Chemical control of convective instabilities

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Abstract
By modifying a physical property of a solution like its density or viscosity, chemical reactions are able to influence and even trigger convective motions. These flows in turn affect the spatiotemporal distribution of the chemical species. A usually non trivial coupling between reactions and flows is then setting in. We present here simple model systems allowing to understand and analyze this chemo-hydrodynamic coupling. We illustrate in particular the possibility for chemical reactions to control or trigger viscous fingering, Rayleigh-Taylor, double diffusive, and convective dissolution instabilities. We discuss laboratory experiments performed to study these phenomena and compare the experimental results to theoretical predictions. We insist in each case on the specificities of the chemo-hydrodynamic patterns and instabilities with regard to those that develop in non reactive systems and unify the different dynamics in terms of the common features of the related spatial mobility profiles.
1. INTRODUCTION

Hydrodynamic instabilities of miscible interfaces are encountered in lots of applications where two fluids are put in contact. They typically develop when gradients of a physical property is present across the interface. As an example, a Rayleigh-Taylor instability can deform the miscible contact line into alternating rising and sinking fingers when a denser solution is put on top of a less dense one in the gravity field. Similarly, a viscous fingering instability can develop when a less viscous fluid displaces a more viscous one in a porous medium. Such instabilities have been much studied both experimentally and theoretically and are well understood nowadays. If the solutions at hand contain chemicals that react such that this reaction changes the density or viscosity in situ, an interplay between reactions and hydrodynamics can come into play (De Wit et al. (2012), De Wit (2016)). We will describe here the major influence that reactions can have on such instabilities. In particular, we will explain how, by changing the physical property controlling the hydrodynamic flow, reactions can be used to control the location and amplitude of the convective motions. As a corollary, flows can also be used to tune the yield and spatio-temporal distribution of the chemicals. This control over the pattern formation in reactive fluids sets up the basis of chemo-hydrodynamic pattern selection at the heart of this review.

Of course, the subject is very vast and flows of reactive fluids are encountered in numerous applications ranging from convective motions in stars or in planet interiors, atmospheric chemistry, engineering applications like combustion, blooming of bacteria in oceanic currents and many more. We will necessarily have to make a choice. Dictated by our own research in the field, we will focus here mainly on reactive interfaces in porous media flows but will, as much as possible, point towards the generality of the chemo-hydrodynamic control described. To do so, we will first introduce the mobility profile that is at the heart of the viscously and buoyancy-driven instabilities we will discuss and the influence of chemical reactions on them. Next, we will describe briefly the properties of the hydrodynamic viscous fingering, Rayleigh-Taylor and double-diffusive instabilities in absence of any reaction to be able to appreciate in a second part the effect of reactions on their dynamics.
2. REACTIVE INTERFACE IN A POROUS MEDIUM

Without any loss of generality, we will consider here an interface between two semi-infinite regions in a homogeneous porous medium with constant permeability $\kappa$. In this system, a solution of reactant $A$ with initial concentration $a_0$ is put in contact along a contact line at initial time with a solution of another reactant $B$ with initial concentration $b_0$. The two solutions are both considered diluted and to have their own viscosity $\mu$ and density $\rho$. In absence of any flow, the two reactants meet by diffusion and a simple chemical reaction

$$A + B \rightarrow C$$

(1)
takes place in the mixing zone, generating the product $C$. We seek to understand how this reactive two-layer stratification can be destabilized by viscosity or density gradients and how the reaction can modify the properties of the hydrodynamic instability. For incompressible flows, the dynamics in porous media can be described by the following system of reaction-diffusion-convection (RDC) equations:

$$\nabla \cdot \mathbf{u} = 0,$$

(2)

$$\nabla p = -\frac{\mu}{\kappa} \mathbf{u} + \rho g,$$

(3)

$$\frac{\partial a}{\partial t} + \mathbf{u} \cdot \nabla a = D_A \nabla^2 a - kab,$$

(4)

$$\frac{\partial b}{\partial t} + \mathbf{u} \cdot \nabla b = D_B \nabla^2 b - kab,$$

(5)

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D_C \nabla^2 c + kab,$$

(6)

where $a, b$ and $c$ denote respectively the concentrations of the two reactants $A, B$ and of the product $C$, $k$ is the kinetic constant, $p$ is the pressure, $\mathbf{u}$ is the velocity field while $D_{A,B,C}$ are the diffusion coefficients of the species $A, B$ and $C$ respectively. Eq.(3) is Darcy’s law relating the velocity field $u$ to the gradient of pressure. The interplay between reactions and hydrodynamics arises thanks to the dependence of either viscosity $\mu$, density $\rho$ or permeability $\kappa$ on the concentrations. The model equations (2-6) need therefore to be complemented in each case by a state equation expressing this dependence, in addition to given initial and boundary conditions for all variables. This state equation will depend on the instability considered but common features can be sketched in terms of the relevant mobility profile.

3. MOBILITY PROFILE

The essence of the chemical control of convective motions lies in the control of the mobility profile at the origin of the hydrodynamic instability. The mobility profile $M(x)$ is here defined as the function describing the way the physical property $M$, which is the motor of the instability, varies along a given spatial coordinate $x$ (Homsy (1987), Manickam & Homsy (1995)). It is a direct consequence of the state equation. In the cases to be analyzed here, $M$ is typically the viscosity $\mu$, the density $\rho$ or the permeability $\kappa$. Figure 1 shows various profiles $M(x)$ that can develop in miscible systems when two different fluids or two solutions of a same solvent but different chemical composition are put in contact. As they usually have different densities or viscosities, there is a jump in this property across
the initial contact line. Upon diffusive mixing, $M$ typically expands as an error function. Figures 1a,b show two examples of such profiles at a given time after contact. Depending on the arbitrary choice of orientation of the $x$ axis, $M$ is then either decreasing (Figure 1a) or increasing (Figure 1b) along $x$. As will be shown later, the typical influence of reactions or of the presence of chemicals with different diffusion coefficients is to modify these reference mobility profiles either by changing the gradient of $M$ or by introducing extrema in $M$ (Figures 1c-f). We will review below how reactions can induce these changes in the mobility profile and what are their consequences on the dynamics. Before doing so, let us first review the hydrodynamic instabilities that can develop in presence of a monotonic mobility profile like those of Figure 1a,b.

Figure 1: Mobility profiles $M(x)$ giving the spatial dependence of a given physical property of the fluid like its viscosity or density as a function of space. (a) Monotonic decreasing along $x$; (b) Monotonic increasing along $x$; (c) Non-monotonic with a maximum; (d) Non-monotonic with a minimum; (e) Non-monotonic with two extrema with amplitude larger than the two end-points values of $M$; (f) Non-monotonic with two extrema with amplitude smaller than the two end-points values of $M$.

4. HYDRODYNAMIC INSTABILITIES

4.1. Viscous fingering

When a less viscous fluid displaces a more viscous one in a porous medium, the interface between the two is unstable towards viscous fingering inducing a fingered deformation of the miscible mixing zone (Saffman & Taylor (1958), Homsy (1987)). In that case, the typical unstable mobility profile is the one of viscosity such as in Figure 1b where the mobility $M$ is the dynamic viscosity $\mu$ and $x$ is the direction of injection. The reverse monotonic decreasing viscosity profile corresponding to a more viscous fluid displacing along $x$ a less viscous one (as in Figure 1a) is on the contrary stable.

4.2. Buoyancy-driven instabilities

When the mobility profile M is giving the spatial dependence of the density $\rho$, various instabilities can develop depending on the orientation of the interface between two different
fluids A and B with regard to the gravity field and on the diffusivities of the species involved. To connect to reactive solutions later, let us consider that, initially a solution of species A is put in contact along a line with a miscible solution of B. Both species contribute to change the density of the solution. If initially, the interface between the two miscible solutions is vertical, the mobility profiles where $\rho$ here are those of Figures 1a,b with $x$ pointing perpendicularly to the gravity field. The stratification always gives convection as the denser solution sinks below the other one, inducing a gravity current (Meiburg & Kneller (2010)).

A Rayleigh-Taylor (RT) instability develops across an initially horizontal interface when a denser fluid lies above a less dense one in the gravity field. If $x$ points downwards along the gravity field, then the unstable density profile in the non reactive case is the one of Figure 1a with $M = \rho$. Regular fingers develop then symmetrically across the interface (Wooding (1969), Fernandez et al. (2002)). However, even in the case of a stratification of less dense fluid on top of a denser one like in Figure 1b, an instability can develop due to double-diffusion (Turner (1979), Radko (2013)). If the lower solute B diffuses faster than the upper solute A, then a double-diffusive (DD) instability destabilizes the interface into density fingering similar to the one of the RT modes while a diffusive layer convection (DLC) mode giving disconnected localized convective zones both above and below the interface is obtained when the upper A diffuses faster than the lower B (Trevelyan et al. (2011), Carballido-Landeira et al. (2013)). In miscible solutions containing two different solutes A and B with different diffusivities, the RT and double-diffusive modes can interact giving rise for instance to specific mixed modes fingers where the difference in diffusion of the species deform the tip of the RT fingers into antennas (Carballido-Landeira et al. (2013)). Similarly, double diffusive effects can control the onset times and intensity of the convective velocity of RT modes (Gopalakrishnan et al. (2018)). All these effects will be at play in reactive systems where different solutes with different diffusion coefficients are involved in the reactions.

To appreciate the effect of simple bimolecular reactions on hydrodynamics, it is important to first understand the properties of reaction-diffusion fronts as their concentration profiles control the mobility profile at the heart of the hydrodynamics.

5. REACTION-DIFFUSION $A + B \rightarrow C$ CHEMICAL FRONTS

The properties of $A + B \rightarrow C$ reaction-diffusion (RD) fronts where the reactant solutions of A and B in respective concentrations $a_0$ and $b_0$, are put in contact at $t = 0$ at a location...
and react according to a bimolecular \( A + B \rightarrow C \) kinetics, have been thoroughly studied since the pioneering work of Gálfi & Rácz (1988). When A and B meet by diffusion, they react producing C in the miscible contact zone. Depending on the relative value of the diffusion coefficients \( D_A \) and \( D_B \) of reactants A and B and on the ratio \( \beta = b_o/a_o \) of their initial concentrations, the front can move in either directions. The reaction front position defined as the location where \( R \) is maximum stays at \( x = 0 \) if A and B have the same diffusion coefficient and \( \beta = 1 \) as shown on Figure 2a. This is related to the fact that the diffusive flux of A towards the reaction front is then the same as the flux of B. If however \( a_o^2 D_A \neq b_o^2 D_B \), these two fluxes differ and the front moves towards the region which has the smallest diffusive flux (Gálfi & Rácz (1988), Jiang & Ebner (1990), Gérard & De Wit (2009)). As an example, for equal diffusion coefficients, the more concentrated solution invades the other one (Figure 2b). On the other hand, when \( \beta = 1 \), the front invades the reactant of lowest diffusion coefficient (Figure 2c). We see thus that, depending on the initial concentrations and nature (and more specifically diffusion coefficient) of the reactants, the RD concentration profiles of reactants A and B and of the product C can evolve symmetrically (Figure 2a) with regard to the initial position of the front or, on the contrary, develop asymmetries (Figure 2b,c). This will, in turn affect the related mobility profiles if the chemical species have an active effect on density or viscosity. Note that, in flow conditions such that the reactants are passively advected, the properties of the RD fronts are recovered in a rectilinear geometry while, in radial geometries, the properties of the front can be tuned by varying the flow rate (Brau et al. (2017), Trevelyan & Walker (2018)). Let us now review what happens if the chemical species are actively changing the flow, starting with the effect on viscosity before addressing the changes in density.

### 6. HYDRODYNAMIC INSTABILITIES IN REACTIVE FLUIDS

#### 6.1. Viscous fingering in reactive systems

In absence of any reaction, viscous fingering (VF) develops when a less viscous fluid displaces a more viscous one in a porous medium. This instability has been thoroughly studied both experimentally and theoretically because, among others, of its ubiquity in oil recovery when a fluid like water or CO\(_2\) displaces the more viscous oil in the soils (Saffman & Taylor (1958), Homsy (1987)). In this context, reactions producing for instance surfactants in situ can modify the local surface tension and affect the Saffmann-Taylor instability between two immiscible fluids (Jahoda & Hornof (2000), Nasr-El-Din et al. (1990), Hornof & Baig (1995), Fernandez & Homsy (2003), Niroobakhsh et al. (2017), Tsuzuki et al. (2019)). We won’t review this particular case as we focus here on miscible systems. VF is also observed in chromatography, a separation technique by which a mixture of chemical components dissolved in a given solvent is separated via dispersion at different speeds in a porous matrix and/or selective adsorption on the solid phase (Guiochon et al. (2006)). If the carrier fluid has a different viscosity than the sample solvent, VF can appear, which is dramatic for the efficiency of separation (Dickson et al. (1997), Broyles et al. (1998), De Wit et al. (2005), Rousseaux et al. (2007, 2011)). In this context, it has been shown that adsorption on the porous matrix of components controlling the viscosity of the fluids can influence the instability (Mishra et al. (2007), Rana et al. (2014, 2018)). As an example, the onset time of the instability can, in some cases, depend nonmonotonically on the retention parameter of the solute adsorption (Hota et al. (2015)).

Note that several works have analyzed VF of miscible autocatalytic fronts able to form
a self-organized interface between the reactants and products of an autocatalytic front (De Wit & Homsy (1999b,a), Swernath & Pushpavanam (2007, 2008), Ghesmat & Azaiez (2009)). The reactions can modify the relative stability of the front and induce the formation of isolated droplets if the reaction is bistable.

The interplay of chemistry and VF occurs through the influence of reactions on viscosity. As, in Darcy’s law, the effect on mobility for viscous fingering comes into the mobility ratio $M = \kappa/\mu$ via changes in viscosity, it is logical that changes in the permeability $\kappa$ via dissolution or precipitation reactions affecting the pore space of the porous matrix will also be able to trigger fingering. This has long been known in studies on infiltration instabilities, where the invading fluid reacts with the solid matrix leading to dissolution of the solid phase and an increase in porosity (Chadam et al. (1986), Daccord & Lenormand (1987), Szymczak & Ladd (2014)). More recent works have shown that precipitation locally decreasing the permeability can also induce fingering (Nagatsu et al. (2014)) leading to beautiful precipitation patterns in various contexts including chemical gardens (Haudin et al. (2014)) or CO$_2$ mineralization reactions (White & Ward (2012), Schuszter et al. (2014)). With no surprise, concomitant changes in viscosity and permeability can induce an interplay between viscous and precipitation-driven fingering giving rise to interesting new patterns (Nagatsu et al. (2008a), Haudin & De Wit (2015)).

Let us here explain in more details the analytical and numerical developments showing how simple $A + B \rightarrow C$ reactions can influence or even trigger VF before explaining how these theoretical predictions allow to rationalize the various experimental observations made.

### 6.2. Viscous fingering of $A + B \rightarrow C$ chemical fronts

#### 6.2.1. Viscosity profiles in reactive systems

If all three species $A$, $B$ and $C$ influence the viscosity, we have in the most general case that $\mu(r, t) = \mu(a, b, c)$ i.e. the viscosity profile depends on the concentration profiles $a(r, t), b(r, t), c(r, t)$ of both reactants $A$, $B$ and product $C$. This state equation couples the RDC equations for the concentrations to the flow equation. The viscosities $\mu_A = \mu(a_0, 0, 0), \mu_B = \mu(0, a_0, 0)$ and $\mu_C = \mu(0, 0, a_0)$ represent the viscosity of the fluid when only one of the chemical species is present in concentration $a_0$. For simplicity, theoretical studies usually assume an exponential relation

$$\mu = \mu_A e^{(R_b + R_c)/a_0} \quad (7)$$

where $R_b$ and $R_c$ are the log mobility ratios defined as

$$R_b = \ln \left( \frac{\mu_B}{\mu_A} \right) \quad \text{and} \quad R_c = \ln \left( \frac{\mu_C}{\mu_A} \right). \quad (8)$$

The parameter $R_b$ compares the viscosity of the two reactant solutions while $R_c$ measures the ratio between the viscosities of the product $C$ and reactant $A$ solutions. In absence of any reaction and if $A$ is injected into $B$, VF takes places when $R_b > 0$ i.e. if the displacing solution of $A$ is less viscous than the displaced solution of $B$.

In the reactive case, as soon as the reactants $A$ and $B$ come into contact via diffusion, the chemical reaction is triggered generating the product $C$ in the reactive zone (Hejazi et al. (2010)). The changes in the concentrations profiles reshape the viscosity profiles which can, depending on the relative values of $R_b$ and $R_c$, become non-monotonic with a maximum if $C$ is sufficiently more viscous than the reactants or with a minimum if $C$ is, on the contrary, decreasing the viscosity (Figure 3).
6.2.2. Theoretical studies. Linear stability analysis (LSA) of these viscosity profiles modified by reactions have discussed two different cases depending whether the reference non-reactive situation is unstable or not. If the underlying non-reactive system is already unstable because a less viscous solution of A displaces a more viscous solution of B ($R_b > 0$, Figure 1b), the reaction modifies the stability properties because, unless $R_c = R_b$ which is the equivalent of the non-reactive case, the presence of the product C modifies the viscosity profile (Hejazi et al. (2010)). The LSA predicts that the reactive situations are always more unstable than their nonreactive counterpart for relatively large time because the gradient of viscosity is steepened by the reaction either in the frontal part of the reaction zone where C pushes B when $R_c < R_b$ or in the rear part of the reaction zone where A pushes C when $R_c > R_b$. Strikingly, the LSA also shows that injecting a more viscous fluid into a less viscous fluid ($R_b < 0$), a situation classically stable in non-reactive systems, can also induce instabilities when the reaction induces non-monotonic viscosity profiles (Hejazi et al. (2010)). Fingering then develops thanks to the local region where the viscosity increases along the displacement direction. This means that, for a non-monotonic profile with a maximum, fingering develops at the back of the extremum while in presence of a minimum, fingering is expected in the frontal part of the extremum.

Nonlinear simulations confirm the predictions of the LSA, in the sense that the reactive cases for $R_b > 0$ are all more unstable than the non reactive situation, with fingering starting earlier and with a smaller wavelength. Different morphologies of fingers are also observed: fingers become thinner and their center of mass is more and more displaced towards the back when $R_c$ is increased above $R_b$ (Figure 4, bottom) (Nagatsu & De Wit (2011), Hejazi & Azaïez (2010b)). In case of a maximum in viscosity, fingers develop backwards in the zone where A pushes the more viscous C while the frontal part where the more viscous C invades the less viscous B is stable. On the contrary, if $R_c$ is sufficiently lower than $R_b$ such that a minimum in viscosity builds up, more active forward fingering is observed where less viscous C pushes B while the rear part is stabilized. More efficient coarsening decreases
Figure 4: Numerical concentration profiles of $C$ at three successive times from left to right showing VF with $R_b = 2$ modified by reactions for $R_c = -2; 0; 2; 4; 6$ from top to bottom.

Figure 5: Numerical concentration profiles of $C$ at three successive times from left to right showing reaction-driven VF for $R_b = -1$ triggered by a maximum for $R_c = 4$ (a) or a minimum for $R_c = -6$ (b) in viscosity when a more viscous solution of A displaces a less viscous solution of B.

the number of fingers while tip splitting events are more often observed (Figure 4, top) (Nagatsu & De Wit (2011), Hejazi & Azaiez (2010b)). Note that even if the viscosity ratio of the displaced and displacing solutions in the unfavorable part of the profile is the same, the situation with a minimum leads to a faster progression of the fingers at the frontal part with a larger mixing zone than for the reverse fingers associated to a maximum in viscosity because the fingers then develop along the flow rather than against it (Mishra et al. (2010a)).

When $R_b < 0$, the non reactive case is stable. Yet, reactions can then trigger fingering when a non-monotonic viscosity profile builds up. In presence of a minimum, forward fingering is observed where the less viscous $C$ pushes $B$ (Figure 5b). In case of a maximum, the unfavorable viscosity jump is located in the trailing zone where $A$ displaces the more viscous $C$ and reverse fingering is favored (Figure 5a). This shows thus that reactions are able to destabilize an otherwise hydrodynamically stable situation (Riolfo et al. (2012)). The case of reversible reactions has also been tackled, showing that, depending on the viscosities of the reactant and product solutions, reversibility may enhance/attenuate the instability (Alhumade & Azaiez (2013)).

Nonlinear simulations have also analysed the effect of $A + B \rightarrow C$ reactions in the case of
a finite size sample of B displaced in a rectilinear geometry within a fluid A. Both interfaces
where A pushes B and B pushes A are then present in the same system (Gérard & De
Wit (2009), Hejazi & Azaiez (2010a)). Such studies also allow to analyze the possible
interaction between the two different interfaces and discuss the influence of reactivity on
the spreading due to VF of finite size pollutant zones. Even if the reactant solutions have
the same viscosity \( R_b = 0 \), such fronts can feature different fingering dynamics when the
product C is more viscous and the diffusion coefficients or initial concentrations of the two
reactants are different (Gérard & De Wit (2009)). Indeed, because of the asymmetry of
the C profile that differential diffusion or concentrations induce (as seen on Figure 2),
the non-monotonic viscosity profile is also asymmetric with different unfavorable viscosity
gradients depending whether A or B is the displacing solution. This evidences the possible
fine tuning of the chemical control of local fingering dynamics via the right selection of
chemical species with specific differences in concentration or diffusion coefficients. With
variable diffusivities, double diffusion effects can, in addition, also come into play (Mishra
et al. (2010b)).

Note that control of the viscosity profile via extrema to obtain stabilization of fingering
or on the contrary destabilization of an otherwise stable displacement can be further tuned
via the addition of nanocatalysts acting on the reaction rate (Ghesmat et al. (2013), Sabet
et al. (2017, 2018), Dastvareh & Azaiez (2019)), or the local production of foams (Kahrobaei
et al. (2017)) which enlarges the range of action. Interestingly, the geometry also matters
as it has been shown that, in a radial injection, the fact that the local speed decreases with
the radius from injection influences the production of C (Brau et al. (2017), Trevelyan
& Walker (2018)) and thus changes the fingering of finite size samples (Sharma et al. (2019)).
In the case where the reactants and the chemical product all have different viscosities, the
wealth of possible different dynamics of course increases (Hejazi & Azaiez (2010a)).

6.2.3. Experimental results. Experiments demonstrating the influence of chemical reactions
on the properties of miscible viscous fingering are typically conducted in Hele-Shaw cells
consisting in 2 glass or plexyglas plates separated by a thin gap in which a host solution of
a reactant B is contained (Nagatsu (2015)). Another reactant A is then injected radially
or rectilinearly into the cell at a constant flow rate and the \( A+B \rightarrow C \) reaction can proceed
in the miscible contact zone between them. If the reaction does not change the viscosity \textit{in situ},
the chemical species are then simply slaved to the flow and the concentration pattern
depends on the initial concentrations and injection flow rate (Nagatsu (2015), Nagatsu

The first experiments on reactive VF where an \( A+B \rightarrow C \) type of reaction is actively
changing the viscosity and hence affects VF have been performed by Nagatsu et al. (2007).
In these experiments, viscous solutions of polymers, the viscosity of which varies with pH,
are displaced by less viscous miscible solutions of various reactants. They observe that
the fingering pattern is changed by the reaction. If the reaction is very fast, an increase
in viscosity induces wider fingers and a suppression of the shielding effect giving a pattern
covering a larger area of the displaced solution. On the contrary, a decrease in viscosity leads
to thinner fingers with a stronger shielding effect (Nagatsu et al. (2007, 2010)). This is in
good agreement with nonlinear simulations performed for fast reactions (Nagatsu & De Wit
(2011), Hejazi & Azaiez (2010b)). Interestingly, for slower reactions, the opposite effect is
obtained i.e. respectively wider and thinner fingers for a decrease (Nagatsu et al. (2009))
and increase of viscosity (Nagatsu et al. (2011)). This can be rationalized by a careful
inspection of the relative times of reactions and of advection at the tip or base of the fingers (Nagatsu (2015)). In some cases, the polymer used can exhibit non-newtonian properties. An astonishing growth of fingers in a spiral way has then been observed (Nagatsu et al. (2008b)). All these results showing how the reaction changes the VF pattern have been obtained by Nagatsu et al. in cases where the less viscous aqueous solution of reactants displaces a more viscous polymeric situation but the mobility profile remains monotonic. The non-reactive reference situation is thus already unstable and reactions change here the gradients of viscosities, favouring or slowing down fingering without however producing any local extremum in viscosity (zones IIa and IIb of Figure 3).

The case of reactions inducing a non-monotonic increasing viscosity profile with a maximum (zone III of Figure 3) has been recently studied in Hele-Shaw cells in the case of a step-growth cross-linking polymerization reaction (Bunton et al. (2017), Stewart et al. (2018)). In absence of reaction, the invading solution is less viscous than the displaced one and VF is obtained. By addition of a reaction initiator in the displacing solution in variable concentration, the amount of the more viscous polymer product can be tuned in the contact zone. The cross-linked reaction product is more viscous which results in a non-monotonic viscosity profile at the interface, affecting flow stability. In particular, the numerically predicted fact that fingers extend preferentially at the back of the reaction zone where the less viscous injected reactant displaces the locally produced more viscous product while the frontal part of the reaction zone is stabilized is recovered in the experiments (Bunton et al. (2017)).

The most striking influence of reactions is however to be able to destabilize an otherwise hydrodynamically stable displacement, i.e. typically when the displacing solution is more viscous or of same viscosity than the displaced solution ($R_b \leq 0$). In absence of reactions, the interface is stable and no fingering can develop. As shown theoretically, extrema in the viscosity profile can however destabilize the displacement. Experimentally, Podgorski et al. have for the first time demonstrated chemically-driven VF when the reaction forms a more viscous elastic micellar product following contact between two reactants solutions of the same viscosity (Podgorski et al. (2007)) (axis $R_c > 0$ for $R_b = 0$). The fingering patterns are different depending whether A is injected into B or vice versa, which can be related to the asymmetry of the underlying viscosity profiles (Gérard & De Wit (2009)). Purely reaction-driven fingering has also been obtained when a viscous polymer reaction displaces a less viscous reactant solution and a maximum (zone IV of Figure 3) or minimum (zone VI of Figure 3) in viscosity is produced (Riolfo et al. (2012)). As predicted theoretically (Hejazi et al. (2010), Nagatsu & De Wit (2011), Hejazi & Azaiez (2010b)), fingers are then seen in experiments to extend backwards in case of a maximum while they progress ahead of the extremum in case of a minimum (Figure 6).

In 3D opaque porous media, reactive fingering has also been analyzed using magnetic resonance. Using the same micelle producing reaction as Podgorski et al. (2007), Rose and Britton have evidenced for the first time in 3D how the in situ production of the more viscous product can destabilize the displacement of reactants solutions of similar viscosity in a packed bed filled with borosilicate glass beads (Rose & Britton (2013)).

We have here reviewed how changes in the mobility profile induced by an $A+B\rightarrow C$ reaction changing locally the viscosity can influence viscous fingering and even trigger fingering in otherwise stable non reactive situations. Note that recent progresses have also studied the interplay of fingering with more complex reactions like nonlinear clock reactions (Escala et al. (2019)) able to trigger sudden large changes of viscosity (Escala et al. (2017)). This is
6.3. Buoyancy-driven instabilities in reactive systems

The density of a given solution is a function of temperature and composition. Let us neglect any heat effect to focus here on compositional effects only. Of course some reactions can be exo- or endothermic and change thus both composition and temperature (Tanoue et al. (2009a,b)) but we will nevertheless neglect heat effect as they have been shown to be negligible in the experiments to be described (Almarcha et al. (2013)). When two solutions, each containing a reactive species, are put in contact in the gravity field, local variations in the density due to the reaction can induce convective motion and mixing.

If all three species A, B and C contribute to changes in density, the coupling between the RDC equations for the concentrations and the flow equation comes from the state equation $\rho(\mathbf{r},t) = \rho(a,b,c)$. If the solutions are diluted enough, the density is assumed to vary linearly with concentrations as:

$$\rho = \rho_0(1 + \alpha_A a + \alpha_B b + \alpha_C c) \quad (9)$$

where $\alpha_i = (\partial \rho / \partial c_i) / \rho_0$ is the solutal expansion coefficient of species $i$ and $c_i$ its concentration. In dimensionless forms, the important parameters of the problem are the Rayleigh numbers $R_{A,B,C}$ of the reactants A, B and product C respectively, expressing the contribution of each species to the dimensionless density

$$\bar{\rho} = R_{A} \bar{a} + R_{B} \bar{b} + R_{C} \bar{c} \quad (10)$$

where the bar denotes a dimensionless variable. In a porous medium with permeability $\kappa$,
the Rayleigh numbers can be defined as

\[ R_i = \frac{\alpha_i a_0 \kappa l_c}{\nu D_A} \]  

(11)

if concentrations are scaled by the initial concentration \( a_0 \) of species \( A \) and where \( l_c \) is the characteristic length of the problem and \( \nu \) the kinematic viscosity of the solvent. A large variety of different density profiles can then develop depending on the boundary conditions, concentrations and diffusion coefficients of the chemical species (Citri et al. (1990)).

First, if the initial contact line between the two solutions is vertical, the front can be influenced by buoyancy-driven convection as soon as the densities of the species are different (Rongy et al. (2008, 2010), Eckert et al. (2012), Tiani et al. (2018)). Depending on the structure of the density profile, one or two convective rolls can deform the front and induce its propagation (Rongy et al. (2008)). In particular, one convection roll is obtained in case of a monotonic density profile like in Figures 1a,b while two counter-rotating vortices are obtained with non-monotonic profiles like those of Figures 1c,d. Here again the mobility profile is the key quantity allowing to predict most of the system’s behavior. Similar considerations can be discussed in case of gradients of surface tension inducing Marangoni effects if the upper phase is in contact with air (see Tiani et al. (2018) for a review).

In the case of a horizontal contact line between solutions of \( A \) and \( B \), buoyancy-driven instabilities influenced or triggered by simple \( A + B \rightarrow C \) reactions can be divided in three main categories depending whether the solutions of \( A \) and \( B \) are miscible, partially miscible or immiscible. Let us review each category successively.

### 6.4. Density fingering of miscible \( A + B \rightarrow C \) fronts

In absence of reaction, the stratification of a solution of \( A \) above a miscible solution of \( B \) develops a Rayleigh-Taylor (RT) instability when the upper layer is denser than the lower one. If the initial stratification is initially statically stable (less dense \( A \) above denser \( B \)), a double diffusive (DD) instability occurs if \( B \) diffuses faster than \( A \) while a diffusive layer convection (DLC) mode can be observed if \( A \) diffuses faster (Trevelyan et al. (2011)).

In reactive systems, we recover the same instabilities when comparing the relative density and diffusivity of the reactant solutions of \( A \) and \( B \). However, the fact that the product \( C \) with different density and diffusivity is generated \textit{in situ} can drastically change the situation. Figure 7 compares experimental patterns that have been obtained in Hele-Shaw cells when putting two miscible solutions of different density in contact along a horizontal line in the gravity field. Note that color indicators can play an active role in the dynamics and change the overall look of the patterns (Almarcha et al. (2010b), Kuster et al. (2011), Mosheva & Shmyrov (2017)), hence Figure 7 has been obtained using a Schlieren technique tracking changes in index of refraction without any use of color indicator. The upper line features mixing between non reactive solutions of salt and sugar (Carballido-Landeira et al. (2013), Gopalakrishnan et al. (2018)) while the lower line shows the effect of an \( A + B \rightarrow C \) neutralisation reaction on the stratification between aqueous solutions of a strong acid and of a strong base (Tanoue et al. (2009a,b), Zalts et al. (2008), Almarcha et al. (2010a, 2011), Lemaigre et al. (2013), Bratsun et al. (2015)). When the upper solution is denser than the lower one, then the initial condition develops a RT instability (Figure 7a) with fingers extending in the non reactive case on average the same distance above and below the initial contact line. If a reaction takes place, the sinking fingers do not develop because
the downward moving denser A is eaten by the reaction and replaced by a salt of lower density (Figure 7d) (Almarcha et al. (2010a), Lemaigre et al. (2013)). Similarly, the local production by reaction of the salt C with different density can break the symmetries of the double diffusive (Figure 7b,e) and DLC convective modes (Figure 7c,f) (Almarcha et al. (2010a), Lemaigre et al. (2013)). In addition, it is observed that the reactive patterns can feature secondary instabilities in time, once enough production of C can trigger for instance the fingered sinking of denser C in the less dense reactant B (Almarcha et al. (2011)) or when differential diffusion effects between the zone rich in C and the lower layer of B (Lemaigre et al. (2013)) come into play. Importantly, even if the reaction is the same i.e. here if the same acid-base neutralisation reaction takes place, the dynamics is extremely sensitive to the nature of the counter-ions which do not participate in the reaction but have a major role in the density profile and diffusion of the chemicals (Almarcha et al. (2011)). Non ideal effects can influence these dynamics. Indeed, if the solutions are not dilute enough, the diffusion coefficients become a function of concentrations which can trigger extrema in non-monotonic density profiles and induce additional local convection (Bratsun et al. (2015)).

Theoretical linear stability analysis (Kim (2014)), nonlinear simulations (Almarcha et al. (2010a), Lemaigre et al. (2013), Kim (2014)) and a classification of all possible density profiles (Almarcha et al. (2011), Lemaigre et al. (2013), Trevelyan et al. (2015)) in the parameter space of the problem can rationalize these experimental observations that chemical reactions can trigger instabilities in otherwise stable situations but also break the symmetry of convective structures and instabilities. To understand this, Figure 8 shows a variety of possible density profiles around $A + B \rightarrow C$ fronts, depending on the relative values of the Rayleigh numbers and diffusion coefficients of the three species. We see that the density profile can feature up to 3 extrema in the reaction zone depending on the values of parameters. These extrema can suppress, trigger and localize convection and act as efficient controller of the flows.

Hejazi an Azaiez have further analyzed numerically the case where the chemical product solution C has both a different density and viscosity than the one of the reactant solutions and a transverse flow is applied parallelly to the initial horizontal interface between the reactants (Hejazi & Azaiez (2012, 2013)). They find that, in presence of the transverse flow, fingers with sharp concentration gradients develop and advance faster downward and that higher chemical production rates are obtained.

Recently, chemo-hydrodynamic patterns involving more complex oscillating reactions have been studied as well. In particular, in the $A + B \rightarrow$ oscillator case, separate reactants of an oscillatory reaction are put in contact along a horizontal miscible interface. The oscillations in concentration develop then in the local reaction zone (Escala et al. (2014)). As they induce local changes in density, an interplay between localized concentration waves and buoyancy-driven convection produces truly genuine chemo-hydrodynamic structures that would exist neither in oscillating RD systems nor in pure hydrodynamics. Such studies pave the way towards analysing patterns merging the self-organising structure of chemical and hydrodynamical systems (Budroni & De Wit (2017)).

### 6.5. Influence of $A + B \rightarrow C$ reactions on convective dissolution

The previous section has described buoyancy-driven convection around miscible $A + B \rightarrow C$ fronts when convection can extend both above and below the initial contact line. An
important application of similar dynamics but in partially miscible systems is currently attracting much attention: the case of convective dissolution relevant to CO$_2$ sequestration, aiming at reducing atmospheric concentrations of this greenhouse gas (Metz et al. (2005)). In this technique, CO$_2$ is injected into soils, typically in saline aquifers well spread around the globe. After injection, CO$_2$ rises up to the impermeable cap rock delimitating the aquifer and a two-layer stratification of CO$_2$ above the salt water is obtained. Upon dissolution of CO$_2$ in water, a denser boundary layer forms which can become unstable towards buoyancy-driven convection (Riaz et al. (2006), Neufeld et al. (2010), Huppert & Neufeld (2014), Slim (2014), Emami-Meybodi et al. (2015), Thomas et al. (2018)). Depending on the chemical composition of the host aquifer, chemical reactions can take place which affect the density profile and hence convection.

We recover in this application the stratification of a phase A above B with local production of C but the boundary condition is different than the one in the miscible case: the host phase is initially filled only with B and A dissolves into B with a given solubility $a_0$ from an upper fixed interface. Due to the importance for climate issues of quantifying
the flux of CO$_2$ that can dissolve in a given host phase, numerous works have focused on analysing convective dissolution in reactive systems. Let us point here the specificities of this partially miscible case and to what extent reactions can favour convective dissolution.

6.5.1. Theoretical modeling. As said above, the model equations are again Eqs.(2-6) with, as for the miscible case discussed in section 6.4, the density being a function of the concentrations $a, b$ and $c$. The only differences are in the initial condition ($a = a_0$ at the partially miscible interface and zero in the bulk while $b = b_0$ and $c = 0$ everywhere) and the boundary condition at the interface where zero velocity, no-flux for B, C and $a = a_0$ are applied (Loodts et al. (2016)).

These specific initial and boundary conditions induce a downward progression of buoyancy-driven fingers generated at the interface. There is thus a change of symmetry with regard to the miscible case. Yet, the analysis of density profiles in the partially miscible case helps again to classify all possible dynamics (Loodts et al. (2016)). If all species have the same diffusion coefficients, the important parameters of the problem are the difference $\Delta R_{CB} = R_C - R_B$ between the Rayleigh numbers of the product C and that of the reactant B and the ratio $\beta = b_0/a_0$ between the initial concentration of reactant B and the solubility of A in the host phase.

While simple reactions consuming A out of the solution stabilize convection (Ghesmat et al. (2011), Andres & Cardoso (2011, 2012), Cardoso & Andres (2014), Ward et al. (2014), Kim & Choi (2014), Kim & Kim (2015), Ghoshal et al. (2018)), various theoretical works have shown that A+B$\rightarrow$C reactions can accelerate or decelerate the convective dynamics with respect to the nonreactive case and that the steady-state dissolution flux of species A varies with the difference $\Delta R_{CB}$ (Loodts et al. (2014, 2015, 2017, 2018), Jotkar et al. (2019), Ghoshal et al. (2019), Ghoshal et al. (2017)). For equal diffusion coefficients, if $\Delta R_{CB} > 0$, the density profiles are monotonic. If C is sufficiently denser than B, the density at the interface is increased which gives rise to enhanced convective dissolution, a regime that we refer to as "destabilizing" with regard to the non reactive reference case (Figure 9). Conversely, if C

Figure 8: Variety of density profiles that can develop around A+B$\rightarrow$C chemical fronts when all three species have different Rayleigh numbers and diffusion coefficients. The vertical dashed line features the position of the initial contact line between the miscible solutions of A and B. From Trevelyan et al. (2015).
is less dense than B ($\Delta R_{CB} < 0$), the density profiles are non-monotonic. The upper part of the profile features density decreasing along gravity which is prone to trigger convection. However, this zone is followed downwards by a stabilizing density barrier that constrains the fingers in a localized zone of space (Loodts et al. (2018), Jotkar et al. (2019), Budroni et al. (2014, 2017)). The resulting nonlinear dynamics can be quite different: in the destabilizing case, long sinking fingers are formed which regularly merge while new fingers appear at the boundary (Figure 10a). The activation of the dynamics by chemistry leads to a very active renewed convection as seen on the space-time map of the density along a line just below the interface (Figure 10b). In the stabilizing case, the minimum in density freezes the fingers above the extremum at a given fixed wavelength (Figure 10c). Only, when the reaction front has traveled a while downward is then merging towards a new larger fixed wavelength obtained (Figure 10d). In all reactive cases, the convective flux of A into the host phase is larger than in the non reactive case (Loodts et al. (2017), Jotkar et al. (2019)). The case of reversible reactions and additional viscosity contrasts has also been tackled, showing even more complex scenarios when C can revert to the reactants once formed (Alhumade & Azaiez (2015)). Differential diffusive effects further enlarge the variety of possible dynamics (Loodts et al. (2018), Kim & Cardoso (2018)). Similarly to viscous fingering, buoyancy effects can also be coupled to changes in permeability via dissolution of the porous matrix or precipitation (J. Ennis-King & Paterson (2007), Ritchie & Pritchard (2011), Hidalgo et al. (2015), Binda et al. (2017), Thomas et al. (2019)). Interestingly, reactions are also able to destabilize the otherwise stable case of a species dissolving in a host phase and decreasing its density. In that case, the upper layer is less dense but, if reactions come into play, non-monotonic density profiles can develop triggering local convection (Bees et al. (2001), Loodts et al. (2015, 2016), Kim & Cardoso (2018)).

6.5.2. Experimental results. Convecting dissolution fingering of CO$_2$ in absence of any reactions has been experimentally evidenced in Hele-Shaw cells using a color indicator tracing
Figure 10: Numerical convective dissolution patterns of density. In the destabilizing case (a), fingers sink downwards rapidly with very active merging and formation of new fingers as seen in the corresponding space-time map (b). In the stabilizing case (c), the non-monotonic density profile with a minimum is giving a density barrier inducing regular fingers with a constant wavelength in time. As seen on the corresponding space-time map (d), these fingers rearrange after a while to yield a new "frozen" pattern but with a larger wavelength (Loodts et al. (2017), Jotkar et al. (2019)).

The pH decrease within the fingers as CO$_2$ acidifies the host aqueous phase (Kneafsey & Pruess (2010, 2011)). Outeda et al. (2014) have studied in such systems the temporal evolution of the mixing zone as well as dispersion curves, and the growth rate of the instability for different pressures in CO$_2$ and different color indicator concentrations. They find that, at earlier time, the growth changes with the concentration of the color indicator and that increasing the pressure destabilizes the system. Using analogous systems, Slim et al. (2013) could quantify the nonlinear dynamics, which is, as seen in simulations, featuring onset of fingering followed by merging and regular formation of new fingers after a while. Here again, color indicators can perturb the dynamics (Thomas et al. (2015)), which is why Schlieren or interferometric optical techniques tracking gradients of index of refraction should be preferred to visualize convection. Experiments confirm that first order reactions where A is consumed while no other species changes the density stabilize the flow (Cardoso...
Bimolecular $A+B \rightarrow C$ reactions where all species participate in density changes can on the contrary either accelerate or decrease convection (Loodts et al. (2014), Budroni et al. (2014, 2017), Wylock et al. (2008, 2011, 2014), Thomas et al. (2015, 2016, 2019), Cherezov & Cardoso (2016)). Strikingly, the acceleration is very sensitive to the nature of all ions present in the host phase which emphasizes the fine tuning that reactions can have on the control of the density profile (Thomas et al. (2016)).

6.6. Effect of $A+B \rightarrow C$ reactions on buoyancy-driven convection in immiscible systems

Spatio-temporal convective patterns can become quite complex in the case of two immiscible solvents put in contact along a horizontal line, each of them containing a reactant. Upon transfer of one reactant from one phase to the other, a wealth of different convective chemo-hydrodynamic patterns can be observed in both the upper and lower layers (Figure 11) (Eckert & Grahn (1999), Eckert et al. (2004), Asad et al. (2010), Schwarzenberger et al. (2012)). The situation is then often complicated by the presence of Marangoni effects due to surface-tension gradients (Bratsun & De Wit (2004)) and modeling needs to account for reaction-diffusion-convection equations in both layers (Bratsun & De Wit (2011)).

7. CONCLUSIONS

Chemical reactions can actively influence or even trigger convective motions when two solutions containing separate reactants are put in contact. We have here reviewed some of their effects on viscous fingering, Rayleigh-Taylor, double diffusive and convective dissolution instabilities. The key in controlling chemically these various hydrodynamic instabilities is in the action that reactions can have on the viscosity or density profiles. Specifically, changes in concentration profiles by the local generation of the product of the reaction after consumption of the reactants and the fact that all species can diffuse at different rates can produce local extrema in the mobility profile that can slow down, favor or generate convection. We have here mainly focused on simple $A+B \rightarrow C$ reactions but more complex reactions giving spatio-temporal complex reaction-diffusion patterns could also be used, increasing then the power of chemical control. In this regard, development of chemo-hydrodynamic pattern selection aiming at predicting the properties of hydrodynamic instabilities in active reactive systems should seek for new dynamics existing only thanks to the active coupling between reaction-diffusion and convective modes. Note that the control strategy suggested here relies mainly on controlling the mobility profile that is independent of the flow equation. In that respect, studying the various chemo-hydrodynamic instabilities discussed here on Darcy’s law in Stokes or Navier-Stokes flows should be an interesting topics for the future.

SUMMARY POINTS

1. Summary point 1. Chemical reactions can influence and even trigger hydrodynamic instabilities by changing the related mobility profile.
2. Summary point 2. Reactions can break the symmetries of convective instabilities and localize the fluid motions.
3. Summary point 3. In viscous fingering, $A+B \rightarrow C$ reactions can enhance or stabilize fingering when a less viscous reactive solution displaces a more viscous one but can
The engine of the dynamics described above is the diffusion coefficient. If the critical value $Ra_c$ exceeded the b.l. becomes unstable. This is in close analogy to the instability of a thermal b.l. on a heated plate [17].

As soon as acid enters the aqueous phase it instantaneously takes place. This is known to be one of the fastest solution reactions since both ions do not undergo conventional thermal buoyancy.

A large part of the released potential energy is provided by solubility and viscosity, this observation gives a hint that 20%. Although the temperature reduction also influences we find a slowing down of the plume rising by at least faster removal of heat through the walls of the cell. Here, the rising of the plumes is both a solutal and a thermal acceleration of the plume ensemble in the first case. Thus, this plume ensemble with and without reaction we find an resembles that of Fig. 2a. If we compare the advancing of formic-acid system displays a plume regime which re-

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2. Future issue 2. A general theory of chemo-hydrodynamic pattern selection should be developed.
3. Future issue 3. Generalization of the chemical control of porous media flows to flows described by Stokes or Navier-Stokes equations can be guided by a classification of the dynamics on the basis of the reference mobility profiles.

DISCLOSURE STATEMENT

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