Bistability between Different Localized Structures in Nonlinear Optics

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We report the observation of different localized structures coexisting for the same parameter values in an extended system. The experimental findings are carried out in a nonlinear optical interferometer and are fully confirmed by numerical simulations. The existence of each kind of localized structure is put in relation to a corresponding delocalized pattern observed. A quantitative evaluation of the range of pump parameter allowing bistability between localized structures is given. The phenomenon reported results to be robust in parameter space.

Localization of structures is a widespread phenomenon occurring in both conservative and dissipative systems. On one hand, the study of this topic aims to understand the general conditions needed for the occurrence of localized structures (LSs) and, on the other hand, to explore the potentialities offered by these objects in view of applications such as information transmission and storage. In this context, LSs in dissipative systems have been demonstrated in many diverse fields, including granular materials, fluid dynamics, electroconvection in liquid crystals, chemistry, and nonlinear optics [1,2].

In this Letter, we present the experimental evidence of bistability between different single peak localized structures in a dissipative system. Bistability of localized excitations has been predicted in some optical systems of both conservative [3,4] and dissipative [5–7] types. To our knowledge, however, experimental evidence of such a phenomenon has not been given so far, neither in optics nor in other contexts.

The two kind of localized structures we find differ in shape and are separated by a discrete gap in their peak intensity. If used as pixels for information storage, these LSs represent three-state variables, instead of the common two-state variables (“bits”) that a common localized structure can encode. Consequently, use of these new structures would lead to an increase of \( \log_2 3 \approx 1.585 \) for the information storable in a given area.

Localized structure bistability is observed in the presence of two modulational instabilities having different critical wave numbers. The question of the interaction between these two instabilities has been investigated in [8] for semiconductor cavities.

Our experimental system is a nonlinear optical interferometer formed by a liquid crystal light valve (LCLV) with optical feedback and is shown in Fig. 1. The LCLV is a transversally extended (\( \approx 2 \times 2 \) cm) electro-optical device, longitudinally formed by the sequence of a nematic liquid crystal cell, a mirror, and a photoconductor [9]. It converts the spatial intensity distribution of a laser beam impinging on its write (photoconductor) side into a spatial phase distribution for the laser field impinging on its read (liquid crystal) side. In a closed loop configuration as the one shown in Fig. 1, the free propagation trough a distance \( L \) and the interference with a reference beam introduced by means of the polarizer \( P \) convert the phase distribution of the reading light into an intensity distribution for the writing light. If a positive feedback is established for some spatial frequency of the laser field sent to the system, the uniform solution loses stability in favor of patterned ones [9–13], and extended as well as localized patterns can be formed. Localized excitations of this kind have been reported in many different systems, including the one shown in Fig. 1. The free propagation through a distance \( L \) results in a change of the system nonlinearity from self-defocusing, as it would be for the LCLV alone, to self-focusing. In the experiments reported here, \( L = 30 \) mm.

FIG. 1. Experimental setup. \( E_0 \), input field; \( O \), microscope objective; \( P_1 \), pinhole; \( A \), iris; \( BS_1 \) and \( BS_2 \), beam splitters; \( LCLV \), liquid crystal light valve; \( L_1 \) and \( L_2 \), lenses of focal length \( f \); \( P \), polarizer; \( FB \), fiber bundle; CCD, videocamera; \( F \), image plane of the LCLV. The overall free propagation of a negative distance \( L \) results in a change of the system nonlinearity from self-defocusing, as it would be for the LCLV alone, to self-focusing. In the experiments reported here, \( L = 30 \) mm.
studied in several different optical devices [13–18], including miniaturized semiconductor resonators [17,18].

The system is described by the equations [9,11]

\[ \tau \frac{\partial \varphi}{\partial t} = - (\varphi - \varphi_0) + I_0^2 \nabla^2 \varphi + f(I_{fb}), \]

\[ I_{fb} = I_0 | e^{i L \nabla^2 / 2k_0} (B e^{i \varphi} + 1 - B) * j_0(q_B \cdot \mathbf{r}) |^2, \]

Here, \( j_0 \) is the spherical Bessel function of zeroth order.

Equation (1) rules the evolution of the phase \( \varphi \) of an initially plane laser beam sent into the system. Here \( \tau \) and \( l_d \) are the relaxation time and the diffusion length of the LCLV, respectively, \( I_{fb} \) is the feedback intensity on the valve, and \( \varphi_0 \) is the phase retardation induced by the valve on the input beam in the absence of feedback. In the following, we denote the homogeneous steady state solution of the system as \( \varphi^{(0)} \). The function \( \varphi^{(0)} \) depends on all the parameters appearing in (1)–(3), including \( \varphi_0 \). Equation (2) describes the conversion of phase into intensity distribution due to the propagation over a distance \( L \) and to the interference between the signal beam of amplitude \( B \) and a reference beam of amplitude \( 1 - B \). \( I_0 \) is the input intensity, and \( j_0(q_B \cdot \mathbf{r}) \) represent a spatial filtering \( * \) denotes a convolution product \( \) introduced by means of the aperture \( A \) in the Fourier plane of the system. \( k_0 \) denotes the optical wave number of the laser light. Equation (3) describes the LCLV nonlinearity. The parameters \( \varphi_{sat} \) and \( \alpha \) represent the saturation phase value and the sensitivity of the LCLV, respectively.

For the present study, we set the system parameters at \( l = 30 \text{ mm}, l_d \approx 18 \text{ \mu m}, B \approx 0.52, \varphi_{sat} = 5, \varphi_0 = \pi, \) and the spatial bandwidth \( q_B = 3.7 \). Here and in the following, the spatial frequencies are expressed in units of the diffractive scale \( q_{diff} = 2\pi/\sqrt{2\lambda l} \), where \( \lambda = 633 \text{ nm} \) is the optical wavelength.

In our system, one type of LS has been shown to exist over very broad ranges of parameters [13,16,19]. It has circular symmetry, with a bright central peak connected to the dark background via a series of small amplitude oscillations along the radial direction. Here, we report another completely different localized solution existing for the same device. The two localized structures, as observed in the experiment, are shown in Fig. 2. Numerical simulations faithfully reproduce the observations.

The most evident feature of the new LS is its triangular symmetry, observed both in the central peak and in the tails. Hence, in the following, we refer to these structures as triangular localized structures (TLSs), and to the ones with circular symmetry as round LSs (RLSs).

A second fundamental difference between TLSs and RLSs is their peak intensity. This can be appreciated in Figs. 2(c) and 2(d). Finally, the size of the central peak is substantially smaller for a TLS than for a RLS, though the overall size of the two structures including the tails are comparable.

The triangular and the round LS shown in Fig. 2 are observed for identical values of all the parameters, indicating bistability between the two structures.

The existence of localized structures with triangular symmetry has been recently predicted in nonlinear optical systems [20]. In these systems, contrary to our case, no bistability between RLSs and TLSs is found.

Each of the two localized structures existing in our system can be switched on by an appropriate addressing pulse. Lower intensity pulses trigger a RLS, higher intensity ones a TLS. In these regards, the observed weak sensitivity to the addressing intensity suggests the existence of large basins of attraction for each localized structure; hence, the noise is not expected to be a serious limit in the use of these structures for information storage and processing tasks.

We are now going to investigate the dependence of the phenomena observed on the pump intensity. The experimental state diagram of the system is shown in Fig. 3, together with its numerical counterpart. Here it is plotted for each LS the peak intensity \( I_{peak} \), measured on the feedback signal \( I_{fb}(x) \).

Starting from a very low value of input intensity and gradually increasing it, the lower uniform solution is the only state observed up to \( I_0 \approx 0.32 \text{ mW/cm}^2 \) (throughout

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**FIG. 2.** (a),(c) Round localized structure; (b),(d) Triangular localized structure.
hexagons. The upper branch loses stability at point is strongly subcritical and results in the formation of portion of the stability balloon. The bifurcation from this better visible in the inset of Fig. 4(b), showing an enlarged

physical units. In the conditions of the experiments here reported, 

the homogeneous steady state (HSS) characteristic of the system

saturates to a uniform high output intensity state.

A useful insight about the origin of the LS observed can be gained from the observation of Fig. 4. Figure 4(a) shows the homogeneous steady state solution \( \varphi^{(0)} \) vs \( \alpha I_0 \), together with the phase maxima of the localized and delocalized structures found in numerical simulations. In Fig. 4(b) shows the instability region of the homogeneous steady state solution \( \varphi^{(0)}(\alpha I_0) \), as a function of spatial wave number \( q \) of the perturbations.

In the rest of the Letter, we mostly discuss numerical results; hence it is convenient to refer to the adimensional parameter \( \alpha I_0 \) as the pump, rather than to the intensity \( I_0 \) in physical units. In the conditions of the experiments here reported, \( \alpha \approx 18 \text{ cm}^2/\text{mW} \).

The lower branch of the HSS destabilizes at \( \alpha I_0 \approx 28 \), for a critical wave number \( q_c \approx 0.71 \). The critical point is better visible in the inset of Fig. 4(b), showing an enlarged portion of the stability balloon. The bifurcation from this point is strongly subcritical and results in the formation of hexagons. The upper branch loses stability at \( \alpha I_0 \approx 20 \), \( q_c \approx 1.15 \), with another subcritical bifurcation leading to the appearance of honeycomb patterns.

Our simulations show that both the hexagons and the honeycombs are unstable within most of their existence region. Indeed, hexagons typically decay toward a collection of spatially well separated RLSs; honeycombs are instead destabilized either towards a collection of TLSs, or towards a situation of developed space-time chaos, depending on the value of \( \alpha I_0 \). Since we simulate the dynamics of the Eqs. (1)–(3), we are able to determine the phase maxima values only for the stable solutions. In order to stabilize the periodic patterns for all the values of \( \alpha I_0 \), we introduced a filter in Fourier space allowing only for the existence of the six fundamental modes needed to create hexagons and honeycombs, plus all their harmonic combination up to the cutoff frequency fixed at 3.7. We have verified that this filter does not alter significantly the value of the phase maxima in the case in which the patterns are already dynamically stable. In this way, we obtain the phase maxima for the delocalized patterns shown in Fig. 4(a).

In the presence of the strongly subcritical patterns here observed, localized structures can be formed in the parameter range where a homogeneous stable steady state coexists, or is not far from, a patterned state [1]. These patterned states can be either stable or unstable for the parameter values at which LS exist.

In our case, the superposition of the RLS and of the hexagons phase maxima seen in Fig. 3 clearly indicates the relation between the localized and delocalized structure. As for the relation between TLSs and honeycombs, the results is less obvious from the data of Fig. 3. We conjecture, however, that such a connection exists also in this case. On one side, our conjecture is based on the general argument which describes the LS as local patches of a patterned solution, embedded in a uniform solution back-
This appears to be a feature nonspecific to our experimental system and suggests that the phenomenon reported here could be observed in systems of a different nature.

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