Stochastic Geometry Modeling of EMF exposure due to Cellular Networks in Urban Environments

Quentin Gontier, Jean-Francois Determe, Luca Petrillo, Trung-Hien Nguyen, Franois Horlin, Joe Wiart, Claude Oestges, Philippe De Doncker,

***Abstract*—Stochastic geometry is used to model global exposure to Electro-Magnetic Fields (EMF) in urban environments. The method is applied to two distinct urban environments: the historic city center of Brussels, Belgium, and the 14th district of Paris, France. It is shown that the proposed model fits experimental values, paving the way to a new methodology to assess general public exposure to EMF.**

***Index Terms*—Cellular Networks, EMF Exposure, Stochastic Geometry.**

I. Introduction

EMF exposure due to cellular networks is difficult to calculate deterministically in a reasonable time. It is also subject to many uncertainties (due to the number of base stations in operation, the environment geometry, the presence of people and vehicles causing shadowing...). But calculating EMF exposure at every point of the area under study is not always required. Instead, statistical values are often looked for, for instance to estimate probability of exceeding some exposure thresholds, or to assess the mean level of exposure. In this paper, we show that stochastic geometry (SG) [1] is a powerful tool to model statistical distributions of the global EMF exposure in an urban environment. Stochastic geometry simulates cellular networks by randomly generating base station (BS) patterns in the area under study. In this letter, we propose to combine SG with a simple propagation model, to obtain the statistical distribution of the global exposure in the area under study. By comparing simulated and experimental data for the center of Brussels and Paris, optimal values for the propagation model parameters will be obtained. It will be shown that SG provides a statistically relevant model for global EMF exposure, that fits experimental distributions.

II. Reference statistical distributions

Statistical distributions for the amplitude of the electric field were experimentally obtained by drive-tests in the historic center of Brussels, Belgium, and in the 14th district in Paris, France. The comprehensive experimental set-up is described in [2]. A spectrum analyzer is mounted on a moving car, taking calibrated measurements from $300\,\mathrm{MHz}$ to $3\,\mathrm{GHz}$. A GPS is used to tag the measurements with position. Measurements are averaged over squared local areas of $2\,\mathrm{m} \times 2\,\mathrm{m}$. This size was heuristically chosen. It is small enough to keep a relevant spatial sampling, but large enough to smooth out fading. In accordance to Brussels regulations, and to the International Commission on Non-Ionizing Radiation Protection (ICNIRP) guidelines [3], measurements are then summed up over the spectrum. They are brought back to a reference frequency, here $900\,\mathrm{MHz}$, to obtain the so-called $900\,\mathrm{MHz}$-equivalent electric field:

$$E_{900\,\mathrm{MHz}} = \sqrt{\sum_{f=300\,\mathrm{MHz}}^{3\,\mathrm{GHz}} \left(\frac{E_f}{E_{L,f}} \; E_{L,900\,\mathrm{MHz}} \right)^2} \tag{1}$$

where E_f is the electric field strength measured at frequency f, $E_{L,f}$ is the reference level for general public exposure at frequency f as defined by ICNIRP [3], and where $E_{L,900\,\mathrm{MHz}} = 41.25$ V/m. In this summation, we only kept frequencies related to cellular services. The $900\,\mathrm{MHz}$-equivalent electric field is so an indicator of global exposure to EMF due to cellular networks.

III. Stochastic geometry approach

In the SG approach, the BS spatial distribution is considered as a random point pattern with constant density λ in a given 2D region P, referred to as the *window*. According to [4], BS patterns in cities such as Paris or Brussels are well modeled by using Poisson Point Processes (PPP): for any P, the number of points falling in P has a Poisson distribution with mean $\lambda \cdot \nu_2(P)$ (where $\nu_2(P)$ is the area of P).

Using the open access databases of BS locations available in Belgium [5] and in France [6], the BS density can be calculated over a given area. It was found equal to $\lambda = 26\ ^{\mathrm{BS}}\!/_{\mathrm{km}^2}$ in Brussels historic center, and to $\lambda = 19\ ^{\mathrm{BS}}\!/_{\mathrm{km}^2}$ in Paris 14th district. As an example, the BS pattern in Brussels is shown in figure 1.

To obtain statistical distributions for $E_{900\,\mathrm{MHz}}$, P is first defined as the area under study, either Brussels, or Paris. PPP realizations with relevant densities are drawn to obtain an ensemble of BS patterns in P. For each draw, $E_{900\,\mathrm{MHz}}$ is computed at the geometric center of P by using the propagation model described here below. Applying ergodicity, statistical distributions for $E_{900\,\mathrm{MHz}}$ are deduced.

To compute the electric field at the center of P we considered a very simple, approximate yet efficient model: we supposed that the $900\,\mathrm{MHz}$-equivalent electric field is deduced

This work was supported in part by the FNRS under MUSE-WINET EOS grant and the FEDER Icity.BRU grant."

Q. Gontier, J-F Determe, L. Petrillo, T-H. Nguyen and Ph. De Doncker are with the Wireless Communications Group, Université libre de Bruxelles, Belgium (e-mail: firstname.lastname@ulb.ac.be)

C. Oestges is with the ICTEAM Institute, Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium (e-mail: claude.oestges@uclouvain.be)

J.Wiart is with LTCI, Telecom Paris, Chaire C2M, Institut Polytechnique de Paris, Palaiseau 91120, France (e-mail: joe.wiart@telecom-paris.fr)

Fig. 1. BS in the historic center of Brussels. $\lambda = 26$ BS/km^2

from the distances to the BSs by an extended Friis law. Assuming that the fields emitted by the different BSs are uncorrelated, $E_{900\,\mathrm{MHz}}$ is obtained as:

$$E_{900\,\mathrm{MHz}} = \sqrt{\sum_{i|BS(i)\subset D(R)} \left(\frac{A_i\,\sqrt{30\ \mathrm{EIRP}}}{(d_i^2+h^2)^{n/4}}\right)^2} \quad (2)$$

where only BSs present in a disc $D(R)$ of radius R (to be defined) at the center of P, are taken into account in the summation. Distance d_i is the distance in the horizontal plane from the center of P to BS i. All other parameters have to be fitted:

- EIRP is the effective isotropic radiated power.
- A_i is a random factor (with dimension $(\mathrm{m}^{n/2-1})$) modeling large scale fading w.r.t to BS i. A_i's follow log-normal distributions with zero mean (in dB) and standard deviation σ_A to be fitted.
- h is the BS height.
- n is the path loss exponent.

In this simple, homogenized model, no discrimination is made over the BS transmit frequencies, heights or EIRPs.

Two approaches were compared for choosing the value of R. First, considering the whole set of BS (Whole Network, WN), R was chosen large enough to contain at least one BS but not too large to avoid high computation times. R was chosen equal to $400\,\mathrm{m}$ in this case. Second, we considered the Nearest Neighbor case (NN) : R was chosen to contain one and only one BS at each draw of the PPP.

For both approaches, parameters have been fitted by minimizing

$$K(\theta) = \left(\frac{Q_1(\theta)}{Q_{1,exp}}-1\right)^2 + \left(\frac{Q_2(\theta)}{Q_{2,exp}}-1\right)^2 + \left(\frac{\mu(\theta)}{\mu_{exp}}-1\right)^2 + \left(\frac{Q_3(\theta)}{Q_{3,exp}}-1\right)^2 + \left(\frac{\sigma(\theta)}{\sigma_{exp}}-1\right)^2 \quad (3)$$

where $\theta = (h, n, \mathrm{EIRP}, \sigma_A)$ is the 4-tuple of parameters and Q_1 is the first quartile, Q_2 the median, Q_3 the third quartile, μ the mean and σ the standard deviation. The minimization of $K(\theta)$ is an exhaustive search onto a regular grid $\mathcal{G} = \mathcal{I}_h \times \mathcal{I}_n \times \mathcal{I}_{\mathrm{EIRP}} \times \mathcal{I}_{\sigma_A}$ with $\mathcal{I}_h = [15; 55]$m, $\mathcal{I}_n = [2; 4]$, $\mathcal{I}_{\mathrm{EIRP}} = [56.0; 74.0]$dBm and $\mathcal{I}_{\sigma_A} = [0.0; 8.0]$dBm.

IV. Results

Statistical parameters of $E_{900\,\mathrm{MHz}}$ distributions are listed in Table 1. They were derived from the measurements (Exp), and from the stochastic geometry approach, either by considering the whole set of BS (WN), or only the nearest BS (NN). The optimal sets of parameters for the propagation model (2) are also listed in this table.

TABLE I

Parameters of the statistical distributions of the equivalent 900MHz electric field in Brussels and Paris. Exp: experimental results, NN: nearest-neighbour model, WN: whole network model. Q_i's are the quartiles, μ the mean, and σ the standard deviation.

	Brussels-center			Paris XIV		
	NN	WN	Exp	NN	WN	Exp
h (m)	50	33		50	34	
n	2.90	3.40		2.70	3.20	
EIRP (dBm)	62.79	70.83		64.52	71.1	
σ_A (dB)	3.53	2.71		3.26	0.00	
Q_1 (V/m)	0.23	0.24	0.23	0.29	0.30	0.28
Q_2 (V/m)	0.38	0.36	0.41	0.46	0.45	0.50
μ (V/m)	0.47	0.48	0.48	0.59	0.59	0.57
Q_3 (V/m)	0.62	0.60	0.61	0.76	0.72	0.72
σ (V/m)	0.35	0.37	0.41	0.43	0.42	0.41
K	0.006	0.021	0.00	0.011	0.020	0.00
$D_{KL}(\mathcal{P}\|\|\mathcal{Q})$	0.0222	0.0568	0.00	0.0165	0.0552	0.00
D_{JS}	0.0219	0.0582	0.00	0.0164	0.0565	0.00
$K_{nsim,m}$	0.94	2.69	0.00	0.84	1.88	0.00
p-value	0.139	≈ 0	1	0.239	≈ 0	1

To estimate goodness-of-fit, the Kullback-Leibler divergence (D_{KL}) [7], [8], and the Jensen-Shannon divergence (D_{JS}) [9] were calculated. The KL and JS divergences strongly depend on the shapes of the histograms, i.e. the number of bins and the mid-position of each bin. Therefore, they cannot be used alone to take a decision on the model validity. The last statistics we used is the two-sample Kolmogorov-Smirnov (KS) distance, based on cumulative distribution functions (CDFs).

Table 1 lists the KS normalized distances $K_{nsim,m}$ under the hypothesis $\alpha = 0.1$ for which $K_{0.1} = 1.224$. The last row indicates the associated p-value, which is the value of α for which $K_{nsim,m} = K_\alpha$. Among all the test-distributions, the optimal distribution almost always corresponds to the distribution with the lower KL, JS and KS distances.For both areas, $K_{nsim,m}$ is smaller than $K_{0.1}$ in the NN case, suggesting that the simulated models give a good approximation of the experimental distributions. In figures 2 and 3, the theoretical and experimental CDFs are compared. As seen, the SG CDFs fit the experimental ones, for both Brussels and Paris. Fitted values for h, n, EIRP and σ_A are realistic on physical grounds. It is worth noting that, in our approach, the BS network is homogenized in the sense that it is supposed that all BSs are located at the same height, emitting the same EIRP, with no discrimination w.r.t. frequency. Under these strong assumptions, results obtained by considering NN only is statistically slightly better than the more tedious model taking into account WN in the computation of the 900-equivalent electric field.

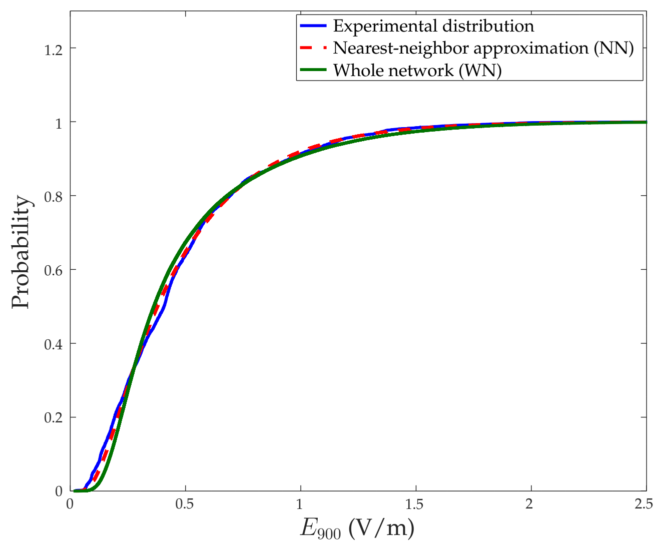

Fig. 2. Cumulative distribution functions of $E_{900\,\mathrm{MHz}}$ in Brussels historic center.

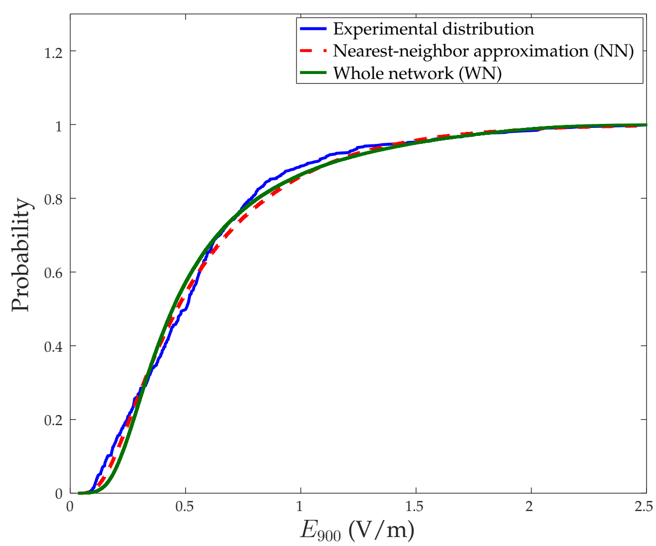

Fig. 3. Cumulative distribution functions of $E_{900\,\mathrm{MHz}}$ in Paris 14th district.

V. Conclusion

In this paper, we showed that it is possible to statistically model the global EMF exposure in urban areas using stochastic geometry. The model was applied and validated to the historic center of Brussels and the 14th district in Paris. It was shown that considering PPPs for the BS spatial pattern in combination with the NN BS for computing the 900-equivalent electric field is a statistically relevant approach.

References

[1] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks*. NoW Publishers, 2009.

[2] T. Lemaire, J. Wiart, and P. Doncker, "Variographic analysis of public exposure to electromagnetic radiation due to cellular base stations: Variographic analysis of bts emf exposure," *Bioelectromagnetics*, vol. 37(10), 2016.

[3] ICNIRP, "Icnirp guidelines for limiting exposure to electromagnetic fields (100khz to 300 ghz)," *Health Physics*, vol. 118(5), pp. 483–524, 2020.

[4] J. S. Gomez, A. Vasseur, A. Vergne, P. Martins, L. Decreusefond, and W. Chen, "A case study on regularity in cellular network deployment," *IEEE Wireless Communications Letters*, vol. 4, pp. 421–424, Aug 2015.

[5] IBPT/BIPT, 2019. [Online]. Available: https://www.ibpt.be/en/operators/radio/antennas-site-sharing/antennas-cartography.

[6] ANFR., 2020. Cartoradio [Online]. Available: https://www.cartoradio.fr/index.html#/cartographie/all/lonlat/2.323026/48.830108.

[7] S. Kullback, *Information Theory and Statistics*. New York: Wiley, 1959.

[8] S. Kullback and R. A. Leibler, "On information and sufficiency," *The Annals of Mathematical Statistics*, vol. 22, no. 1, pp. 79–86, 1951.

[9] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.