Square waveforms in edge-emitting diode laser subject to polarization-rotated optical feedback

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\textbf{ABSTRACT}

The response of a diode laser resulting from an incoherent delayed optical feedback is considered from numerical and experimental perspectives. We concentrate on a class of solutions that appear as regular square waveforms. A two-field model is used and the bifurcation diagram of these square-wave regimes is studied. Conditions under which they typically appear are determined. The roles of various parameters are examined, particularly with regard to the gains and losses of the two polarization modes. Numerical results are in close agreement with experiments.

\textbf{1. INTRODUCTION}

Several papers have appeared in the last few years discussing and extending the mathematical treatment of incoherent feedback. In this particular system composed of a stripe diode laser that lases naturally in the fundamental (horizontal) polarization mode, the field is rotated into the orthogonal polarization, delayed in an external cavity and reinjected into the diode laser. The original treatment by Otsuka and Chen\textsuperscript{1} is described in terms of a single field in which the intensity equation takes on the usual form giving rise to amplification due to the coupling with the carriers. However, the rotated and delayed intensity appears only in the carriers equation, suppressing the gain and inducing complex dynamics due to the significant time delay. Subsequent experiments\textsuperscript{2} have shown that the correct treatment requires equations for both polarization fields, in addition to the equation of the carriers. Similar results and conclusions were obtained by Heil \textit{et al.}\textsuperscript{3} based on a set of extended equations in which the fundamental (horizontal) mode is dominant due to the larger gain available in the diode laser. As a result, without reinjection to the vertical mode this mode will not survive due to the natural competition between the two modes. However, some earlier work\textsuperscript{2} assumes that the explanation of the experimental results relies on a model in which the two modes have different losses, thus suppressing the vertical mode due to its larger losses over that of the natural mode. At this time, it is not apparent which of the two mechanisms, whether differential losses or gain, is operating in this particular system. Since this mechanism has not been explored before experimentally or numerically, and in particular since it has not been included in any detailed model, we will take the opportunity to discuss it further in this paper and determine some of its consequences.

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2. EXPERIMENT

In this experiment we have used a SDL-5401 index guided Fabry-Perot MQW diode laser. The laser was temperature stabilized and operated at 817.9 nm. The threshold of the laser is 18.48 mA and all the data for this experiment were taken at a pump current of 28.02 mA. The experimental setup consists of the diode laser and an external cavity in which a polarization rotator and elements that control the strength of the feedback are present. The light is collimated by a lens, passes through a nonpolarizing beam splitter, and continues through a Faraday rotator. The rotator is simply a Faraday isolator with its input polarizer removed but retaining the output polarizer oriented at 45° from the horizontal. Since the natural lasing field from the diode laser is in the horizontal polarization, the field after emerging from the rotator, now polarized at 45°, encounters a rotatable linear polarizer used for attenuation, and is retroreflected from a highly reflective mirror that forms one of the ends of the cavity. The reflected field passes again through the polarizer and enters the Faraday rotator, assuring its 45° polarization orientation. The additional 45° rotation results in a vertical polarization, orthogonal to its original polarization direction, and this beam is reinjected into the diode laser after a delay equal to \( \tau \), the round trip time of the external cavity. This particular configuration excludes feedback from multiple roundtrips, since any vertically polarized light reflected from the laser facet back into the cavity is extinguished by the Faraday's output polarizer.

The light reflected from the nonpolarizing beam splitter, 30% of the incident beam, is directed to the detection equipment. Using this signal, polarization-resolved detection is accomplished by using a second nonpolarizing 50% beam splitter. The two resulting beams enter similar detection arms, consisting of linear polarizers configured to pass either horizontal or vertical signals and density filters to limit the power incident on two photodetectors. The ac signals from these 8.75 GHz detectors (Hamamatsu C4258-01) are amplified by 23 dB using wideband (10 kHz - 12 GHz) amplifiers, and are captured and analyzed by use of either a digital storage oscilloscope (LeCroy 8600, 6 GHz analog bandwidth) or a microwave spectrum analyzer (Agilent E4405B).

In this experimental setup we observe square waveforms that are predicted numerically, as well as more complicated periodic waves. Two such waveforms are captured and displayed in Fig. 1. Figures 1(a) and 1(b) show the horizontal polarization mode and the vertical polarization mode respectively when the external cavity length is 39 cm. This gives a round trip time of 2.60 ns. The roundtrip feedback power is 37.6%. A very regular square wave with damped relaxation oscillations is evident. Additionally, the vertical and the horizontal polarization modes are out of phase, and exhibit a period of 5.85 ns, slightly longer than twice the round trip.
of the external cavity (by 0.65 ns), as intuitively expected and consistent with the numerical findings.

To further demonstrate that these solutions have periods near 4L/c, Figs. 1(c) and 1(d) show the horizontal and the vertical modes when the cavity length is extended to 104 cm. The square waveform period has increased correspondingly, and the relaxation oscillations are still apparent when the wave switches polarizations. Additionally, RF spectra confirm all these observations, showing a strong fundamental frequency of the square at twice the period of the external cavity followed predominantly by the odd harmonics as expected from classical Fourier theory. The first even harmonic is over 30 dB lower than the fundamental.

3. MODEL

The model is very similar to one presented in Heil et al.\textsuperscript{3} in which the horizontal mode $E_1$ interacts with the carrier with a gain $g_1$, and the vertical mode $E_2$ with gain $g_2$. However, in addition we assign $\tau_1$ to be the lifetime of the cavity of the mode $E_1$, and $\tau_2$ for the $E_2$ mode. The equations therefore are:

\begin{equation}
\frac{dE_1}{dt} = \frac{1}{2} (1 + i\alpha) [g_1(N - N_0) - \frac{1}{\tau_1}] E_1,
\end{equation}

\begin{equation}
\frac{dE_2}{dt} = \frac{1}{2} (1 + i\alpha) [g_2(N - N_0) - \frac{1}{\tau_2}] E_2 + r E_1 (t - \delta),
\end{equation}

\begin{equation}
\frac{dN}{dt} = J - \frac{N}{\tau_s} - g_1(N - N_0) |E_1|^2 - g_2(N - N_0) |E_2|^2.
\end{equation}

The rest of the parameters appearing in these equation are: $\alpha$ is the linewidth enhancement factor, $\tau_2$ is the carriers lifetime, $r$ is the strength of the feedback, $\tau$ is the delay in the external cavity of the polarization rotated field $E_1$ and $J$ is the pumping current. The set of Eqs. (1) - (3) can be simplified and rendered dimensionless by carrying out a few simple transformations. In particular, we redefine the time by $s = t/\tau_1$, that is we normalize it to the cavity lifetime time of the horizontal mode, where $\tau_1$ is proportional to the losses associated with this mode. Similarly the lifetime or losses associated with the vertical mode is defined in the analogous manner. Naturally the delay is expressed in a dimensionless quantity $\tau = \delta/\tau_1$. Additionally, we define

\begin{equation}
Z = \frac{1}{2} \left[ \tau_1 g_1(N - N_0) - 1 \right]
\end{equation}

and redefine the fields as $\sqrt{\frac{g_1}{2}} E_1 \rightarrow E_1$ and $\sqrt{\frac{g_2}{2}} E_2 \rightarrow E_2$. We then obtain the following two equations for the fields

\begin{equation}
\frac{dE_1}{ds} = (1 + i\alpha) Z E_1,
\end{equation}

\begin{equation}
\frac{dE_2}{ds} = (1 + i\alpha) k(Z - \beta) E_2 + \eta E_1 (s - \tau)
\end{equation}

where $k \equiv g_2/g_1$ is the ratio of the two gains, $\eta \equiv \sqrt{k} r \tau_1$ is the normalized feedback strength, and $\beta \equiv \frac{1}{2} (\frac{g_1 N_0}{g_2})^2 > 0$ represents the differential losses. The last parameter depends on a combination of gains and losses in such a manner as to remain always larger than zero making $E_1$ the fundamental lasing mode when there is no feedback available. The carrier equation takes the form

\begin{equation}
T \frac{dZ}{ds} = P - Z - (1 + 2Z)(|E_1|^2 + |E_2|^2)
\end{equation}

where $T \equiv \tau_s/\tau_1$ and $P \equiv \frac{g_1}{2} (J \tau_s - N_0) - 1/2$. The definition of the current at threshold is obtained form the equation $P(J_{th}) = 0$. Thus factor $k$ has been eliminated from Eq. (7) but remains in front of the gain in Eq. (6) and it can not be eliminated. In an amplitude and phase decomposition in which the calculations in this section are performed we define $E_j = A_j e^{i\phi_j}$, $j = 1, 2$, and obtain
\[ \frac{dA_1}{ds} = ZA_1 \]
\[ \frac{dA_2}{dt} = k(Z - \beta)A_2 + \eta A_1(s - \tau) \cos(\Phi) \]
\[ \frac{d\Phi}{ds} = \alpha(Z(s - \tau) - kZ + k\beta - \eta \frac{A_1(s - \tau)}{A_2} \sin(\Phi) \]
\[ T \frac{dZ}{ds} = P - Z - (1 + 2Z)(A_1^2 + A_2^2) \]

where the phase is defined by \( \Phi(s) = \phi_1(s - \tau) - \phi_2(s) \). The steady state is easily computed and gives
\[ A_1^2 = \frac{PK^2\beta^2(1 + \alpha^2)}{\eta^2 + k^2\beta^2(1 + \alpha^2)} \]
and
\[ A_2^2 = \frac{P\eta^2}{[\eta^2 + k^2\beta^2(1 + \alpha^2)]]} \]

The form of the steady states depend only on the combination of \( k/\beta \), but the dynamics may depend on the individual values of these two parameters.

In this paper we will present some of the square wave solutions and their dependence on the gain and losses of this system. In previous calculations we have assumed that the gains of the two modes are identical, and have therefore considered only cases for which \( k = 1 \), for various values of \( \beta \). In particular, we found that the square wave solutions dominate when the differential loss are fairly small \( \beta = O(10^{-2}) \). We have assumed that

![Figure 2: Bifurcation diagram for \( k = 1.0 \) and \( \beta = 0.03 \).](image)
Figure 3. Time series of the modes (a) horizontal and (b) vertical as a function of normalized time for $k = 1.0$, $\beta = 0.03$ at $\eta = 0.14$.

the rest of the parameters are representative of an SDL semiconductor laser. We have fixed $T = 150$, and $\alpha = 2.0$. The differential losses are set to $\beta = 0.03$ and the pumping level to $P = 0.5$. We have examined a large range of normalized delays from $\tau = 100$ to as large as $10,000$ and we have found essentially the same behavior. In the following we will keep the parameters $k/\beta = 0.03$ and examine the behavior of the square wave solutions as one of the parameters is varied. For all of the calculations the delay is set to $\tau = 1000$, and we have kept all the rest of the parameters to the values mentioned earlier in this paragraph. For all of the calculations a small amount of noise was included with a mean correlation of $10^{-6}$.

In Fig. 2 we show a bifurcation diagram for $k = 1.0$, $\beta = 0.03$ for a system in which the diode laser has the same gain for the two modes but different differential losses. The dynamics emerge from a Hopf bifurcation with oscillations at the relaxation frequency. The dynamics lead into a torus showing quasiperiodicity at two frequencies, relaxation and external cavity frequency. The square wave emerges at about $\eta = 0.063$, and it exhibits strong oscillations at the relaxation frequency. These oscillations are decreased markedly as the
feedback is increased and at about \( \eta = 0.08 \) the square wave remains essentially unchanged in a further increase of the strength of the feedback.

The square wave dynamics are shown in Fig. 3 at a convenient value of \( \eta = 0.14 \). It is clear that the amplitudes of both modes show decaying relaxation oscillations as they switch between steady state plateaus. Also notice that \( A_1 \) switches between the off state (A) and a state of nonzero amplitude (B) of about 0.7 close to a value equal to \( \sqrt{P} \). Additionally, the duration of the zero state (A) is longer than the nonzero steady state (B) whose duration is very close to the round trip time of the external cavity. Therefore, the period of the square wave is longer than twice the round trip time of the delay. It should be noticed that the mode amplitude of \( A_2 \) actually switches among three values, the zero state (A) and two nonzero states, one approximately at 0.7 almost equal to \( \sqrt{P} \), state (B), and one at 0.65, state (C), both exhibiting decaying relaxation oscillations. Comparing with the experimental time series we find that the period of the square wave is in very good agreement with the numerical simulations, and the waveform of the horizontal polarizations also is in good agreement with the calculated time series of Fig. 3(a). However, the time series of the horizontal polarization mode does not have the additional plateau of the steady state (C) exhibited in Fig. 3 (b). Also note that the experimental time series show much stronger relaxation oscillations that are not present in the numerical computations.

A visual confirmation can be seen in the Fig. 4 in which we plot the trajectory in the plane of the inversion \( Z \) versus \( A_2 \). The two nonzero steady states of \( A_2 \) are states (B) and (C) as shown by the arrows. They are approached by decaying relaxation oscillations indicated by the spirals into the steady states, and the zero state (A), is indicated by \((Z, A_2) = (0, 0)\) and it is approached along its stable manifold.

As the value of \( k \) is decreased to \( k = 1/3 \), and \( \beta = 0.09 \), we find that the square waves exhibit stronger relaxation oscillations for practically all values of \( \eta > 0.10 \) as shown in the bifurcation diagram of Fig. 5. In fact it appears that the amplitude of the relaxation oscillations is larger as the feedback is increased but eventually it saturates when \( \eta \) becomes sufficiently large, keeping the relaxation oscillations too a rather large amplitude. The time series of the two modes are shown in Fig. 6 at \( \eta = 0.14 \).
Figure 5: Bifurcation diagram for $\beta = 0.09$, and $k = 1/3$.

Figure 6. Time series of the modes (a) horizontal and (b) as a function of the normalized time for $\beta = 0.09$, $k = 1/3$ at $\eta = 0.14$. 
It is clear that for these values of gain ratio and differential losses, the second steady state of $A_2$ denoted by (C) has decreased markedly in duration, and it consists of a single rather strong pulsation of relaxation oscillations. This can also be exhibited rather nicely by plotting the trajectory in the plane of the inversion versus the amplitude of the horizontal polarization $A_2$ shown in Fig. 7. The arrow by (C) indicates the single pulse of relaxation oscillation. The duration of the zero steady state and the nonzero steady states are almost equal and therefore the period of the square wave is slightly longer than twice of the roundtrip of the external cavity. The time series of the square wave shown in Fig. 8 can be compared and contrasted with the experimental time series in Fig. 1. The experimental data shown in Figs. 1(b) and 1(d) actually do contain a single oscillation in the same position as in Fig. 6, but it is of very small amplitude.

We can obtain a better agreement with the experiment if we can decrease the amplitude of this pulsation. To this end, we find that if the feedback is increased to $\eta = 0.35$ the resulting time series shows a very good agreement with the experimental time series. This time series is shown in Fig. 8 and, indeed, the arrow in Fig. 8(b) shows a small pulsation. The decaying relaxation oscillations in the steady states (9) have a larger amplitude than the experiment but the frequency filter present in the detection of the time series should average out some of the more strong pulsations. Results of such further detailed calculations would be published elsewhere.

The trajectory in the plane of $Z$ and $A_2$ is shown in Fig. 9. The arrow denoted by (C) shows the position of the steady state that is not visited anymore by the trajectory. The second arrow points at the single of small amplitude relaxation oscillation.

Note that both Hopf bifurcations shown in Fig. 2 and Fig. 5 at the same values of $k/\beta = 0.03$ are independent of the individual values of $\beta$ and $k$, and appear at $\eta_H = 0.026$. Several more such bifurcation calculations at a selected set of values of these quantities confirm these observations.
Figure 8. Time series of the modes (a) horizontal and (b) as a function of the normalized time for $\beta = 0.09$, $k = 1/3$ at $\eta = 0.35$.

4. SUMMARY

In summary, we have presented a series of experimental observations of the existence of square waves in the incoherent feedback system. Several such observations confirm the periodic square wave nature of such waveforms, along with a variety of complex solutions whose periods are associated with twice the external cavity roundtrip time. Similar observations, that is, such as polarization self-modulation have been seen in VCSELs. This previous work has typically employed a quarter wave plate as the polarization-rotating element, which allows mutual coupling between polarization modes as well as multiple roundtrips due to reflections from the laser's front facet. In this paper, however, we have considered edge-emitting, Fabry-Perot diode lasers in which the feedback from an external cavity couples asymmetrically only from the natural, horizontal polarization mode to the non-lasing, orthogonal mode.
Figure 9: Plot of the trajectory in the plane of $A_2$ and $Z$, for $k = 1/3$, and $\beta = 0.09$ at $\eta = 0.35$.

The numerical simulations using a two-polarization model has taken into account the differential losses as well as the gain asymmetry between the two modes. It was found that the square waveforms solutions dominate for small values of the product of the gain and differential losses. A few time series as well as bifurcations to square wave solutions were presented in this paper and were compared with experimental time series and observations. The comparison is in very good agreement as long as the differential losses are small, gain ratios about equal and for conditions in which the feedback is fairly large. The secondary relaxation oscillations shown by the arrow in Fig. 8(b) can be totally eliminated by selecting a smaller differential loss but at the expense of larger amplitude of relaxation oscillations. In future publications we will discuss further details, such as the effect of noise to the stability of the square wave solutions along with an possible analytical treatment of the square wave solutions.

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