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Abstract

We propose an axiomatic approach to characterize normative criteria for the evaluation of lifetime income distributions according to the opportunity egalitarian perspective (Roemer, 1998). In a setting in which both individual incomes and predetermined circumstances are variable over time, we adopt a norm-based approach to the measurement of inequality, and propose two different benchmark distributions, referring respectively to the ex ante and the ex post versions of equality of opportunity. We first aggregate over time, thereby characterizing measures of interetemporal individual inequality of opportunity, and then aggregate the individual measures into a societal measure. Our individual measure results to be a weighted average of individuals' opportunity gap experienced in each period. Our aggregate measure is an average of a concave transformation of the individual intertemporal opportunity gap and can be interpreted as an intertemporal inequality of opportunity index. We apply our framework to evaluate the Korean distribution of income from an intertemporal and opportunity egalitarian perspective.

Keywords: lifetime income inequality; intertemporal distributions; equality of opportunity; opportunity gap; Korea.

JEL codes: D31, D63, I32, J62.

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1 Introduction

1.1 Motivation

Understanding the origins of inequality and its trend over time has become the core of an increasing number of contributions in different branches of the economic literature. At the same time these themes have also climbed on top of the policy agenda, especially after the 2007 financial crisis followed by the 2011 sovereign debts crisis. Economists and policymakers have started to investigate the role of inequality in determining such macroeconomic events as well as the distributive consequences of these events. And now there is even a more urgent need to assess the distributive implications of the COVID-19 pandemic, which may have exacerbated existing disparities and opened new distributional tensions.

One approach to analyze the roots of economic inequality that has proved to be particularly successful in recent years is the equality of opportunity (EOp) approach, according to which one should distinguish between outcome inequalities that are due to factors beyond the individual control, such as, for example, gender, social origin, colour of skin, and outcome inequalities due to factors which lie within the sphere of individual responsibility and control. The equality of opportunity theory postulates that the former inequalities (due to so-called circumstances) are unfair and should be eliminated as much as possible, while the latter inequalities (due to individual effort or responsibility) should be considered acceptable. Beyond theoretical reasoning proposed by prominent political philosophers such as Rawls (1971), Sen (1985), Arneson (1989), Cohen (1989) and Dworkin (1981a,b), the equality of opportunity approach rests on some compelling empirical evidence that people indeed disapprove inequalities that are rooted in factors beyond individual control: elicited preferences for redistribution show that individuals are more willing to accept income differences which are due to effort rather than exogenous circumstances (Fong, 2001; Cappelen *et al.*, 2007, 2013; Alesina *et al.*, 2017). Moreover, the distinction between effort-based and circumstances-based inequalities may offer a solid argument against the view that defends existing outcome inequalities as a necessary price to pay for incentivize individuals in a market economy.

Along this line of reasoning, recent contributions in the literature have proposed to distinguish between 'good' and 'bad' inequalities (see Aiyar & Ebeke (2018); Ferreira & Özler (2018); Marrero & Rodríguez (2013)). The idea is that income differences that arise from differential rewards to effort might be efficiency enhancing and associated with faster economic growth; while other kinds of differences, arising from unequal opportunities associated to predetermined circumstances, might be detrimental to growth.

Mainly inspired by the philosophical debate on the responsibility sensitive egalitarian justice, Roemer (1993, 1998) and Fleurbaey (1994, 2008) have proposed formal economic models of equality of opportunity. Theoretically, EOp is composed by two independent principles: first, people should be compensated for unequal circumstances (compensation principle). A prominent formulation of this principle (ex ante compensation) postulates that the value of opportunity sets should be equalized across people with different circumstances. The principle is ex ante in the sense that opportunity sets are evaluated before the individual level of effort is revealed. An alternative expression (ex post compensation) starts from the identification of individual effort and requires that individuals with the same effort should obtain the same outcomes, regardless of circumstances. Hence, while the ex post approach focuses on the inequality between achievements of individuals exerting the same level of effort, the ex ante approach focuses on the inequality between individual opportunity sets. The second principle inherent to the concept of EOp is that individuals should be rewarded for differential efforts (reward principle). While there are different formulations of this idea, one prominent version is the principle of utilitarian reward, stating that inequalities in outcomes among individuals with the same circumstances are a matter of indifference. Following the seminal contributions by Van De Gaer (1993), Roemer (1993) and Fleurbaey (1994) there is now a rich theoretical and empirical literature that has proposed different approaches and methodologies to measure the degree of inequality of opportunity (IOp): see Ferreira & Peragine (2016) and Roemer & Trannoy (2015) for recent surveys. However, most of existing contributions, both in the theoretical and the empirical literature, are set up in a static context and propose cross-sectional and unitemporal measures. In other words, the existing literature does not discuss dynamics of IOp.[1] On the other hand, the time dimension is crucial when making individual and social welfare evaluations. In fact, over the last decades, increasing discontent has been expressed with distributional analysis based on observations of income for a single period (year). The line of reasoning is that high annual inequality might occur side by side with little or no inequality in long-term incomes, if individuals' positions in the annual income distributions change over time. Moreover, income fluctuations over time may affect the individual welfare 'per se', and hence need to be accounted for when making welfare comparisons. This has led to a spur of research on inequality and social welfare in long-term income according to the traditional Equality of Outcome view (Bourguignon *et al.*, 2007; Piketty & Saez, 2014; Aaberge & Mogstad, 2015; Aaberge *et al.*, 2020).

The present paper fills a gap in the existing literature: we propose a framework for the measurement of inequality of opportunity which is able to account for the individuals' income streams, thereby introducing a lifetime or intertemporal perspective in the equality of opportunity literature. Our framework includes the possibility that both circumstances and effort change over time. This assumption is novel in the literature, which is instead usually concerned with circumstances at birth, hence fixed over time, and makes our framework truly intertemporal.

1.2 Methodology

We propose an axiomatic methodology for the characterization of our social rankings. Moreover, we adopt the norm-based approach (Cowell, 1985; Magdalou & Nock, 2011), according to which a measure of inequality is derived by looking at the distance between the actual distribution and a benchmark (or norm) distribution. We propose two different benchmark distributions, which refer respectively to the ex ante and the ex post versions of equality of opportunity. Once a norm distribution is defined, our strategy follows a two-step procedure. In the first step we derive a measure of intertemporal inequality of opportunity at the individual level, where the individual inequality of opportunity in each period is defined as a

[1] Two exceptions to this are Aaberge *et al.* (2011) and Roemer & Ünveren (2017) that will be discussed in Section 1.3.

function of the distance between the observed and the norm individual incomes. Then, for each individual, we aggregate this measure across time. In doing so, this framework allows for the possibility that both circumstances and effort change over time.

In the second step, we aggregate the individual evaluations and derive a measure of intertemporal inequality of opportunity at a societal level, which turns out to be equivalent to the average of a concave transformation of the individual intertemporal measure. We show that, under particular conditions, our measure corresponds to (the negative equivalent of) the average across time of the mean logarithmic deviation of the time specific distributions.

In addition to characterizing families of intertemporal indexes, we also propose an intertemporal opportunity version of the generalized Lorenz partial ordering, which provides suitable dominance conditions that can be used for robust social comparisons.

Finally, we provide an empirical application of the measurement tools characterized in the paper by analysing the Korean distribution of incomes from an intertemporal and opportunity egalitarian perspective. We use the KLIPS (Korean Labor and Income Panel Study), from 2001 to 2014, a rich but still very unexplored dataset. Our paper provides the first analysis of equality of opportunity for income in Korea.[2] We show that although South Korea is known as one of the most growing and progressive countries, it still suffers from some degree of unfairness. However, the country seems to be on the right path for improving equality of opportunity over time. Indeed, South Korea fared well in dealing with the global financial crisis since, over time, the country did not worsen intertemporal inequality of opportunity. This trend is clear when implementing the proposed intertemporal approach and results to be less neat when looking at each single year. Moreover, the intense South Korea's GDP growth results to have been opportunity inequality improving for the new generations which receive a fairer remuneration of effort.

[2]Lee & Cho (2017) is the only other contribution that we are aware of that analyzes inequality of opportunity in Korea, but with a focus on education and wages.

1.3 Relation to the literature

Our paper is especially related to Aaberge *et al.* (2011), who propose an evaluation of lifetime income distributions from an opportunity egalitarian perspective. Their approach is based on long-term incomes, in the spirit of the *permanent income hypothesis à la Milton Friedman*: their first step consists of aggregating the income stream of each individual into an interpersonal comparable measure of permanent income. To this end, they draw on intertemporal choice theory and use a measure of permanent income which incorporates the costs of and constraints on making inter-period income transfers. Once a distribution of permanent incomes is obtained, they apply a rank dependent approach (Yaari (1988)) to derive measures of opportunity inequality and social welfare. We depart from Aaberge *et al.* (2011) in two respects: first, we allow for time varying circumstances, while they assume fixed circumstances over time; second, following Fleurbaey & Schokkaert (2009), we implement a fairness gap approach to the measurement of inequality of opportunity, which allows us to characterize axiomatically individual, in addition to social, measures of inequality of opportunity. Our work is also related to Almås *et al.* (2011), who propose a criterion to rank distributions according to EOp that builds upon the concept of individual fairness gap. However, they employ a static framework, i.e., they do not account for the time dimension, and they only consider the ex post approach to EOp. Moreover, they do not provide an axiomatic characterization of the proposed measure.

A recent application of EOp in a dynamic setting has been proposed by Roemer & Ünveren (2017). They show the impact of the educational system on the long run effect of educational policy aimed at removing IOp. While Roemer & Ünveren (2017) assess EOp in a stationary state, which can be considered as the end of a (long) time span, we suggest to assess opportunity inequality in between the beginning and the end of this period, in order to give more weight to the history of each individual.

The paper is organised as follows. Section 2 introduces the preliminary notation. Section 3 defines the norm distributions. Section 4 characterizes a measure of individual and societal intertemporal inequality of opportunity. Section 5 develops the empirical application and Section 6 concludes.

2 Preliminaries

We observe for $T \in \mathbb{N}_{++}$ periods a population of $N \in \mathbb{N}_{++}$ individuals. The income level of an individual $i \in \{1, \ldots, N\}$ at time $t \in \{1, \ldots, T\}$ is denoted $x_{it} \in \mathbb{R}_+$ and is assumed to be an increasing function of his circumstances and effort at time t. Hence, an individual i with circumstances $c_{j,t} \in \{c_{1,t}, \ldots, c_{n,t}\} = \mathcal{C}$ and effort $e_{k,t} \in \{e_{1,t}, \ldots, e_{m,t}\} = \mathcal{E}$ at time t will have income $x_{it} = f_t(c_{j,t}, e_{k,t})$.

We do not impose any structure on the elements of $\mathcal{C}$ and $\mathcal{E}$; they can contain numbers, vectors or sets: this has no impact on our framework. However, to simplify the exposition, we assume that each combination of circumstances and effort $(c_{j,t}, e_{k,t})$ occurs only once for each time period and that circumstances ad effort sets are countable with fixed cardinality so that $|\mathcal{C}| = n$, $|\mathcal{E}| = m$ and $nm = N$.

Let us shorten the notation by denoting with $x_{jk,t}$ the income of an individual with circumstances $c_{j,t} \in \mathcal{C}$ and effort $e_{k,t} \in \mathcal{E}$ at time t. Under these assumptions, for all time periods t, we can write the unitemporal income distribution $X_t \in \mathbb{R}_+^{n \times m}$, in the following matrix form.

$$X_t = \begin{bmatrix} x_{11,t} & \cdots & x_{1k,t} & \cdots & x_{1m,t} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{j1,t} & \cdots & x_{jk,t} & \cdots & x_{jm,t} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{n1,t} & \cdots & x_{nk,t} & \cdots & x_{nm,t} \end{bmatrix}$$

This matrix describes the income distribution in the population in a period t. Each row $(\mathbf{x}_{j\cdot,t})$ represents the income distribution of individuals sharing the same circumstance $c_{j,t}$ (called *type* j *of* t), while each column $(\mathbf{x}_{\cdot k,t})$ contains the income of individuals exerting the same level of effort $e_{k,t}$ (called *tranche* k *of* t).

Notice that time can change the way circumstances (and effort) influence outcomes; consider, for example, the impact of gender on expected income which, although relevant, seems to be lower today with respect to fifty years ago. This positive aspect of the reality is captured by our time-dependent income functions f_t, $t \in$

$\{1, \ldots, T\}$. Other ways of defining the income generating process are possible,[3] but their implementation would push our normative assessment toward other directions that we consider out of the scope of this paper.

The lifetime or intertemporal income distribution is represented by a matrix $X \in \mathbb{R}_+^{N\times T}$ such that each column $(\mathbf{x}_{\cdot t})$ represents a unitemporal income distribution, each row $(\mathbf{x}_{i\cdot})$ is the intertemporal income stream of a single individual and the element $x_{it} \in X$ is the income of agent i at time t.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1t} & \cdots & x_{1T} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{it} & \cdots & x_{iT} \\ \vdots & & \vdots & & \vdots \\ x_{N1} & \cdots & x_{Nt} & \cdots & x_{NT} \end{bmatrix}$$

The matrix representing the intertemporal income distribution is therefore obtained by listing the elements of the unitemporal income distributions $\left(X_t \in \mathbb{R}_+^{n\times m}\right)$ defined above, in such a way that each column t of X contains all the elements of X_t, and in each row of X we have the income levels of a unique individual across time.[4] Therefore, $x_{it} \in X$ is the income of individual i at time t which, assuming she has circumstances $c_{j,t}$ and effort $e_{k,t}$, corresponds to the element $x_{jk,t}$ in X_t.

Let, then, $\tilde{X} \in \mathbb{R}_+^{N\times T}$ denote the norm or benchmark distribution such that, for all $\tilde{x}_{it} \in \tilde{X}$, $\tilde{x}_{it}$ is the norm income level that an individual i at time t would enjoy if the society were to fully achieve equality of opportunity. As for the intertemporal income distribution, each column of $\tilde{X}$ can be rewritten as a matrix $\tilde{X}_t \in \mathbb{R}_+^{n\times m}$ whose element $\tilde{x}_{jk,t}$ is the norm income of an individual with circumstance $c_{j,t}$ and effort $e_{k,t}$.

[3] Alternatively, we could write f_t as function of the entire history of the individual up to t with contemporary and previous efforts and circumstances influencing the income at each period. From a mathematical point of view, this is an harmless modification of our framework. However, this would generate normative concerns on the way we should consider past circumstances and effort.

[4] Let us denote $Vec(X_t)$ the vectorization of the matrix X_t, i.e. a linear transformation which converts X_t into a N-dimensional column vector. Then, each column t of X is a permutation of $Vec(X_t)$. Such permutation guarantees that in each row of X we have the income stream of a unique individual.

The norm-based approach, roughly speaking, requires to evaluate a given distribution on the basis of a measure of the distance between observed incomes and norm incomes[5] (see Cowell (1985), Fleurbaey & Schokkaert (2009) and Magdalou & Nock (2011)). In what follows, we first discuss how to define a norm distribution according to the opportunity egalitarian perspective; then, we proceed with the derivation of a measure of intertemporal inequality of opportunity.

3 Norm distributions

Let us call $\mathcal{X}$ the set of all the possible norm distributions that we can assign to $X \in \mathbb{R}_+^{N\times T}$:

$$\mathcal{X} = \left\{ \tilde{X} : \tilde{X} \in \mathbb{R}_+^{N\times T} \right\}$$

Then, $\mathcal{X}$ is the set of all $N \times T$ non-negative real matrices. Absent any normative principle, $\tilde{x}_{it} \in \tilde{X} \in \mathcal{X}$ is the norm income of an individual $i \in \{1, \dots, N\}$ at time $t \in \{1, \dots, T\}$.

Let us recall that $x_{jk,t}$ and $\tilde{x}_{jk,t}$ denote, respectively, the actual and norm income of an individual with circumstances $c_{j,t} \in \mathcal{C}$ and effort $e_{k,t} \in \mathcal{E}$. Moreover, to shorten the notation, let us impose $j, h \in \{1, \dots, n\}$ and $k, r \in \{1, \dots, m\}$ in this section.

Consider the following restrictions on the possible norm distributions. Two requirements we can impose to a norm distribution are feasibility and efficiency. While the former constraint ensures that the total income in the society is sufficient to achieve the norm, the latter ensures that in the redistribution process no resources are wasted. Under these restrictions, each unitemporal norm distribution will be obtained as a result of a redistribution of the total income at the considered period. We call $\mathcal{X}_F$ the subset of distributions where feasibility and efficiency are satisfied:

[5]In the present paper we will use interchangeably the terms ‘norm’, ‘optimal’ and ‘fair’ distribution or individual income, as opposed to actual distribution or individual income.

$$\mathcal{X}_F = \left\{ \mathcal{X} : \sum_{j,k=1}^{n,m} \tilde{x}_{jk,t} = \sum_{j,k=1}^{n,m} x_{jk,t}, \ \forall t \right\} \tag{1}$$

In order to identify a norm distribution, the first opportunity egalitarian principle we introduce is *ex post compensation.* This principle calls for *some* degree of aversion to income inequality among individuals with the same effort, i.e., inequality aversion within tranches. Here we interpret this principle in a strong way, by requiring that individuals exerting the same level of effort should receive the same income. Hence we can restrict $\mathcal{X}_F$ to the set of norms that satisfy ex post compensation:

$$\mathcal{X}_{Ex-P} = \{ \mathcal{X}_F : \tilde{x}_{jk,t} = \tilde{x}_{hk,t}, \ \forall j, h, k, t \}$$

The second principle to consider is *reward*, which is concerned with the respect of income differences among individuals with the same circumstances. One way of expressing this principle consists in excluding the possibility of within type redistributions; i.e., prohibiting income transfers between individuals with the same circumstances. This reward principle, which we call *no within type redistribution* (NWTR), allows us to further restrict the set $\mathcal{X}_{Ex-P}$. Indeed, absent the possibility of redistributing within types, the only way of realizing ex post compensation is via within tranches redistribution. This implies that (under feasibility and efficiency) the optimal income of an individual has to coincide with the average income of his tranche. The following subset of $\mathcal{X}_{Ex-P}$ satisfies this requirement:

$$\mathcal{X}_{Ex-P}^{NWTR} = \left\{ \mathcal{X}_{Ex-P} : \tilde{x}_{jk,t} = \frac{1}{n} \sum_{h=1}^{n} x_{hk,t}, \ \forall j, k, t \right\}$$

This set is a singleton and its unique element is the feasible norm that satisfies ex post compensation and no within type redistribution. We call this optimum *ex post opportunity egalitarian distribution*, or simply *Ex post EOp* distribution, and we formalize it as follows.

Definition 1. For all $X \in \mathbb{R}^{N \times T}$ the *Ex post EOp distribution* is $\tilde{X} \in \mathbb{R}^{N \times T}$ such that, for all $i \in \{1, \ldots, N\}$ and $t \in \{1, \ldots, T\}$, if $e_{k,t} \in \mathcal{E}$ is the effort level of individual i at t, then $\tilde{x}_{it} \in \tilde{X}$ is

$$\tilde{x}_{it}^{Ex-post} = \frac{1}{n} \sum_{j=1}^{n} f_t \left(c_{j,t}, e_{k,t} \right) \tag{2}$$

with $c_{j,t} \in \mathcal{C}$.

The Ex post EOp distribution is such that, at each period, individual income is the same for the same level of effort. Notice that, the Ex post EOp distribution $\tilde{X}$ is obtained from X through a series of within tranche progressive transfers, at each t. Therefore, it is an element of $\mathcal{X}_{Ex-P}^{NWTR}$. In other words, the Ex post EOp distribution is the matrix that represents the collection of optimal reallocations of each period's total income. In particular, each column of $\tilde{X} \in \mathcal{X}_{Ex-P}^{NWTR}$ is computed independently of the others and satisfies the budget constrain imposed by Eq. 1.

An alternative interpretation of the compensation principle is the ex ante one, which requires the reduction of inequality between the individual opportunity sets. In the standard formal EOp framework, the individual opportunity set is represented by the income distribution of the type the individual belongs to. Hence, reducing the inequality between opportunity sets amounts at reducing the inequality between types' income distributions. Fleurbaey & Peragine (2013) show that we can refine this principle by evaluating individual opportunity sets by the types' average income and then by imposing a strong ex ante compensation which requires equality of such types average incomes. This compensation principle, called *ex ante utilitarian compensation* (Fleurbaey & Peragine, 2013), allows us to restrict $\mathcal{X}_F$ as follows:

$$\mathcal{X}_{Ex-A} = \left\{ \mathcal{X}_F : \frac{1}{m} \sum_{k=1}^{m} \tilde{x}_{jk,t} = \frac{1}{m} \sum_{k=1}^{m} \tilde{x}_{hk,t}, \forall j, h, t \right\}$$

Therefore, $\mathcal{X}_{Ex-A}$ is the set of optimal distributions where there is equality between the types average incomes. However, no criteria is imposed on the way income should be distributed within each type. This issue is addressed by different versions of the reward principle (see again Fleurbaey & Peragine (2013)). We propose the following *proportional reward* (PR) version, according to which income redistributions are allowed as far as each individual holds the same share of the type income. This reward principle preserves the fair (relative) inequality in the within type distribution. We can also interpret it as a solidarity principle which asks individuals in each type to contribute, in proportion of their income, to the between type redistribution needed to achieve ex ante utilitarian compensation. The following subset of $\mathcal{X}_{Ex-A}$ satisfies this proportional reward principle:

$$\mathcal{X}_{Ex-A}^{PR} = \left\{ \mathcal{X}_{Ex-A} : \frac{\tilde{x}_{jk,t}}{\sum_{k=r}^{m} \tilde{x}_{jr,t}} = \frac{x_{jk,t}}{\sum_{k=r}^{m} x_{jr,t}}, \forall j, k, t \right\}$$

This set is a singleton, and its unique element is the feasible fair distribution for X that satisfies ex ante utilitarian compensation and proportional reward at each t. Under these normative constraints, the optimal counterpart for a given $x_{jk,t}$ is

$$\tilde{x}_{jk,t} = x_{jk,t} + \frac{x_{jk,t}}{\sum_{k=1}^{m} x_{jk,t}} \underbrace{\left(\frac{1}{n} \sum_{h=1}^{n} \sum_{k=1}^{m} x_{hk,t} - \sum_{k=1}^{m} x_{jk,t} \right)}_{\text{type-compensation}} \tag{3}$$

In other words, the optimal income of each individual coincides with his current income plus a fraction of the type specific transfer implied by ex ante utilitarian compensation, which is *type-compensation.* Hence, by ex ante utilitarian compensation, the total income is equally distributed between types in order to equalize the type means, thereby originating type specific transfers (which can be of course positive or negative); then, within each type, the type specific transfer is distributed among individuals proportionally to their share of the total type income, as required by proportional reward. We can rewrite the previous equation in the following simpler version.

$$\tilde{x}_{jk,t} = x_{jk,t} \; \frac{\frac{1}{nm} \sum_{h=1}^{n} \sum_{k=1}^{m} x_{hk,t}}{\frac{1}{m} \sum_{k=1}^{m} x_{jk,t}}$$

Therefore, at any of the considered periods, the fair income of an individual according to ex ante utilitarian compensation and proportional reward is a rescaling of the income for the ratio between the average total income and the average type income. Interestingly, the norm distribution obtained using this rescaling operation coincides with the 'standardized distribution' used in the literature to capture the 'between group' inequality and to decompose overall inequality into a 'between' and 'within' group inequality components.[6] In this sense, we are providing an additional normative justification for this standard technique in the literature.[7]

We call the obtained norm *ex ante opportunity egalitarian distribution*, or simply *Ex ante EOp* distribution, and we formalize it as follows.

Definition 2. For all $X \in \mathbb{R}^{N \times T}$ the *Ex ante EOp distribution* is $\tilde{X} \in \mathbb{R}^{N \times T}$ such that, for all $i \in \{1, \ldots, N\}$ and $t \in \{1, \ldots, T\}$, if $c_{j,t} \in \mathcal{C}$ is the circumstance of agent i at t, with income $x_{it} \in X$, then $\tilde{x}_{it} \in \tilde{X}$ is

$$\tilde{x}_{it}^{Ex-ante} = x_{it} \; \frac{\frac{1}{nm} \sum_{h=1}^{n} \sum_{k=1}^{m} f_t \left(c_{h,t}, e_{k,t}\right)}{\frac{1}{m} \sum_{k=1}^{m} f_t \left(c_{j,t}, e_{k,t}\right)} \tag{4}$$

for all $c_{j,t} \in \mathcal{C}$.

The Ex ante EOp distribution is such that, at each time, the value of an individual opportunity set does not depend on his circumstances. To obtain the Ex ante EOp, for each type j and time t, we first compute the type-compensation $\Delta_{c_{j,t}}$ that type j should receive in order to realize equality between expected incomes. Then we *distribute* this amount within type proportionally to the individual income.[8]

4 Ranking criteria

4.1 Unitemporal individual equality of opportunity

Given our opportunity egalitarian criteria, for each income distribution $X \in \mathbb{R}_{+}^{N \times T}$ there exists an optimal distribution $\tilde{X} \in \mathbb{R}_{+}^{N \times T}$ that assigns a unique optimal income to any individual in the population. We then follow a norm based approach

[6]See Foster & Shneyerov (2000). After Checchi & Peragine (2010), the application of this decomposition technique to the EOp framework has become a common practice in the EOp literature.

[7]See Appendix A1.

[8]In the empirical analysis, in addition to implement the Ex post EOp and Ex ante EOp optimal distributions, we will also implement a traditional egalitarian distribution defined as a distribution where each individual receives an income equal to the average income in the population.

(Cowell & Kuga, 1981; Magdalou & Nock, 2011) to develop our measure of inequality of opportunity. In particular, in this section, we follow Magdalou & Nock (2011) in considering divergence measures.[9] Hence, for each pair of actual and optimal incomes of individual $i \in \{1, \dots, N\}$ at $t \in \{1, \dots, T\}$ the function $d : \mathbb{R}_+^2 \to \mathbb{R}$ measures the divergence or gap between them. We assume it to be continuous and twice differentiable.[10] To characterize our measure of inequality of opportunity at the individual level, we impose the following properties on the function d.

- *Gap Normalization* (GN): For all $x_{it} \in X$ and $\tilde{x}_{it} \in \tilde{X}$, $d(x_{it}, \tilde{x}_{it}) = 0$ if and only if $x_{it} = \tilde{x}_{it}$.

This axiom normalizes the gap to zero if the actual and the norm income of an individual coincide. Another natural requirement for a divergence measure is the following: if two agents have the same optimal income, the one with higher actual income is more distant. Vice versa, among two agents with the same actual income, the one with higher optimal income has more negative divergence.

- *Gap Monotonicity* (GM): For all $x_{it}, x_{jt} \in X$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}$, $d(x_{it}, \tilde{x}_{it}) \geq d(x_{jt}, \tilde{x}_{it})$ if and only if $x_{it} \geq x_{jt}$, and $d(x_{it}, \tilde{x}_{it}) \geq d(x_{it}, \tilde{x}_{jt})$ if and only if $\tilde{x}_{it} \leq \tilde{x}_{jt}$.

Given our intertemporal perspective, we find desirable to assess opportunity inequality in relative terms.[11] The following scale invariance property requires the gap to be invariant to multiplication of both its arguments for the same scalar.

- *Gap Scale invariance* (GS): For all $x_{it} \in X$, $\tilde{x}_{it} \in \tilde{X}$ and $\lambda \in \mathbb{R}_+$, $d(\lambda x_{it}, \lambda \tilde{x}_{it}) = d(x_{it}, \tilde{x}_{it})$.

[9]Even if we will use them as synonymous, in mathematical terms divergence differs from distance as it does not necessary need to be symmetric and satisfy the triangular inequality.

[10]Smoothness of the divergence or distance measure is a standard assumption in the norm based approach (Cowell & Kuga, 1981; Magdalou & Nock, 2011).

[11]We can also conduct an assessments in absolute terms. In this case, we should simply substitute the following scale invariance with a translation invariance property.

From an individual and normative perspective, benefiting from an amount of income that is higher or lower than the optimal amount does make a difference. As also suggested in Cowell & Kuga (1981) the direction in which the actual income deviates with respect to the norm one is informative and normatively relevant. Among two individuals that experiment the same deviation from the fair income, but in different directions, we cannot consider a positive divergence less fair than a negative one. In a framework in which the gap fully describes the situation of an individual, the following property suggests the evaluator to give (weakly) higher priority to those individual with negative divergence that pay the price of the unfairness.

- *Gap Asymmetry* (GA): For all $x_{it}, x_{jt} \in X$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}$, if $x_{it} \geq \tilde{x}_{it} = \tilde{x}_{jt} \geq x_{jt}$ and $x_{it} - \tilde{x}_{it} = \tilde{x}_{jt} - x_{jt}$, then $d\left(x_{it}, \tilde{x}_{it}\right)$ - $d\left(\tilde{x}_{it}, \tilde{x}_{it}\right) \leq d\left(\tilde{x}_{jt}, \tilde{x}_{jt}\right) - d\left(x_{jt}, \tilde{x}_{jt}\right)$.

The following proposition characterizes our measure of the divergence between actual and norm income.

Proposition 1. *For all $x_{it} \in X$ and $\tilde{x}_{it} \in \tilde{X}$, $d : \mathbb{R}^2_+ \rightarrow \mathbb{R}$ satisfies gap normalization (GN), gap monotonicity (GM), gap scale invariance (GS) and gap asymmetry (GA) if and only if there exists a strictly increasing and concave $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that*

$$d\left(x_{it}, \tilde{x}_{it}\right) = g\left(\frac{x_{it}}{\tilde{x}_{it}}\right) \tag{5}$$

with $g\left(1\right) = 0$.

Proof. Appendix B. □

This proposition suggests to measure the divergence between the actual and norm income, of each individual at each time, using a concave monotone transformation of their ratio.

Let us call $g_{it} = g(x_{it}/\tilde{x}_{it})$ the *(unitemporal) opportunity gap* of individual $i \in \{1, \ldots, N\}$ at time $t \in \{1, \ldots, T\}$. Given a function g as in the previous proposition, we can construct the intertemporal opportunity gap distribution $G \in \mathbb{R}^{N \times T}$ which is a matrix such that each T-dimensional row $\mathbf{g}_{i\cdot}$ is the agent i intertemporal opportunity gap distribution and each of its N-dimensional column $\mathbf{g}_{\cdot t}$ is the time t opportunity gap distribution of the society. The intertemporal opportunity gap distribution can be written as follows.

$$G = \begin{bmatrix} g_{11} & \cdots & g_{1t} & \cdots & g_{1T} \\ \vdots & & \vdots & & \vdots \\ g_{i1} & \cdots & g_{it} & \cdots & g_{iT} \\ \vdots & & \vdots & & \vdots \\ g_{N1} & \cdots & g_{Nt} & \cdots & g_{NT} \end{bmatrix}$$

The individual intertemporal opportunity gap distributions, i.e. the rows of G, are the arguments of the individual intertemporal opportunity gap we axiomatize in the following section.

4.2 Intertemporal individual equality of opportunity

In this section we characterize a continuous and twice differentiable aggregator $\gamma : \mathbb{R}^T \to \mathbb{R}$ of the opportunity gaps of each individual across time, which we label *individual intertemporal opportunity gap*. We first propose three minimal axioms that γ has to satisfy, which define its structure. Then, we introduce other axioms we would like to impose to model the effect of time on γ.

The first axiom we impose is a criterion for normalizing our aggregator.

- *Intertemporal Normalization* (IN) - For all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ and $x \in \mathbb{R}$, if $g_{it} = x$ for all $t \in \{1, \dots, T\}$, then $\gamma(\mathbf{g}_{i\cdot}) = x$.

According to this normalization axiom, if an individual's opportunity gaps are equal in all periods, it is reasonable to argue that this individual's intertemporal opportunity gap can be appropriately represented by the unitemporal opportunity gap.

We would like our individual intertemporal opportunity gap to be sensitive to the intensity of the opportunity gap in each period. The following monotonicity axiom imposes this property.

- *Intertemporal Monotonicity* (IM) - For all $\mathbf{g}_{i\cdot}, \mathbf{g}_{j\cdot} \in \mathbb{R}^T$, if $g_{it} \geq g_{jt}$ for all $t \in \{1, \ldots, T\}$, then $\gamma(\mathbf{g}_{i\cdot}) \geq \gamma(\mathbf{g}_{i\cdot})$.

The third axiom requires separability between the effects of different opportunity gaps on the intertemporal individual opportunity gap.

- *Intertemporal Independence* (II) - For all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ and $s, t \in \{1, \ldots, T\}$, if $s \neq t$, then $\frac{\partial^2 \gamma(\mathbf{g}_{i\cdot})}{\partial g_{it} \partial g_{is}} = 0$.

With the independence axiom, we impose a twice differentiable function γ to perform a linear aggregation of the opportunity gaps. We introduce this property as a benchmark, which we will modify afterwards in order to consider alternative ways of evaluating the time dimension.

The axioms above characterize a family of additive individual intertemporal opportunity gaps as stated in the following proposition.

Proposition 2. *The individual intertemporal opportunity gap function $\gamma : \mathbb{R}^T \to \mathbb{R}$ satisfies intertemporal normalization (IN), intertemporal monotonicity (IM) and intertemporal independence (II) if and only if there exist twice differentiable functions $\omega_t : \mathbb{R}_+ \to \mathbb{R}_+$, $t = \{1, \ldots T\}$, such that, for all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$,*

$$\gamma(\mathbf{g}_{i\cdot}) = \sum_{t=1}^{T} \omega_t(t) g_{it} \tag{6}$$

with $0 < \omega(t) < 1$, for all t, and $\sum_{t=1}^{T} \omega_t(t) = 1$.

Proof. Appendix B. □

Following this proposition, the individual intertemporal opportunity gap is a weighted average of the individual (unitemporal) opportunity gaps, with weights that only depend on time. We can therefore state the following corollary.

Corollary 1. *The individual intertemporal opportunity gap function $\gamma : \mathbb{R}^T \to \mathbb{R}$ satisfies (IN), (IM) and (II) if and only if it can be written as a weighted average of the opportunity gaps such that, for all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$,*

$$\gamma(\mathbf{g}_{i\cdot}) = \sum_{t=1}^{T} \alpha(t, T) g_{it} \tag{7}$$

where the function α *is such that* $0 < \alpha(t,T) < 1$ *and* $\sum_{t=1}^{T} \alpha(t,T) = 1$.

Proof. Appendix B. □

Eq. 7 defines the family of individual intertemporal opportunity gap measures we refer to throughout this paper. This particular structure allows us to isolate the weight function $\alpha(t,T)$ and operate on it to specify the effect of the time component in the intertemporal assessment. The remaining of this section focuses on how to model the time component of γ.

Let us consider the following individual intertemporal opportunity gap distributions: $\mathbf{g}_{i\cdot} = (x,x,x)$, $\mathbf{g}_{j\cdot} = (x+\delta, x, x-\delta)$ and $\mathbf{g}_{k\cdot} = (x-\delta, x, x+\delta)$, with $x \in \mathbb{R}_-$ and $\delta \in \mathbb{R}_+$. It is clear that i, j and k have different opportunity gap histories: while i has constantly a less-than-fair income, k has a worse situation at the beginning of the considered period but ends up having a more-than-fair income. If we consider these as opportunity gap distributions over the entire life-span, we can say that, for example, k had a harder childhood than i and an opportunity egalitarian assessment should be more concerned with what happens in the initial stages of life where opportunities are shaped. The following early gap axiom applies this idea to our framework.

- *Early period gap* (EG) - For all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ and $t,s \in \{1,\ldots,T\}$, if $t < s$, then $\frac{\partial\gamma(\mathbf{g}_{i\cdot})}{\partial g_{it}} \geq \frac{\partial\gamma(\mathbf{g}_{i\cdot})}{\partial g_{is}}$.

Let us now consider the situation of individual j. He started the observation period with a better situation than the current one (assuming that the last observed opportunity gap refers to the present). Knowing that an opportunity egalitarian policy cannot be retro-active, and although we do not want to disregard the history of each individual, we may want to give priority to those who have more recently obtained a level of income lower than the optimal one, rather than helping people that seems to be more able to recover from an initial situation of unfairness as, for example, agent k. The following axiom formalizes this idea.

- *Late period gap* (LG) - For all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ and $s,t \in \{1,\ldots,T\}$, if $t < s$, then $\frac{\partial\gamma(\mathbf{g}_{i\cdot})}{\partial g_{it}} \leq \frac{\partial\gamma(\mathbf{g}_{i\cdot})}{\partial g_{is}}$.

Consider now other two individual intertemporal opportunity gap distributions $\mathbf{g}_{p\cdot} = (0, x, x, 0)$ and $\mathbf{g}_{q\cdot} = (x, 0, 0, x)$, with $x \in \mathbb{R}_-$ and the two following waiting schemes $\alpha = (0.4, 0.3, 0.2, 0.1)$ and $\alpha' = (0.1, 0.2, 0.3, 0.4)$.[12] Independently of the time-weights we implement, agents p and q will be considered equal in terms of intertemporal opportunity gap. However, if intertemporal IOp has to exist, it is unfair that the individuals who bare its negative consequences are constantly the same. Therefore, the history of p with consecutive less-tan-fair income periods is less desirable than the one of q in which the 'bad periods' are more spread-out. The following axiom formalizes this concern for persistence of opportunity deprivation.

- *Opportunity gap persistence* (GP) - For all $\mathbf{g}_{i\cdot}, \mathbf{g}_{j\cdot} \in \mathbb{R}^T$ such that $g_{is} = g_{iu} < 0$ with $g_{it} = 0$ for all $t \neq s, u \in \{1, \ldots, T\}$, and $g_{jr} = g_{jv} < 0$ with $g_{jt} = 0$ for all $t \neq r, v \in \{1, \ldots, T\}$. If $g_{is} = g_{jr}$, $g_{iu} = g_{jv}$, $1 \leq r < s < u < v \leq T$ and $s - r = v - u$, then $\gamma(\mathbf{g}_{i\cdot}) \leq \gamma(\mathbf{g}_{j\cdot})$.

In the example discussed above, this axiom imposes $\gamma(\mathbf{g}_{p\cdot}) \leq \gamma(\mathbf{g}_{q\cdot})$.
As stated in the following proposition, when combined with the basic axioms (II, IM and IN), these alternative assessments of the time component define two subsets of measures.[13]

Proposition 3. *The individual intertemporal opportunity gap $\gamma : \mathbb{R}^T \to \mathbb{R}$, satisfying monotonicity (M), independence (I) and normalization (N), satisfies also:*

(i) early period gap (EG) and opportunity gap persistence (GP) if and only if $\alpha(t, T)$ is concave and decreasing in t;

(ii) late period gap (LG) and opportunity gap persistence (GP) if and only if $\alpha(t, T)$ is concave and increasing in t.

Proof. Appendix B. □

[12] As we will see in Proposition 3 α and α' satisfy respectively EG and LG.

[13] See Hoy & Zheng (2011) for a similar approach in the context of lifetime poverty measurement.

According to this Proposition, to emphasize the early stage of life, we should assess the time dimension via a decreasing and concave weighting scheme as in the following example, that will be used in the empirical analysis:

$$\alpha_{EC}(t,T) = \frac{\sqrt{\frac{T-t+1}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{T-t+1}{T}}}$$

Conversely, to emphasize the late periods, we should use an increasing, but still concave, weighting scheme as the following example:

$$\alpha_{LC}(t,T) = \frac{\sqrt{\frac{t}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{t}{T}}}$$

We should notice that, thanks to the intertemporal independence axiom (II) imposed above, our aggregation across time isolates the *position* effect - tackled by (GP) - of an opportunity gap from its *intensity* effect - captured by (IM).

In the literature on inequality measurement we often refer to the Lorenz curve as a tool to define dominance conditions between distributions. Analogous partial orderings have been defined in the EOp framework: see Peragine (2004); Aaberge *et al.* (2011). In this paper, we can follow the same logic and construct partial orderings of societies $\mathbf{s} = (\gamma(\mathbf{g}_{1\cdot}), \ldots, \gamma(\mathbf{g}_{N\cdot})) \in \mathbb{R}^N$ described by the individual intertemporal opportunity gap $\gamma(\mathbf{g}_{\mathbf{i}\cdot})$ of each individual i in the society. Since in our framework the sign and magnitude of the opportunity gaps are relevant, we define the *intertemporal opportunity generalized Lorenz curve* as

$$gL(k/N, \mathbf{s}) = \frac{1}{N}\sum_{i=1}^{k}\gamma(\mathbf{g}_{\mathbf{i}\cdot})$$

for all increasingly ordered vectors $\mathbf{s} \in \mathbb{R}^N$ and $k \in \{1, \ldots, N\}$.[14] We can now rank societies by referring to the generalized Lorenz ranking of their distribution of individual intertemporal opportunity gaps. Therefore, for all increasingly ordered

[14]This Lorenz curve can be seen as a generalization of the unfairness Lorenz curve proposed in Almås *et al.* (2011).

social intertemporal opportunity gap distributions $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^N$, if $gL(k/N, \mathbf{s}) \geq gL(k/N, \mathbf{s}')$ for all $k \in \{1, \ldots, N\}$, then the distribution $\mathbf{s}'$ shows at least as much intertemporal IOp as $\mathbf{s}$.

4.3 From individual to societal intertemporal equality of opportunity

Given all the individual intertemporal opportunity gaps constructed as in the previous section, we can define the vector $\mathbf{s} \in \mathbb{R}^N$ which contains all the $\gamma(\mathbf{g}_{i\cdot})$ derived from the intertemporal opportunity gap distribution $G \in \mathbb{R}^{N \times T}$. To simplify the notation, let $\gamma(\mathbf{g}_{i\cdot}) = \gamma_i$ for all $i \in \{1, \ldots, N\}$. Therefore, $\mathbf{s} = (\gamma_1, \gamma_2, \ldots, \gamma_N)$ denotes the distribution of individuals' intertemporal inequality of opportunity in the population. We denote by $\Gamma : \mathbb{R}^N \to \mathbb{R}$ the societal aggregator and we assume that it is a twice differentiable function and satisfies the following properties.

- *Aggregate Normalization* (AN) - For all $\mathbf{s} \in \mathbb{R}^N$, if $\gamma_i = 0$ for all $i \in \{1, \ldots, N\}$, then $\Gamma(\mathbf{s}) = 0$.

This axiom normalizes our measure to be equal to zero when every individual in the population is in an intertemporally fair situation which coincides with an intertemporal opportunity gap equal to zero.

- *Aggregate Monotonicity* (AM) - For all $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^N$, if $\gamma_i \geq \gamma_i'$ for all $i \in \{1, \ldots, N\}$, then $\Gamma(\mathbf{s}) \geq \Gamma(\mathbf{s}')$.

This axiom imposes our aggregate measure to be sensitive to the signs and magnitude of each intertemporal opportunity gap in the society.

- *Decomposability* (D) - For all $\mathbf{s} \in \mathbb{R}^N$ and $i, j \in \{1, \ldots, N\}$, if $i \neq j$, then $\frac{\partial^2 \Gamma(\mathbf{s})}{\partial \gamma_i \partial \gamma_j} = 0$.

Decomposability is a quite standard property imposed to aggregators of social phenomena. In our case, its importance is emphasized by the potential need of distinguishing individuals with income higher than the optimal one from individuals with income lower than the optimal one.

- *Population invariance* (P) - For all $\mathbf{s} \in \mathbb{R}^N$ and $p \in \mathbb{N}_{++}$, if $\mathbf{s}' = (\mathbf{s}, \ldots, \mathbf{s}) \in \mathbb{R}^{pN}$, then $\Gamma(\mathbf{s}) = \Gamma(\mathbf{s}')$.

Population (replication) invariance is necessary for comparing populations with different sizes. We prefer this invariance property to one based on a critical value (Hoy & Zheng, 2011; Bossert *et al.*, 2012) that could be 0 in our context. The reason for this choice can be explained by considering that $\mathbf{s} = (a, 0, 0, b)$, with $a, b \in \mathbb{R}$ describes a society that is able to guarantee intertemporal fairness for two individuals: a society that we consider fairer than $\mathbf{s}' = (a, b)$.

- *Anonymity* (A) - For all $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^N$ and all permutation functions $\Pi : \mathbb{R}^N \rightarrow \mathbb{R}^N$, if $\mathbf{s} = \Pi(\mathbf{s}')$, then $\Gamma(\mathbf{s}) = \Gamma(\mathbf{s}')$.

With the anonymity axiom, we impose our social opportunity gap to be independent from the identity of the individuals, so that the criteria to discriminate between two members of the population have to be based on their individual intertemporal opportunity gaps.

- *Aggregate Pigou-Dalton* (APD) - For all $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^N$ if there exist $\delta \in \mathbb{R}_+$ such that $\gamma_i > 0 > \gamma_j$, $\gamma_i' = \gamma_i - \delta$ and $\gamma_j' = \gamma_j + \delta$, with $\mathbf{s}$ and $\mathbf{s}'$ coinciding everywhere else, then $\Gamma(\mathbf{s}) \leq \Gamma(\mathbf{s}')$.

Following APD, redistributing resources from an individual with positive intertemporal opportunity gap to one with negative intertemporal opportunity gap should increase Γ. A higher value of our social evaluation function Γ has to be considered as an improvement in terms of intertemporal equality of opportunity. This axiom compensates the effect of (AM) by emphasizing the presence and the cost of having in the society individuals with negative intertemporal opportunity gaps.

The axioms proposed above characterize a measure of societal intertemporal inequality of opportunity, as shown in the following proposition.

Proposition 4. $\Gamma : \mathbb{R}^N \rightarrow \mathbb{R}$ *satisfies aggregate normalization (AN), aggregate monotonicity (AM), population invariance (P), decomposability (D), anonymity*

(A) and aggregate Pigou-Dalton (APD) if and only if, for all $\mathbf{s} \in \mathbb{R}^N$*,*

$$\Gamma(\mathbf{s}) = \frac{1}{N} \sum_{i=1}^{N} \sigma(\gamma_i) \tag{8}$$

with $\gamma_i = \gamma(\mathbf{g}_{i\cdot})$ *defined in Proposition 2, for all* $i \in \{1, \ldots, N\}$*, and* $\sigma : \mathbb{R} \to \mathbb{R}$ *twice differentiable, increasing and non-convex such that* $\sigma(0) = 0$*.*

Proof. Appendix B. □

Societal intertemporal inequality of opportunity can be measured by the average of non-convex transformations of the individual intertemporal opportunity gaps. Such measure aggregates the γs giving more weight to individuals with observed incomes lower than their own optimum. Γ is a social evaluation function that allows us to compare and order different states of the world according to their respect for the EOp allocation criteria that define the optimal distributions.

Two additional features characterize Eq. 8. First, independently of the implemented norm - Definition 1 or 2 - Γ is always negative (see Appendix A.2 for a deeper discussion). Therefore, we should read Eq. 8 as an intertemporal measure of inequality of opportunity that is zero if there is full equality of opportunity intertemporally and deceases (becomes more negative) as inequality of opportunity increases. In other words, Proposition 4 provides a complete ranking of intertemporal opportunity gap distributions such that, for all $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^N$, if $\Gamma(\mathbf{s}) > \Gamma(\mathbf{s}')$ then $\mathbf{s}$ shows lower intertemporal inequality of opportunity. This ranking is coherent with the partial one defined in the previous section through the generalized Lorenz curve.[15] In particular, by a well known result in the literature (Shorrocks, 1983) we have that, for all social intertemporal opportunity gap distributions $\mathbf{s}, \mathbf{s}'$, if $gL(k/N, \mathbf{s}) \geq gL(k/N, \mathbf{s}')$ for all $k \in \{1, \ldots, N\}$, then $\Gamma(\mathbf{s}) \geq \Gamma(\mathbf{s}')$ for any specification of Eq. 8.

A second interesting feature of Eq. 8 arises under the particular case in which $\sigma(\gamma_i) = \sum_{t=1}^{T} \alpha(t, T) \ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$ so that:

[15] Another possibility to extend the generalized Lorenz ranking of social opportunity gap distributions consists in restricting the focus to the generalized Lorenz curve of the intertemporal opportunity deprived individuals $\mathbf{s}^-$ in order to perform a greater number of comparisons through an *almost generalized Lorenz dominance* as in Zheng (2018).

$$\Gamma = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\alpha\left(t,T\right)\left(\ln x_{it} - \ln \tilde{x}_{it}\right)\right] = -\sum_{t=1}^{T}\alpha\left(t,T\right)\underbrace{\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{\tilde{x}_{it}}{x_{it}}\right)}_{\text{MLD}} \qquad (9)$$

Eq. 9 highlights the link between this particular specification of Γ and the mean log deviation (MLD): a standard and widely implemented inequality index. In particular, Eq. 9 is a time-weighted average of the MLD in each of the considered period, where the norm $\tilde{x}_{it}$ coincides with the ones defined in Section 3. The relevance of this link is strengthened by the fact that Eq. 9 is a *path independent* version of Eq. 8. Indeed, Eq. 9 is a case in which our approach of aggregating across time first and then across individuals coincides with the alternative one that aggregates first across individual. Notice also that, if we implement an egalitarian norm distribution such that $\tilde{x}_{it} = \frac{1}{N}\sum_{i=1}^{N} x_{it}$ for all $i \in \{1, ..., N\}$ and $t \in \{1, ..., T\}$,[16] our aggregate measure becomes (the opposite of) a time-weighted average of the inequality in each period, where the standard MLD is implemented as inequality index.

The function σ in Proposition 4 imposes a structure on the way we should transform the individual intertemporal opportunity gaps. A flexible specification for this function is $\sigma(\gamma) = \gamma - \rho^{-\gamma} + 1$ where $\rho \geq 1$ can be interpreted as the opportunity inequality aversion parameter.

The following section implements the proposed framework to assess intertemporal EOp in South Korea.

5 Intertemporal inequality of opportunity in South Korea

South Korea is among the most developed countries in the world, especially if one looks at its per capita GDP and speed of technological innovation. According to

[16] This norm distribution reflects standard outcome egalitarian principle and will be used in the empirical application to illustrate the different results that can be obtained when different perspectives to the analysis of inequality are adopted.

the most recent World Bank data that refer to 2018, per capita GDP in this country is about $31,362$ computed in current US $ and its annual growth rate fluctuated around the 5% in the last twenty years, although it has been stabilizing around 2.5% in the last five years. According to the OECD, South Korea is second ranked in terms of investment in research and development as a share of GDP among other advanced countries (Israel ranks first): in 2018, South Korea spent about 4.5% of its GDP in R&D. The World Bank also ranks South Korea as the east Asia's most egalitarian society and, in general, more egalitarian than other western countries like France, the U.K. and Canada. It is, thus, of interest to shade light on the fairness aspects of this country. We do this by applying our measurement framework.

5.1 Data

Our empirical analysis is based on the KLIPS (Korean Labor and Income Panel Study), which is a Korean census conducted every year on a sample of about 5,000 households. Started in 1998, it collects data at both household and individual level and it is one of the few panel surveys that contains information on individual socio-economic background. For our analysis we use 14 waves ranging from 2001 to 2014.

The unit of observation is the individual, in particular we consider all individuals aged between 20 and 65 and interviewed in each wave. The measure of living standards is equivalized disposable household income, expressed in constant 2005 prices, using country and year-specific price indexes, and adjusted for differences in household size by dividing incomes by the square root of the household size.[17] KLIPS surveys all incomes as after-tax income, and the household income is obtained as the sum across each household members of the following components: financial income (interest from banks and financial institutions, interest from private loans, gain from securities and bond transactions, dividends, etc.), real-estate income (rent, gain from real estate transactions, rent from land lease, premium money, etc.), social insurance income (amount of one-time benefit payment), transfer income (receipts of National Basic Livelihood Protection payments, other gov-

[17] Consumer Price Indexes are taken from Korea National Statistical Office KOSTAT.

ernment subsidies, social group subsidies, family/ relatives' support, etc.), other incomes (insurance payment receipts, severance pay, gifts or inheritances, other celebratory/ condolence money, lottery, racetrack winnings, disaster compensations). This variable is recorded in 10,000 KRW (Korean Won). Individuals with zero sampling weights are excluded since our measures are calculated using sample weights designed to make the samples nationally representative.

A fundamental step to operationalize our measurement framework is the identification of the vector of observable circumstances. This is a normative choice, subject to the constraint of data availability. Our data contain information on a small set of basic circumstances, but nonetheless of prominent importance. For each wave, in fact, we can observe the following: gender, birth place, parental education, parental support.

Birth place is categorized following the major administrative divisions of the country. The first category is represented by individuals born in the special city - namely Seoul. The second category is represented by individuals born in one of the metropolitan cities (self-governing cities that are not part of any province) - namely Busan, Daegu, Incheon, Gwangju, Daejeon, Ulsan - or in the autonomous metropolitan city - namely Sejong. The third category is represented by individuals born in other provinces - namely Gyeonggi, Gangwon, North Chungcheong, South Chungcheong, North Jeolla, South Jeolla, North Gyeongsang, South Gyeongsang, Jeju - or outside South Korea. Parental education - measured by the highest educational attainment between mother and father - is also coded into 3 categories: individuals whose parents have elementary education or no education; individuals with at least one parent with middle/secondary education; individuals with at least one parent having attained tertiary education. The last circumstance used is parental support that is a binary variable indicating whether or not the individual received any material/financial support from the parent(s) during the year preceding the survey. Notice that differently from gender, birth place, and parental education - circumstances that are fixed over time - parental support is a variable circumstance. In particular, in our sample it results that around 60% of individuals have experienced a change in this circumstance at least once over the time horizon considered and around 45% at least twice, which makes even more meaningful the application of our framework that is flexible enough to account for the possibility that circumstances may vary over time.

Individuals with missing information for one or more circumstances are excluded from the analysis, therefore, our final sample is composed by 3,061 observations.[18]

As explained above, in order to compute our measure, one needs to identify the optimal distributions. In this empirical analysis, this is done by implementing to our data the ex post and ex ante criteria defined in Section 3. To implement the ex post approach we approximate the individual effort with the position in the relative type distribution after dividing it in 10 quantiles.[19]

We also compare the opportunity egalitarian approach with the outcome egalitarian one. To this scope we construct the egalitarian optimal distribution, according to which the fair income for all individuals is set at the average income for each period considered.

For the sake of exposition, the aggregate indexes are computed on a measure of individual opportunity gap at each time t given by $g_{it} = \ln(\frac{x_{it}}{\tilde{x}_{it}})$. We then consider different classes of indexes by considering two main features: the different concerns with respect to the time in which the gap is experienced by each individual and the degree of inequality aversion in the social intertemporal opportunity gap distribution.

Therefore, we estimate the following six measures of social intertemporal inequality of opportunity.

- A measure that weights equally both the period in which the gap in experienced and the individuals experiencing that gap:

$\Gamma_{1,Lin} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T} g_{it}$.

- A measure that expresses neutrality with respect to the period in which the gap in experienced but that attaches higher weight to the individuals experiencing higher intertemporal inequality of opportunity by choosing a value of 1.5 for the parameter capturing inequality aversion:

$\Gamma_{1.5,Lin} = \frac{1}{N}\sum_{i=1}^{N}(\gamma_i - 1.5^{-\gamma_i} + 1)$ with $\gamma_i = \frac{1}{T}\sum_{t=1}^{T} g_{it}$.

[18]Some descriptive statistics are available in Appendix C.

[19]This approach follows Roemer's (Roemer, 1998) identification to approximate effort.

- A measure that expresses more concern with earlier periods but that weights equally the intertemporal inequality of opportunity of each individual:
$\Gamma_{1,EG} = \Gamma_{1,Lin} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\sqrt{\frac{T-t+1}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{T-t+1}{T}}}g_{it}$.
- A measure that expresses more concern with earlier periods and that attaches higher weight to the individuals experiencing higher intertemporal inequality of opportunity:
$\Gamma_{1.5,EG} = \Gamma_{1,Lin} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}(\gamma_i - 1.5^{-\gamma_i} + 1)$ with $\gamma_i = \sum_{t=1}^{T}\frac{\sqrt{\frac{T-t+1}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{T-t+1}{T}}}g_{it}$.
- A measure that attaches more weight to later periods of life but that weights equally the intertemporal opportunity inequality of each individual:
$\Gamma_{1,LG} = \Gamma_{1,Lin} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\sqrt{\frac{t}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{t}{T}}}g_{it}$.
- A measure that expresses both more concern with later periods and with the individuals experiencing higher intertemporal inequality of opportunity:
$\Gamma_{1.5,LG} = \Gamma_{1,Lin} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}(\gamma_i - 1.5^{-\gamma_i} + 1)$ with $\gamma_i = \sum_{t=1}T\frac{\sqrt{\frac{t}{T}}}{\sum_{t=1}^{T}\sqrt{\frac{t}{T}}}g_{it}$.

5.2 Results

Table 1 reports the results of our estimates for the whole sample and the whole 14 years-period considered.

It immediately comes out that all entries of Table 1 have value lower than 0: although South Korea is known as one of the most growing and progressive country, it still suffers from some degree of inequality of opportunity. Moreover, no matter the approach used, inequality of opportunity is always higher with late period relevance. As expected, each index of ex ante and ex post intertemporal inequality of opportunity is less negative than the corresponding indexes obtained implementing the standard egalitarian perspective. This is in line with the canonical unitemporal and aggregated perspective to the measurement of inequality of opportunity, which states that not all the inequalities that we observe are deemed to be objectionable, but only the inequalities due to different opportunities - those captured here by the ex ante and ex post approaches. Hence, inequality of opportunity is usually interpreted as one component of overall outcome inequality, specifically as that part of outcome inequality that is explained by circumstances outside the in-

Table 1: Korea intertemporal fairness, 2001-14

	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2692	-0.0082	-0.0707
$\Gamma_{1.5,Lin}$	-0.4129	-0.0123	-0.1041
$\Gamma_{1,EG}$	-0.2494	-0.0076	-0.0570
$\Gamma_{1.5,EG}$	-0.3817	-0.0113	-0.0832
$\Gamma_{1,LG}$	-0.2881	-0.0089	-0.0835
$\Gamma_{1.5,LG}$	-0.4454	-0.0133	-0.1242

Source: Authors' elaborations based on KLIPS, 2001-2014.
Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

dividual control. In Table 1 the contribution of unequal opportunities to outcome inequality, either measured with the ex ante or the ex post approach, remains quite constant independently of the degree of interindividual inequality aversion captured by the value assigned to the parameter ρ (1 or 1.5) and independently of which period of life matters more.

Previous empirical literature has shown that the economic crisis of 2007 affected the distribution of individual incomes. Data on the GDP trend provided by World Bank and OECD witness that this country experienced a deep contraction between 2008 and 2009 (about -10.5%), which was however soon recovered in 2010 (about $+20\%$), bringing the country to a higher GDP per capita than before the crisis. We then ask whether the economic crisis played any role in shaping the trend of inequality of opportunity in South Korea. To this aim we compare two subperiods: 2001-07 and 2008-14.

This exercise is also useful to illustrate how our framework can be used, in addition to evaluate single distributions of income streams, to make comparisons across distributions.

Table 2 shows that the intertemporal measures referring to the two subperiods are characterized by features similar to the indexes applied on the whole period. That

Table 2: Korea intertemporal fairness, 2001-07 vs 2008-14

	2001-07		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.3204	-0.0107	-0.1058
$\Gamma_{1.5,Lin}$	-0.5107	-0.0160	-0.1634
$\Gamma_{1,EG}$	-0.3083	-0.0106	-0.0959
$\Gamma_{1.5,EG}$	-0.4898	-0.0160	-0.1481
$\Gamma_{1,LG}$	-0.3311	-0.0108	-0.1144
$\Gamma_{1.5,LG}$	-0.5346	-0.0165	-0.1793
	2008-14		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2180	-0.0058	-0.0358
$\Gamma_{1.5,Lin}$	-0.3373	-0.0088	-0.0531
$\Gamma_{1,EG}$	-0.2091	-0.0056	-0.0312
$\Gamma_{1.5,EG}$	-0.3231	-0.0085	-0.0459
$\Gamma_{1.5,LG}$	-0.2265	-0.0100	-0.0402
$\Gamma_{1.5,LG}$	-0.3520	-0.0090	-0.0602

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

is, the indexes take always values different from 0 implying that both subperiods where characterized by some degree of unfairness. Moreover, inequality of opportunity is always higher when later periods of life are given more relevance than earlier periods. The egalitarian, ex ante and ex post approaches are consistent in ranking the second period as better-off than the first one: in all cases inequality (of opportunity) is less intense in 2008-14 than in 2001-07. It is then possible to infer that South Korea fared well in dealing with the global financial crisis as over time the country did not worsen intertemporal inequality of opportunity.

The indexes adopted so far have also the advantage of providing a complementary picture of the one that would emerge from adopting the alternative and standard unitemporal perspective. Table 3 reports the estimates of unitemporal inequality of opportunity for each year considered in the analysis. The indexes are computed aggregating across individuals the opportunity gap experienced by each individual in each specific year, hence these indexes ignore the individual dynamics. To be more specific, for each year t unitemporal inequality of opportunity is measured by $\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. We observe that while the values of the indexes are consistent with the multitemporal perspective (they are all different from 0, with the ex ante and ex post measures being smaller than the egalitarian one), the trend is overall less clear-cut. In particular, when initial (2001) and final (2014) periods are compared, inequality of opportunity improves according to all three approaches; however, when we focus on the whole period-by-period evolution there are both upward and downward variations that we want to take into account by using the intertemporal approach proposed in this paper.

We deepen our investigation by implementing the fairness generalized Lorenz curve to the two subperiods as done in Figure 1; this device can help shading some light on how different segments of the distribution fared in terms of intertemporal inequality of opportunity and establishing more robust dominance between the two periods. For the sake of brevity, we report the generalized Lorenz curve only for the case of time neutrality. Thus, the curves are constructed on a measure of individual intertemporal inequality of opportunity given by $\frac{1}{T}\sum_{t=1}^{T}\ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Figure 1 confirms that the 2008-14 period generalized Lorenz dominates the 2001-07, the dominance is especially evident around the higher middle part of the distribution and especially for the ex ante generalized Lorenz curve, the only curve that shows one instance of intersection between the two subperiods compared at the very bottom of the distribution and tends to converge at the very top.

South Korea has been characterized by a rapid economic growth process as well as by a rapid process of integration in the world economy. After the Korean war, end of 50s, per capita GDP was at the same level of some poorest African countries. In our analysis we can distinguish two cohorts of individuals: one which was involved in the initial phase of South Korean development, and one that was born in the same but more developed country. To investigate whether this has an impact on equality of opportunity, we divide the sample into two cohorts: the cohort of individuals aged between 20 and 40 in 2001 and the cohort

Table 3: Evolution over time

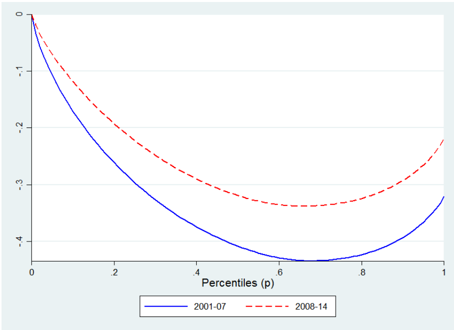
Year	**Egalitarian**	**Ex ante**	**Ex post**
2001	-0.4487	-0.0123	-0.2040
2002	-0.2881	-0.0094	-0.0885
2003	-0.3291	-0.0077	-0.1160
2004	-0.3259	-0.0139	-0.1051
2005	-0.2822	-0.0079	-0.0746
2006	-0.2997	-0.0139	-0.0832
2007	-0.2690	-0.0096	-0.0689
2008	-0.2393	-0.0072	-0.0504
2009	-0.2934	-0.0060	-0.0872
2010	-0.2222	-0.0075	-0.0275
2011	-0.2099	-0.0045	-0.0261
2012	-0.1810	-0.0041	-0.0182
2013	-0.1803	-0.0062	-0.0192
2014	-0.1904	-0.0052	-0.0222

Source: Authors' elaborations based on KLIPS, 2001-2014.

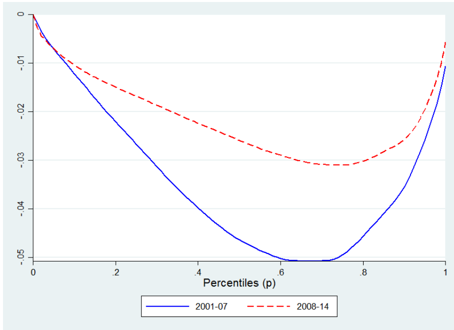
Note: For each year, fairness is measured by $\frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Figure 1: Intertemporal unfairness generalized Lorenz curves, 2001-07 vs 2008-14

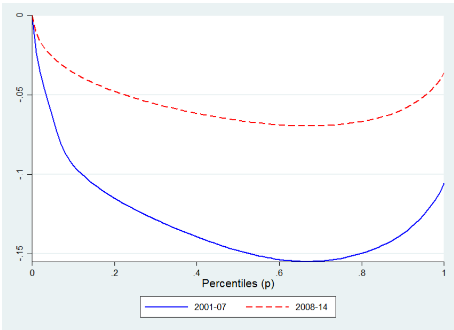
(a) Egalitarian



(b) Ex ante



(c) Ex post



Source: Authors' elaborations based on KLIPS, 2001-2014.
Note: The generalized Lorenz curves are constructed on a measure of individual interetemporal fairness given by $\frac{1}{T}\sum_{t=1}^{T}\ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

of individuals older than 40. Because the optimal distributions are computed on the whole sample, the index of fairness computed on the two subsamples may take positive value. This is indeed the case for the younger cohort when the analysis is performed using the ex ante approach, as it can be observed in Table 4. Interestingly, in terms of distributions ranking, the comparison between the two cohorts depends on the optimal benchmark implemented. According to the standard outcome perspective that is endorsed in the egalitarian benchmark, the older cohort performs better than the younger cohort as the indexes corresponding to the latter subsample are always lower than the indexes corresponding to the former. According to the inequality of opportunity perspective endorsed in the ex ante and ex post approach, the sign of the dominance is inverted; that is, the younger cohort shows more fairness than the older one. Therefore, while the income of an individual in the younger cohort tends to be lower, it is on average less dependent on exogenous factors than the income of the older cohort, their effort seems also to be remunerated in a fair manner. The use of the unfairness generalized Lorenz curve reported in Figure 2 highlights the difference between the standard outcome and the opportunity egalitarian perspective. In panel a) the focus is on the standard outcome perspective. Here we notice that for the bottom 10% the two curves overlap almost perfectly, while they tend to diverge for the rest of the distribution, with the older cohort's curve lying always above than that of the younger cohort's. In panel b) and c) a different picture can be drawn. For the bottom half of the distribution, the curve of the older cohort slightly dominates the curve of the younger cohort, the two curves then intersect - specifically at the 40th percentile according to the ex ante approach and at the 50th percentile for the ex post approach - being the younger cohort's curve to dominate the other curve for the rest of the distribution.

We conclude our analysis by performing some robustness checks related to different degree of (i) gap asymmetry, (ii) time concern, (iii) inequality aversion. The results, are reported respectively in Appendix D, E and F and show that our conclusions are robust.

Table 4: Korea intertemporal fairness, 2001-14 by cohort

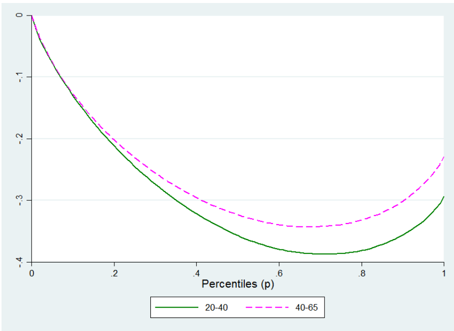
Younger cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2937	0.0012	-0.0622
$\Gamma_{1.5,Lin}$	-0.4486	0.0010	-0.0922
$\Gamma_{1,EG}$	-0.2781	0.0002	-0.0493
$\Gamma_{1.5,EG}$	-0.4232	-0.0004	-0.0724
$\Gamma_{1,LG}$	-0.3090	0.0024	-0.0744
$\Gamma_{1.5,LG}$	-0.4761	0.0025	-0.1118
Older cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2290	-0.0238	-0.0849
$\Gamma_{1.5,Lin}$	-0.3545	-0.0341	-0.1235
$\Gamma_{1,EG}$	-0.2023	-0.0203	-0.0697
$\Gamma_{1.5,EG}$	-0.3135	-0.0290	-0.1009
$\Gamma_{1,LG}$	-0.2538	-0.0275	-0.0983
$\Gamma_{1.5,LG}$	-0.3952	-0.0393	-0.1445

Source: Authors' elaborations based on KLIPS, 2001-2014.

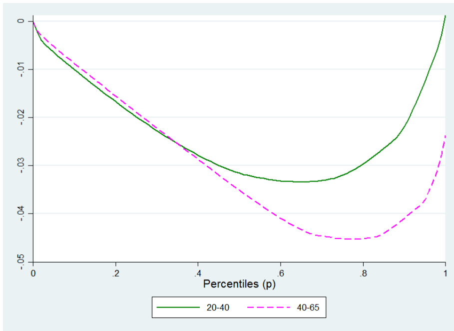
Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Figure 2: Intertemporal fairness generalized Lorenz curves, cohorts 20-40 vs 40-65

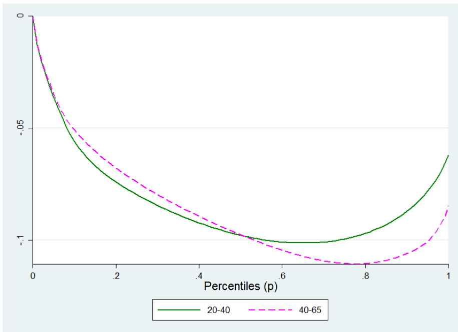
(a) Egalitarian



(b) Ex ante



(c) Ex post



Source: Authors' elaborations based on KLIPS, 2001-2014.
Note: The generalized Lorenz curves are constructed on a measure of individual interetemporal fairness given by $\frac{1}{T}\sum_{t=1}^{T} \ln\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

6 Conclusions

The theoretical and empirical literature on the measurement of inequality of opportunity has been very florid in the last decades. A vast variety of tools has been proposed and applied to different data sets.

However, the temporal aspect has been almost entirely neglected by the existing literature. In this paper we have proposed one possible answer to fill this gap, by deriving a new set of measures of intertemporal inequality inspired by the EOp view. We have followed a two-step procedure, that is, we have first derived axiomatically a measure of intertemporal inequality of opportunity at the individual level, and then we have aggregated this measure into a measure of societal inequality of opportunity. In both stages of aggregation, we have adopted the opportunity egalitarian approach and in particular the opportunity gap methodology proposed by Fleurbaey & Schokkaert (2009). As the opportunity gap is measured as a distance between each individual's outcome and a benchmark outcome, we have proposed two benchmark distributions that endorse, respectively, the ex ante end the ex post versions of the opportunity egalitarian principles; for the sake of comparison, we have also proposed one benchmark distribution that endorses standard outcome egalitarian principles. We have then shown that for the latter case our measure is equivalent to the negative of the Mean Log Deviation and, thus, that our measure can be interpreted as an intertemporal generalization of a standard measures of outcome inequality.

Last, we have applied our measurement tool to study inequality of opportunity in South Korea, by using the KLIPS dataset, a rich source of data on the Korean population, which provides information not only on individuals' standard of livings but also on a different set circumstances and for a considerable number of years. Our analysis shows that although South Korea is known as one of the most growing and progressive countries, it still suffers from some degree of unfairness. However, the country seems to be on the right path for improving equality of opportunity over time. Indeed, South Korea fared well in dealing with the global financial crisis since, over time, the country did not worsen intertemporal fairness. This trend is clear when implementing the proposed intertemporal approach and results to be less neat when looking at each single year. Moreover, the intense South Korea's

GDP growth results to have been opportunity inequality improving for the new generations which receive a fairer remuneration of effort.

The proposed norm-based approach to assess intertemporal EOp shows interesting features that will allow the empirical literature to deepen the EOp assessment by looking at the intertemporal perspective. Thus, this paper opens room for new empirical assessment of intertemporal EOp in other countries. From a purely normative perspective, we should emphasize the role of the income generating function. Changing our assumptions on the income generating process would impact both the definition and the measurement of EOp generating new normative issues that will be addressed in future research.

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APPENDIX A

Norm distributions: additional results

A.1 - Link between the ex ante norm and the between group inequality

In Section 3 we claim that Eq. 3 and Eq. 4 coincide. In this appendix we better discuss this equivalence in a more general framework. Let n be the number of types and $m(j)$ the number of individuals in type j. Then, we can rewrite the claim as follows. If

$$A = x_{jk,t} + \frac{x_{jk,t}}{\sum_{i=1}^{m(j)} x_{jk,t}} \left(\frac{1}{n} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t} - \sum_{k=1}^{m(j)} x_{jk,t} \right)$$

and

$$B = x_{jk,t} \frac{\frac{1}{\sum_{h=1}^{n} m(h)} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t}}{\frac{1}{m(j)} \sum_{k=1}^{m(j)} x_{jk,t}}$$

then $A = B$.

After some rearrangement and simplification of the equation $A = B$ we obtain

$$\frac{1}{n} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t} = \frac{m(j)}{\sum_{h=1}^{n} m(h)} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t}$$

If types have the same number of individuals we can set $m(j) = m$ for all $j \in \{1, .., n\}$ and obtain the desired equality.

If the number of agent in each type is different, then we need to control for such heterogeneity across types by considering expected rather than total incomes in A. Hence, we redefine

$$A = x_{jk,t} + \frac{x_{jk,t}}{\frac{1}{m(j)}\sum_{i=1}^{m(j)} x_{jk,t}} \left(\frac{1}{\sum_{h=1}^{n} m(h)} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t} - \frac{1}{m(j)} \sum_{k=1}^{m(j)} x_{jk,t} \right)$$

After similar simplifications and rearrangements we get

$$\frac{1}{\sum_{h=1}^{n} m(h)} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t} = \frac{1}{\sum_{h=1}^{n} m(h)} \sum_{h=1}^{n} \sum_{k=1}^{m(h)} x_{hk,t}$$

as desired.

We can hence conclude that the combination of ex ante utilitarian compensation and proportional reward in Section 3 offers a normative justification for the standard income rescaling that removes within group inequality. This rescaling is equivalent to a redistributive policy that equalizes the values of the opportunity set of each group and preserves the relative position of each individual in the within group ranking.

A.2 - Sign of the social intertemporal equality of opportunity

In this section we look at the sign of Eq. 8 that we rewrite as

$$\Gamma = \frac{1}{N} \sum_{i=1}^{N} \sigma \left(\sum_{t=1}^{T} \alpha(t,T)\, g\left(\frac{x_{it}}{\tilde{x}_{it}} \right) \right) \tag{10}$$

with g and σ increasing and concave and such that $g(1) = \sigma(0) = 0$.

Claim: Equation 10 is never positive.

Proof. We begin this proof by recalling the following property of concave functions.

Remark- Let f be a concave function and let $\lambda_1, ..., \lambda_N$ be non-negative real numbers such that $\lambda_1 + ... + \lambda_N = 1$, then

$$f(\lambda_1 x_1 + ... + \lambda_N x_N) \geq \lambda_1 f(x_1) + ... + \lambda_N f(x_N)$$

Therefore, by concavity of g, and monotonicity and σ we have

$$\frac{1}{N}\sum_{i=1}^{N} f\left(\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right) = \frac{1}{N}\sum_{i=1}^{N}\sigma\left(g\left(\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right)\right) \geq \Gamma$$

for $f = \sigma \circ g$ increasing, concave and such that $f(1) = 0$. We therefore have an upper-bound for Γ so that our equality index is always negative if

$$\frac{1}{N}\sum_{i=1}^{N} f\left(\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right) \leq 0$$

Notice now that, by concavity of f we have

$$\frac{1}{N}\sum_{i=1}^{N} f\left(\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right) \leq f\left(\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right)$$

Therefore, we need to check

$$f\left(\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}}\right) \leq 0$$

which holds if and only if

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{x_{it}}{\tilde{x}_{it}} \leq 1$$

We can rearrange the previous inequality as

$$\sum_{t=1}^{T}\alpha\left(t,T\right)\frac{1}{N}\sum_{i=1}^{N}\frac{x_{it}}{\tilde{x}_{it}} \leq 1$$

We claim that

$$\frac{1}{N}\sum_{i=1}^{N}\frac{x_{it}}{\tilde{x}_{it}} = 1 \tag{11}$$

for all $t \in \{1, \ldots, T\}$.

If this is true, since $\sum_{t=1}^{T}\alpha\left(t,T\right) = 1$ the condition will hold with equality. Therefore, we need to check Eq. 11 for a generic period t. To simplify the notation, we can omit the subscript t in the rest the proof and rewrite Eq. 11 as

$$\frac{1}{N}\sum_{i=1}^{N}\frac{x_i}{\tilde{x}_i} = 1 \tag{12}$$

We will analyse separately the three norm implemented in the empirical application.

Egalitarian

Let $\tilde{x}_i = \mu$ for all $i \in \{1,..,N\}$, where μ is the average income. Then Eq. 12 becomes

$$\frac{1}{N}\sum_{i=1}^{N}\frac{x_i}{\mu} = \sum_{i=1}^{N}\frac{x_i}{\sum_{i=1}^{N} x_i} = 1$$

as desired.

Ex post

Let us consider the more general case in which there may be a non-fixed number of individuals in each tranche. Let us indicate with $n(k)$ the number of individuals that, at the considered period, are exerting effort e_k with $k \in \{1,..,m\}$. Of course, $\sum_{k=1}^{m} n(k) = N$.

The norm distribution for the ex post approach is such that for each tranche $k \in \{1,...,m\}$ and $j \in \{1,...,n(k)\}$ we have

$$\tilde{x}_{jk} = \frac{1}{n(k)}\sum_{j=1}^{n(k)} x_{jk} = \mu_k$$

Therefore, Eq. 12 becomes

$$\frac{1}{\sum_{k=1}^{m} n(k)}\sum_{k=1}^{m}\sum_{j=1}^{n(k)}\frac{x_{jk}}{\mu_k} = 1$$

Notice that

$$\frac{1}{\sum_{k=1}^{m} n(k)} \sum_{k=1}^{m} \sum_{j=1}^{n(k)} \frac{x_{jk}}{\mu_k} = \frac{1}{\sum_{k=1}^{m} n(k)} \sum_{k=1}^{m} \frac{n(k)}{\sum_{j=1}^{n(k)} x_{jk}} \sum_{j=1}^{n(k)} x_{jk} = 1$$

as desired.

Ex ante

Let us consider the more general case in which there may be a variable number of individuals in each type. Let us indicate with $m(j)$ the number of individuals that at the considered period have circumstances c_j with $j \in \{1, .., n\}$. Of course, $\sum_{j=1}^{n} m(j) = N$.
The norm distribution for the ex ante approach is such that, for each individual $k \in \{1, .., m(j)\}$ in type $j \in \{1, .., n\}$ we have

$$\tilde{x}_{jk} = x_{jk} \frac{\mu}{\mu_j}$$

where μ is the average total income and μ_j is the average income of type j. Therefore, Eq. 12 becomes

$$\frac{1}{\sum_{j=1}^{n} m(j)} \sum_{j=1}^{n} \sum_{k=1}^{m(j)} \frac{x_{jk}}{x_{jk} \frac{\mu}{\mu_j}} = 1$$

Notice that

$$\frac{1}{\sum_{j=1}^{n} m(j)} \sum_{j=1}^{n} \sum_{k=1}^{m(j)} \frac{x_{jk}}{x_{jk} \frac{\mu}{\mu_j}} = \frac{1}{\sum_{j=1}^{n} m(j)\mu} \sum_{j=1}^{n} \sum_{k=1}^{m(j)} \mu_j = \frac{1}{\sum_{j=1}^{n} m(j)\mu} \sum_{j=1}^{n} m(j)\mu_j = 1$$

as desired. Indeed, $\sum_{j=1}^{n} m(j)\mu$ is the product between the number of individuals and the average income, hence it coincides with the total income. Moreover, $\sum_{j=1}^{n} m(j)\mu_j$ is a weighted sum of the average type incomes, with weights that depend on the number of individuals in the type; hence it coincides with the total income. □

APPENDIX B

Proofs

Proposition 1

Proof. We begin the proof by stating the following lemma.

Lemma 1. *For all* $x_{it}, x_{jt} \in X$ *and* $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}$, $d : \mathbb{R}^2_+ \to \mathbb{R}$ *satisfies Gap Monotonicity (GM) and Gap Scale invariance (GS) if and only if*

$$d\left(x_{it}, \tilde{x}_{it}\right) \geq d\left(x_{jt}, \tilde{x}_{jt}\right) \iff \frac{x_{it}}{\tilde{x}_{it}} \geq \frac{x_{jt}}{\tilde{x}_{jt}}$$

Proof Lemma 1. By Gap Scale invariance (GS)

$$d\left(x_{it}, \tilde{x}_{it}\right) = d\left(\frac{x_{it}}{\tilde{x}_{it}}, 1\right)$$

and

$$d\left(x_{jt}, \tilde{x}_{jt}\right) = d\left(\frac{x_{jt}}{\tilde{x}_{jt}}, 1\right)$$

Therefore,

$$d\left(x_{it}, \tilde{x}_{it}\right) \geq d\left(x_{jt}, \tilde{x}_{jt}\right) \iff d\left(\frac{x_{it}}{\tilde{x}_{it}}, 1\right) \geq d\left(\frac{x_{jt}}{\tilde{x}_{jt}}, 1\right)$$

By Gap Monotonicity (GM),

$$d\left(\frac{x_{it}}{\tilde{x}_{it}}, 1\right) \geq d\left(\frac{x_{jt}}{\tilde{x}_{jt}}, 1\right) \iff \frac{x_{it}}{\tilde{x}_{it}} \geq \frac{x_{jt}}{\tilde{x}_{jt}}$$

□

Following Lemma 1, there must exist an increasing function $g : \mathbb{R}_+ \to \mathbb{R}$ such that, for all $x_{it} \in X$ and $\tilde{x}_{it} \in \tilde{X}$, $d\left(x_{it}, \tilde{x}_{it}\right) := g(y)$ for $y = (x_{it}/\tilde{x}_{it})$.

By Gap Normalization (GN), g has to be such that $g\left(x_{it}/\tilde{x}_{it}\right)=0$ if and only if $x_{it}=\tilde{x}_{it}$.

By Gap Asymmetry (GA), for all $x_{it}, x_{jt} \in X$ and $\tilde{x}_{it}, \tilde{x}_{jt} \in \tilde{X}$, if $x_{it} \geq \tilde{x}_{it}=\tilde{x}_{jt} \geq x_{jt}$ and $x_{it}-\tilde{x}_{it}=\tilde{x}_{jt}-x_{jt}$, then

$$d\left(x_{it}, \tilde{x}_{it}\right)-d\left(\tilde{x}_{it}, \tilde{x}_{it}\right) \leq d\left(\tilde{x}_{jt}, \tilde{x}_{jt}\right)-d\left(x_{jt}, \tilde{x}_{jt}\right)$$

Let us set $\tilde{x}_{it}=\tilde{x}_{jt}=\tilde{x}$ and $x_{it}-\tilde{x}=\tilde{x}-x_{jt}=\delta \in \mathbb{R}_{+}$. Applying Lemma 1 we can rewrite this inequality as follows

$$g\left(\frac{x_{it}}{\tilde{x}}\right)-g\left(\frac{\tilde{x}}{\tilde{x}}\right) \leq g\left(\frac{\tilde{x}}{\tilde{x}}\right)-g\left(\frac{x_{jt}}{\tilde{x}}\right)$$

$$g\left(\frac{\tilde{x}+\delta}{\tilde{x}}\right)-g\left(\frac{\tilde{x}}{\tilde{x}}\right) \leq g\left(\frac{\tilde{x}}{\tilde{x}}\right)-g\left(\frac{\tilde{x}-\delta}{\tilde{x}}\right)$$

only if g is also concave.

□

Proposition 2

Proof. We begin this proof by stating the following lemma proved in Hoy & Zheng (2011).

Lemma 2. *For any twice differentiable function $f(\mathbf{x})$ with $\mathbf{x} \in \mathbb{R}^n$, the conditions $\frac{\partial f(\mathbf{x})}{\partial x_i \partial x_j}=0$ for all $i \neq j \in \{1, \ldots, n\}$ are satisfied if and only if $f(\mathbf{x})$ can be written as*

$$f(\mathbf{x})=f_1(x_1)+f_2(x_2)+\ldots+f_n(x_n)$$

for some twice differentiable functions $f_1, f_2 \ldots f_n$.

(If)

It is easy to see that Eq. 6 satisfies intertemporal normalization (IN), intertemporal monotonicity (IM) and intertemporal independence (II).

(Only if)

Following Lemma 2, independence (II) implies that there exist some twice differentiable functions $f_1, f_2 \dots f_T$ such that, for all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$,

$$\gamma(\mathbf{g}_{i\cdot}) = f_1(g_{i1}) + f_2(g_{i2}) + \ldots + f_T(g_{iT}) \tag{13}$$

By Proposition 1, for all $g_{it} \in G$, $g_{it} = g(x_{it}/\tilde{x}_{it})$, for a twice differentiable, strictly increasing and concave $g : \mathbb{R}_+ \to \mathbb{R}$, $x_{it} \in X$ and $\tilde{x}_{it} \in \tilde{X}$.

Let $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ be such that $\mathbf{g}_{i\cdot} = (e, e, \ldots, e)$, with $e \in \mathbb{R}$. Then, there must exist $\delta \in \mathbb{R}$ such that $g(\delta) = e$. By normalization (IN), we can rewrite Eq. 13 as

$$\gamma(\mathbf{g}_{i\cdot}) = \sum_{1 \le t \le T} f_t(g(\delta)) = e \tag{14}$$

By totally differentiating Eq. 14 with respect to δ we have

$$\sum_{1 \le t \le T} f_t' \, g' = 0 \tag{15}$$

Notice that $g' > 0$. Moreover, by monotonicity (IM), it must be $f_t' \geq 0$ for all $t \in \{1, \ldots, T\}$. [20] Therefore, for Eq. 15 to hold, it must be $f_t' = 0$ for all $t \in \{1, \ldots, T\}$. In other words, for all $t \in \{1, \ldots, T\}$, f_t is a real number that does not depend on the value of the function g.

Remark - Let $v \in \mathbb{R}$ be a given real number then, for all $\beta \in \mathbb{R}$, we can write $\beta = \omega v$, for a given $\omega \in \mathbb{R}$.

Since for all $t \in \{1, \ldots, T\}$, $f_t(g_{it})$ is a real number, there must exist a list of real numbers $\omega_1, \omega_2, \ldots, \omega_T$ such that, for all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$

$$\gamma(\mathbf{g}_{i\cdot}) = \sum_{1 \le t \le T} \omega_t \, g_{it} \tag{16}$$

Moreover, if $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$ is such that $\mathbf{g}_{i\cdot} = (e, e, \ldots, e)$, with $e \in \mathbb{R}$, then Eq. 16 becomes

[20] Assume not, i.e. there exist f_p, $p \in \{1, \ldots, T\}$, such that $f_p' < 0$. Let $\mathbf{g}_{i\cdot}, \mathbf{g}_{j\cdot}$ be such that $g_{ik} = g_{jk}$ for all $k \neq p \in \{1, \ldots, T\}$ and $g_{ip} > g_{jp}$. Then, applying Eq. 13 we have $\gamma(\mathbf{g}_{i\cdot}) < \gamma(\mathbf{g}_{k\cdot})$. A contradiction to monotonicity (M).

$$\sum_{1\leq t\leq T}\omega_t\, e = e \tag{17}$$

Notice that Eq. 17 holds if and only if

- $\sum_{t=1}^{T}\omega_t = 1$ - this condition can be obtained by dividing both sides of Eq. 17 by e:
- and $\omega_t \in (0,1)$ for all $t \in \{1,\ldots,T\}$ - this condition is obtained by observing that:
  - $\omega_t > 1$ implies $\sum_{t\in T}\omega_t > 1$. A contradiction.
  - $\omega_t < 0$ contradicts monotonicity (IM). Indeed, we would have $\frac{\partial\gamma(\mathbf{g}_{i\cdot})}{\partial g_{it}} = \omega_t < 0$. A contradiction to (IM).
  - $\omega_t = 0$ contradicts normalization (IN). Indeed, if $T = 1$, then $\gamma(\delta) = \omega_1\delta = 0$, for all $\delta \in \mathbb{R}_{++}$. A contradiction to (IN).
  - $\omega_t = 1$ is possible only for $T = 1$.

Finally, observe that the list of real numbers $\omega_1, \omega_2, \ldots, \omega_T$ can be expressed as a list of constant twice differentiable functions $\omega_1(1), \omega_2(2), \ldots, \omega_T(T)$. □

Corollary 3.1

Notice that, for all $\mathbf{g}_{i\cdot} \in \mathbb{R}^T$,

$$\sum_t \omega_t(t)\, g_{it} = \sum_t \omega_t(t)\cdot\sum_t \frac{\omega_t(t)\, g_{it}}{\sum_t \omega_t(t)} = \sum_t \omega_t(t)\cdot\sum_t\left(\frac{\omega_t(t)}{\sum_t \omega_t(t)}\cdot g_{it}\right)$$

with $1 \leq t \leq T$. Since $\sum_t \omega_t(t) = 1$, we can rewrite Eq. 6 as

$$\gamma(\mathbf{g}_i) = \sum_{t=1}^{T}\alpha(t,T) g_{it}$$

with $\alpha(t,T) = \frac{\omega_t(t)}{\sum_{t=1}^{T}\omega_t(t)} \in (0,1)$.

Proposition 3

Proof. It is easy to check this proposition in the parts that define α increasing or decreasing. We only need to prove its concavity by (GP). Set $g_{is} = g_{iu} = x < 0$. Opportunity gap persistence axiom requires

$$\alpha(s,T)x + \alpha(u,T)x \leq \alpha(r,T)x + \alpha(v,T)x$$

dividing by $x < 0$ and rearranging

$$\alpha(u,T) - \alpha(v,T) \geq \alpha(r,T) - \alpha(s,T)$$

since $s - r = v - u = k$

$$\alpha(u,T) - \alpha(u+k,T) \geq \alpha(s-k,T) - \alpha(s,T)$$

with $u \geq s$, the inequality is satisfied if and only if $\alpha(t,T)$ is concave in t. □

Proposition 4

Proof. (If)

It is easy to see that Eq. 8 satisfies aggregate normalization (AN), aggregate monotonicity (AM), population invariance (P), decomposability (D), anonymity (A) and aggregate Pigou-Dalton (APD).

(Only if)

Following Lemma 2, decomposability (D) implies the existence of some twice differentiable functions $f_1, f_2 \ldots f_N$ such that, for all $\mathbf{s} \in \mathbb{R}^N$,

$$\Gamma(\mathbf{s}) = f_1(s_1) + f_2(s_2) + \ldots + f_N(s_N) \tag{18}$$

Anonymity (A) imposes that, for all $z \in \mathbb{R}$, we must have $f_i(z) = f_j(z)$ for all $i, j \in \{1, \ldots, N\}$. Therefore, we can rewrite Eq. 18 as

$$\Gamma(\mathbf{s}) = \sum_{i=1}^{N} f(s_i) \tag{19}$$

for a twice differentiable function $f : \mathbb{R} \to \mathbb{R}$ and $s_i \in \mathbf{s}$. Aggregate monotonicity (AM) implies $f' \geq 0$, aggregate Pigou-Dalton (APD) implies $f'' \leq 0$ and aggregate normalization (AN) implies $f(0) = 0$.

Since, for all $s_i \in \mathbf{s}$, $f(s_i) \in \mathbb{R}$, we can rewrite $f(s_i) = \beta\sigma(s_i)$, for a given twice differentiable, increasing and concave $\sigma : \mathbb{R} \to \mathbb{R}$ and $\beta \in \mathbb{R}$. We can set $\beta = \frac{1}{N}$, so that we can write:

$$\Gamma(\mathbf{s}) = \frac{1}{N} \sum_{i=1}^{N} \sigma(s_i) \tag{20}$$

Therefore, by population invariance (P), if $\mathbf{z} = (\mathbf{s}, \mathbf{s}, \ldots, \mathbf{s}) \in \mathbb{R}^{pN}$, then

$$\Gamma(\mathbf{z}) = \frac{1}{pN} \sum_{i=1}^{pN} \sigma(z_i) \tag{21}$$

Assume not, i.e.

$$\Gamma(\mathbf{z}) = \frac{1}{\alpha} \sum_{i=1}^{pN} \sigma(z_i)$$

for $\alpha \neq pN$. Then, by construction of $\mathbf{z}$, we can rewrite

$$\Gamma(\mathbf{z}) = \frac{1}{\alpha} p \sum_{i=1}^{N} \sigma(s_i)$$

with $s_i \in \mathbf{s}$. Population invariance (P) imposes $\Gamma(\mathbf{z}) = \Gamma(\mathbf{s})$, satisfied if and only if

$$\frac{1}{\alpha} p \sum_{i=1}^{N} \sigma(s_i) = \frac{1}{N} \sum_{i=1}^{N} \sigma(s_i)$$

which holds if and only if $\alpha = pN$. Therefore, if $\alpha \neq pN$, then $\Gamma(\mathbf{z}) \neq \Gamma(\mathbf{s})$. A contradiction.

□

APPENDIX C

Descriptive statistics

Table 5: Descriptive statistics

Year	Mean Income	Gini Coefficient	Individuals with parental support (%)
2001	855.71	0.3801	18.41
2002	1,041.78	0.3583	16.64
2003	1,071.81	0.3654	19.48
2004	1,151.41	0.3795	22.94
2005	1,183.96	0.3667	22.01
2006	1,304.48	0.3758	22.70
2007	1,365.33	0.3640	24.94
2008	1,391.66	0.3637	22.96
2009	1,326.76	0.3591	20.02
2010	1,439.10	0.3566	16.44
2011	1,465.08	0.3461	16.80
2012	1,489.93	0.3261	17.51
2013	1,530.43	0.3251	22.12
2014	1,606.73	0.3314	19.66
Female (%)	55.54		
Parents with no/elementary education (%)	58.07		
Parents with medium/secondary education (%)	34.52		
Parents with higher education (%)	7.41		
People born in Seul (%)	9.79		
People born in other metropolitan city (%)	12.54		
People born in other province or outside Korea (%)	77.66		
Sample size	3,061		
People in younger cohort (%)	57.50		

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: The top part of the table reports for each year the sample mean income and Gini coefficient. It also reports the percentage of individuals that receive a material and/or a financial support from parents, which is the only circumstances that can vary over time. The bottom part of the table reports information of the sample characteristics that do not change over time.

APPENDIX D

Intertemporal fairness using higher degrees of gap asymmetry

Table 6: Intertemporal fairness, 2001-14

	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2938	-0.0090	-0.0773
$\Gamma_{1.5,Lin}$	-0.4547	-0.0135	-0.1141
$\Gamma_{1,EG}$	-0.2722	-0.0083	-0.0622
$\Gamma_{1.5,EG}$	-0.4200	-0.0124	-0.0911
$\Gamma_{1,LG}$	-0.3144	-0.0098	-0.0911
$\Gamma_{1.5,LG}$	-0.4910	-0.0146	-0.1363

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 7: Intertemporal fairness, 2001-07 vs 2008-14

	2001-07		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.3496	-0.0117	-0.1154
$\Gamma_{1.5,Lin}$	-0.5654	-0.0176	-0.1802
$\Gamma_{1,EG}$	-0.3364	-0.0117	-0.1047
$\Gamma_{1.5,EG}$	-0.5421	-0.0176	-0.1633
$\Gamma_{1,LG}$	-0.3613	-0.0116	-0.1286
$\Gamma_{1.5,LG}$	-0.5931	-0.0176	-0.1981
	2008-14		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.3280	-0.0063	-0.0391
$\Gamma_{1.5,Lin}$	3715	-0.0096	-0.0582
$\Gamma_{1,EG}$	-0.2282	-0.0061	-0.0340
$\Gamma_{1.5,EG}$	-0.3558	-0.0093	-0.0503
$\Gamma_{1,LG}$	-0.2472	-0.0065	-0.0438
$\Gamma_{1.5,LG}$	-0.3880	-0.0099	-0.0661

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 8: Intertemporal fairness, by cohorts

Younger Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2729	0.0014	-0.0679
$\Gamma_{1.5,Lin}$	-0.4236	0.0010	-0.1011
$\Gamma_{1,EG}$	-0.2404	0.0002	-0.0537
$\Gamma_{1.5,EG}$	-0.3730	-0.0005	-0.0794
$\Gamma_{1,LG}$	-0.3057	0.0026	-0.0812
$\Gamma_{1.5,LG}$	-0.4779	0.0027	-0.1228
Older Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,Lin}$	-0.2022	-0.0260	-0.0926
$\Gamma_{1.5,Lin}$	-0.3211	-0.0372	-0.1353
$\Gamma_{1,EG}$	-0.1576	-0.0221	-0.0761
$\Gamma_{1.5,EG}$	-0.2537	-0.0317	-0.1104
$\Gamma_{1,LG}$	-0.2454	-0.0300	-0.1073
$\Gamma_{1.5,LG}$	-0.3894	-0.0430	-0.1584

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{2.5}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 9: Intertemporal fairness, 2001-14

	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.5,Lin}$	-0.1704	-0.0052	-0.0440
$\Gamma_{1.5,EG}$	-0.1577	-0.0047	-0.0354
$\Gamma_{1.5,LG}$	-0.1828	-0.0056	-0.0521

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{10}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 10: Intertemporal fairness, 2001-07 vs 2008-14

		2001-07	
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.5,Lin}$	-0.2053	-0.0067	-0.0670
$\Gamma_{1.5,EG}$	-0.1974	-0.0067	-0.0607
$\Gamma_{1.5,LG}$	-0.2130	-0.0067	-0.0728
		2008-14	
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.5,Lin}$	-0.1386	-0.0037	-0.0224
$\Gamma_{1.5,EG}$	-0.1327	-0.0035	-0.0194
$\Gamma_{1.5,LG}$	-0.1442	-0.0038	-0.0252

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{10}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 11: Intertemporal fairness, by cohorts

Younger Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.5,Lin}$	-0.6153	0.0006	-0.0388
$\Gamma_{1.5,EG}$	-0.7480	-0.0001	-0.0306
$\Gamma_{1.5,LG}$	-0.4790	0.0012	-0.0457
Older Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.5,Lin}$	-0.5738	-0.0146	-0.0526
$\Gamma_{1.5,EG}$	-0.6988	-0.0125	-0.0431
$\Gamma_{1.5,LG}$	-0.4440	-0.0170	-0.0611

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed on a measure of individual fairness at time t given by $log_{10}\left(\frac{x_{it}}{\tilde{x}_{it}}\right)$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

APPENDIX E

Intertemporal fairness using a higher degree of time concern

Table 12: Intertemporal fairness, 2001-14

	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,EG}$	-0.2584	-0.0082	-0.0632
$\Gamma_{1.5,EG}$	-0.3956	-0.0123	-0.0924
$\Gamma_{1,LG}$	-0.2792	-0.0079	-0.0774
$\Gamma_{1.5,LG}$	-0.4299	-0.0117	-0.1145

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed using $\alpha = \sqrt[4]{\frac{T-t+1}{T}}$ for $\Gamma_{1,EG}$ and $Gamma_{1.5,EG}$ and $\alpha = \sqrt[4]{\frac{t}{T}}$ for $\Gamma_{1,LG}$ and $Gamma_{1.5,LG}$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 13: Intertemporal fairness, 2001-07 vs 2008-14

2001-07			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,EG}$	-0.3137	-0.0107	-0.1003
$\Gamma_{1.5,EG}$	-0.4987	-0.0160	-0.1547
$\Gamma_{1,LG}$	-0.3259	-0.0106	-0.1102
$\Gamma_{1.5,LG}$	-0.5224	-0.0159	-0.1713
2008-14			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,EG}$	-0.2137	-0.0057	-0.0334
$\Gamma_{1.5,EG}$	-0.3297	-0.0086	-0.0492
$\Gamma_{1,LG}$	-0.2224	-0.0059	-0.0380
$\Gamma_{1.5,LG}$	-0.3447	-0.0089	-0.0567

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed using $\alpha = \sqrt[4]{\frac{T-t+1}{T}}$ for $\Gamma_{1,EG}$ and $Gamma_{1.5,EG}$ and $\alpha = \sqrt[4]{\frac{t}{T}}$ for $\Gamma_{1,LG}$ and $Gamma_{1.5,LG}$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 14: Intertemporal fairness, by cohorts

Younger Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,EG}$	-0.2853	0.0008	-0.0551
$\Gamma_{1.5,EG}$	-0.4346	0.0002	-0.0813
$\Gamma_{1,LG}$	-0.3019	0.0019	-0.0686
$\Gamma_{1.5,LG}$	-0.4629	0.0018	-0.1024
Older Cohort			
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1,EG}$	-0.2023	-0.0203	-0.0697
$\Gamma_{1.5,EG}$	-0.3135	-0.0290	-0.1009
$\Gamma_{1,LG}$	-0.2538	-0.0274	-0.0983
$\Gamma_{1.5,LG}$	-0.3952	-0.0393	-0.1445

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Aggregated indexes constructed using $\alpha = \sqrt[4]{\frac{T-t+1}{T}}$ for $\Gamma_{1,EG}$ and $Gamma_{1.5,EG}$ and $\alpha = \sqrt[4]{\frac{t}{T}}$ for $\Gamma_{1,LG}$ and $Gamma_{1.5,LG}$. Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

APPENDIX F

Intertemporal fairness using a higher degree of unfairness aversion

Table 15: Intertemporal fairness, 2001-14

	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.75,Lin}$	-0.4895	-0.0142	-0.1195
$\Gamma_{1.75,EG}$	-0.4511	-0.0130	-0.0949
$\Gamma_{1.75,LG}$	-0.5322	-0.0154	-0.1440

Source: Authors' elaborations based on KLIPS, 2001-2014.

Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 16: Intertemporal fairness, 2001-07 vs 2008-14

	2001-07		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.75,Lin}$	-0.6299	-0.0186	-0.1962
$\Gamma_{1.75,EG}$	-0.6023	-0.0187	-0.1779
$\Gamma_{1.75,LG}$	-0.6694	-0.0185	-0.2183
	2008-14		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.75,Lin}$	-0.4015	-0.0102	-0.0612
$\Gamma_{1.75,EG}$	-0.3840	-0.0099	-0.0526
$\Gamma_{1.75,LG}$	-0.4208	-0.0105	-0.0702

Source: Authors' elaborations based on KLIPS, 2001-2014.
Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).

Table 17: Intertemporal fairness, by cohorts

	Younger Cohort		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.75,Lin}$	-0.5303	0.0006	-0.1064
$\Gamma_{1.75,EG}$	-0.4985	-0.0010	-0.0830
$\Gamma_{1.75,LG}$	-0.5677	0.0022	-0.1306
	Older Cohort		
	Egalitarian	**Ex ante**	**Ex post**
$\Gamma_{1.75,Lin}$	-0.3545	-0.0341	-0.1235
$\Gamma_{1.75,EG}$	-0.3135	-0.0290	-0.1009
$\Gamma_{1.75,LG}$	-0.3952	-0.0393	-0.1445

Source: Authors' elaborations based on KLIPS, 2001-2014.
Note: Egalitarian refers to the index computed using the egalitarian benchmark; Ex ante EOp refers to the index computed using the ex ante benchmark (see Section 3); Ex post refers to the index compute using the ex post benchmark (see Section 3).