Fast self-pulsing through nonlinear incoherent feedback

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We consider a double-pass ring cavity with nonlinear incoherent optical feedback and analyze its response when it is driven by a continuous laser beam. This particular cavity is equivalent, in the temporal domain, to a simple spatial-pattern-generating system made from a Kerr slice and a feedback mirror. After formulating the evolution equations, we investigate the behavior of small-amplitude solutions and obtain an expression for the round-trip gains. We then explore the important effect of dispersion in the nonlinear medium. Finally, we show that stable modes are possible by solving numerically the full nonlinear equations.

There is a strong analogy between the transverse dynamics of light in spatially extended geometries and the propagation dynamics of light in waveguides. While in the former case diffraction and nonlinearity typically conspire to produce spatial patterns, in the latter, dispersion and nonlinearity lead to temporal self-modulation of continuous waves, as with counterpropagating dispersive waves and in the modulational-instability laser. One especially simple system leading to spatial instabilities is the Kerr-slice system, this corresponds to the diffraction taking place within the thin slice, the effect of which has so far been neglected.

The proposed scheme is designed such that light makes one and a half round trips in the cavity. This can be understood by removing the nonlinear element in Fig. 1, then, with the chosen configuration, the two quarter-wave plates act as a single half-wave plate. The other hand polarizing beam splitters PBS, let only x-polarized waves through. Light enters at point I with linear in-plane polarization e_x. After the first passage through the two quarter-wave plates, the input wave becomes e_z polarized and is therefore reflected by PBS_y. At the next passage, the polarization reverts to the e_x direction, and the wave exits the cavity at point O.

Fig. 1. Setup: λ/4, quarter-wave plates; NL, nonlinear section; disp, dispersive section; PBS, PBS_y, polarizing beam splitters; e_x, polarization; e_z, polarization; Mirrors; 0, 1, 2, round-trip numbers; A–F and I, sections referred to in Eqs. (1)–(6). The beam enters at point I and acquires a phase proportional to P(t) in the nonlinear medium. The phase oscillation becomes essentially an amplitude oscillation at the output of the dispersive medium. The modulated beam transmits its amplitude oscillation to the entering beam during its second pass into the nonlinear medium and finally exits the system at O.

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coupling can also be achieved in cavities longer than the linear phase accumulated over the nonlinear section, 

\[ g(P_i, \Omega) \]

Fig. 2. Instability gain. See text for parameter values. According to Eqs. (10), all maximal values of \( |g| \) should be equal. Here the dispersion inside the nonlinear medium is taken into account such that the gain is modulated by the dashed curve, resulting in the continuous curve [relation (11)]. In the chosen configuration the MI is limited to the peak about 100 GHz, where the gain is greater than unity.

Between the two quarter-wave plates the linear states of polarization along the \( x \) and \( y \) directions become circular states, respectively \( e_x \) and \( e_y \), where \( e_x = (e_x, \pm i e_y) / \sqrt{2} \). We assume that the nonlinearity is of the isotropic Kerr type such that only cross-phase modulation, but no energy transfer, exists between the two circular polarizations. In this way, light makes only one and a half round trips inside the cavity, and no energy is accumulated in the system. Moreover, since the coupling is incoherent, we can restrict our attention to the temporal envelope of the beam and safely neglect the phase of the carrier wave. Note that the incoherence of the coupling between circular polarization components is a consequence of the isotropy of the Kerr effect; incoherent coupling can also be achieved in cavities longer than the coherence length of light in them.

If we now assume no dispersion inside the nonlinear element, a simple expression of the instability gain can be obtained. Let us write the vectorial envelope of the electric field in an arbitrary section \( Z \) of Fig. 1 as \( \mathbf{u}^Z = u_x e_x + u_y e_y = u_x e_x + u_y e_y \). Furthermore, let \( \gamma \) be the Kerr coefficient, \( \sigma \) the coefficient of cross-phase modulation, and \( L_{NL} \) the length of the nonlinear section. The dispersive section, on the other hand, is characterized by group-velocity dispersion \( \beta_D^2 \) and length \( L_D \); finally, \( \tau \) denotes the roundtrip time. Given constant input power \( P_i \), the following relations hold between successive sections of Fig. 1:

\[ \mathbf{u}^A(t) = \sqrt{P_i} e_x + u^F(t - \tau) e_y, \]

\[ \mathbf{u}^B = u_x A e_x + u_y A e_y, \]

\[ \mathbf{u}^C = u_x B e_x + u_y B e_y, \]

\[ \mathbf{u}^O = u_x D e_x + u_y D e_y, \]

\[ \mathbf{u}^F = \mathcal{F}^{-1}[\mathcal{F}(u_x e_y) \exp(i \phi_D)] e_y, \]

where \( \phi_D = \gamma L_{NL} (|u_x|^2 + \sigma |u_y|^2) / 2 \) is the dispersive phase that affects the Fourier amplitude at frequency \( \Omega/(2\pi) \).

Considering the equations above, we first note that \( |u_x|^2 = P_i \) and that \( |u_y|^2 = |\mathbf{u}^O|^2 \). Consequently, denoting the output power by \( P_o(t) \), we have \( \phi_o(t) = \gamma L_{NL} [P_i + \sigma \mathbf{u}^O(t)] \). Next, from a direct substitution of the above relations into one other, we find that

\[ P_o(t) = P_i \mathcal{F}^{-1} \left[ \mathcal{F} \left( \exp[i \sigma \gamma L_{NL} P_o(t - \tau)] \right) \right. \]

\[ \times \exp(i \beta_D^2 L_D \Omega^2/2)) \right] . \]

Equation (7) admits of the constant solution \( P_o = P_i \), which is consistent with our comment that there is no energy transfer between the two states of polarization. If we now introduce a small perturbation \( P_o(t) = P_i [1 + \epsilon \cos(\Omega t)] \), from Eq. (7) that

\[ P_o(t + \tau) \sim P_i [1 + g(P_i, \Omega) \epsilon \cos(\Omega t)]. \]

The coefficient

\[ g(P_i, \Omega) = -2 \sigma \gamma P_i L_{NL} \sin(\beta_D^2 L_D \Omega^2/2) \]

acts as an amplification factor. The maximum absolute value of \( g(P_i, \Omega) \) is given by

\[ |g_{max}| = 2 \sigma \gamma P_i L_{NL}, \quad \Omega_{max} = \left[ \frac{\pi(2N + 1)}{|\beta_D^2 L_D \Omega^2|} \right]^{1/2}, \]

with \( N = 0, 1, 2, \ldots \). From Eqs. (10) we note that the maximum gain is reached at frequencies that are independent of \( P_i \), while \( |g_{max}| \) is independent of \( \beta_D^2 \), so the two can be tuned independently. Moreover, the maximal gain is independent of \( N \). The system is unstable if \( |g_{max}| > 1 \), and modes with arbitrarily large frequencies can grow. Practically, it is the dispersion of the nonlinear medium that limits the bandwidth of the system.

Taking dispersion inside the nonlinear section into account considerably complicates the analysis. It requires that one consider the detailed evolution of a perturbation as it circles inside the cavity. We find that

\[ \mathbf{u}(t) = u_x B e_x + u_y B e_y, \]

\[ \mathbf{u}^F = \mathcal{F}^{-1}[\mathcal{F}(u_x e_y) \exp(i \phi_D)] e_y, \]

where \( \phi_D = \gamma L_{NL} (|u_x|^2 + \sigma |u_y|^2) / 2 \) are the nonlinear phases accumulated over the nonlinear section, \( \mathcal{F} \) denotes the Fourier transform, and \( \phi_D = \beta_D^2 L_D \Omega^2/2 \) is the dispersive phase that affects the Fourier amplitude at frequency \( \Omega/(2\pi) \).

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where the positions of the gain maxima are determined nonlinear and dispersive effects act separately, such that the double-pass character of the cavity ensures that the oscillation frequency $\Omega_{oe}$. From the condition $|g|=1$, we find that

$$\Omega_{oe}^2 = \frac{12\sigma^2 \gamma P_1}{\beta_2^{NL} g_{\text{max}}^2} \left(1 - \frac{1}{g_{\text{max}}}\right).$$

(12)

From Eq. (12), the maximum achievable bandwidth

$$\Omega_{\text{bw}} = \frac{4}{3} \sigma \sqrt{\gamma P_1 / \beta_2^{NL}}$$

(13)

is obtained for $L_{NL}$ such that $|g_{\text{max}}| = 3/2$.

We have tested our predictions by simulating the system over a wide range of parameter values. Although the theory is mathematically valid for small values of $\beta_2^{NL}L_{NL}$, it remains accurate for the following realistic set of parameters: $L_{NL} = 15$ km, $\gamma = 1$ W km, $\sigma = 2$, $\beta_2^{NL} = 0.2$ ps$^2$/km, $L_D = 320$ m, $\beta_2^D = 25$ ps$^2$/km, and $P_1 = 21$ mW. This leads to a maximum instability gain at 100 GHz. The corresponding gain curve for these parameter values is plotted in Fig. 2, and corresponding numerical results are shown in Fig. 3. These were obtained by integration of two coupled nonlinear Schrödinger equations describing the evolution of a long pulse (400 ps) rather than a true continuous wave (cw). At the center of the pulse, the electric field can be assimilated to a cw, and we have verified that the long-term response to a cw is already reached after 1000 cavity round trips. Experimentally, both cw and pulsed pumping can be used; with the latter, care must be taken to synchronize the pumping with the round-trip time.

It is worth emphasizing that even for rather large values of $\beta_2^{NL}$, as in Fig. 3 where $\beta_2^D L_D - \beta_2^{NL} L_{NL}$, the double-pass character of the cavity ensures that nonlinear and dispersive effects act separately, such that the positions of the gain maxima are determined only by the dispersion accumulated in the dispersive section ($\beta_2^D L_D$). By contrast, in a classic ring cavity, a mean-field theory would apply, where the two effects act simultaneously—and the maximal gain would depend on $\beta_2^D L_D + \beta_2^{NL} L_{NL}$. As a consequence, the instability presented here differs clearly from that reported in Refs. 4 and 5, despite the common dependence of the oscillation frequencies of these two systems on their respective geometrical sizes. Finally, we should mention that polarization modulation instability could appear in the nonlinear section, even though we did not observe it in our numerical simulations.

In conclusion, we have presented a double-pass cavity that is the temporal analog of the simple Kerr-slice system with incoherent feedback. Our analysis shows the existence of a self-pulsed regime, similar to that of the MI laser, with the main difference that the oscillation frequency is robust against power fluctuations of the pump beam. We have provided expressions for the oscillation frequency and the gain factor associated with the instability. Numerical simulations show that the predictions of the perturbative analysis remain accurate for larger oscillations. Further study of the system should reveal ranges of parameters leading to complex multimode dynamics, as observed in the equivalent spatial system. Such studies should help to determine the best conditions for a tunable oscillator in practical experiments.

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