

LETTER TO THE EDITOR

Semiconductor lasers coupled face-to-face

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We show that for a large coupling time, semiconductor lasers coupled face-to-face exhibit a fast dynamics and a slow stairs-like periodic modulation. The effect can be explained by the nonlinear response of semiconductor lasers to external injection and a breakup of subnanosecond synchronization.

Keywords: semiconductor lasers, instabilities, chaos

Coupled lasers have attracted attention in recent years because they can exhibit dynamical effects useful to a wide range of problems. Experimental results on the dynamics of mutually coupled lasers were first reported for solid-state lasers [1]. The timescales over which the relaxation mechanisms operate in solid-state lasers are such that, in general, the delay induced by the field propagation between the two lasers is negligible. This is no longer true when coupling semiconductor lasers: the delay becomes an essential parameter. This feature, together with the phase–amplitude coupling characteristic of semiconductor lasers, is critical in experiments on synchronized chaos and results in a spontaneous symmetry breaking which appears as a time lag between the dynamics of two identical lasers [2–6].

It was also shown that an asymmetry in the lasers forms a leading/lagging configuration where the leading laser synchronizes the lagging one, but not the converse [3]. Another property of mutually coupled semiconductor lasers is localized synchronization, where one of the lasers exhibits large amplitude oscillations whereas the other laser exhibits small amplitude oscillations. This effect was predicted for nearly identical solid-state lasers [7]. Experiments with two semiconductor lasers showed that even when the lasers are pumped at different levels and the lower-pumped laser drives the other laser, localized synchronization occurs and leads to relaxation at the same frequency in both lasers but with different amplitudes [8].

In this letter, we report on a specific dynamical scenario for two semiconductor lasers coupled face-to-face, if the coupling time is greater than the coherence time of the lasers output.

We show that, due to the nonlinear response of semiconductor lasers to external injection, there is a regime where the output of each laser is a staircase, each step having a duration equal to twice the coupling time, followed by an abrupt drop-off. This pattern can be periodic or chaotic. There are periodic solutions with different numbers of stairs: in this letter, we focus on the regime with only two stairs, but states with up to seven stairs have been observed. Numerical results suggest that, if the delay is larger than the laser coherence time, the number of stairs as well as the nature of the dynamical state—periodic or chaotic—is independent of the delay and therefore of the number of steady states.

In our model two identical single mode semiconductor lasers (L_1 and L_2) are coupled in a face-to-face configuration: the output of each laser is injected into the other laser. We assume that there is no self-coupling caused by reflections from the front facet of one laser back into the other laser. Laser rate equations describing this system can be derived for the laser fields $A_{1,2}(t)$ coupled to the nonlinear gains $F_{1,2}$. The coupling terms are written as a delayed electric field of $L_1(L_2)$ added to the equations for the field of $L_2(L_1)$. After a suitable normalization, the equations become [9, 10]

$$\frac{dA_{1,2}}{dt} = (1 + i\alpha)F_{1,2}A_{1,2} + \eta A_{2,1}(t - \tau)e^{\mp i\nu t - i\nu_2,1\tau}, \quad (1)$$

$$T \frac{dF_{1,2}}{dt} = P - F_{1,2} - (1 + 2F_{1,2})|A_{1,2}|^2, \quad (2)$$

where $T = \tau_s \gamma_p$, $t = \gamma_p t'$ where t' is the physical time and γ_p denotes the modal field losses. P is the excess pumping rate above threshold and ν_j the free running frequency of the laser j .

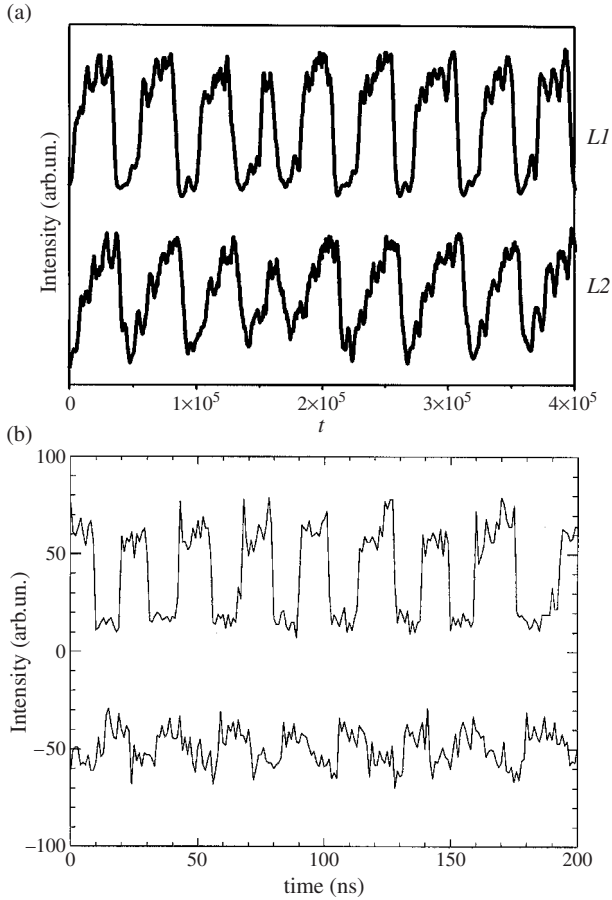


Figure 1. Outputs of single mode semiconductor lasers coupled in a face-to-face configuration. (a) Model simulation using equations (1) and (2). The parameters are $P = 10^{-3}$, $T = 10^3$, $\gamma_p = 1$ THz, $\alpha = 5$, $\eta = 0.0075$, $\tau = 5 \times 10^3$ and $\nu = 0.01$. (b) Experimental results. The delay is 5 ns (separation of 1.5 m). The detuning is $\nu = 15$ GHz. Upper trace is for L_1 .

In the field equations, α is the linewidth enhancement factor and the coupling is characterized by the same dimensionless coefficient $\eta = \kappa/\gamma_p$, where κ is the attenuation of laser field before being injected in the other laser. $A_{1,2}(t - \tau)$ is the field delayed by one coupling time τ and $\nu_{1,2}\tau$ is the phase mismatch. The detuning between the two lasers is $\nu = \nu_1 - \nu_2$, and we limit our analysis to the situation where both lasers are tuned to the centres of the lasing lines. The coupling time τ is chosen to be much greater than the coherence time of the lasers output. In the nonlinear gain equations, J is the pumping current and τ_c is the carrier lifetime. Justifications of the model can be found in [10].

The parameters for the numerical simulations are $P = 10^{-3}$, $T = 10^3$, $\alpha = 5$, $\eta = 0.0075$, $\nu = 0.01$. We found that the phase mismatch does not influence qualitatively the results and thus we keep $\nu_{1,2}\tau = 1 \pmod{2\pi}$. The results of numerical simulations are shown in figure 1(a). The outputs of both lasers are time averaged to mimic the effect of a slow electronic recording device with a 400 GHz bandwidth.

The averaged output displays a low-frequency stairs-like modulation shifted by the coupling time τ . A small asymmetry in the parameters does not affect the low-frequency stairs-like profile. One laser (L_1 in this example) anticipates the temporal

evolution of L_2 , and can be considered as leading. Changing the sign of the frequency detuning leads to a permutation of role between the leading and the lagging lasers. Most important is that the formation of the low frequency pulses does not depend on the distance between two lasers, which appears in equations (1) and (2) via the coupling, or delay time τ .

The experimental results, shown in figure 1(b), were obtained with two VCSELs emitting at 850 nm, separated by 1.5 m which amounts to a delay of 5 ns. L_1 has a threshold of 2.75 mA and operates $1.04\times$ above it. L_2 has a threshold of 2.8 mA and operates $1.08\times$ above it. The detuning is $\nu = -15$ GHz. The isolated lasers operate in steady state. The laser output was detected after passing through a linear polarizer. Rotating the polarizer by 90° did not affect the switching pattern, implying that it is not polarization switching which is observed but the result of the lasers interaction.

The theoretical (figure 1(a)) and the experimental (figure 1(b)) timetraces are nearly regular sequences of the low-frequency pulses with duration proportional to twice the delay time. The durations of the pulses presented in figure 1(a) are 2τ and 6τ , and the durations of the pulses in figure 1(b) are 2τ and 4τ . The distance between the pulses of the output of one laser corresponds to the duration of the pulse in another laser. In this letter we do not discuss the possible irregularity in the pulse durations and focus our attention on the formation of stair-like modulation.

To analyse this behaviour, we plot in figure 2 a single pulse of the lasers output with and without averaging, (figures 2(a), (b) and (c), (d), respectively) and consider the three time domains labelled 1, 2, 3. Unaveraged time series taken from each of the three steps shown in figure 2 are plotted in figure 3 (left column) with their corresponding power spectra (right column). The coupling time for figure 2 is taken much larger than in figure 1 in order to enlarge the number of relaxation oscillations in each domain for detailed spectral analysis.

The fast dynamics generated by the GHz semiconductor laser relaxation oscillations varies on a much shorter timescale than the coupling time. Under this condition we can analyse, to a first approximation, our problem as that of a semiconductor laser with injection, a topic that has been extensively studied in recent years and includes a rich set of phenomena such as four-wave mixing, steady state locking, intensity oscillations and chaos [11]. The nonlinear response of semiconductor lasers to an external injection explains the formation of stairs-like profiles. Each step (or stair) in the lasers output is formed by the lasers interacting during one coupling time. Therefore, the stairs of the two lasers are shifted by one delay time and have a length of two delay times, i.e. one roundtrip time.

During the first step (labelled 1 in figure 2) the small amplitude signal $L_1(1)$ is the chaotically modulated output of L_1 (figure 3(a)), comparable in magnitude to the laser output in the absence of coupling, and the corresponding power spectrum consists of a broad set of frequencies (figure 3(b)). The amplitude of the uncoupled L_1 output is indicated, for reference, in figure 2(a) by an arrow. L_1 sends this signal to L_2 , and after a coupling (delay) time, L_2 reacts to the signal with the output $L_2(1)$ whose amplitude of modulation is greater than the input signal (figure 3(c)). The spectrum of $L_2(1)$ is much simpler than that for $L_1(1)$ and has a dominant peak at the relaxation oscillation frequency ω_r (figure 3(d)). Therefore

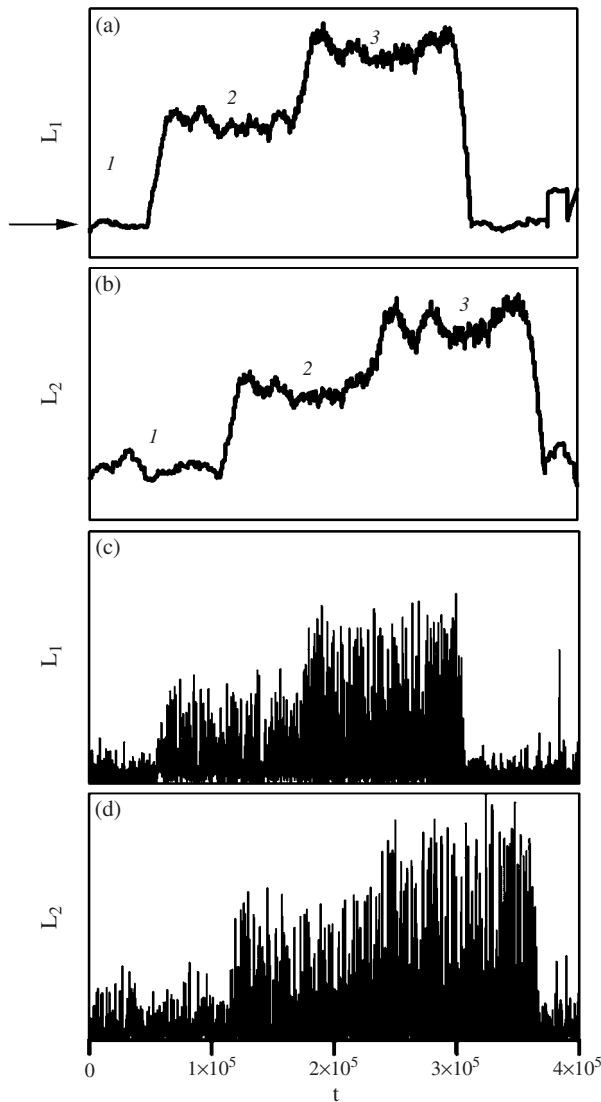


Figure 2. Single pulse blow up of lasers intensities (numerical simulations). (a), (b) With time averaging. (c), (d) Without time averaging. The parameters are the same as in figure 1(a), except for $\tau = 6 \times 10^4$.

the first step can be interpreted as a response of L_2 to a small amplitude noisy signal from L_1 .

During the next step, L_2 has an output modulated at relaxation oscillation frequency ω_r and this signal, which is labelled $L_2(1)$, reaches L_1 . In this step L_1 will react, after yet another coupling time, with output $L_1(2)$ which also has a greater amplitude of modulation than the input (figure 3(e)), forming the first stair. The response $L_1(2)$ has a dominant peak at the third harmonic $3\omega_r$ (figure 3(f)). Frequency entrainment in optically injected SL and the appearance of subharmonics (ω_r/n) and superharmonics ($n\omega_r$) are known phenomena and have been reported previously [12].

The following traces, $L_2(2)$, $L_1(3)$ and $L_2(3)$, are very similar and differ only by the increasing amplitude of oscillations. The power spectra are nearly identical and indicate that the lasers are frequency locked. Thus, it can be concluded that the lasers are synchronized on a subnanosecond scale. A similar result was obtained experimentally in the

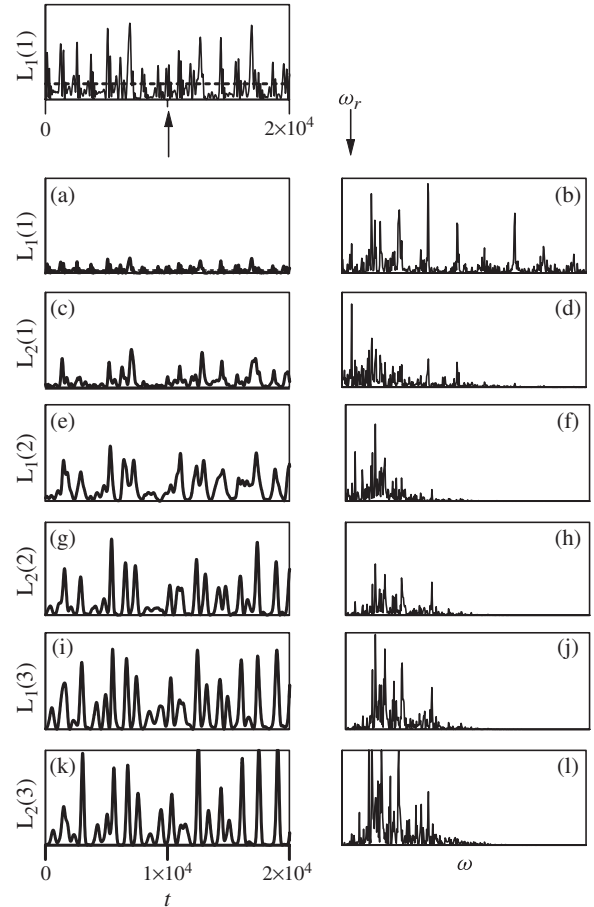


Figure 3. Time traces and rf spectra of the leading (L_1) and the lagging (L_2) lasers for the parameters of figure 2. The labels are the same as in figure 2. The vertical axis in (a) is five times smaller than the vertical axes for the remaining time traces.

regime of delay times comparable to the relaxation time [3]. In these steps, the difference between the lasers chaotic inputs and chaotic outputs results from the asymmetry caused by the detuning between the lasers. The asymmetric response of the semiconductor lasers on the blue detuned or red detuned injection frequencies is caused by the nonzero α factor.

After some steps, forming a staircase profile, the input to L_1 reaches the critical value $L_2(3)$ and the output decreases sharply to a quasi-steady state, i.e. small amplitude fluctuations around a steady state. This state has been already described as $L_1(1)$. The drop-off results from the subcritical breakup of the subnanosecond synchronization and occurs as a hard bubbling transition [16] to this metastable steady state. A similar hysteresis between synchronized and unsynchronized branches of solutions has been already reported for unidirectionally coupled multimode SL [13]. It is also known in the theory of coupled nonlinear oscillators described by Kuramoto equations [14]. Fully analytic results on the synchronization condition are given in [15].

In conclusion, we find that mutually coupled semiconductor lasers can exhibit a slow stairs-like periodic modulation caused by the mutual coupling, when each laser operates as a nonlinear modulator for the other laser. During this process, the two lasers are synchronized on a subnanosecond scale.

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