{12} is the decay rate of the quantum coherence between the ground levels. The rate γ_{34} is associated with the coherence between levels $|3\rangle$ and $|4\rangle$ and equals $\gamma_{\perp d}+\gamma_{\perp p}$. The relaxation transition probabilities between the states $|1\rangle$ and $|2\rangle$ are assumed to be small and hence are disregarded in the equations. Typical values of the relaxation rates are given in Table I [17]. The detuning parameters are $\Delta_\eta=\omega_{4\eta}-\omega_p$ and $\delta_\eta=\omega_{3\eta}-\omega_d$. From now on, we restrict the analysis to the most favorable conditions to reach inversionless amplification, i.e., $S_1=-S_2=-1/\sqrt{2}$ and the fields are detuned halfway between their corresponding transitions: $\Delta_1=-\Delta_2=\delta_1=-\delta_2=\omega_{21}/2$. Consequently, by symmetry, if ρ_{11} and ρ_{22} are initially equal, they remain equal for all times. We therefore set $\rho_{11}=\rho_{22}$ in the remainder of this paper. Moreover, we assume that the detunings are negligible compared to the polarization decay rates,

$$|\Delta_\eta|=|\delta_\eta|\ll\gamma_{\perp d},\gamma_{\perp p}. \tag{11}$$

Finally, we consider a weak probe field, meaning that we neglect its nonlinear contribution in Eqs. (4)–(10).

III. CONTINUOUS WAVE AND ADIABATIC PULSES

In the continuous-wave and adiabatic pulse propagation regimes, the field intensities are best described by the two stimulated transition rates

$$J_d\equiv\frac{|\Omega_d|^2}{\gamma_{\perp d}},\quad J_p\equiv\frac{|\Omega_p|^2}{\gamma_{\perp p}}.$$

As it will appear further in this section, J_d is the rate at which population is trapped in the dark state $S_2|1\rangle-S_1|2\rangle$ under the influence of the driving field. Population trapping is therefore a stimulated process in the present situation. The general steady-state response of the system in the weak probe limit can be found in [17]. If $J_d\ll\gamma_{34}$, the polarizations are

$$P_d=-i\Omega_d\frac{n_{13}-\mathrm{Re}\,\rho_{12}}{\gamma_{\perp d}}, \tag{12}$$

$$P_p=-i\Omega_p\frac{n_{14}-\mathrm{Re}\,\rho_{12}}{\gamma_{\perp p}(1+J_dJ_{\mathbf{s}}^{-1})}, \tag{13}$$

$$J_{\mathbf{s}}=\gamma_{34}\gamma_{\perp p}\gamma_{\perp d}^{-1}. \tag{14}$$

The real part of the low-frequency coherence, Reρ_{12}, accounts for quantum interference and should be maximized in order to obtain inversionless amplification. As we shall show, Reρ_{12} tends to a maximum value equal to n_{13} under the influence of the drive field. The absorption of the drive field is thus suppressed while the probe polarization becomes proportional to $i(\rho_{44}-\rho_{33})$. This means that the ultimate condition on γ amplification rests on population inversion between states $|3\rangle$ and $|4\rangle$. From this fact, the spontaneous decay of ρ_{33} acts in favor of γ amplification by increasing the population inversion $\rho_{44}-\rho_{33}$.

A. Adiabatic pulses

We first examine the dynamical response of the medium to a drive pulse of duration τ and average input intensity $\langle J_d\rangle\equiv\tau^{-1}\int_{-\infty}^{\infty}J_d(0,t')dt'$ such that

$$\gamma_{12},\omega_{12},\gamma_{\parallel d},\gamma_{\parallel p}\ll\tau^{-1},\langle J_d\rangle\ll\gamma_{\perp d},\gamma_{\perp p},\gamma_{34}. \tag{15}$$

Prior to the arrival of the drive and probe pulses, the atoms are distributed according to $\rho_{11}^0=\rho_{22}^0=(2+r)^{-1}$, $\rho_{44}^0=(2+r)^{-1}r$, $\rho_{33}^0=\rho_{12}^0=\sigma_{ij}^0=0$. In the weak probe limit, the population in level $|4\rangle$ is unaffected by the probe field and remains constant. Substituting Eq. (12) in Eqs. (4)– (10) and making use of approximation (15), one finds that the real part of the low-frequency coherence Reρ_{12} tends to n_{13} as

$$\frac{n_{13}-\mathrm{Re}\,\rho_{12}}{n_{13}^0-\mathrm{Re}\,\rho_{12}^0}=\exp\left(-4\int_{-\infty}^{t}J_d(z,t')dt'\right)\equiv\mathcal{K}(z,t). \tag{16}$$

The function $\mathcal{K}$ measures the degree of quantum coherence produced by the driving field. It indicates that the system tends to a stationary state where Re$\rho_{12}=n_{13}$ at a rate $\mathcal{K}^{-1}\partial\mathcal{K}/\partial t=-4J_d(z,t)$. In this sense, the creation of the dark state is similar to a stimulated process, since its rate is proportional to the number of photons in the drive field. Maximum quantum interference is produced if $\mathcal{K}\ll 1$. The details of this process are most easily understood in the basis formed by the dark state $|-\rangle=S_2|1\rangle-S_1|2\rangle$ and its orthogonal counterpart $|+\rangle=S_1|1\rangle+S_2|2\rangle$. Initially, there is equipartition of the population in the two states $|-\rangle$ and $|+\rangle$. The drive field J_d excites population from state $|+\rangle$ to state $|3\rangle$, leaving $|-\rangle$ unaffected. As the $|+\rangle\rightarrow|3\rangle$ transition saturates, the fraction of the ground-state population prepared in the dark state $|-\rangle$ is maximized. This induces an increase of the low-frequency coherence Reρ_{12}. By substituting the time-dependent solution of Eqs. (4)–(10) into Eqs. (1) and (2), one finds

$$\left(\partial_z+\frac{n_d}{c}\partial_t\right)J_d=-\kappa_d J_d\frac{\mathcal{K}}{2+r}, \tag{17}$$

$$\left(\partial_z+\frac{n_p}{c}\partial_t\right)J_p=\kappa_p J_p\frac{2r-1-(1-2J_d\gamma_{34}^{-1})\mathcal{K}}{2(2+r)(1+J_dJ_{\mathrm{s}}^{-1})}$$
$$\simeq\kappa_p J_p\frac{2r-1-\mathcal{K}}{2(2+r)}, \tag{18}$$

where $\kappa_d=\kappa_d'/\gamma_{\perp d}$ and $\kappa_p=\kappa_p'/\gamma_{\perp p}$. The propagation equation for the driving field J_d was analyzed in [4]. It has the solution

$$J_d(z,t)=\frac{J_d(0,t-n_dz/c)}{1+\left[\exp\left(\frac{2}{2+r}\kappa_d z\right)-1\right]\mathcal{K}(0,t-n_dz/c)}, \tag{19}$$

which is used to solve Eq. (18). If the refractive indices n_d and n_p are equal, one has the analytical solution for the probe pulse

$$\frac{J_p(z,t)}{J_p(0,t-n_pz/c)}=\frac{\exp\left(\frac{r-1/2}{2+r}\kappa_p z\right)}{\left\{1+\left[\exp\left(\frac{1}{2+r}\kappa_d z\right)-1\right]\mathcal{K}(0,t-n_dz/c)\right\}^{\kappa_p/(2\kappa_d)}}. \tag{20}$$

We illustrate this solution in Fig. 2. If the coherence between the ground levels is maximum, $\mathcal{K}\ll 1$, it follows from Eq. (20) that

$$J_p(z)\simeq J_p(0)\exp\left(\frac{r-1/2}{2+r}\kappa_p z\right).$$

Therefore, r must at least exceed 1/2 in order to obtain amplification, which means $\rho_{44}^0>0.2$. Although this result is significant from the point of view of lasing without inversion, it might not be sufficient in practice. Indeed, if one can bring 20% of the population in level 4 without damage for the amplifying medium, then one can probably create an inversion of population, too. However, the situation can be improved if the pulse duration is long enough for the population in level 3 to decay spontaneously to the ground levels, as already pointed out in the beginning of this section. Indeed, a stronger reduction of γ absorption can be reached if the level 3 population contributes to quantum interference between the γ transitions. This motivates us to study the limit of continuous-wave propagation.

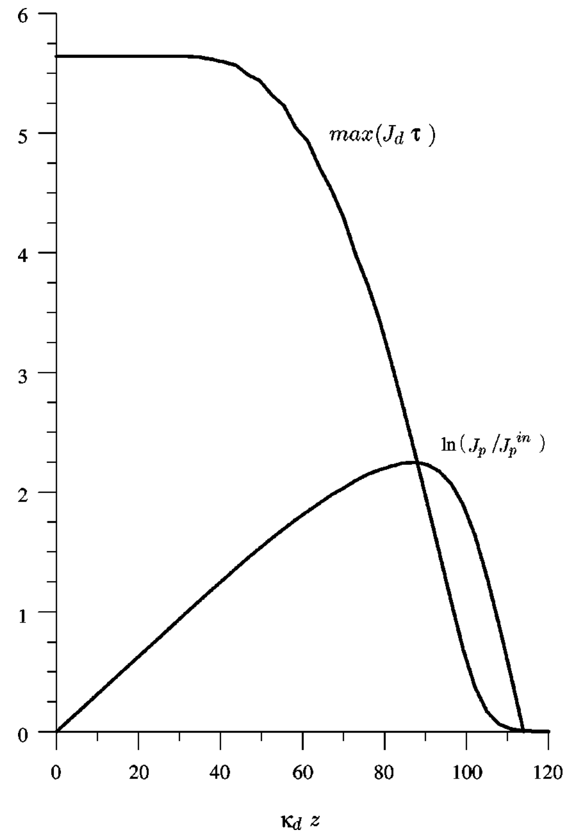

FIG. 2. Peak intensity as a function of the propagation distance in the adiabatic pulse regime. Incoherent excitation $r=0.6$, propagation constants are taken equal $\kappa_p=\kappa_d$. Input condition: $\int_{-\infty}^{+\infty}J_d(0,t)dt=10$. The other parameters are defined in Table I.

B. Continuous wave

Under approximation (11) and in steady-state, the ground state coherence and the population inversion of the atomic transitions are related by [17]

$$\mathrm{Re}\,\rho_{12}=(1-\mathcal{G})n_{13}, \tag{21}$$

where

$$\mathcal{G}\equiv\frac{\gamma_{12}J_d+J_{12}^2}{J_d^2+2\gamma_{12}J_d+J_{12}^2},\quad J_{12}=\sqrt{\gamma_{12}^2+\omega_{21}^2}. \tag{22}$$

If the drive field increases, the value of $\mathcal{G}$ decreases from 1 to 0, eventually leading to a maximum low-frequency coherence Re$\rho_{12}\simeq n_{13}$. To have $\mathcal{G}\ll 1$ requires $J_d\gg J_{12}$ or, equivalently, $J_d\gg\gamma_{12}$ and $J_d\gg\omega_{21}$. These are necessary conditions for the populating rate J_d to balance the depopulating rate γ_{12} of the dark state and compensate the optical detunings $\delta_{1,2}$. The propagation equation for the drive and probe intensities are obtained by substituting the steady-state solution of Eqs. (4)–(10) in the right-hand sides of Eqs. (1) and (2),

$$\frac{dJ_d}{dz}=-\kappa_d J_d\frac{\mathcal{G}}{2+r+(3+r)\mathcal{G}J_d\gamma_{\parallel d}^{-1}}, \tag{23}$$

$$\frac{dJ_p}{dz}=\kappa_p J_p \frac{r-\mathcal{G}-(1-r)\mathcal{G}J_d\gamma_{\parallel d}^{-1}}{(1+J_dJ_{\mathbf{s}}^{-1})[2+r+(3+r)\mathcal{G}J_d\gamma_{\parallel d}^{-1}]}. \quad (24)$$

Considering the numerator of Eq. (24) and noting that $\mathcal{G}\rightarrow \gamma_{12}/J_d$ in the strong driving-field limit $J_d \gg J_{12}$, the minimum incoherent excitation needed for the probe amplification is deduced,

$$\lim_{J_d\rightarrow\infty}[r_{\min}-\mathcal{G}-(1-r)\mathcal{G}J_d\gamma_{\parallel d}^{-1}]=0,$$

hence

$$r_{\min}=\frac{\gamma_{12}}{\gamma_{12}+\gamma_{\parallel d}}. \quad (25)$$

The value of $r_{\min}$ decreases with increasing $\gamma_{\parallel d}$, which confirms the beneficial role of level $|3\rangle$ spontaneous decay in inversionless amplification. Alternatively, one can consider propagation in a medium subjected to uniform incoherent excitation r and calculate the minimum value of the drive intensity $J_d^{\min}$ such that the probe field is amplified,

$$r-\mathcal{G}-(1-r)\mathcal{G}J_d\gamma_{\parallel d}^{-1}=0\rightarrow J_d=J_d^{\min}(r)\equiv\frac{k_1+k_2}{k_3},$$

with

$$k_1=(1-r)\left(2\gamma_{\parallel d}+\frac{J_{12}^2}{\gamma_{\parallel d}}\right)-\gamma_{12},$$

$$k_2=\left\{4(1-r)\left[r-(1-r)\frac{\gamma_{12}}{\gamma_{\parallel d}}\right]J_{12}^2 +\left[(2r-1)\gamma_{12}-(1-r)\frac{J_{12}^2}{\gamma_{\parallel d}}\right]^2\right\}^{1/2},$$

$$k_3=2r-2(1-r)\frac{\gamma_{12}}{\gamma_{\parallel d}}.$$

In addition, dz/dJ_d and dJ_p/dJ_d are analytically integrable with respect to J_d. The maximum amplification length z^* and the corresponding probe intensity $J_p^{\max}$ can thus be deduced from the input intensities J_d^{in}, J_p^{in}, and the incoherent excitation r,

$$\kappa_d z^*=\left(\frac{3+r}{\gamma_{\parallel d}}+\frac{2+r}{\gamma_{12}}\right)(J_d^{in}-J_d^{\min})+(2+r) \times\left(\ln\frac{J_d^{in}}{J_d^{\min}}-\frac{\omega_{21}^2}{\gamma_{12}^2}\ln\frac{1+\gamma_{12}J_d^{in}J_{12}^{-2}}{1+\gamma_{12}J_d^{\min}J_{12}^{-2}}\right), \quad (26)$$

$$\frac{\kappa_d}{\kappa_p}\ln\frac{J_p^{\max}}{J_p^{in}}=l_1\ln\frac{J_d^{in}}{J_d^{\min}}+l_2\ln\frac{1+J_d^{in}J_{\mathbf{s}}^{-1}}{1+J_d^{\min}J_{\mathbf{s}}^{-1}}+l_3\ln\frac{1+\gamma_{12}J_d^{in}J_{12}^{-2}}{1+\gamma_{12}J_d^{\min}J_{12}^{-2}}. \quad (27)$$

The coefficients l_j in Eq. (27) are given by

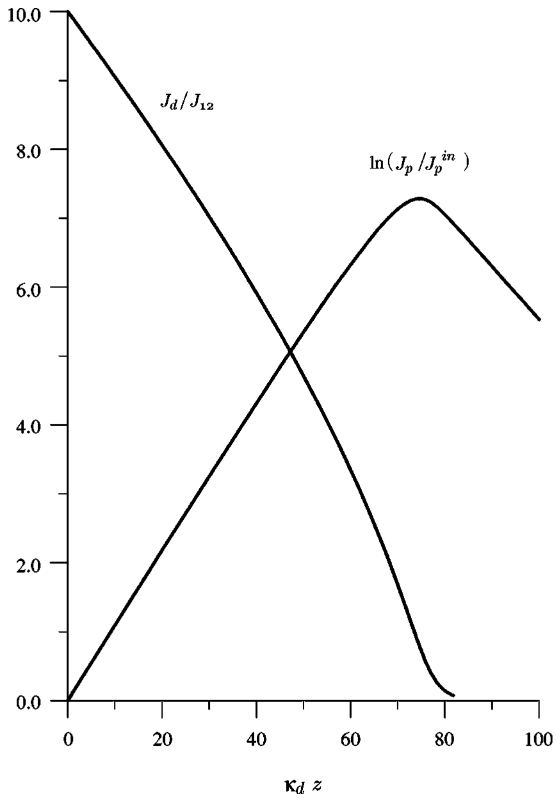

FIG. 3. Steady-state intensity versus the distance of propagation. The drive-field intensity is rescaled to J_{12}. Input condition: $J_d^{in}=10J_{12}$. Same parameters as in Fig. 2.

$$l_1=-1+r,\quad l_2=(1-r)\left(1-\frac{J_{\mathbf{s}}}{\gamma_{\parallel d}}\right)+\frac{J_{\mathbf{s}}^2-J_{\mathbf{s}}\gamma_{12}}{J_{\mathbf{s}}\gamma_{12}-J_{12}^2},$$

$$l_3=-\frac{rJ_{\mathbf{s}}\omega_{21}^2}{\gamma_{12}(J_{\mathbf{s}}\gamma_{12}-J_{12}^2)}.$$

This solution is illustrated in Fig. 3. Since the criterion of an efficient driving is $J_d \gg J_{12}$, the stimulated transition rate J_d is scaled to J_{12}. To interpret the formulas (26) and (27), let us suppose that the input drive-field intensity is much larger than J_{12} and $J_d^{\min}$. The first term in the right-hand side of Eq. (26) is then dominant, indicating that the optimal length scales linearly with the input drive intensity. This results from the fact that each segment of the medium requires a certain amount of optical intensity in order to be trapped in the dark state. Once in the dark state, the medium does not interact with the drive beam anymore. However, it still amplifies the probe beam through the population that is not trapped in the dark state. Supposing also for simplicity that $J_d^{in} \ll J_{\mathbf{s}}$, the right-hand side of the probe propagation equation (24) becomes independent of J_d and the probe intensity grows exponentially up to $z=z^*$. This yields

$$J_p^{\max}=J_p(z^*)\sim J_p(0)\exp\left[\frac{\kappa_p}{\kappa_d}\left(\frac{r}{\gamma_{12}}-\frac{1-r}{\gamma_{\parallel d}}\right)(J_d^{in}-J_d^{\min})\right].$$

This last formula shows that the total gain on the probe field depends exponentially on the input drive intensity, in the weak-probe limit.

IV. SHORT PULSES

Another regime of propagation exists if the pulse duration is shorter than all other characteristic times. The electronuclear scheme then becomes an effective V scheme. The propagation of short pulses in such a scheme was reported in [20]. To be more specific, let us introduce the bright state $|+\rangle=(|1\rangle-|2\rangle)/\sqrt{2}$. The population in this state is given by $\rho_{++}\equiv(\rho_{11}+\rho_{22}-\rho_{12}-\rho_{21})/2$. Neglecting all incoherent terms in Eqs. (4)–(10), the density-matrix equations reduce to

$$\frac{d}{dt}(\rho_{33}-\rho_{++})=-4\Omega_d\,\mathrm{Im}\,P_d,$$

$$\frac{d}{dt}\mathrm{Im}\,P_d=\Omega_d(\rho_{33}-\rho_{++}).$$

These equations show that the drive field induces Rabi oscillations between level $|3\rangle$ and the effective ground state $|+\rangle$. In the weak probe limit, a temporal window of inversion is opened which is exploitable by probe pulses of suitable shapes, as fully described in [20]. An advantage of this dynamical regime is the lossless and distortion-free propagation of the drive pulse if it has the appropriate shape for self-induced-transparency (SIT). It was numerically shown in [20] that eventually all the drive photons can be converted into probe photons, with the conservation of energy being assured by the incoherent excitation. Accordingly, the amplification condition is again that there is a population inversion between levels $|3\rangle$ and $|4\rangle$. However, SIT limits the amount of energy that can be transferred to the probe in a single pulse. If the drive pulse area well exceeds 2π, the drive pulse will break up in multiple 2π pulses, each giving rise to a separate probe pulse. As a result, since the velocity of each pulse depends on its peak intensity, light can be expected to exit the amplifying medium in a very irregular fashion.

V. CONCLUSION

We have studied the propagation of a bichromatic field through an electronuclear medium that is capable of amplification without inversion. Three regimes were identified: continuous wave, adiabatic pulse, and short pulse propagation. Continuous waves and adiabatic pulses share the same mechanism of inversionless amplification. However, amplification in the continuous-wave regime is more efficient because it is reinforced by the spontaneous decay from the state $|3\rangle$ whereas adiabatic pulse amplification is not. The maximum extractable power in the γ-radiation scales exponentially with the input drive intensity, the optimal amplification length being proportional to this input intensity.

Comparing the continuous-wave propagation to the short pulse propagation, the ultimate amplification condition is in both cases to have a population inversion between the upper states $|3\rangle$ and $|4\rangle$. While population is driven irreversibly to the dark state in the cw regime at the rate J_d, it flows periodically between states $|+\rangle$ and $|3\rangle$ in the latter regime, at the frequency Ω_d. The exchange of population between states $|+\rangle$ and $|3\rangle$ in the course of Rabi oscillation temporarily establishes an inversion $\rho_{44}^0-\rho_{33}^0$ on the probed transition. Although short pulse propagation benefits from the distortionless law of SIT propagation, the maximum amount of energy that can be obtained in a single probe pulse is limited by the 2π area of the drive pulse. This limitation does not exist in the cw regime.

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