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Abstract

We propose a theory of small campaign contributions driven by an electoral motive, i.e., the desire to influence election outcomes. Though small donors take as given the actions of others, strategic interactions induce patterns consistent with empirical findings, e.g., election closeness and underdog effects. We also study different forms of campaign finance laws, and show why caps should be combined with a progressive tax on contributions. Next, we introduce large donors and show how several conclusions in the literature may be modified by the interaction with small donors. Throughout, we discuss the empirical implications of our findings.

Keywords: Campaign contributions, Small donors, Campaign finance laws, Elections, Income inequality

JEL codes: D71, D72, H31

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An informed public of small contributors "would make the millions feel that it was their government, as it is; and that you and your administration were beholden to the many, not to the few." — Lincoln Steffens to Theodore Roosevelt, September 21, 1905 (Doris Kearns Goodwin, *The Bully Pulpit*, p. 417)

1 Introduction

The role of campaign contributions in elections is a central issue in democracies. Yet both popular and academic discussions have mostly concentrated on large donors, despite the fact that small donors account for a large fraction of total contributions. In the 2012 U.S. presidential campaign for instance, the Federal Election Commission reported that out of a total campaign spending of about \$1.3 billion for the main candidates, small contributions (less than \$200 each) added up to \$621 million, and those between \$200 and \$1000 to another \$243 million.[1] The numbers tilted even further towards small contributions in the 2016 presidential race: Bernie Sanders, for example, raised 202 million dollars from small contributions, out of a total campaign budget of 223 million, while Hillary Clinton and Donald Trump also each received more than 2 million from small donors.

Small donors are important in other countries as well. In Canada, they represent about a third of total funds raised for recent campaigns. This figure is similar in the United Kingdom, where a significant share of party funding comes from membership dues and small donations. The UK Labour party, for example, reported £19.2 million in donations and £9.5 million in membership dues in 2015.[2] In Germany, they likewise represented about 53% of campaign resources in the 2012 cycle, about half that amount reflecting party membership dues.[3] Small contributions account for such a significant fraction of total funding due to the considerable number of small donors.

To the best of our knowledge, there is no formal model of small campaign contributions in the literature. Thus far, the focus has been on large donors with a *policy influence motive* for contributing.[4] For small donors, a *consumption motive* is put forward almost

[1] http://www.fec.gov/disclosurep/pnational.do;jsessionid=5E34A548A5EEB1D08BBECEA07049DF53.worker1 and http://www.fec.gov/disclosurep/pnational.do

[2] http://search.electoralcommission.org.uk/Api/Accounts/Documents/17488

[3] Most of the rest being public funding, while medium and large contributions represented about 9% of the total.

[4] The leading theoretical model is that of Grossman and Helpman (1994, 1996), although the empirical literature finds mixed support for an influence motive (Stratmann, 1992; Ansolabehere, de Figueiredo and Snyder, 2003; Gordon, Hafer, and Landa 2007; Chamon and Kaplan 2013, DellaVigna *et al.* 2016, Avis 2018). Hence, it is not clear to what extent large contributions "buy" policy favors.

by default.[5] The basic reasoning is that when individual contributions are small, donors cannot be motivated either by an attempt to "buy" influence or by any effect their contributions may have on election outcomes (an *electoral motive*).

Yet, as we discuss at length in the next section, there are compelling reasons why an electoral motive for small donors deserves closer attention. Moreover, developing a model of electorally-motivated contributions allows us to generate predictions that can then be tested against patterns of contributions observed in real life. Indeed, we find strong support for our results in the empirical literature.

In this paper, we therefore propose a theory of small campaign contributions motivated by the desire of either donors (Section 3) or candidates (Section 5) to influence election outcomes. By "small," we mean that a donor takes as given both the policy of candidates and the behavior of other donors. We show nonetheless that electoral considerations produce strategic interactions: total contributions determine the influence of money on outcomes, and hence individual best responses. As a result, the comparative statics on individual and total contributions are quite different than those implied by theories that ignore such interactions. These differences can help explain a number of empirical observations that seem otherwise anomalous.

Given that small donors are the main focus of this paper, we start with a model of *small donors only.* After laying out the small-donors model of a two-candidate race in Section 3.1, we characterize the equilibrium in Section 3.2. Importantly, we find that contributions increase when the support for the two candidates is more even, that is, a "closeness effect," and that relative contributions for the advantaged party are smaller than their underlying advantage, that is, an "underdog effect." Although these effects are seen in empirical findings on individual contributions, they are not implications of a consumption motive for contributions.

In addition, we study the effects of income and income inequality on the contributions to the two candidates. As with a consumption motive, donations are predicted to increase in income. However, strategic interactions also imply that relative changes in income inequality have significantly different effects if they affect supporters of the leading candidate or the trailing one.

[5] Ansolabehere *et al.* (2003) have stressed this view, arguing that the "tiny size of the average contribution made by private citizens suggests that little private benefit could be bought with such donations" (p117). They support their claim with the finding that "income is by far the strongest predictor of giving to political campaigns and organizations, and it is also the main predictor of contributing to nonreligious charities" like other normal consumption goods.

In Section 3.3, we analyze the effects of various campaign finance laws on the behavior of small donors. We find that a cap on individual contributions affects all donors, including those not directly hit by the cap. The cap generally favors the candidate with the largest number of donors and works against the one with the richest donors, but these effects are not necessarily monotonic. Furthermore, we examine the effects of public subsidy and taxation schemes. We find that matching subsidies have limited or no effects, and that it is possible to eliminate the effects of income inequalities on campaign contributions by implementing a progressive tax on contributions.

We then extend the analysis to include large donors who choose donations prior to small donors, where we contrast large-donor behavior under an *electoral motive* –as for small donors– and under an *influence motive* –as typically considered in the literature. We find not only that our results when there are only small donors are robust to the presence of large donors, but also that the presence of small donors may affect a large donor's behavior. We identify a new *indirect cost of contributing* under the influence motive, which may induce large donors to moderate their contributions, as well as their request for favors. The importance of the two motives moves in opposite directions with election closeness. As a consequence, our model predicts that policy favors should be more prevalent in lopsided elections. We also find that capping small contributions during the electoral campaign, while not effectively capping large donors (e.g., because they can make their contributions through other, uncapped conduits such as super PACs) has highly perverse effects. Such a cap, even if it barely affects their contribution *levels*, limits the small donors' room for maneuver, effectively reducing the indirect cost. This boosts large donors' requests and favors extraction.

At various places throughout the paper, we describe how existing empirical results call for incorporating an electoral motive in the literature's conceptual framework, and conversely how our theoretical findings may inform future empirical research. First, different motives for contributions produce qualitatively different donor responses, which could be leveraged to further understanding of small donors' motivations (see, e.g., Ansolabehere *et al.* 2003 and Barber *et al.* 2017). Second, our results also show how estimates of the income elasticity of contributions (see, e.g., Gordon *et al.* 2007, and Bonica and Rosenthal 2018) may be influenced by aggregate, equilibrium, responses. We also find that the direction of some effects crucially depends on whether the candidate is ahead or behind, as well as on the specifics of the income shock. Third, estimates of the effects of changes in campaign finance laws (such as caps on individual contributions) on electoral outcomes (see, e.g., Lott 2006, and Stratmann and Aparicio-Castillo 2006) are also subtle. Our model predicts that such effects are non-monotonic and may change sign depending on the source of the difference in popularity between candidates. Finally, caps that only bind for some specific donors may also substantially affect the behavior of other, uncapped, donors.

2 On the Electoral Motive

Starting with Ansolabehere *et al.* (2003), significant empirical effort has been invested in assessing the motives behind contributions, either those of individual donors or corporations (see, e.g., Bertrand *et al.* 2014 and DellaVigna *et al.* 2016). For small donations, it has generally been argued that each contribution is far too small relative to the total to have any effect in influencing either policy choices or election outcomes. Given the "almost zero" effect small donations are perceived to have, a consumption motive has thus been put forward. An electoral motive has been largely omitted from these analyses, basically by default, rather than due to empirical evidence of its absence.

We argue in this paper that the electoral motive should not, however, be dismissed out of hand. In fact, there is substantial empirical evidence supporting the electoral motive. First, in surveys, donors overwhelmingly list "to affect an election outcome" as an important motive for giving (Brown *et al.* 1995; Francia *et al.* 2003; Barber 2016a). Second, numerous studies find ideological proximity to be a strong determinant of contributor behavior in different types of contests (see, e.g., McCarty, Poole, and Rosenthal 2006; Claasen 2007; Bonica 2014; Barber 2016a; Barber, Canes-Wrone, and Thrower 2017). We will see that the distance between the ideological positions of donors and candidates does also matter for donors who care only about election outcomes.[6] Last but not least, as discussed extensively in Section 3.2, three key predictions of our "rational-instrumental" donor model are in line with empirical patterns in the literature: (i) *closeness effect*: donations are significantly and positively affected by the (perceived) closeness of the election; (ii) *underdog effect*: relative contributions to the front-runner are always smaller than her intrinsic advantage; (iii) *income effect*: donations are increasing in the wealth of donors. While one cannot reject that these patterns may be consistent with another theory of

[6] A related observation from Barber *et al.* (2017, p.283) is that contributions are made to legislators who "will represent their professional interests, rather than due to expectations of legislative access or an unsophisticated response to networking." This too is consistent with an electoral motive rather than simply a consumption motive for giving.

small campaign contributions, they show that electoral considerations must be part of that theory. This is exactly what we do in Section 5, where we develop a demand-driven model of small contributions that delivers the exact same predictions (more details below).

Why would electoral considerations be so important given the almost zero effect of individual small donations on the electoral outcome? There is of course a distinction between the comparative static effects we identify and the magnitude of these effects. Electoral considerations, and hence the *qualitative* predictions of our model, are identical whether we view donors as fully rational or as behavioral. In contrast, *quantitative* implications are highly sensitive to the details of the utility and/or cost functions. We argue that electorally-motivated behavioral factors may well yield significantly larger empirical effects. For example, donors may overestimate the effect of their contributions on the electoral outcome by orders of magnitude, which would be in line with the results of surveys about donors' motivation.

Another possible explanation is that the electoral motive operates through the fundraising behavior of candidates. In Section 5, we formally show the equivalence between the model developed in the core of the paper and a model in which "naive" donors respond to their candidate's fund-raising effort. The relevant assumption is that candidates believe that money is crucial to winning an election. This certainly seems to be the case: as Jesse Unruh said when he was Speaker of the California State Assembly in 1966, "Money is the mothers' milk of politics," a view clearly supported by the enormous effort candidates put into fund-raising. Voters respond to such efforts: Magleby *et al.* (2018), for example, document large spikes in donations on days that candidates increase their fund-raising effort. More generally, these are known as "moneybomb" events, a highly lucrative grassroots fund-raising effort over a brief period.[7]

A remaining important issue is whether money actually matters for electoral outcomes. The empirical literature can be divided into two sets of studies. The first focuses on the effect of specific campaign spending (e.g., TV ads) with recent studies, with well-defined identification strategies, finding positive and significant effects (see, e.g., Da Silveira and De Mello 2011; Kendall *et al.* 2015; Larreguy *et al.* 2018; Spenkuch and Toniatti 2018; and Bekkouche and Cage 2020). The second analyzes the effects of total spending. Here, however, the evidence is mixed: spending by challengers appears more effective than spending

[7] Note however that such a demand-driven model does not seem to fully capture the small contributions phenomenon. Magleby *et al.* (2018, p. 357) find that "A third of those [small donors] who donated in 2008 reported they gave without being asked to do so, 37 percent for Obama, and 25 percent for McCain."

by incumbents and, for the latter, no consensus has been reached as to whether the effect of money is economically significant (see, e.g., Levitt 1994; Erikson and Palfrey 1998, 2000; Gerber 2004; Benoit and Marsh 2008; Stratmann 2009; Bombardini and Trebbi 2011; and Kawai and Sunada 2015). A simple way to reconcile the apparent contradiction between these two sets of studies is provided by Sprick Schuster (2020). Using detailed transaction-level data on candidate disbursements, he finds systematic differences in the way incumbents and challengers allocate their campaign resources. In particular, incumbents spend a smaller share of their total spending than challengers on "messages to voters" (i.e., advertising and events), and a larger share on other types of spending, such as refunding of contributions and donations to other campaigns. These latter types of spending have arguably no effect on their chances of winning their own race.

3 Small Donors

We begin by setting out our theory of electorally-motivated small contributions. Hence, this section considers a simple model with *small donors only.* In Section 4, we embed this model of small donors model into a dynamic game in which a large donor has first mover advantage. We will see that the results of the simple model extend to the generalized model, although with additional and novel insights on the interactions between small and large donors.

3.1 The Model

There are two candidates, A and B, who need funding to finance their electoral campaigns. Our key assumption is that money affects the outcome of the election (see discussion at the end of Section 2). We summarize this through a *contest success function*.[8] This captures the idea that these funds are used to finance activities such as get-out-the-vote efforts (see Enos and Fowler, 2016) or advertising (as for example in Baron, 1994, Prat, 2002, Coate 2004a,b, and Morton and Myerson, 2012), which may increase a candidate's vote totals. As Cmar (2005) put it, "a political campaign is almost never successful unless its resources are comparable to those of its opponents – and the most important of these resources is money."

[8]This is inspired by an increasingly large literature: see Tullock (1980), Hirshleifer (1989), Baron (1994), Skaperdas and Grofman (1995), Esteban and Ray (2001), Epstein and Nitzan (2006), Konrad (2007), Jia *et al.* (2013), Herrera *et al.* (2014, 2016), among others.

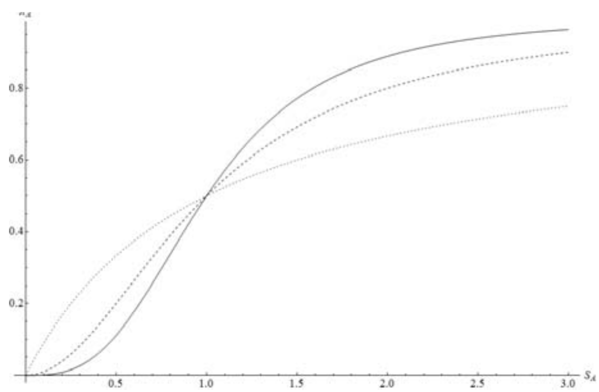

Figure 1: $\pi_A (S_A, S_B)$ for $S_B = 1$ and $\gamma = 1$ (dot), $\gamma = 2$ (dash), or $\gamma = 3$ (solid)

Formally, given total contributions $\mathbf{S} = \{S_A, S_B\} \in \mathbb{R}_+^2$ by small donors, A's probability of winning the election is given by:

$$\pi_A (\mathbf{S}) = \frac{S_A^\gamma}{S_A^\gamma + S_B^\gamma} = \frac{1}{1 + \left(\frac{S_B}{S_A}\right)^\gamma}, \tag{1}$$

for $\max\{S_A, S_B\} > 0$, and $\pi_A (\mathbf{S}) = x \in [0, 1]$ for $S_A = S_B = 0$ (see Esteban and Ray, 1999). Note that π_P is everywhere concave in S_P for $\gamma \leq 1$. Values of $\gamma > 1$ capture the presence of setup costs: π_P is then convex for $S_P < \bar{S}_P \equiv \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma}} S_{-P}$. In words, P's campaign must reach $\bar{S}_P$ for additional contributions to have maximal effect. Figure 1 illustrates the shape of π_A for $\gamma = 1$ (dotted line), $\gamma = 2$ (dashed), and $\gamma = 3$ (solid) when $S_B = 1$.

Small donors simultaneously and non-cooperatively select which amount s_P^i to contribute to their preferred candidate, with $\sum_i s_P^i = S_P$.[9] Each small donor i has a two-dimensional type $\left(p^i, y^i\right)$, where $p^i \in \{A, B\}$ identifies who is his preferred candidate/party and y^i represents i's income (which will influence his willingness to contribute.) There are n_P donors of type P, distributed in income classes $y^1 < ... < y^G$ according to some (discrete) cumulative distribution function $F_P\left(y^i\right)$ with $F_P(0) = 0$, and $F_P\left(y^G\right) = 1$. The fraction of type-P donors with income y^i is denoted $f_P\left(y^i\right) = F_P\left(y^i\right) - F_P\left(y^{i-1}\right) \geq 0$.

Given our focus on the electoral motive, we consider small donors who contribute to influence the election outcome. Each small donor contributes some amount $s_P^i \in (0, \bar{s}]$,

[9] A similar setup with many individual players investing resources to collectively fight over an issue has been pioneered by Katz *et al.* (1990) for rent-seeking, and by Esteban and Ray (1999, 2001) for conflict situations.

where $\bar{s}$ is the legal contribution limit. We thus concentrate on the intensive margin.[10] Letting the costs of contributions being increasing and convex, *i.e.* $\rho > 1$ (we will focus on the case $\rho = 2$ in most of the paper), the objective function of a small donor is:[11]

$$\max_{s_P^i} \pi_P(\mathbf{S}) v_P - \frac{\left(s_P^i\right)^\rho / \rho}{\left(y^i\right)^\theta}, \tag{2}$$

where v_P is the utility that a small contributor attaches to his candidate P being elected. The parameter θ allows for potentially different contribution costs across income classes: for $\theta = 0$, the cost of contributing is independent of income. For $\theta > 0$, (total and marginal) costs are strictly decreasing in y^i. Such a negative effect of the income on the cost would emerge naturally from a model in which (i) donors have a given budget y^i that they allocate between contributions and other goods, and/or (ii) the marginal utility of income decreases as income increases. This is exactly what, e.g., Layard, Mayraz, and Nickell (2008) find in the data.

3.2 Equilibrium Analysis

For now, we abstract from potential contribution caps and show that there exists only one candidate *pure strategy Nash equilibrium* of this contribution game, and we identify sufficient conditions for existence. We then show that total contributions (1) increase in the closeness of the election, (2) display a partial underdog effect, and (3) increase with income and income inequalities. Formally, the first two observations mirror existing results on voter turnout. The novelty lies more in identifying how they match empirical patterns of contribution behavior, both in observational and experimental settings. The comparative statics about income and income inequalities are more novel from a theoretical standpoint. They also find support in the data.

As a first step on the way to solving for the equilibrium, we derive a small donor's best response given the other donors' contributions. To this end, we take first order conditions of the utility function (2) with respect to the individual donor's contribution, s_P^i:

[10] Some donors could be at a corner solution, contributing exactly zero when they expect to have too low an effect on the election, and move to an interior solution when this effect increases in magnitude. In essence, this extensive margin is the focus of the turnout models discussed in Section 3.2. For the sake of simplicity, and since these effects are known, we abstract from them here.

[11] Note that convex costs can be microfounded easily. For instance, consider a model in which donors have a given budget y^i that they allocate between contributions and other goods. Convex costs then follow automatically from the utility function being concave in the consumption of the other goods, even if the utility function were linear in contributions.

$$\text{For types } A \quad : \qquad s_A^i = \left(\frac{d\pi_A}{dS_A}\left(y^i\right)^{\theta} v_A\right)^{\frac{1}{\rho-1}} \tag{3}$$

$$\text{For types } B \quad : \qquad s_B^i = \left(\frac{d\pi_B}{dS_B}\left(y^i\right)^{\theta} v_B\right)^{\frac{1}{\rho-1}}, \tag{4}$$

where we have used the fact that $dS_P/ds_P^i = 1$. The term $d\pi_P/dS_P$ embodies the electoral motive for small donors. Put simply, donors with an electoral motive contribute more when they perceive that their contribution has a higher impact on their candidate's probability of winning. The other two elements in the best response function are discussed below.

Central to the electoral motive is the fact that it generates non-trivial strategic interactions: while each individual increases her contribution when $d\pi_P/dS_P$ increases, the combined responses of all donors also feed back into $d\pi_P/dS_P$. To evaluate these two-way interactions, we aggregate small donors' best responses into their total contribution. Adding up individual best responses and simplifying yields:

$$S_P \;=\; \sum_{i\in\{s_P\}}\left(\frac{d\pi_P}{dS_P}\left(y^i\right)^{\theta} v_P\right)^{\frac{1}{\rho-1}} = \left(W_P\;\gamma\pi_A\left(1-\pi_A\right)\right)^{\frac{1}{\rho}}, \tag{5}$$

$$\text{where } W_P \equiv n_P\ \left(v_P\right)^{\frac{1}{\rho-1}}\ \sum_{i=1}^{G} f_P\left(y^i\right)\times\left(y^i\right)^{\frac{\theta}{\rho-1}} = n_P v_P \bar{Y}_P.$$

We will interpret W_P as the candidate's *intrinsic support* among small donors, reflecting their numbers (n_P), intensity of preferences (v_P), and distribution of income ($\bar{Y}_P$). We label A the candidate who is *ahead* and B the candidate who is *behind*, in the sense that $W_A \geq W_B$.

Using this notation, we find that:

Proposition 1 *Whenever a pure strategy equilibrium exists, it is unique and characterized by the aggregate contributions:*

$$\left(S_A^*, S_B^*\right) = \left(\left(\gamma\omega W_A^{\rho-1}\right)^{\frac{1}{\rho}}, \left(\gamma\omega W_B^{\rho-1}\right)^{\frac{1}{\rho}}\right),$$

with $\omega \equiv \frac{\left(W_B/W_A\right)^{\gamma\left(1-\frac{1}{\rho}\right)}}{\left(1+\left(W_B/W_A\right)^{\gamma\left(1-\frac{1}{\rho}\right)}\right)^2}$ *representing the closeness of the election.*[12] *The associated*

[12] Note that ω is a direct transformation of $\pi_A^*\left(1-\pi_A^*\right)$, but expressed in terms of the primitives of the model. It is increasing in W_B/W_A, $\forall W_B/W_A < 1$, and decreasing in W_B/W_A, $\forall W_B/W_A > 1$. It is thus maximized in $W_B = W_A$, or $\pi_A^* = 1/2$.

winning probabilities are:

$$\pi_P^* = \frac{(W_P)^{\gamma\frac{\rho-1}{\rho}}}{(W_A)^{\gamma\frac{\rho-1}{\rho}} + (W_B)^{\gamma\frac{\rho-1}{\rho}}}. \tag{6}$$

Two sufficient conditions for the existence of a pure strategy equilibrium are: (1) $\gamma \leq \rho$ and, if $\gamma > \rho$, (2) W_A/W_B not too large.

The proof of the first part of the proposition is straightforward: it consists of substituting for π'_P in (5), and solving for the contributions consistent with best responses. The second part is about ensuring that second order conditions are satisfied. What we find is that the solution to the first problem is always unique, but it is only an equilibrium when either (1) the problem does not display non-convexities *or* (2) the race is not too lopsided.

The equilibrium winning probabilities in (6) show that a candidate can only benefit from having higher intrinsic support W_P among her donors.[13] This can result from receiving contributions from more donors, from higher income donors and/or donors with more intense preferences. The effect on the size of the campaign ($S_A^* + S_B^*$) and on individual contributions is however less than straightforward. We summarize them by studying three important implications of Proposition 1. In the remainder of the paper, we restrict our attention to the case $\rho = 2$ for the ease of readability.

3.2.1 Election Closeness

The first corollary of Proposition 1 is that under the electoral motive election closeness matters for contributions:

Corollary 1 *Small donors' contributions s_P^i increase in* election closeness*, as measured by $\left|\sqrt{W_A/W_B} - 1\right|$ or $|\pi_A^* - 1/2|$.*

This is similar to formal results in the literature on voter turnout, which say that voters should be more likely to vote when they are more likely to affect the election outcome, *i.e.* when the election is close (see *e.g.* Palfrey and Rosenthal 1985, Castanheira 2003, Feddersen and Sandroni 2006, or Herrera *et al.* 2014, 2016).

[13] In a different context, Esteban and Ray (2001) show that this is partly due to the shape of the cost function, and partly to winning the election acting as a public good. We use the qualifier "partly" because they focus on the case in which $\gamma = 1$. For that value of γ, Esteban and Ray (2001, Proposition 3) identify that free-riding effects cannot dominate collective action when payoffs are similar to that of a purely public good, as we have here.

Empirically, this effect of (perceived) election closeness appears quantitatively important: combining survey data on US donors with FEC data, Barber *et al.* (2017, p.17) shows that "a standard deviation increase [in competitiveness] raises the likelihood a donor gives to that campaign by 43%." Jacobson (1980, 1985) studies how the expected closeness of a US congressional election, proxied by the winner's share of the two-party vote in the last election, affected campaign contributions between 1972 and 1982. In line with Corollary 1, he finds that the closer the race, the larger are contributions to both the challenger and the incumbent. Culberson *et al.* (2019) focuses on small contributions and finds similar results for US House elections between 2006 and 2010. To control for hidden heterogeneity, Mutz (1995) and Fuchs *et al.* (2000) study the dynamics of a given campaign to see how shocks to perceived closeness and other events influencing the marginal effect of contributions affect donor behavior. They consistently find that, when the race between the front-runner and the runner-up narrows, contributions to both candidates increase.

3.2.2 Underdog Effect

A second implication of Proposition 1 is that equilibrium contributions are affected by free riding. The fact that A is ahead implies that the problem is more salient among A-donors:

Corollary 2 *In any equilibrium, the ratio of small contributions for A and B displays an* underdog effect:

$$\frac{S_A^*}{S_B^*} = \sqrt{\frac{W_A}{W_B}} \quad \left(< \frac{W_A}{W_B}\right).$$

That is, relative contributions for A by small donors are always smaller than A's intrinsic advantage among small donors.

This underdog effect results from individual free riding among small donors: since A is ahead among small donors, each A-donor will tend to contribute less *ceteris paribus* than a B-donor.[14]

The underdog effect has also been identified in theoretical models of turnout (see *e.g.* Palfrey and Rosenthal 1985, Castanheira 2003, Feddersen and Sandroni 2006 and, for models that use the contest success function, Herrera *et al.* 2014, 2016, and Kartal 2015).[15]

[14] This is because the marginal return of contributing should be lower for the leading candidate, which is actually the exact finding of Erikson and Palfrey (2000). They find that the effect of contributions on the election outcome is larger for the trailing candidate unless the race is close.

[15] In voting models, the underdog effect results from pivot probabilities being higher for the underdog (see among others Castanheira (2003), Myatt (2015), Agranov *et al.* (2014)).

We are not aware of a similar finding regarding political contributions; to the contrary, the main rationale for strategic contribution is the policy influence which would predict a *bandwagon effect* (comparatively more contributions to the advantaged candidate) while the "pure" consumption effect would predict no effect. As explained by Mutz (1995, p1019), "[i]n fact, many studies of bandwagon phenomena have ended up demonstrating strong underdog patterns rather than movement in the direction of majority opinion."

The most direct piece of evidence of an underdog effect comes from the field experiment reported in Rogers and Moore (2014) and Rogers, Moore, and Norton (2017, pp. 1298-1300). They contacted more than 660,000 people on the fund-raising list of the *Democratic Governors Association* and invited contributions to the campaign of Charlie Crist, the Democratic candidate for governor in Florida in 2014. They divided the sample in two, and sent two variants of an otherwise identical e-mail: one depicted the candidate as leading in the polls, the other one as trailing behind. Their overall result is that people are more motivated to support the candidate when he is presented as losing in the polls. In particular "the losing message increased the number of donations among past donors by 33% and raised 76% more money" (Rogers and Moore 2014, p16) and "controlling for donor status, [recipients of the winning message] gave less money than [recipients of the losing message]" (Rogers *et al.* 2017, p1299).

Another type of evidence comes from the analysis of candidate fund-raising strategies. As explained by Mutz (1995, p1019): "Outside the context of direct-mail fund-raising, it is also common for candidates to vie for the 'underdog' role for similar reasons (see Adams 1983). [...] In the face of an imminent threat, [supporters] may be prompted to give money by news that their candidate is threatened or losing ground." Rogers and Moore (2014) report similar findings in both the Obama and Romney campaigns: "when the campaign messages communicate that the race is close, the majority of those messages assert that the candidate is losing" (p24).

This is not to say that there is no evidence of any bandwagon effect for other types of donors, PACs in particular (see *e.g.* Stratmann 1992), or for multicandidate races in which some candidates' viability may be in doubt. In primaries for instance, most donors want to focus on the top two or three candidates (Hall and Snyder 2014). This temporarily creates significant bandwagon effects when the names mentioned for the top two or three change over the course of the campaign, while the underdog effect remains dominant for the frontrunner (Mutz 1995, Fuchs *et al.* 2000, Feigenbaum and Shelton 2013).[16]

Finally, Bonica (2016, Figure 2) compares the behavior of small donors from other donor types, in particular from Corporate PACs. Small individual contributions disproportionately flow to underdogs: depending on the election cycle, only 48 to 55% of their funds go to the winner, instead of 80-90% for Corporate PACs, to be compared to an average vote share of 60 to 65% — which could be used as a rough proxy for $W_A/(W_A+W_B)$.[17]

3.2.3 Income and Income Inequality

The literature typically approaches the issue of income and contributions from a different angle: the focus is on how income skews policies towards the rich and unduly favors the party with the richest supporters (see *e.g.* Coate 2004a,b, and Feddersen and Gul 2015). In this section, we instead focus on how income inequality influences campaign contributions by small donors when platforms have already been chosen—the case of large, early donors influencing platforms, with consequences on small donors' contributions, is analyzed in Section 4.

The political science literature provides a number of interesting studies that directly or indirectly estimate the income elasticity of contributions. There is a lot of evidence that political participation in general, and campaign contributions in particular, is heavily tilted towards high-income citizens (see e.g. Schlozman, Verba and Brady 1995, 2012, Bonica *et al.* 2013, Malbin 2013). Gordon *et al.* (2007) also report positive income elasticities for the individual contributions of executives—note that, in the absence of strategic interactions, their estimate would be a direct measure of θ in our model; this would be as high as 5 according to their main estimation (Table 1). Bonica and Rosenthal (2018) instead obtain wealth elasticities "quite close to zero" for democrats, and slightly below 1 for republicans. Although their interpretation is different, Ansolabehere *et al.* (2003, p122) find an income elasticity slightly above one, and income growth explains about 80% of the observed increase in contributions.

Arguably, all these empirical findings depict (*i*) *equilibrium* behavior, with the proviso

[16] In a companion paper (Bouton *et al.* 2018), we analyze elections with more than two candidates, and find that the electoral motive produces a bandwagon effect for longshot candidates (these are abandoned by instrumental donors when perceived as having too low a chance of winning the election), *and* an underdog effect for the top-two candidates.

[17] Authors' computation based on Bonica's dataset (Bonica, Adam. 2016. Database on Ideology, Money in Politics, and Elections: Public version 2.0. Stanford, CA: Stanford University Libraries. <https://data.stanford.edu/dime>). We thank Moritz Hennicke for his thorough work on these data.

that free-riding effects may bias estimates of θ downward; (ii) of very wealthy individuals, due to lack of data on smaller donors. Yet, two remarks are in order: first, remember that the definition of a "small donor" in the model has more to do with timing–small donors move too late to influence either other donors or the candidates' platforms–than with income. Second, as we will see in Section 4, our model predicts similar income effects for large donors, independently of their motive.

Another finding is that income inequality tends to stimulate total contributions, with a spillover effect on the other group of donors:

Proposition 2 *If and only if the income elasticity of contributions is larger than 1, a mean-preserving spread:*
(1) of the A-donors' income distribution increases S_A^* *and decreases* S_B^*.
(2) of the B-donors' income distribution increases both S_A^* *and* S_B^*.

We need two elements to clarify the intuition behind this result. The first is the connection between a mean-preserving spread and the income elasticity of contributions. If and only if θ is strictly larger than 1, individual contributions become a convex function of individual income. Then, within-group inequality quite intuitively increases intrinsic support W_P. Less straightforward is the equilibrium spillover effect on the other group: Lemma 1 in the appendix shows that a same increase in W_P has opposite spillover effects depending on whether P is Ahead or Behind, because of its effect on election closeness.

Combining these two elements produces the proposition: increasing inequality within the group of A-donors increases their willingness to pay, and therefore A's lead. This in turn depresses contributions by B-donors. A same increase among B-donors instead makes the election closer, which increases contributions by A-donors. The same mechanisms can be applied to between-group inequality. For instance, if A-donors become richer, B-donors will contribute less.

These spillover effects are new and, to the best of our knowledge, no empirical work studies the reactions of small donors to income inequality. Our results indicate that, because "income inequality" is not a sufficient statistic to identify the direction of all these effects, empirical work may benefit from carefully distinguishing between the different shocks to the overall income distribution.

3.3 Campaign Finance Laws: Effects on Small Donors

Campaign finance laws are, generally speaking, meant to limit the influence of money in politics (see, *e.g.*, Ashworth 2006, Coate 2004a,b). One rationale is that contributions buy policy influence outside of any direct effect on voting, that is, trading contributions for policy favors in a "quid pro quo" (see section 4). Such a rationale, as important as it might be for large and early contributions, plays essentially no role for small contributions.[18]

A second rationale to limit campaign spending is that it is like an "arms race" – what is crucial in the end is the level of total contributions *relative to* those of one's opponent. Hence, the level of money ratchets up without giving either candidate a relative advantage but draining resources nonetheless. Our small donor model captures well that feature of campaign spending.[19]

A third argument is that a donor's influence on elections is determined by the size of her contribution, so that larger contributions have undue effect on electoral outcomes. In that context, contribution caps are meant to ensure that the "voices of small donors" are also heard–this is sometimes referred to as the "equalization" argument.[20] This is central to our paper, and we show here that this can happen even in the absence of the quid pro quo component that we analyze in Section 4.2.

The main take away of our analysis is that, due to the strategic interactions highlighted in Section 3.2, campaign finance laws can have unintended consequences. Among other things, small donors will be affected even if they are not capped directly. To the best of our knowledge, such indirect effects have been ignored in the literature. Further, we identify when aggregate effects go the opposite direction of the direct effect of the cap.

[18] Coate (2004a) considers such negative welfare effects of contributions because they buy policymaker influence. In his setup, contribution limits may increase social welfare not only because they reduce such influence, but also – and because of this – such limits increase the information value of activities that contributions finance.

[19] Another important factor is incumbency, which typically provides a substantial exogenous advantage, that a challenger may find easier to overcome with money. See *e.g.* Lott (2006) and Bonneau and Cann (2011).

[20] The debate about campaign finance in the United States, as reflected in U.S. Supreme Court decisions, has been largely framed in terms of issues of 'freedom of speech'. In the famous Buckley v. Valeo decision, a majority held that limits on campaign spending and individual contributions in the Federal Election Campaign Act of 1971 were unconstitutional because they violated the First Amendment provision on freedom of speech, the argument being that a restriction on spending "necessarily reduces the quantity of expression". Arguments in favor of restrictions have also relied on such considerations. In Austin v. Michigan Chamber of Commerce (1990) the court had upheld previous limits on corporate spending, writing "Corporate wealth can unfairly influence elections." Analogously, Justice Stevens, in the minority dissent in Citizens United, reiterated the "unfair influence" argument, writing that "unregulated expenditures will give corporations 'unfair influence' in the electoral process and distort public debate in ways that undermine rather than advance the interests of listeners."

3.3.1 Caps on Individual Contributions

The diversity of possible effects is illustrated in the following two propositions: the effects of contribution caps can go in exactly opposite directions, depending on whether the advantage of A results from a larger number of donors (Proposition 3) or from wealthier donors (Proposition 4). Moreover, the effects need not be monotonic. From Proposition 1, we can pinpoint s_P^G, the equilibrium contribution by a donor of type P with income y^G. We have:

Proposition 3 *Consider the case of identical income distributions and preference intensity* (v_P) *for* A*- and* B*-donors, but* $n_A > n_B$*. In that case:*
(1) π_A *will be* ***lowest*** *when the cap is higher than* $\max\left\{s_A^G, s_B^G\right\}$*;*
(2) π_A *will be* ***highest*** *when the cap constrains all donors*;
(3) Depending on the shape of the income distribution, the effects of varying the cap can be non-monotonic.

The main driver of the difference between (1) and (2) is the underdog effect (Corollary 2). With $n_A > n_B$, free riding implies that an A-donor with income y^i contributes less than a B-donor with the same income. A binding cap must therefore constrain B-donors more than A-donors. Candidate A is thus better off with a cap than with no cap, and best off when the cap is binding for all donors.

However, this still does not imply that the effects of a cap are monotonic, as illustrated in Figure 2.[21] Indeed, capping high-income donors stimulates contributions by low-income donors *and* impacts closeness – remember that closer elections stimulate contributions in all groups. Thus, while the direct effect of the cap favors A (B-donors being more constrained), indirect effects tend to work in the opposite direction, and may dominate.

As figure 2 indicates, indirect equilibrium effects dominate for intermediate caps. In the example, this is due to the fact that small and comparatively large contributions both represent a significant fraction of the total (initially 50%), with no intermediate contributions. This proxies what we typically observe in actual data, where there is a huge number of very small contributions, and another mass at higher levels (typically

[21] The simulation behind Figure 2 builds on a two-group income distribution with $y_l = 3$ and $y_h = 10$; while we set $\gamma = \rho = 2$, and $v_P = \theta = 1$. The number of low- and high-income donors are: $n_{A,l} = 60 > n_{B,l} = 30$ and $n_{A,h} = 20 > n_{B,h} = 10$. That is, both income classes are willing to contribute about the same amount (this proxies actual values in the 2015-16 US presidential elections), but there are twice as many A- as B-donors, implying that $W_A = 380$ and $W_B = 190$.

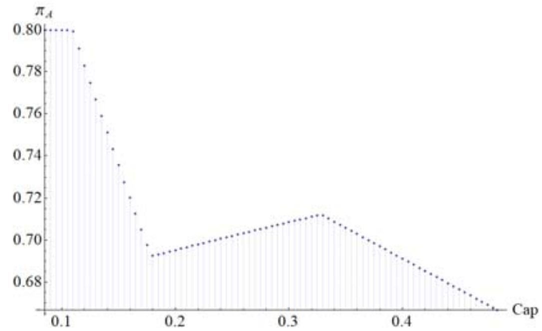

Figure 2: Simulated effect of an individual contribution cap (horizontal axis) on the probability of A winning the election (vertical axis) when $n_A > n_B$ but the income distribution is identical across donor groups.

bunched at legal limits). Technically, when we move from lax to tighter caps, *i.e.* from right to left in the figure, the cap initially binds for high-income donors only, which corrects for the underdog effect among large contributions, but also increases the weight of small contributions in the total. When the cap is intermediate (caps between 0.18 and 0.33 in the figure) the underdog effect has been fully addressed among high-income donors, but has been reinforced among low-income donors. Since the latter represent an increasing fraction of the total, tighter caps now handicap A. In contrast, both lax (above 0.33) and tight (below 0.18) caps primarily reduce the underdog effect, which benefits A.

Now, contrast these results with the case in which the advantage of A is due to *higher donor income*, rather than a numerically larger donor base:

Proposition 4 *Consider the case in which A and B have equal popular support ($n_A = n_B$) and preference intensity, but A-donors benefit from higher income, by a factor $k > 1$ ($f_A\left(ky^i\right) = f_B\left(y^i\right)$, $i = 1, ..., G$). In that case, the effects of a cap are the* opposite *of the ones in Proposition 3:*
*(1) π_A will be **highest** when the cap is larger than* $\max\left\{s_A^G, s_B^G\right\}$*;*
*(2) π_A will be **lowest** when the cap constrains all donors*;
(3) Depending on the income distribution, the effects can be non-monotonic.

The intuition and the mechanism of the proof are similar to those of the previous proposition, with the difference that, if A-donors are richer but no more numerous than B-donors, they must be the first constrained.[22] Hence, there are more type-A than type-B

[22] This numerical example also builds on two income classes in each donor group: $y_{A,l} = 6$ and $y_{A,h} = 20$, $y_{B,l} = 3$ and $y_{B,h} = 10$; $\gamma = \rho = 2$, and $\theta = 1$. Thus A-donors have twice the income of B's, while their

constrained donors, and any unconstrained A-donor contributes more than the equivalent B-donor. The logic is the same as above, although closeness and free-riding effects now work in the opposite direction: for high levels of the cap, *i.e.* $\bar{s} \in [0.51, 0.7]$, the cap only binds A high-income donors. This reduces the gap between contributions by A and B high-income donors. The slope reversal for $\bar{s} \in [0.25, 0.51]$ happens when both A and B high-income donors are constrained: the only effect left is the equilibrium response of low-income donors, who weigh increasingly more in the total. Here, a marginal tightening of the cap favors A because low-income A-donors are richer. The local maximum at 0.25 is reached when low-income A-donors start being capped, and the global minimum for $\bar{s} \in (0, 0.22]$ is reached when all donors are capped. Then, the income differences that favored A no longer define contributions.

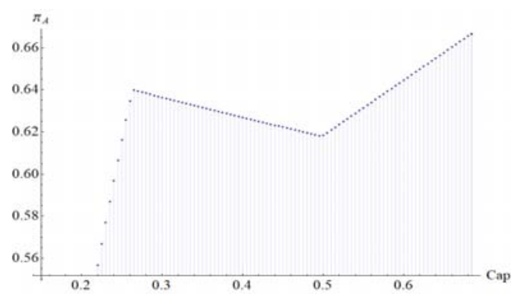

Figure 3: Simulated effect of an individual contribution cap (horizontal axis) on the probability of A winning the election (vertical axis) when $y_A^i = 2y_B^i$ but the number of donors is identical across donor groups.

The empirical literature on the effects of caps on individual contributions finds seemingly contradictory evidence. Stratmann and Aparicio-Castillo (2006) find that, for elections to US state Assemblies (lower house of a bicameral legislature) between 1980 and 2001, caps on individual contributions led to closer elections.[23] Lott (2006) finds the opposite result for elections to US state Senates (upper house) from 1984 to 2002: caps led to more lopsided elections.[24]

numbers are identical: $n_{P,l} = 30$ and $n_{P,h} = 10$, $\forall P$. Hence, as in the previous example, $W_A = 380$ and $W_B = 190$.

[23] They also find that both the share and the absolute level of total contributions going to the incumbent decrease significantly. This is also in line with the result in Proposition 4. Stratmann (2006) find that, for the same elections, campaign spending by candidates (both incumbents and challengers) are more effective, and converge one towards the other, in elections with campaign contribution limits. This is also in line with what our model predicts when the cap on contribution has a positive (or nill) effect on the closeness of the race. Indeed, the marginal effect of contributions increase when the total contributions to both parties go down (because of the free-riding effect), and their returns become more equal when $S_A \rightarrow S_B$.

[24] Similarly, Bonneau and Cann (2011) find that, in US state supreme court elections from 1990 to 2004, campaign finance restrictions (more broadly defined) hurt challengers more than incumbents.

Propositions 3 and 4 suggest avenues to reconcile these findings. First, empirical studies inevitably focus on the effects of "local" changes in caps on contributions. But, Propositions 3 and 4 show that such local effects need not be monotonic. Estimates as in Stratmann and Aparicio-Castillo (2006) and Lott (2006) may thus have opposite signs simply because the specific cap changes under study affect different parts of the distribution of donors. Second, these propositions also highlight how the effects of caps on individual contributions change sign depending on the main source of differences in support for the candidates. Our model thus suggests to explore in more details these sources for US state legislature elections. For instance, do we observe significant differences in the median number and value of individual donations for the candidates in those elections?

3.3.2 Campaign Subsidies and Taxes

Consider now the effects of campaign subsidies. We focus here on *matching subsidies*, where for each dollar of contributions, the government adds (or subtracts in the case of a tax on contributions) m dollars.[25,26] Total small contributions then become:

$$\tilde{S}_A = \sum_{i=1}^{n_A} (1+m)\, s_A^i \text{; and } \tilde{S}_B = \sum_{i=1}^{n_B} (1+m)\, s_B^i. \tag{7}$$

We find that:

Proposition 5 *A matching subsidy m that applies to contributions by small donors has no effect on their behavior, nor on the outcome of the election.*

The rationale behind this proposition is rather straightforward given winning probabilities follow a contest success function. Since the matching subsidy modifies each (and hence total) small contributions by the same proportion m for both candidates, it has no effect on the *relative* position of the two candidates, nor on election probabilities. Matching subsidies may affect outcomes for other specifications of the contest success function, but the mechanism behind Proposition 5 makes clear why a general matching subsidy

[25] Ashworth (2006) considers a situation that complements our analysis: in his model, incumbents may have an unfair advantage in fundraising, and matching subsidies are then a way to correct the situation. Yet, as he shows, welfare effects may be less than straightforward even in such a situation.

[26] In a previous version of the paper that omitted large donors, we also studied the effect of *block subsidies*, where the government gives a lump-sum to both candidates' campaigns. We found that these may increase or decrease small donors' total contributions (See Bouton *et al.* 2018).

will not have a major effect. Analogously, there is no reason to anticipate that it should systematically increase or decrease individual contributions.

On the other hand, a matching subsidy that only applies to contributions below a certain level will generally have an effect.[27] If the aggregate amount of matched contributions (including the matched subsidy) rises, contributions of those above the matching threshold will decrease. The overall impact on the election can then go either way, depending on which candidate has the largest support among those who contribute below the threshold.

Turning to *taxes* on contributions, making them dependent on the size of the contribution acts like a *negative* conditional matching subsidy. Since contributions depend positively on income, this would be like a differential tax on contributions, that is a function of income. Such a tax has the possibility of reducing–or even eliminating–the effect of income on contributions:

Proposition 6 *A tax on small contributions equal to* $\left[\left(y^i\right)^{\theta/2} - 1\right] s_P^i$ *removes the effect of income inequalities from equilibrium small contributions.*

The tax considered in Proposition 6 increases with income in such a way that all donors, rich and poor, eventually face the same marginal cost of contribution. As a consequence, the size of individual contributions depends only on preference intensity (and the features of the electoral environment, such as the closeness of the race).

Though such a tax may seem distant from what is observed in current campaign finance regulations across countries, a regulation broadly mimicking such a policy was actually in place in the U.S. between 1972 and 1986 (Cmar 2005). It is still in line with existing tax laws, for example in the U.S., in the following sense. Suppose campaign contributions were deductible from income tax liabilities (including perhaps a subsidy as in the footnote 27, that is, "negative deductibility"), but where the allowed deduction was a decreasing function of income. In the United States, for example, allowed itemized deductions as a whole fall with income for high income taxpayers, with deductions in specific categories differentially limited by income. Suppose further that an income-adjusted deductibility specifically for political contributions as described in the sentence above were combined with an increase in tax rates overall. The net effect would be a tax on campaign contributions that increases with the size of the contribution. For examples of similar tax

[27] In New York City campaigns, for example, donations up to $175 from New York City residents are matched at a rate of 6:1. In 2013, small donations and matching funds accounted for 71 percent of the individual contributions in the city's elections. See https://nyccfb.info/program/impact-of-public-funds

incentives in various U.S. States, see Cmar (2005) and Magleby *et al.* (2018).

The next question is of course the political feasibility of such a change. Any proposal framed as a tax on contributions that increases with income would have little prospect of being adopted in the U.S. In contrast, deductibility of contributions that gets phased out as income increases should be far more politically viable. For a thorough discussion of the feasibility and implementability of such a federal tax (or tax credit), see Rosenberg (2002) and Cmar (2005).

4 Large Donors

Large contributions are empirically important, so an obvious question is whether the results on small donors are robust to the presence of larger donors. "Large" refers not so much to size, but (because of their size and especially their timing) their known influence –unlike individual small contributions– on other donors and on policy platforms. We therefore now introduce "large" donors who make contributions in an early stage of the campaign. There are therefore two stages in the campaign: in the first, "large" donors make contributions; in the second, "small" donors decide how much to contribute, after observing large contributions. This timing allows large donors to influence the behavior of small donors.

The sequential nature of large and then small donations is also consistent with the observation that collecting a large number of small contributions requires a more developed infrastructure (and hence financing the construction of this infrastructure) than collecting a small number of large contributions. (See, for example, the discussion in Magleby *et al.* 2018). Thus, it is not surprising that even though small contributions play a major role in campaign finance today, candidates tend to tap wealthier donors first. As Magleby *et al.* (2018, p. 248) write, "In fact, almost every campaign in 2008 and 2012 received larger donations earlier and smaller donations later." (See also their figure 8.2 on the declining average donation size as a campaign progresses.) Similarly, EMILY's list recognized at its founding that early money was crucial to a campaign, where successful fund-raising early in the race aids in attracting other donors later on. EMILY is an acronym for Early Money is Like Yeast, *i.e.* it makes the dough rise:[28] early and large contributions allow a campaign to "get off the ground".

[28] https://www.emilyslist.org/pages/entry/our-history

Large donors may be driven by at least two different motives: an electoral motive, similar to small donors, and an influence motive, that is, a desire to affect a candidate's policy platform. Hence, first-stage contributions may affect small donor decisions in two ways: (i) positively, as seed, "campaign-starting," large donations may magnify the effect of small donations; (ii) negatively, if the influence effect moves policy away from the small donors' preferred policy.[29] To disentangle the effects of these two motives, we first analyze them in isolation, before discussing their combined effects. In each case, the focus is mainly on the interactions with small donors.

4.1 Large Donor: Electoral Motive

We formalize the above by considering a "large" donor who must choose her level of contributions R_A to candidate A. Letting y^l denote the income of the large donor, her optimization problem is:

$$\max_{R_A} \pi_A\left(R_A; S_A, S_B\right) \times \Delta - \frac{\left(R_A\right)^2/2}{\left(y^l\right)^{\theta}}, \tag{8}$$

where $\Delta := u_l\left(A\right) - u_l\left(B\right)$ is the large donor's utility differential between the policy of A and that of B. We treat the equivalent contribution R_B as given for simplicity.[30]

This large donor is a Stackelberg leader: she moves before small donors, and will have to take account of the small donors' reaction to her contribution. A priori, we think of early contributions as complementary to late contributions, in particular because they come in the form of building a fundraising infrastructure. But one may instead argue that funds are fungible that early and late contributions are instead perfect substitutes. To capture this range of possibilities, we propose to let the contest success function become:

$$\pi_P\left(Q\right) = \frac{Q_P^{\gamma}}{Q_A^{\gamma} + Q_B^{\gamma}}, \text{ with } Q_P := q\left(R_P, S_P\right). \tag{9}$$

This is essentially the same specification as in Section 3.1, but with aggregated contributions Q_P according to an aggregator function q, with $\partial q/\partial R_P > 0$ and $\partial q/\partial S_P > 0$. This

[29] Another possible negative effect arises if large and small donations are substitutes. We explore this effect in Section 4.3.

[30] We could also study the strategic interactions between this rich donor, and another one who supports B, as in the literature on "lobby competition " (Grossman and Helpman 1994), or consider multiple donors on each side of the political spectrum. However interesting, such interactions have already been studied and integrating all of them into this model would blur our focus, which is on the interactions with small donors and whether it modifies or reinforces previous findings.

specification has the advantage of being highly flexible: the case of perfect substitutes translates into $Q_P = R_P + S_P$. The case of complements translates into an elasticity of substitution below 1.

In this section, we focus on the Cobb-Douglas aggregator function, for two reasons. First, with an elasticity of substitution equal to one, it separates all aggregator functions in which R_P and S_P are complements, from those in which they are substitutes. Second and not least, it produces tractable closed-form solutions for small donors:[31]

$$q\left(R_P, S_P\right) = R_P^{\alpha} S_P^{1-\alpha}. \tag{10}$$

By constrast, the cases of perfect substitutes and of perfect complements do not produce such tractable solutions. We return to the general case in Section 4.3.

4.1.1 Equilibrium Analysis and Campaign Finance Laws

Solving by backward induction, it is easy to show that the equilibrium in small donors contributions, for a given R_A, is:

$$\begin{aligned} \left(S_A^*, S_B^*\right) &= \left(\sqrt{\left(1-\alpha\right)\gamma\pi_A^*\pi_B^*\, W_A}, \sqrt{\left(1-\alpha\right)\gamma\pi_A^*\pi_B^*\, W_B}\right), \\ \text{with} &: \pi_A^* = 1/\left(1 + \left[\left(\frac{R_A}{R_B}\right)^{\alpha}\left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma}\right) = 1 - \pi_B^*. \end{aligned}$$

The problem of the large donor to candidate A then becomes:

$$\max_{R_A} \frac{\Delta}{1 + \left[\left(\frac{R_A}{R_B}\right)^{\alpha}\left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma}} - \frac{R_A^2}{2\left(y^l\right)^{\theta}},$$

and her first order condition:

$$\frac{R_A^2}{\left(y^l\right)^{\theta}} = \alpha\Delta\gamma\pi_A\left(1-\pi_A\right).$$

In other words, her FOC is similar to the one we found for small donors. Hence:[32]

[31] We will see that the small donor problem becomes separable from the large donor problem with the Cobb-Douglas. Section 4.3 shows that when the aggregator function displays strategic substitutabilities (respectively complementarities), the equilibrium level of S_P would be decreasing (resp. increasing) in R_P.

[32] Beyond the Cobb-Douglas case, if R and S become substitutes, then R_A will be lower than in the Cobb-Douglas case, since every additional dollar of contribution by the rich translates into a reduction in

Proposition 7 *When the large donor's contributions are only motivated by the electoral motive, there exists a unique equilibrium* $R_A^* > 0$ *such that:*
(i) R_A^* *is increasing in* Δ *and* y^l*;*
(ii) R_A^* *is decreasing in* $\frac{W_A}{W_B}$ *if and only if* $\frac{R_A^*}{R_B} < \left(\frac{W_B}{W_A}\right)^{\frac{1-\alpha}{2\alpha}}$*, i.e.* $\pi_A^* < 1/2$.

In essence, this proposition shows that the results on small donors are robust to the presence of strategic large donors. For instance, the effects of preference intensity (δ) and income (y^l) are identical to the small donors' case. The effect of $\frac{W_A}{W_B}$ is also similar but with a twist: what matters is the combined effects of $\frac{R_A^*}{R_B}$ and $\frac{W_A}{W_B}$ on election closeness. When $\frac{R_A^*}{R_B} < \left(\frac{W_B}{W_A}\right)^{\frac{1-\alpha}{2\alpha}}$, A is the *overall* runner-up. In this case, an increase in A's advantage among small donors makes the election closer, which stimulates initial contributions by the large donor. The opposite holds when $\frac{R_A^*}{R_B} > \left(\frac{W_B}{W_A}\right)^{\frac{1-\alpha}{2\alpha}}$, in which case A is the overall front-runner. This effect is closely related to the underdog effect for small contributions discussed above. We are not aware of any empirical work exploring this interaction.

From Proposition 7, it is also straightforward that a cap on small contributions could lead to an increase in contributions by the large donor. By Corollary 1, this happens when the cap makes elections closer. Conversely, combining Propositions 1 and 7 shows that capping large donors will impact small contributions, and the direction of the effect will again depend on whether it makes the election closer. For instance, a cap on R_A leads to an increase in small donations (to both candidates) when A has more support among both small and large donors.

4.2 Large Donor: Influence Motive

We now consider a polar opposite assumption where large donors have only an influence motive (and no electoral motive) in order to isolate its effects. To this end, we temporarily assume away the direct effects of R_A on electoral prospects that we just discussed. We reintroduce them in Section 4.3, with the general form of the aggregator function $q(R_P, S_P)$.

contributions by small A donors. Conversely, if the function q displays complementarities between R and S, contributions by the rich A stimulate additional contributions by the small A, which increases the rich's "return on contribution".

4.2.1 Conceptual setup

There is a significant literature on the "trade" of contributions for policy influence (Grossman and Helpman 1994, Prat 2002, Coate 2004a,b, Drazen and Limão 2008, among others). Our interest is not in the mechanism of such a trade *per se* –the papers just mentioned model this– but in how such influence buying may modify the behavior of small donors.

The real-world relevance of such effects turn on two observations: first, the extent to which participants in the electoral process perceive that large contributors "bias" policy away from their bliss point; and, second, the possibility that influence-buying by large donors affects small-donor behavior. On the first, there can be no doubt of this, and it has been widely accepted as a part of existing models of influence buying.[33]

On the second point, candidates rejecting large donations in favor of small ones is a clear trend. For example, in the 2018 US campaign cycle, over 140 candidates campaigning for Congress "pledged to at least reject corporate PAC funding, and a handful of them have rejected all PAC money".[34] Uniformly, they chose to rely on small donations. Note, however, that the argument is more than simply relying on small donations. By rejecting PAC money these candidates wanted to signal that if elected, their policy choices would not be influenced by large donors wanting to buy policy favors. And, crucially, the expectation is that such a signal *would* encourage small donors, as evidence strongly suggests. The discussion in the article referenced in the previous footnote is but one example. Conversely, the effectiveness of the charge by small-donation financed campaigns that opponents have been bought by large donors (for example, in a wine cellar in Napa Valley[35]) provides evidence of a strong reaction by supporters of opposing candidates.

4.2.2 Formalization of the Influence Motive

To formalize such effects, we assume that contributions R_A induce candidate A to modify her plaform by providing policy favors in return. Formally, Δ is now strictly increasing in R_A. For tractability, assume the following functional form:

[33] Becker (1983) is perhaps the first model, albeit in reduced form. Grossman and Helpman (2001), Prat (2002), Coate (2004a,b) and others all stress how special interest group contributions bias policy away from social welfare.

[34] https://eu.usatoday.com/story/news/politics/2018/06/17/more-democrats-limiting-even-rejecting-special-interest-donations/702907002/

[35] https://www.nytimes.com/2019/12/19/us/politics/wine-cave.html

$$\Delta(R_A) = \delta + \phi R_A^{\beta}, \text{ with } \beta \in (0,1), \tag{11}$$

and where $\delta > 0$ captures how much the large donor prefers A over B before any policy favor, and ϕ captures the sensitivity of the utility differential to R_A.

To better understand the sensitivity of candidate platforms –and hence the realized value of Δ– to R_A, think of a "policy possibility frontier" $PPF(\Delta(R_A), v_A(R_A))$ representing the policy distortions due to influence-buying at the expense of the rest of the electorate. Letting $v_A := u_A^i(A) - u_A^i(B)$ be the utility differential for small A-donors of electing A instead of B, this frontier represents the trade-off between this utility differential and policy favors to the large donor. The PPF implies that this differential must weakly fall as R_A buys additional favors: $dv_A/dR_A \leq 0$. By the same token, small B donors dislike A even more when A distorts her policy away from small donors. Hence, the utility differential in favor of B increases weakly: $dv_B/dR_A \geq 0$.

The sensitivity ϕ of the realized utility differential Δ will then depend on the point on the PPF arising from the bargaining over policy favors between the large donor and the candidate. It would reflect strategic behavior of the candidate in balancing the desires of large donors with those of small donors or voters, or the bargaining power of the influence-buying donor (as in Drazen and Limão 2008). For tractability, we assume that ϕ is constant.

Given this section's focus on a pure influence motive, we revert to defining winning probabilities as in (1): $\pi_A = S_A^\gamma/(S_A^\gamma + S_B^\gamma)$. From Proposition 1, it is straightforward to show that:

$$\begin{aligned} \pi_A^*(R_A) &= \left(1 + \left(\frac{S_B^*}{S_A^*}\right)^\gamma\right)^{-1} = \left(1 + \left(\frac{W_B}{W_A}\right)^{\frac{\gamma}{2}}\right)^{-1}, \text{ with } W_P = n_P \bar{Y}_P v_P(R_A) \\ &= \left(1 + \left(\frac{n_B \bar{Y}_B}{n_A \bar{Y}_A}\right)^{\frac{\gamma}{2}} \lambda(R_A)\right)^{-1}, \text{ with } \lambda(R_A) = \left(v_B(R_A)/v_A(R_A)\right)^{\frac{\gamma}{2}}. \end{aligned} \tag{12}$$

The novelty is that v_B/v_A is now increasing in R_A: this connects influence buying by the large donor, and the small donors' stake in the election.

Let $\lambda'(R_A) \equiv d\lambda(R_A)/dR_A\,(> 0)$ denote the first derivative of λ, and focus on the *relative* change in valuations by small donors, *i.e.* $\lambda'(R_A)/\lambda(R_A)$. The higher is this relative change, the "more reactive" small donors are to influence buying. For the sake of interpretability in representing this reaction, we summarize it parametrically as:

$$\bar{\lambda} := \lambda'(R_A)/\lambda(R_A), \ \forall R_A.$$

4.2.3 Equilibrium Analysis

We are now in a position to characterize the equilibrium and prove some comparative statics. Some of these comparative statics are in line with previous results in the literature, while those reflecting the interaction of large and small donors are novel:

Proposition 8 *For π_A^* as defined in* (12)*, there is a unique equilibrium $R_A^* > 0$ such that:*
(i) R_A^* is weakly increasing in ϕ, and y^l;
(ii) R_A^* is weakly decreasing in δ, $\bar{\lambda}$, and $\frac{n_B \bar{Y}_B}{n_A \bar{Y}_A}$.

To understand the comparative statics, it is useful to examine the first order condition associated with (8):

$$\frac{R_A}{(y^l)^\theta} = \pi_A^* \frac{\partial \Delta(R_A)}{\partial R_A} + \frac{d\pi_A^*}{dR_A} \Delta(R_A). \tag{13}$$

The LHS of (13) is the direct marginal cost of contributions, which is weakly decreasing in the large donor's income (y^l). This drives the result that R_A^* is weakly increasing in y^l.

On the RHS of (13), the first term captures the direct benefit of contributing to A: conditional on A winning, a higher contribution translates into a more valuable policy. This term captures the standard prediction for large donors: everything else equal, the more likely one's preferred candidate is to win, the larger is the donor's value of moving her platform in the desired direction.[36] This constrast with the underdog effect for purely electorally-motivated donors. This result finds empirical support in Bonica (2016, figure 2): PACs contribute almost exclusively, and lobbies and Fortune 500 executives largely, to ex post winners. Remember that, by contrast, barely 48 to 55% of small contributions flow to ex post winners (see Section 3.2.2).

However, everything else is *not* equal, and this modifies the behavior of the large donor. This is captured by the second term on the RHS of (13), which identifies a novel *indirect cost of contributing*. It stems from the small donors' adverse reaction to A's targeted favors. Interestingly, this reaction reinforces the "standard" bandwagon effect discussed

[36] In a model with strategic candidates, the value of the parameter ϕ would also be endogenous, and could fall when the candidate is almost sure to win. The effect would then be hump-shaped. Given our focus on donors, we do not explore this potentially interesting effect further.

above: remember that the effect of a small contribution is maximal when the election is close, *i.e.* when π_P^* is close to 1/2. Thus, a given reaction by small donors has a larger electoral impact when the election is close. The larger the impact, the less the large donor contributes in equilibrium.

This indirect cost of R_A affects many of our comparative statics. For instance, it is the sole driver of the effect of δ, *i.e.*, how much the large donor prefers A over B before any policy favor. The higher δ, the *less* such a donor contributes in equilibrium because a defeat of A becomes more costly. She then would moderate, but not eliminate, her favor extraction effort in order not to alienate small donors. The effect of $\bar{\lambda}$, the percentage change in intrinsic support in favor of the opponent by small donors, goes through a similar mechanism. The larger $\bar{\lambda}$, the bigger the electoral pushback resulting from favor buying, hence the negative effect on R_A^*.

The effects of ϕ and $\frac{n_B \bar{Y}_B}{n_A \bar{Y}_A}$ are a combination of the two terms on the RHS of (13). Consider the effect of ϕ, the parameter capturing the effectiveness of the contributions of the influence-buying donor. A higher ϕ means a higher direct return on contributing (higher first term), but also a higher indirect cost of contributing because the small donors' response is also amplified. Proposition 8 shows that the first effect always dominates. For the effect of $\frac{n_B \bar{Y}_B}{n_A \bar{Y}_A}$, which measures the balance in support for the two candidates among small donors, the two terms reinforce one another. A higher $n_B \bar{Y}_B / n_A \bar{Y}_A$ reduces the probability that A wins (smaller benefit of contributing), and increases the effect of the reaction by small donors.

This indirect cost of large contributions is also relevant for Tullock's (1980) "missing money" puzzle. The self-moderation of large donors meant to limit the electoral "backlash" that we identify here may be an additional reason why money ostensibly used to buy influence is not more abundant in electoral politics. Even if there is a significant direct marginal benefit of contributing (as suggested by various empirical estimates, see Avis (2018) and references therein), large donors may limit their contributions because of the indirect cost. This indirect cost also complements the results in Coate (2004): the negative electoral effect of favors to large donors can come from a direct reaction of voters (as in Coate) or through a reaction of small donors who then affect voters (as in our model).

4.2.4 Campaign Finance Laws

What are the effects of campaign contribution caps when we include large donors with a first-mover advantage and only driven by the influence motive? Here we consider the case in which caps on individual contributions constrain small donors only. That is, large donors can find ways to influence policy that are not affected by legal limitations, for example, via lobbying expenditures or contributions to political action committees (PACs). We discuss the case in which caps constrain large donors in the next section.

Suppose that the income distribution is identical for the small A and the small B donors: $f_A(y^g) = f_B(y^g)$ for every income group g. Candidate A draws her advantage from having a larger number of small donors. In that case:

Proposition 9 *Consider a cap on individual contributions by small donors. As the cap starts binding, contributions R_A^* by the large donor increase discontinuously.*

The intuition is that as soon as the cap starts binding on small B donors, the latter can no longer react to R_A by further increasing their own contributions. Thus, although the just-binding cap does not affect the *level* of small contributions, it does reduce the *sensitivity* of small B donors: this produces a discrete drop in $\bar{\lambda}$. Hence, the optimal contribution of the large donor not only rises; it does so by a discrete amount.

This result is stronger than the critique that if large donors can circumvent limitations on contributions, such limitations will not reduce their disproportionate influence. It shows that, once the interactions between large and small donors are taken into account, some caps may have an effect *opposite* to the intended one. They may *increase* influence buying, a possibility that cannot be captured by analyses that ignore large-small interactions.

4.3 Large Donors: Both Motives Combined

These electoral and influence motives can now be combined to get a sense of how the results in Proposition 7 interact with that of Propositions 8–9. We saw that the direction of some of the predictions are reversed between these. Thus, intuitively, while key mechanisms remain valid, eventual predictions are *a priori* less clear-cut.

Formally, let us return to a generic function $q(R_P, S_P)$, as introduced in (9). The equilibrium behavior of small donors can then be *implicitly* determined through the FOC:

$$S_P = \sqrt{W_P \, \gamma\pi_A \left(1-\pi_A\right) \, \eta_{S_P}^{Q_P}}, \quad (14)$$
$$\text{where } \eta_{S_P}^{Q_P} \equiv \frac{\partial Q_P}{\partial S_P}\frac{S_P}{Q_P},$$

where η_X^Y is the elasticity of variable Y with respect to X. The only difference with the FOC in the initial model with small donors only (Section 3.2) is that, there, $\eta_{S_P}^{Q_P}$ was equal to 1. In the Cobb-Douglas case (Section 4.1), it was $1-\alpha$. By contrast, in this generalized setup, $\eta_{S_P}^{Q_P}$ can be larger or smaller than 1, and may vary with the contributions of the large donor. In particular, complementarities would imply that $\eta_{S_A}^{Q_A}$ is increasing in R_A. The case of substitutes would imply that it is decreasing in R_A.

Moving to the first stage of the contribution game, the large donor A must take account of both direct and indirect effects of her contribution:

$$\frac{R_A}{(y^l)^\theta} = \pi_A\frac{\partial\Delta(R_A)}{\partial R_A} + \frac{d\pi_A}{dR_A}\Delta(R_A), \quad (15)$$
$$\text{where} \quad : \quad \frac{d\pi_A}{dR_A} = \frac{\gamma\pi_A\left(1-\pi_A\right)}{R_A}\left[\eta_{R_A}^{Q_A} + \eta_{S_A}^{Q_A}\eta_{R_A}^{S_A} - \eta_{S_B}^{Q_B}\eta_{R_A}^{S_B}\right]. \quad (16)$$

The left-hand side of (15) is, as before, the marginal cost of contributions, which is decreasing in the income of the large donor, y^l, when $\theta > 0$. The first term on the right-hand side captures the direct benefits of buying favors. The second term is the (now direct and indirect) effects of R_A on the probability of winning, given $\Delta\left(R_A\right)$. Turn to (16): the first term between square brackets captures the direct electoral effects of the large donor's contribution. The second and third terms capture the small donors' reactions, mediated through the impact of their contributions on the probability of winning ($\eta_{S_P}^{Q_P}$).

This shows when contributions by the large donor potentially increase or decrease relative support for A. The Cobb-Douglas case implied that the second and third terms added up to zero. If instead the aggregator function q displays complementarities, larger contributions by A would reduce (or possibly reverse) the underdog effect identified for electorally motivated contributions. That is, the second and third terms would add up to a positive amount, inducing higher contributions R_A^* than in the Cobb-Douglas case. The opposite holds in the case of strategic substitutabilities.

Inspection of (15) and (16) leads to additional insights. For instance, note that $\frac{d\pi_A}{dR_A}$ is maximized in $\pi_A = 1/2$, and converges to 0 as $\pi_A \to 1$. Thus, the electoral motive becomes of second order importance for large donors as the election tilts more in favor of A. The influence motive thus gains in relative importance when the election is lopsided. It will also gain in importance when there are caps on individual contributions by small donors. As we have noted before, such a cap reduces–potentially eliminates–the ability of small donors to push back against the favors obtained by large donors.

Further, the general case identifies how a cap on large donors may either help or hurt the electoral prospects of a candidate. Consider a cap specifically on what large donors can give. In a lopsided election, this should mainly reduce the favors granted by A: this was certainly one of the intents of the McCain-Feingold Act.[37] Such a reduction would make A more attractive to small donors, who would therefore increase their relative support for A. If the increase in support from small donors more than offset the loss of support from large ones, the cap would then actually boost A's electoral prospects. By contrast, the same cap is more likely to hurt her electoral prospects when the election is close, since large donors then already moderate their favor extraction efforts, and contributions have a large effect on the probability of election.

5 A Demand-Side Model of the Electoral Motive

One may argue that modeling small donors as highly calculating and perfectly informed actors lacks realism. In particular, small donors may miscalculate the impact of their contribution, or be responding to basic psychological motivations or to their candidates' requests (Mutz 1995, Rogers *et al.* 2017). For example: donors may mechanically react to media attention and/or candidate fund-raising efforts. In this section, we show that our results about small donors' behavior (Section 3.1) are fully consistent with such behavioral motivations. What is more, the amount of effort candidates put into raising funds from small donors suggests that candidates are quite aware of this.

Here, we show that a reasonable functional representation of behavioral responses leads to the same first-order conditions, and hence identical results as in the model with instrumental small donors. Hence, whether individual behavior is driven by rational donors who are instrumentally motivated, or by the strategic behavior of candidates, the implications of electoral considerations for contributions as identified in the previous sections hold.

[37] Many other countries, such as France in 1995, banned contributions by corporations and other legal entities, but not private citizens (Law 95-65 of the 19th of January 1995, modifying Art. 52-8 of the electoral code. See: https://bit.ly/2Um4GCu).

To formalize this point, we assume in this section that small donors mechanically respond to party requests for contributions. Candidates, on their side, need to exert a costly effort in order to induce their supporters to contribute to their campaign. This change in perspective transforms our model into a "demand-side" model in which candidates, who care about winning the election, are the strategic actors, rather than a "supply-side" model in which donors were the strategic actors.

Consider n_P donors of type P, distributed in income classes $y^1 < ... < y^G$ according to the distribution function $F_P\left(y^i\right)$, that satisfies the same assumptions as in Section 3.1. We assume that donor i reacts mechanically to his candidate's (costly) fund-raising effort, denoted e_P^i. His contribution s_P^i is increasing and concave in both e_P^i and y^i. We represent this functionally by:

$$\text{For types } A \quad : \quad s_A^i = \left(\left(y^i\right)^\theta v_A \; e_A^i\right)^{\frac{1}{2}} \tag{17}$$

$$\text{For types } B \quad : \quad s_B^i = \left(\left(y^i\right)^\theta v_B \; e_B^i\right)^{\frac{1}{2}}, \tag{18}$$

where θ parameterizes the donors' elasticity of contributions exactly like in the instrumental model. The Cobb-Douglas specification is chosen both for simplicity and to relate to the main model.

Candidates choose e_P^i to maximize their probability of winning net of the cost of fund-raising (where, for simplicity, we let the cost of soliciting a donor be e_P^i):

$$P \text{ maximizes } \quad : \quad \frac{S_P^\gamma}{S_A^\gamma + S_B^\gamma} - \sum_i e_P^i,$$
$$s.t. \; S_P = \sum_i s_P^i.$$

It follows that:

$$e_P^{i*} = \left(\frac{d\pi_P/dS_P}{2}\right)^2 \left(y^i\right)^\theta v_P.$$

Substituting these equilibrium levels of candidate effort into the donors' contribution functions (17) and (18) yield:

$$s_A^{i*} = \frac{d\pi_A/dS_A}{2}\left(y^i\right)^\theta v_A,$$
$$s_B^{i*} = \frac{d\pi_B/dS_B}{2}\left(y^i\right)^\theta v_B,$$

which is identical (but for the factor $\frac{1}{2}$) to (3) and (4).

In other words, there exists some form of response by behavioral donors and strategic candidates such that the equilibrium level of individual and aggregate contributions are the same as with strategic donors and passive candidates. Hence, although it is a perfectly valid empirical question to ask, "How rational are small donors?", allowing them to be "behaviorally motivated" rather than fully rationally instrumental does not qualitatively change our findings on how electoral motives (here on the part of candidates) influence individual contributions, nor on how economic variables and legal constraints would influence total contributions and the feedback loops between aggregate and individual contributions.

6 Conclusions

Small contributions to political campaigns have become extremely important. Conventional wisdom is that such contributions are a consumption good to the donors. In large part this is a conclusion by default, the basic reasoning being that because each donation is so small relative to total campaign donations, small donors cannot be motivated either by an attempt to buy influence nor by any effect they may have on election outcomes. In this paper, we instead argue that contributions by small donors can be better explained by an electoral motive, either on the part of donors (for instrumental or behavioral reasons), or on the part of candidates.

Our model of small donors predicts patterns of contributions that are in line with a number of empirical findings in the literature, and that contrast with explanations of contributions relying on a simple consumption motive or on an influence motive. There is, for instance, a "closeness" effect in which equilibrium contributions increase when the support for the two candidates is more even, as well as an "underdog effect", whereby equilibrium relative contributions for the advantaged candidate are smaller than their underlying advantage. These are in contrast to a "bandwagon" effect under an influence motive, and no predicted effect in the simple consumption motive. The model also makes novel predictions about the effects of increases in income inequality on campaign contributions and election outcomes depending on the source of inequality.

Our model gives insights into the effects of campaign finance laws. We find that a cap on individual contributions affects all donors, including those not directly hit by the cap. This introduces complications for empirical analyses. The cap generally favors the candidate with the largest number of donors and works against the candidate with the richest donors, but these effects are not necessarily monotonic. Instead, matching subsidies have limited, or no effects. It is possible to eliminate the effects of income inequalities on campaign contributions by implementing an income-contingent tax on contributions.

We also study the interactions between small and large donors. A large donor contributes at an early stage of the campaign, when candidate platforms are potentially still fluid, and before small donors make their decisions. We allow for the donor to be motivated both by a desire to curry favors, that is, an influence motive, and to influence the outcome of the election, that is, an electoral motive. This augmented model produces various additional insights. First, we identify a new indirect cost of contributing that arises specifically from the interactions with small donors. This induces large donors to moderate their contribution, and their request for favors. Second, we find that policy favors should be more prevalent in lopsided elections. Third, due to interactions between small and large donors, cap on contributions can have additional unintended effects. For instance, capping small contributions during the electoral campaign, while not effectively capping large donors may end up boosting the large donors' requests and favor extraction.

We view this paper as a first step to better understanding small political contributions by moving away from the common view that they must be a consumption good for the donors. As discussed in the paper, we believe an electoral motive for such contributions is not only theoretically sensible, it also can better explain several empirical regularities, as well as provide some guidance to further empirical work.

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Appendix

Appendix 1. Proofs for Section 3.2

Lemma 1 *S_P^* is increasing in W_P, $\forall P \in \{A, B\}$. For $S_A^* > S_B^*$, S_A^* is increasing in W_B, whereas S_B^* is decreasing in W_A. For $S_A^* < S_B^*$, S_A^* is decreasing in W_B, and S_B^* is increasing in W_A.*

Proof of Lemma 1. The proof focuses on the case $S_A^* > S_B^*$. By symmetry, the complementary case amounts to a labeling swap between A and B.

From Proposition 1 and the definition of ω, we have:

$$S_A^* = \left(\gamma\omega W_A^{\rho-1}\right)^{\frac{1}{\rho}} \text{ and } S_B^* = \left(\gamma\omega W_B^{\rho-1}\right)^{\frac{1}{\rho}}$$

Taking derivatives and simplifying yields:

$$\frac{\partial S_A^*}{\partial W_A} > 0 \Leftrightarrow \pi_A^* < \frac{1}{2}\left(1 + \frac{\rho}{\gamma}\right) \text{ and } \frac{\partial S_A^*}{\partial W_B} > 0 \Leftrightarrow W_A^{\gamma\frac{\rho-1}{\rho}} > W_B^{\gamma\frac{\rho-1}{\rho}}.$$

The latter is always satisfied. $\frac{\partial S_A^*}{\partial W_A}$ is necessarily positive for $\gamma \leq \rho$. For $\gamma > \rho$, we need to invoke the second order condition for equilibrium existence: we saw that it can be approximated by: $\pi_A^* - \pi_B^* < 1/\gamma$ in the proof of Proposition 1. Substituting for π_B^*, this condition becomes: $\pi_A^* < \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)$. Since $\rho > 1$, condition garantees that $\frac{\partial S_A^*}{\partial W_A} > 0$.

Next,

$$\frac{\partial S_B^*}{\partial W_B} \propto W_A^{\gamma\frac{\rho-1}{\rho}}(\rho+\gamma) + W_B^{\gamma\frac{\rho-1}{\rho}}(\rho-\gamma) \text{ and } \frac{\partial S_B^*}{\partial W_A} \propto W_B^{\gamma\frac{\rho-1}{\rho}} - W_A^{\gamma\frac{\rho-1}{\rho}},$$

where the former is always positive and the latter always negative. ■

Proof of Proposition 1. We are focusing on pure strategies. Even when the pure strategy equilibrium does not exist, there must be a mixed strategy equilibrium (MSE), since payoff functions are continuous and bounded above. We are not interested in such MSE, because they are not realistic in our context: even if a mixed strategy may be reasonable at the individual donor level, total contributions would remain asymptotically deterministic by the law of large numbers.

Differentiating the probability of winning (1) with respect to an individual contribution s_P^i yields:

$$\pi_A' \equiv \frac{\partial \pi_A}{\partial s_A^i} = \frac{\gamma}{S_A}\pi_A(1-\pi_A) = \frac{\gamma}{S_A}\pi_A\,\pi_B \text{ and,} \tag{19}$$

$$\pi_B' \equiv \frac{\gamma}{S_B}\pi_A\,\pi_B. \tag{20}$$

Plugging (19) and (20) into (5), then taking the ratio between S_A and S_B shows that $\frac{S_A}{S_B} = \left(\frac{W_A}{W_B}\right)^{\frac{\rho-1}{\rho}}$ in a pure strategy equilibrium. Substituting for S_B when we solve for the equilibrium value of S_A as a function of the parameters W_A, W_B, and γ, we find that there is a unique solution to that problem:

$$S_A = W_A \times (\pi'_A)^{1/(\rho-1)} = W_A \times \left(\frac{\gamma}{S_A} \times \frac{S_A^\gamma}{S_A^\gamma + S_B^\gamma} \times \frac{S_B^\gamma}{S_A^\gamma + S_B^\gamma}\right)^{1/(\rho-1)}$$

$$= W_A \times \left(\frac{\gamma}{S_A} \times \frac{S_A^\gamma}{S_A^\gamma + \left(S_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma} \times \frac{\left(S_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma}{S_A^\gamma + \left(S_A (W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma}\right)^{\frac{1}{\rho-1}}$$

$$= W_A \times \left(\frac{\gamma}{S_A} \times \frac{\left((W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma}{\left(1 + \left((W_B/W_A)^{\frac{\rho-1}{\rho}}\right)^\gamma\right)^2}\right)^{\frac{1}{\rho-1}} = W_A \times \left(\frac{\gamma}{S_A} \times \omega\right)^{\frac{1}{\rho-1}} = \left(\gamma\omega W_A^{\rho-1}\right)^{\frac{1}{\rho}}.$$

S_B^* is derived following the same steps, and from the fact that $\frac{x^y}{(1+x^y)^2} = \frac{x^{-y}}{(1+x^{-y})^2}$. The latter implies that ω is identical for A and for B.

Second, equilibrium existence of a pure strategy equilibrium depends on the second order conditions being satisfied for this vector of total contributions. After some simplifications, the SOC for type-A donors can be expressed as:

$$-\gamma\frac{\pi_A^*\pi_B^*}{S_A^2}\left(1 + \gamma\left(\pi_A^* - \pi_B^*\right)\right) < (\rho - 1)\frac{\left(s_A^i\right)^{\rho-2}}{\left(y^i\right)^\theta},$$

which is always satisfied since $\pi_A^* \geq \pi_B^*$. A similar condition must hold for B donors:[38]

$$-\gamma\frac{\pi_A^*\pi_B^*}{S_B^2}\left(1 + \gamma\left(\pi_B^* - \pi_A^*\right)\right) < (\rho - 1)\frac{\left(q_B^i\right)^{\rho-2}}{\left(y^i\right)^\theta}. \tag{21}$$

Noting that $\pi_A^*\pi_B^* = \omega$, we can rewrite this condition as follows:

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left(\left(y^i\right)^\theta\ \pi'_B\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^\theta} S_B^2 = (\rho - 1)\frac{\left(\pi'_B\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}(\gamma\omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}}$$

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left(\frac{\gamma\omega}{S_B}\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}(\gamma\omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}}$$

$$\gamma\omega\left(\gamma\left(\pi_A^* - \pi_B^*\right) - 1\right) < (\rho - 1)\frac{\left((\gamma\omega/W_B)^{\frac{\rho-1}{\rho}}\right)^{1-\frac{1}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}(\gamma\omega)^{\frac{2}{\rho}} W_B^{\frac{2(\rho-1)}{\rho}} = (\rho - 1)\frac{\gamma\omega W_B}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}$$

$$(\gamma - 1 \geq)\ \gamma\left(\pi_A^* - \pi_B^*\right) - 1 < (\rho - 1)\frac{\sum n_P f_P\left(y^j\right)\left(y^i\right)^{\frac{\theta}{\rho-1}}}{\left(y^i\right)^{\frac{\theta}{\rho-1}}}(> \rho - 1).$$

[38] Second order condition amounts to looking at different points of the contest function for A and for B donors. Since A donors perceive a higher winning probability than B, their SOC is automatically satisfied: they are in the concave part of the CSF. Instead, B donors may be in a spot in which the CSF is convex. That is, a slight decrease in their contribution base would also decrease their individucal incentives to contribute. For sufficiently high values of γ, this would reinforce the drop in individual incentives so markedly that total contributions may be driven to 0. In that case, there is no pure strategy equilibrium. The proposition shows that this can never happen if γ is no larger than ρ, or –for γ larger– if the contribution bases are not too asymmetric.

This is automatically satisfied for $\rho \geq \gamma$ (since $\pi_A^* - \pi_B^* \leq 1$), and when $\pi_A^* - \pi_B^* \leq 1/\gamma$ for any other value of ρ and γ. ■

Proof of Proposition 2. Remember that $W_P \equiv (v_P)\ n_P\ \sum_{i=1}^{G} f_P\left(y^i\right) \times \left(y^i\right)^\theta$. A mean-preserving spread of the income distribution is such that $\sum_{i<\bar{y}_P} \Delta f_P\left(y^i\right) \times y^i = -\sum_{i>\bar{y}_P} \Delta f_P\left(y^i\right) \times y^i$, where $\bar{y}_P$ is the subgroup with mean income in group P, and $\Delta f_P\left(y^i\right)$ is the change in density of each income class. If and only if $\theta > 1$, this implies that $\left|\sum_{i<\bar{y}_P} \Delta f_P\left(y^i\right) \times \left(y^i\right)^\theta\right| < \left|\sum_{i>\bar{y}_P} \Delta f_P\left(y^i\right) \times \left(y^i\right)^\theta\right|$ and hence that W_P increases. Applying the proof of Lemma 1 in the appendix then demonstrates the result. ■

Proof of Proposition 3. First, we show that, for any given level of total contributions S_A and S_B, the marginal return of contributing to A is larger than that of contributing to B iff $S_A < S_B$:

$$\begin{aligned}
\frac{\partial \pi_A}{\partial S_A} &= -\pi_A^2 (-\gamma) (S_A/S_B)^{-\gamma-1} (1-\alpha) S_A/S_B \frac{1}{S_A} \\
&= \gamma \pi_A^2 \left((S_A/S_B)^{1-\alpha}\right)^{-\gamma} (1-\alpha) \frac{1}{S_A} \\
&= \gamma \pi_A \pi_B (1-\alpha) \frac{1}{S_A} \\
\frac{\partial \pi_B}{\partial S_B} &= \frac{\gamma \pi_A \pi_B}{S_B},
\end{aligned}$$

and hence $\frac{\partial \pi_A}{\partial S_A} > \frac{\partial \pi_B}{\partial S_B}$ iff $S_A < S_B$.

Next, remember that $y^i \in \left[\underline{y}, \bar{y}\right]$ with $\underline{y} > 0$ and $\bar{y}$ positive and finite, and that we are still focusing on the case $\rho = 2$. In that case, there exist two cutoffs s_0 and s_1 for the cap on individual contributions $\bar{s}$, such that: $\forall \bar{s} > s_1$, no small donor is constrained and $\forall \bar{s} < s_0$ all small donors are constrained. By Proposition 1, for $\bar{s} > s_1$, the ratio of total small contributions must be:

$$\frac{S_A^*}{S_B^*} = \left(\frac{W_A}{W_B}\right)^{\frac{1}{2}} = \left(\frac{n_A}{n_B}\right)^{\frac{1}{2}},$$

and winning probabilities are the ones in Proposition 1:

$$\pi_A^* = 1/\left(1 + \left[\left(\frac{n_A}{n_B}\right)^{\frac{1}{2}}\right]^{-\gamma}\right).$$

For $\bar{s} < s_0$, all small donors contribute $\bar{s}$. Therefore, $S_A = n_A \bar{s}$ and $S_B = n_B \bar{s}$. The contribution ratio is then $\frac{n_A}{n_B}$, and it is immediate to derive that A's winning probability is then

$$\pi_A^0 = 1/\left(1 + \left[\frac{n_A}{n_B}\right]^{-\gamma}\right).$$

For $\bar{s} \in (s_0, s_1)$, S_A must always be strictly larger than S_B, otherwise, individual best responses would be such that $s_A^*\left(y^i\right) \geq s_B^*\left(y^i\right)$, $\forall y^i$, which would in turn imply $S_A > S_B$, a contradiction

If follows that:

(1) there is a (possibly empty) set of income levels y^i such that neither A nor B-donors are capped: $s_A^i < s_B^i$
(2) there is a non-empty set of income levels y^i such that A-donors are uncapped and B-donors are capped: $s_A^i < s_B^i = \bar{s}$
(3) there is a (possibly empty) set of income levels y^i such that both A and B-donors are capped, $s_A^i = s_B^i = \bar{s}$.

Parts (1) and (2) imply that $\pi_A(\bar{s})$ must be strictly less than π_A^0. The fact that proportionately more B-donors than A-donors are capped when $\bar{s} > s_0$ implies that their joint contribution capacity is reduced more than A's. This amounts to letting W_B drop because of a reduction in top B incomes. Following Proposition 1, this increases $\pi_A(\bar{s})$ above π_A^*. The proof of non-monotonicity is provided by the example in the main text. ■

Proof of Proposition 4. Define $y_A^i = k y_B^i \ \forall i = 1, ..., G$, and order income groups such that $y_P^i < y_P^{i+1}$. Remember that, for any two donors i and j who support the same candidate and are unconstrained by the cap, we must have: $s_P\left(y_P^i\right)/s_P\left(y_P^j\right) = \left(y_P^i/y_P^j\right)^\theta$. The equilibrium is thus fully characterized by two income cutoff levels $\bar{y}_A(\bar{s})$ and $\bar{y}_B(\bar{s})$ and two "lowest contribution levels" $s_A\left(y_A^1\right)$ and $s_B\left(y_B^1\right)$ such that:

$$\begin{aligned} \text{for } y_P^i &< \bar{y}_P(\bar{s}), \ s_P\left(y_P^i\right) = s_P\left(y_P^1\right)\left(y_P^i/y_P^1\right)^\theta, \\ \text{for } y_P^i &> \bar{y}_P(\bar{s}), \ s_P\left(y_P^i\right) = \bar{s}. \end{aligned}$$

First, we show that $s_A\left(y_A^i\right) > s_B\left(y_B^i\right)$ for all unconstrained donors of some income group i, and hence that more A- than B-donors will be constrained. To prove this, note that a necessary condition for the fraction of constrained A-donors to be smaller than that of B-donors is to have $\bar{y}_A(\bar{s}) > k\bar{y}_B(\bar{s})$. This would require that $s_B(\bar{y}_B(\bar{s})) > s_A(k\bar{y}_B(\bar{s})) = k^\theta s_A(\bar{y}_B(\bar{s}))$, and hence $s_B\left(y^i\right) > k^\theta s_A\left(y^i\right)$ for any $y^i < \bar{y}_B(\bar{s})$. But this leads to a contradiction: such contributions would aggregate into $Q_A(\bar{s}) < Q_B(\bar{s})$, which would produce best-response contributions $s_B(\bar{y}_B(\bar{s})) < s_A(k\bar{y}_B(\bar{s}))$, because of free riding.

This establishes that $s_A\left(y_A^i\right) \geq s_B\left(y_B^i\right)$ for all $i = 1, ..., G$, and the inequality must be strict for some i. Then, following the same steps as for the proof of Proposition 3 leads to Proposition 4. ■

Proof of Proposition 5. First, we note that (7) can be rewritten as:

$$\tilde{S}_P = (1+m) S_P.$$

Plugging that into candidate A's probability of winning, we get:

$$\begin{aligned} \pi_A\left(\tilde{\mathbf{S}}\right) &= \left(1 + \left[\frac{(1+m) S_A}{(1+m) S_B}\right]^{-\gamma}\right)^{-1}, \\ &= \left(1 + \left[\frac{S_A}{S_B}\right]^{-\gamma}\right)^{-1} = \pi_A(\mathbf{S}) \end{aligned}$$

As a consequence, incentives, and therefore the equilibrium, are the same for any $m \lesseqgtr 0$. ■

Proof of Proposition 6. With this tax, the cost of contributing s_P^i for a donor with income y^i becomes:

$$\left(s_P^i + \left[\left(y^i\right)^{\theta/2} - 1\right] s_P^i\right)^2 / \left[\left(y^i\right)^{\theta} 2\right] = \left(s_P^i\right)^2 /2,$$

which is independent of y^i. ■

Appendix 2. Proofs for Section 4

Proof of Proposition 8. Plugging (11) and (12) in the FOC (13), and rearranging, we obtain:

$$\phi\beta R_A^{\beta-1} - \frac{\sqrt{\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}}^{\gamma}\lambda\left(R_A\right)}{1+\sqrt{\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}}^{\gamma}\lambda\left(R_A\right)}\bar{\lambda}\left[\delta + \phi R_A^{\beta}\right] - \frac{R_A}{\left(y^l\right)^{\theta}}\left(1+\sqrt{\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}}^{\gamma}\lambda\left(R_A\right)\right) = 0. \qquad (22)$$

R_A^* is such that this condition is satisfied and the LHS is locally strictly decreasing in R_A. There is at least one solution to (22) because the LHS goes to ∞ when $R_A \to 0$, and goes to $-\infty$ when $R_A \to \infty$. The equilibrium is generically unique because, even if there were multiple solutions to (22), the large donor would pick the global maximum.

Comparative statics follow. First, R_A^* is strictly decreasing in δ: following an increase in δ, the second term of (22) becomes more negative. Thus the LHS must increase to restore equality, which requires R_A^* to decrease. The proof follows the same steps for $\bar{\lambda}$, $\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}$, and y^l.

Second, R_A^* is strictly increasing in ϕ. To prove this, note that (22) implies:

$$\phi\beta R_A^{\beta-1} - \frac{\sqrt{\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}}^{\gamma}\lambda\left(R_A\right)}{1+\sqrt{\frac{n_B\bar{Y}_B}{n_A\bar{Y}_A}}^{\gamma}\lambda\left(R_A\right)}\bar{\lambda}\phi R_A^{\beta} > 0,$$

since the LHS of this expression amounts to adding strictly positive terms to (22). Hence, an increase in ϕ must increase the LHS of (22), implying that the LHS must decrease to restore equality. This requires R_A^* to increase. ■

Proof of Proposition 9. Start from the equilibrium vector of contributions: $\{R_A^*, S_A^*, S_B^*\}$, with $S_B^* = \sum_g n_B^g s_B^*\left(y^g; R_A\right)$. Since $S_B^* < S_A^*$, we have that $s_B^*\left(y^g\right) > s_A^*\left(y^g\right)$, $\forall g$. The sum of these contributions determine the winning probability:

$$\pi_A^* = \left(1 + \left(S_B^*/S_A^*\right)^{\gamma}\right)^{-1} \;\; (> 1/2).$$

In the absence of a cap on individual contributions, the response of these S_P^* with respect to R_A is:

$$\frac{\partial S_P^*}{\partial R_A} = \sum_g n_P^g \frac{\partial s_P^*\left(y^g; R_A\right)}{\partial R_A},$$

with $\frac{\partial s_B^*\left(y^g;R_A\right)}{\partial R_A} > 0$. In the proof of Proposition 8, we found that the stronger these responses, the less the large donor to A contributes in equilibrium.

Now, set the cap on individual contributions at $\bar{s} = s_B^*\left(y^G\right)$, such that, at $R_A = R_A^*$, this cap does not modify small donor contributions. Yet, n_B^G donors now have a response $\partial s_B\left(y^G\right)/\partial R_A = 0$ for any $R_A > R_A^*$. Hence $\frac{\partial S_B^*}{\partial R_A}$ drops by a discrete amount, proportional to n_B^G. This reduces the marginal cost of R_A by the same discrete amount (this corresponds to a discrete drop in $\bar{\lambda}$ in (22)). Accordingly, R_A^* must increase discontinuously. ■

Proof of Proposition 7. The FOC of the large donor to A is:

$$R_A^2 = \alpha\gamma\delta\left(y^l\right)^\theta \frac{\left[\left(\frac{R_A}{R_B}\right)^\alpha \left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma}}{\left(1+\left[\left(\frac{R_A}{R_B}\right)^\alpha \left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma}\right)^2}. \tag{23}$$

Note that, when R_A tends to 0, the LHS tend to 0 and the RHS tends to infinity. When R_A tends to infinity, the LHS tends to infinity and the RHS tends to 0. This directly implies that $R_A^* > 0$.

After some simple but tedious manipulation, (23) becomes

$$\alpha\delta\left(y^l\right)^\theta \gamma\left[\left(\frac{1}{R_B}\right)^\alpha \left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma} = R_A^2\left(R_A^{\alpha\gamma} + \left[\left(\frac{1}{R_B}\right)^\alpha \left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}\right]^{-\gamma}\right)^2.$$

Note that the LHS does not depend on R_A and the RHS increases in R_A. Hence, R_A^* is strictly increasing in δ and $\left(y^l\right)^\theta$

We now want to determine the effect of $W := \left(\frac{W_A}{W_B}\right)^{\frac{1-\alpha}{2}}$ on $R := \frac{R_A}{R_B}$. Let's rewrite (23) with this notation:

$$R^2 = \frac{\alpha\gamma\delta\left(y^l\right)^\theta}{R_B^2} \frac{\left[R^\alpha W\right]^{-\gamma}}{\left(1+\left[R^\alpha W\right]^{-\gamma}\right)^2}.$$

which boils down to

$$R = \sqrt{\frac{\alpha\gamma\delta\left(y^l\right)^\theta}{R_B^2} \frac{\left[R^\alpha W\right]^{-\gamma}}{\left(1+\left[R^\alpha W\right]^{-\gamma}\right)^2}}.$$

Differentiating both sides with respect to W obtains:

$$\frac{\partial R}{\partial W} = -\frac{1}{2}\frac{\frac{\alpha\gamma\delta\left(y^l\right)^\theta}{R_B^2}\frac{\gamma}{(W(R)^\alpha)^{\gamma+1}}\frac{(R)^\alpha + \alpha W(R)^{\alpha-1}R'}{\left(\frac{1}{(W(R)^\alpha)^\gamma}+1\right)^2} - 2\frac{\alpha\gamma\delta\left(y^l\right)^\theta}{R_B^2}\frac{\gamma}{(W(R)^\alpha)^\gamma(W(R)^\alpha)^{\gamma+1}}\frac{(R)^\alpha + \alpha W(R)^{\alpha-1}R'}{\left(\frac{1}{(W(R)^\alpha)^\gamma}+1\right)^3}}{\sqrt{\frac{\frac{\alpha\gamma\delta\left(y^l\right)^\theta}{R_B^2}}{(W(R)^\alpha)^\gamma\left(\frac{1}{(W(R)^\alpha)^\gamma}+1\right)^2}}}$$

which boils down to

$$\frac{\partial R}{\partial W} = \frac{\gamma}{2}\frac{R}{W}\frac{1-(WR^\alpha)^{-\gamma}}{1+\frac{\gamma\alpha}{2}+(WR^\alpha)^{-\gamma}\left(1-\frac{\gamma\alpha}{2}\right)}$$

Note that $1 - \frac{\gamma\alpha}{2} > 0$ because $\gamma \leq 2$ and $\alpha < 1$. The sign of $\frac{\partial R}{\partial W}$ is thus the same as that of

$1-(WR^{\alpha})^{-\gamma}$. Hence, R_A^* is decreasing in W if and only if

$$1<(WR^{\alpha})^{-\gamma},$$

which boils down to

$$\frac{R_A^*}{R_B}<\left(\frac{W_B}{W_A}\right)^{\frac{1-\alpha}{2\alpha}} \text{ or } \pi_A^*<1/2.$$

■