Synchronization of two unidirectionally coupled multimode semiconductor lasers

E. A. Viktorov

Institute for Laser Physics, 199034 St. Petersburg, Russia

Paul Mandel

Optique Nonlinéaire Théorique, Université Libre de Bruxelles, Campus Plaine CP 231, B-1050 Bruxelles, Belgium

(Received 31 May 2001; published 10 December 2001)

We study unidirectionally coupled multimode semiconductor lasers. The transmitter is subject to a weak-to-moderate optical feedback. Multimode chaotic output exhibits antiphase dynamics and multiple coexisting attractors. As a result of antiphase dynamics, total output power synchronization does not require modal output synchronization. The route to synchronization is found to be via intermittency. We also report on synchronization collapse, even for identical lasers.

DOI: 10.1103/PhysRevA.65.015801  PACS number(s): 42.55.Px, 42.60.Mi

Research on laser synchronization has been boosted by potential applications in secure information transmission. Several communication schemes based on synchronizing a transmitter and a receiver were successfully developed to demonstrate that a chaotic laser output can be used as an information carrier [1,2]. A simplified model to synchronize ring laser cavities was analyzed in Ref. [3]. However, a cryptoanalytic echo-detection attack on the transmitted signal has shown the weakness of the echo-feedback laser schemes [4]: it was shown that slow and low-dimensional chaos allows the recovery of the message when the laser parameters are known. This suggests that semiconductor lasers with delayed feedback should have an advantage because of the high-dimensional chaos [5] and the gigahertz frequency range of the output. Similar motivations are developed in Ref. [2]. Experiments based on single-mode semiconductor lasers have demonstrated the possibility of synchronization [6,7] and the feasibility of a masking scheme [8]. Synchronized chaotic-mode hopping was demonstrated in Ref. [9].

Most of the above schemes are of the feedback/feedback type, meaning that the transmitter and the receiver are similar lasers with similar feedback loops. They are also unidirectional: the field propagates only from the transmitter to the receiver. The need to multiplex the transmission suggests using a multimode laser. Numerical simulations have confirmed that multichannel communication can be realized [10,11]. Our model follows the first successful experiment of a feedback/injection setup for multimode synchronization [12]. In this configuration, the output of the transmitter (which is a semiconductor laser with external feedback) is unidirectionally injected into the receiver semiconductor laser. This provides a more robust system than the feedback/injection configuration as synchronization is less sensitive to external perturbations.

In this paper, we report on the synchronization of two unidirectionally coupled multimode semiconductor lasers in the feedback/injection configuration. We show that due to the antiphase dynamics, total output synchronization does not require modal output synchronization. We also show that the route to synchronization in the chaotic regime is via a form of intermittency. Finally, we report on the possibility of synchronization collapse for identical lasers operating with identical or different parameters.

The transmitter is a multimode semiconductor laser with coherent optical feedback. The receiver is a similar multimode semiconductor laser but with an injected field instead (Fig. 1). The setup is such that the field, which is fed back into the transmitter and the field injected into the receiver are identical to meet the synchronization condition derived in Ref. [5,7]. Laser rate equations describing this system can be derived for the modal fields $E_m(t) \propto \int E(x,t) \phi_m(x) dx$ coupled to the carrier moments or nonlinear gains $N_m \propto \int N(x,t) \phi_m^2(x) dx$, proportional to the grating created by the field in a lasing cavity with eigenmode $\phi_m(x)$. After a suitable normalization, the equations become [13,14]:

$$\frac{dA_m^r}{d(\gamma_p t)} = (1 + i \alpha)F_m^r A_m^r + \eta A_m^r(t-\tau)e^{-i\Omega_m^r},$$

(1)

$$T_m \frac{dF_m^r}{d(\gamma_p t)} = P^r - F_m^r - (1 + 2F_m^r) \sum_n \beta_{mn} |A_n|^2,$$

(2)

$$\frac{dA_m^f}{d(\gamma_p t)} = (1 + i \alpha)F_m^f A_m^f + \eta A_m^f(t-\tau)e^{-i\Omega_m^f},$$

(3)

FIG. 1. Schematic setup of unidirectional coupling. The transmitter is a semiconductor laser (SLT) with external cavity, the receiver is a similar semiconductor laser (SLR) but without external cavity. The mirror (M) and the attenuator (A) form the external feedback for the transmitter and the external injection for the receive. An optical isolator (ISO) prevents feedback for the receiver.
Hopf bifurcation, leading to a periodic regime featuring an increased linewidth enhancement factor. The feedback and the injection are characterized by the same dimensionless attenuation coefficient \( \eta \), \( A_m^{r,t}(t-\tau) \) is the field delayed by one round-trip time \( \tau \) and \( \Omega_m^{r,t} \tau \) is the phase mismatch after one round trip. In the nonlinear gain equations, \( J^{r,t} \) is the pumping current and \( \tau_s \) is the carrier lifetime. The cross-saturation parameters \( 0<\beta_{nm}^r<1 \) measure the free carrier grating in the transmitter and in the receiver. \( \gamma, k, \) and \( \beta_{nm}^r \) are assumed to be mode independent and \( \beta_{nm}^r = \beta_{nm}^r \) with \( m+n \neq 0 \) (\( \beta_{nm}^r = 1 \)).

We consider equal delays for the feedback to the transmitter and the injection to the receiver for simplicity. The fixed parameters for the numerical simulations are \( N=3 \), \( P^{r,t}=10^{-3} \), \( T=10^3 \), \( \gamma_p=1 \) THz, \( \alpha=5 \), and \( \beta^r=0.666 \). We find that the phase mismatch does not influence the results qualitatively and keep \( \Omega_m^{r,t} \tau=1 \) mod(\( 2\pi \)). We characterize the synchronization level by the difference between the intensities of the transmitter and the receiver, normalized to the pump of the transmitter, \( D_n(t)=\left[|A_n^t(t)|^2-|A_n^r(t)|^2\right]/P^r,D(t)=\sum_n D_n(t) \).

For the transmitter laser, experiments have demonstrated the importance of multimode operation in the low-frequency fluctuation (LFF) regime. In particular they indicate that the modal intensities are not in phase: the characteristic nonoptical frequencies are controlled by the external cavity round-trip time, but the oscillation phases differ from mode to mode [15–17]. In two recent papers [13,14], we have extended the Lang-Kobayashi model [18] to the multimode regime and predicted that the LFF can be interpreted as a chaotic itinerancy among coexisting unstable attractors.

A \( N \)-mode semiconductor laser with a weak optical feedback can be destabilized in two ways: a standard self-pulsing instability where all modes are in phase, and a \((N-1)\)-degenerate Hopf bifurcation. The degenerate Hopf bifurcation can generate coexisting periodic and quasiperiodic attractors that display antiphase dynamics [19]. In the LFF regime, these attractors are unstable and subdivide the chaotic dynamics. The total number of coexisting attractors can be greater than \( N! \) and the complexity of the chaotic behavior can be very high. A typical example of the total and modal outputs is shown in Fig. 2.

We now consider the coupled feedback/injection system. First, let all parameters of the two lasers be identical so that the system is completely symmetric in terms of the laser parameters. We consider the response of the receiver as \( \eta \) is increased. The steady state is destabilized by a degenerate Hopf bifurcation, leading to a periodic regime featuring different phases from mode to mode. The multimode output exhibits antiphase dynamics [19,20]. A secondary Hopf bifurcation leads to a stable quasiperiodic output for both lasers [13,14]. In these two regimes, the output of the transmitter and the receiver are synchronized. If the strength of the feedback to the transmitter exceeds a critical value, the multimode output becomes chaotic. The number of unstable periodic and unstable quasiperiodic attractors involved in the chaotic motion is large, and chaos is well developed. It is known that the necessary condition for a stable synchronized chaotic state is a negative value of all transverse Lyapunov exponents, including exponents for all invariant sets [21]. This condition is not fulfilled, and the system is completely desynchronized for \( \eta=0.0003 \) [Fig. 3(a)], although the two lasers are identical.

Increasing the strength of the feedback leads to a temporal synchronization by means of an intermittent process between laminar \( [D(t) = 0] \) and chaotic behavior for \( D(t) \) [Fig. 3(b–c)]. The intermittency between desynchronized and synchronized states means that there are invariant sets (periodic cycles, invariant manifolds) within the global chaotic attractor for which the transverse Lyapunov exponents satisfy the stability conditions, i.e., they are negative. The transmitter and the receiver are synchronized when the system operates on these cycles, and desynchronized otherwise. The system becomes completely synchronized when it operates in the LFF regime. Thus, we can report on a route to synchronization via intermittency, though we did not find any law for the duration of the laminar domains. An observation of bistabili-
FIG. 3. $D_{tot}(t)$ is the difference between total outputs of the transmitter and the receiver, normalized to the pump of the receiver. The change of feedback strength $\eta$ leads from desynchronization to synchronized regime via intermittency. The fixed parameters are $P_{r}^{\text{r}}=10^{-3}$, $T=10^{3}$, $\gamma_{p}=1$ THz, $\alpha=5$, $\beta_{r}^{\text{s}}=0.666$, and $\tau=6$ ns. Feedbacks are: (a) $\eta=3\times10^{-4}$; (b) $\eta=4\times10^{-4}$; (c) $\eta=2\times10^{-3}$. The range of the vertical axis is the same for all three figures.

It is important to assess the role of asymmetry since some parameters are difficult to control. This also gives a hint on the possible influence of noise on synchronization. We introduce an asymmetry in the system by using unequal cross-saturation parameters $\beta' \neq \beta''$ for the transmitter and the receiver. This choice is motivated by the fact that these parameters are the least accessible to an experimental control.

The loss of synchronization shown in Fig. 5(a–c) occurs in two very different manners: short- and long-time synchronization collapses. Short-time synchronization collapse results in sharp peaks for the output difference. Numerically, it is observed when the transmitter switches between two chaotic quasiperiodic attractors. The peak in $D(t)$ means that there is a short transient during which the receiver is either on the same attractor but with a phase delay or on a different attractor. A different situation occurs in the long-term synchronization collapse that is observed when the transmitter switches from a chaotic quasiperiodic regime to a chaotic periodic regime while the receiver settles down in a quasi-steady state, i.e., very small amplitude fluctuations around a steady state. The details of the modal behaviors and the difference between chaotic quasiperiodic and chaotic periodic regimes are displayed in Fig. 2. The long-term collapse does not occur immediately: first there is a short transient during which the system remains synchronized. When there is synchronization, the receiver acts as an amplifier of the transmitter signal, though it is a laser operating above threshold. Therefore, its output depends crucially on the dynamics of the transmitter. On the contrary, in the long-term synchronization collapse, both lasers being driven only slightly above
threshold ($P_r \ll 1$), the receiver operates at very low output intensity ($I \sim P$), as if the transmitter’s output was of no relevance to its dynamics: it operates around the steady state corresponding to the absence of injected signal. This suggests that the injected signal acts like a noise source for the receiver during the long-term synchronization collapse, since the transmitter is in a chaotic regime. Indeed, in the long-time desynchronization, the output of the receiver displays the transmitter is in a chaotic regime. Indeed, in the long-term synchronization collapse, since the injected signal acts like a noise source for the receiver during the long-term synchronization collapse, since the injection oscillation frequency of the receiver without injection. This important shift in frequency due to an injected signal has recently been experimentally observed and characterized [24]. In our model, this shifted relaxation oscillation frequency is observed in the receiver during the synchronization collapse but not in the synchronized regime. It appears again if noise is added to the receiver in the synchronized regime.

After the collapse, the system synchronizes again when the output of the transmitter is back to the chaotic quasiperiodic attractor. Note that the synchronization collapse can be canceled by tuning the pump of the receiver in the low pump regime ($P_r = 8 \times 10^{-3}$). This also reduces significantly the level of the remaining short-time desynchronization that can be described as a bursting phenomenon [21] occurring when the system is temporarily trapped on the unstable periodic cycle embedded in the chaotic quasiperiodic attractor.

Our main conclusion is that when dealing with multimode semiconductor lasers, synchronization is not a simple property for a feedback/injection configuration: (i) total output synchronization does not require modal output synchronization, (ii) even if the two lasers are identical, loss and absence of synchronization can be observed. We have connected these features to the structure of the phase space, which is characterized by many coexisting attractors that eventually originate from the degeneracy of a Hopf bifurcation induced by the feedback in the transmitter laser.

This research has been supported by the Fonds National de la Recherche Scientifique and the Inter-University Attraction Pole program of the Belgian government.